Approximately Optimal Mechanism Design (Preprint)*

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1 The Optimal Mechanism Design Paradigm: Success Stories

Optimal mechanism design enjoys a beautiful and well-developed theory, and also a number of killer applications. Let's review two famous examples.

1.1 Example: The Vickrey Auction

In the Vickrey or second-price single-item auction (Vickrey, 1961), there is a single seller with a single item; assume for simplicity that the seller has no value for the item. There are n bidders, and each bidder i has a valuation v_i that is unknown to the seller. The Vickrey auction is designed to maximize the welfare, which in a single-item auction just means awarding the item to the bidder with the highest valuation. This sealed-bid auction collects a bid from each bidder, awards the item to the highest bidder, and charges the second-highest price. The point of the pricing rule is to ensure that truthful bidding is a dominant strategy for every bidder. Provided every bidder follows its dominant strategy, the auction maximizes welfare ex post (that is, for every valuation profile).

In addition to being theoretically optimal, the Vickrey auction has a simple and appealing format. Plenty of real-world examples resemble the Vickrey auction. In light of this confluence of theory and practice, what else could we ask for? To foreshadow what lies ahead, we mention that when selling multiple non-identical items, the generalization of the Vickrey auction is much more complex.

1.2 Example: Myerson's Auction

What if we want to maximize the seller's revenue rather than the social welfare? Since there is no single auction that maximizes revenue ex post, the standard approach here is to maximize the expected revenue with respect to a prior distribution over bidders' valuations. Assume bidder i's valuation is drawn independently from a distribution F_i that is known to the seller. For the moment, assume also that bidders are homogeneous, meaning that their valuations are drawn i.i.d. from a known distribution F.

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Myerson (1981) identified the optimal auction in this context, which is a simple twist on the Vickrey auction — a second-price auction with a reserve price r.[1] Moreover, the optimal reserve price is simple and intuitive—it is the *monopoly price* $\text{argmax}_p[p \cdot (1 - F(p))]$ for the distribution F, the optimal take-it-or-leave-it offer to a single bidder with valuation drawn from F. Thus, to implement the optimal auction, you don't need to know much about the valuation distribution F—just a single statistic, its monopoly price.

Once again, in addition to being theoretically optimal, Myerson's auction is simple and appealing. It is more or less equivalent to an eBay auction, where the reserve price is implemented using an opening bid. Given this success, why do we need to enrich the traditional optimal mechanism design paradigm? As we'll see, when bidders' valuations are not i.i.d., the theoretically optimal auction is much more complex and no longer resembles the auction formats that are common in practice.

1.3 The Optimal Mechanism Design Paradigm

Having reviewed two well-known examples, let's zoom out and be more precise about the optimal mechanism design paradigm. The first step is to identify the design space of possible mechanisms, such as the set of all sealed-bid auctions. The second step is to specify some desired properties. In this survey, we focus only on cases where the goal is to optimize some objective function that has cardinal meaning, and for which relative approximation makes sense. We have in mind objectives such as the seller's revenue (in expectation with respect to a prior) or social welfare (ex post) in a transferable utility setting. The goal of the analyst is then to identify one or all points in the design space that possess the desired properties—for example, to characterize the mechanism that maximizes the welfare or expected revenue.

What can we hope to learn by applying this framework? The traditional answer is that by solving for the optimal mechanism, we hope to receive some guidance about how to solve the problem. With the Vickrey and Myerson auctions, we can take the theory quite literally and simply implement the mechanism advocated by the theory. More generally, one looks for features present in the theoretically optimal mechanism that seem broadly useful. For example, Myerson's auction suggests that combining welfare maximization with suitable reserve prices is a potent approach to revenue-maximization.

There is a second, non-traditional answer that we exploit explicitly when we extend the paradigm to accommodate approximation. Even when the theoretically optimal mechanism is not directly useful to the practitioner, for example because it is too complex, it is directly useful to the analyst. The reason is that the performance of the optimal mechanism can serve as a benchmark, a yardstick against which we measure the performance of other designs that stand a chance of being implemented.

2 The Optimal Mechanism Design Paradigm: Failure Modes

The Vickrey and Myerson auctions are exceptions that prove a rule: theoretically optimal mechanisms are generally too complex to be used in practice. "Complexity" can take many forms,

[1] That is, the winner is the highest bidder with bid at least r, if any. If there is a winner, it pays either the reserve price or the second-highest bid, whichever is larger.

including excessive computation, excessive communication, or unrealistic informational assumptions. We next illustrate this point with three examples. These examples motivate the alternatives to optimal mechanisms described in Sections 4–6.

2.1 Optimal Single-Item Auctions (Excessive Information)

We now return to expected revenue-maximization in single-item auctions, but allow *heterogeneous* bidders, meaning that each bidder i's private valuation v_i is drawn independently from a distribution F_i that is known to the seller. Myerson (1981) characterized the optimal auction, as a function of the distributions $F_1, \ldots, F_n$. We assume for simplicity that each distribution F_i has bounded support and a density function f_i.

The trickiest step of Myerson's optimal auction is the first one, where each bid b_i is transformed into a *virtual bid* $\varphi_i(b_i)$, defined by

$$\varphi_i(b_i) = b_i - \frac{1 - F_i(b_i)}{f_i(b_i)}.$$

The exact functional form in this equation is not important for this survey, except to notice that computing $\varphi_i(b_i)$ requires knowledge of the distribution, namely of $f_i(b_i)$ and $F_i(b_i)$.

Given this transformation, the rest of the auction is straightforward. The winner is the bidder with the highest positive virtual bid (if any). To make truthful bidding a dominant strategy, the winner is charged the minimum bid at which it would continue to be the winner.[2]

When all the distributions F_i are equal to a common F, and hence all virtual valuation functions φ_i are identical, the optimal auction simplifies and is simply a second-price auction with a reserve price of $\varphi^{-1}(0)$, which turns out to be the monopoly price for F. In this special case, the optimal auction requires only modest distributional knowledge (the monopoly price). In general, the optimal auction does not simplify further than the description above. A major impediment to implementing such a "virtual welfare maximizer" is that accurate distributional details are not always available – this widely-accepted criticism is known as Wilson's doctrine (Wilson, 1987). Second, even if such details are available, the corresponding optimal mechanism can be too inscrutable for real-world deployment. For example, on the second point, an optimal single-item auction might award the item to a low bidder over a high bidder (even if the latter clears its reserve).

2.2 Welfare-Maximizing Multi-Item Auctions (Excessive Communication)

In the standard setup for allocating multiple items via a combinatorial auction, there are n bidders and m non-identical items. Each bidder has, in principle, a different private valuation $v_i(S)$ for each bundle S of items it might receive. Thus, each bidder has 2^m private parameters. In this example, we assume that the objective is to determine an allocation $S_1, \ldots, S_n$ that maximizes the social welfare $\sum_{i=1}^{n} v_i(S_i)$.

The Vickrey auction can be extended to the case of multiple items; this extension is the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973). The VCG mechanism is a direct-revelation mechanism, so each bidder i reports a valuation $b_i(S)$ for each bundle

[2]We have only described the optimal auction in the special case where each distribution F_i is *regular*, meaning that the virtual valuation functions φ_i are nondecreasing. The general case "monotonizes" or "irons" the virtual valuation functions and then applies the same three steps (Myerson, 1981). (Monotonicity is essential for incentive-compatibility.)

of items S. The mechanism then computes an allocation that maximizes welfare with respect to the reported valuations. As in the Vickrey auction, suitable payments make truthful revelation a dominant strategy for every bidder.

Even with a small number of items, the VCG mechanism is a non-starter in practice, for a number of reasons (Ausubel and Milgrom, 2006). For example, the VCG mechanism, as a direct-revelation mechanism, solicits 2^m numbers from each bidder. This is an exorbitant number: roughly a thousand parameters already when $m = 10$, roughly a million when $m = 20$. In modern spectrum auctions, m might be in the hundreds or larger.[3]

2.3 Welfare-Maximization with Single-Minded Bidders (Excessive Computation)

If bidders' preferences are easy to communicate, does the VCG mechanism become easy to implement? For example, suppose each bidder i is *single-minded* and only cares about a (publicly known) subset T_i of items (Lehmann et al., 2002). Bidder i has a private value v_i for every superset of T_i, and 0 for every other set. This is a single-parameter environment, so communication between the bidders and the mechanism is not an issue. "All" the VCG mechanism has to do is compute a welfare-maximizing allocation (with respect to the reported valuations) and appropriate prices.

The problem is that, for single-minded bidders and many other examples of succinctly described valuations, it is difficult to compute a welfare-maximizing allocation in a reasonable amount of time (less than a year, say). The problem is that the number of candidate solutions grows *exponentially* with the number n of bidders. A subset W of bidders can all receive their desired subsets simultaneously if and only if if $T_i \cap T_j = \emptyset$ for distinct $i, j \in W$ (since no item can be allocated more than once). With n bidders, there are 2^n possibilities for W. For modestly large n (at least 50, say), there is no hope of checking them all in a reasonable amount of time.[4] For some computational problems with exponentially many candidate solutions, there is a clever algorithm that shortcuts to the optimal solution while examining only a tiny fraction of the possibilities. For "NP-hard" optimization problems, including the problem of welfare-maximization with single-minded bidders, the exponential scaling appears fundamental, with no clever shortcut in sight.

3 Approximately Optimal Mechanism Design

3.1 Benchmarks and Approximate Optimality

The examples in Section 2 demonstrate that, for many different reasons, it is not always feasible to implement the theoretically optimal mechanism. To give better design guidance in such settings, we have no choice but to take a different approach. This brings us to the main theme of this survey: using the relaxed goal of *approximate optimality* to make new progress on fundamental but challenging mechanism design problems.

To study approximately optimal mechanisms, we again begin with a design space and an objective function. Often the design space will proxy for the set of "plausibly implementable mechanisms," and is accordingly limited by side constraints such as a "simplicity" constraint. For example,

[3] Cramton (1998) writes: "The setting of spectrum auctions is too complex to guarantee full efficiency."

[4] Auctions for online advertising can have dozens or even hundreds of participants. The reverse auction in the FCC Incentive Auction (Section 6.2) had thousands of participants.

we later consider mechanisms with a restricted number of pricing parameters, with low-dimensional bid spaces, and with limited computational power.

The new ingredient of the paradigm is a *benchmark*. This is a target objective function value that we would be ecstatic to achieve. Generally, the working hypothesis will be that no mechanism in the design space realizes the full value of the benchmark, so the goal is to get as close to it as possible. In the examples we discuss, where the design space is limited by a simplicity constraint, a natural benchmark is the performance achieved by an unconstrained, arbitrarily complex mechanism. The goal of the analyst is to identify a mechanism in the design space that approximates the benchmark as closely as possible.

A typical positive result in approximately optimal mechanism design identifies a mechanism in the desired design space that always guarantees an objective function value (social welfare, expected revenue, etc.) that is at least an α percentage of the benchmark value. (The closer α is to 100%, the better.) A typical negative result proves that there is no mechanism in the design space with such a guarantee (for some fixed percentage α).

3.2 Goals of Approximately Optimal Mechanism Design

What is the point of applying this design paradigm? The first goal is exactly the same as with the traditional optimal mechanism design paradigm. Whenever you have a principled way of choosing one mechanism from many, you can hope that the distinguished mechanism is literally useful or highlights features that are essential to good designs. The approximation paradigm provides a novel way to identify candidate mechanisms.

There is a second reason to use the approximately optimal mechanism design paradigm, which has no analog in the traditional approach. The approximation framework allows the analyst to quantify the cost of imposing side constraints on a mechanism design space. For example, if there is a simple mechanism with performance close to that of the best arbitrarily complex mechanism, then this fact suggests that simple solutions might be good enough. Conversely, if every point in the design space is far from the benchmark, then this provides a forceful argument that complexity is an essential feature of every reasonable solution to the problem. Our second case study (Section 5) is a particularly clear example of this perspective.

3.3 Coming Up: Three Case Studies

Sections 4–6 describe three such instantiations, each addressing a different drawback of theoretically optimal mechanisms. First, we study expected revenue-maximization in single-item auctions, with bidders that have independent but not necessarily identically distributed valuations. Virtual welfare maximizers are an overparameterized class of auctions, and selecting the right one requires detailed distributional knowledge. We use the approximation paradigm to understand fundamental trade-offs between optimality and simplicity.

Our second case study concerns the problem of selling multiple non-identical items to maximize the social welfare. The theoretically optimal mechanism is well known (the VCG mechanism) but suffers from several drawbacks that preclude direct use. We apply the approximation paradigm to identify when mechanisms with low-dimensional bid spaces can perform well, and when high-dimensional bid spaces are necessary for non-trivial welfare guarantees.

Our final case study concerns settings where computation is the primary obstacle to optimality. Multi-unit auctions are one canonical example. We use the approximation framework to identify

mechanisms that guarantee near-optimal social welfare and are also computationally efficient.

An enormous amount of research over the past twenty years, largely but not entirely in the computer science literature, can be viewed as instantiations of the approximately optimal mechanism design paradigm. The case studies in this survey are representative but far from exhaustive. The book of Hartline (2017) is a good source for additional examples.

3.4 In Defense of Approximation Ratios

The positive results in our three case studies have the form: "the objective function value of the simple mechanism M is always at least an α percentage of that of the (complex) optimal mechanism." In some cases, α will be close to 100%, and the utility of the guarantee is self-evident. In most settings, however, the best-possible approximation guarantee is bounded away from 100%. What use is an approximate guarantee of, say, 63%?[5]

In the authors' experience, researchers tend to fixate unduly on and take too literally the numerical values in approximation guarantees. There are several points to keep in mind:

1. Both of the primary motivations for applying the approximately optimal mechanism design paradigm (Section 3.2) strive for qualitative rather than quantitative insights. This holds both for identifying mechanism features that are potentially useful in practice, and for assessing the cost of a simplicity side-constraint on the mechanism. For example, if the pursuit of a best-possible approximation guarantee justifies a widely-used mechanism or guides the analyst to an interesting new mechanism, is the exact numerical value of the guarantee so important?

2. A reader who, against our advice, insists on interpreting approximation guarantees literally, is likely to ask: "what about the other 37% of the welfare or expected revenue being left on the table?" But in all of the canonical applications of approximately optimal mechanism design, the benchmark of full optimality is only a utopia in the analyst's mind, and not one of the available options. For example, in a multi-item auction with more than a few items, it is flat-out impossible to implement a welfare-maximizing mechanism like the VCG mechanism. The choice is not whether to implement an optimal mechanism; it's whether to implement a suboptimal mechanism that has a good approximation guarantee or one that doesn't. While the mechanism with the best-possible approximation guarantee may or may not be the best one to implement in practice, it is always worth considering.

3. Approximation guarantees are usually "worst case," meaning that they hold for every possible setting (e.g., for an arbitrary valuation profile, or in expectation for an arbitrary prior distribution). An approximately optimal mechanism usually performs better than its worst-case guarantee in most settings of interest. For example, a mechanism with a worst-case guarantee of 50% might well achieve at least 90% of the benchmark value on "typical" inputs. In some cases, this property can be proved formally by establishing better approximation guarantees under additional assumptions about the setting; in other cases, the argument is best made through simulations.

4. Is a number like "63%" big or small? As in real life, the answer depends on the context.[6] The best way to justify theoretically an approximation guarantee is to prove a matching

[5] For the most part, we focus on relative approximation guarantees, which have the advantage of canceling out units of measurement. Absolute approximation guarantees are also meaningful in some settings (e.g., Theorem 4.2).

[6] A professional basketball team that wins 63% of its games is good but not great, while a baseball team with the

impossibility result, stating that there is no mechanism in the class of interest with a superior guarantee. With a few exceptions, like "large market"-type results (Section 7.3), optimal approximation guarantees tend to be bounded well away from 100% (e.g., 50% or 63%).

Whatever the merits of approximately optimal mechanism design, the unimplementability of optimal mechanisms in complex settings is real and will not go away (cf., footnotes 3 and 14). Any theorist who wants to reason seriously about such settings must work with an alternative to the classical optimal mechanism design paradigm. Approximation is by no means the only possible alternative (see also Section 7), but it is one of the most successful approaches to date.

Finally, like any general analysis framework, the approximation paradigm can be abused and should be applied with good taste. In settings in which the paradigm does not give meaningful results, different modeling or benchmark choices should be made, or a completely different analysis framework should be considered.

4 Case Study #1: Simple vs. Optimal Results

When bidders are heterogeneous, with different valuation distributions, the expected revenue-maximizing single-item auction can be complex and highly dependent on the details of the distributions (Section 2.1). Are there simpler auctions that perform almost as well? Section 4.1 studies approximation guarantees for the Vickrey auction supplemented with reserve prices. Section 4.2 presents t-level auctions, which offer a smooth trade-off between simplicity and optimality. Section 4.3 discusses the state-of-the-art for multi-item auctions.

4.1 Vickrey with Reserves

Recall the single-item auction setting of Section 2.1. There are n bidders, with bidder i's private valuation v_i drawn independently from a distribution F_i (with density f_i) that is known to the seller. We assume that every distribution is regular, meaning that the corresponding virtual valuation function is nondecreasing.[7] The optimal auction is a virtual welfare maximizer, and it computes a virtual bid for each bidder, awards the item to the bidder with highest positive virtual bid (if any), and charges the lowest winning bid that the bidder could have made. This auction depends in a detailed way on the distributions $F_1, \ldots, F_n$.

Virtual welfare maximizers are a rich class of auctions, parameterized by the virtual valuation functions $\varphi_1, \ldots, \varphi_n$. Intuitively, there is an infinite number of degrees of freedom in specifying such an auction. A natural and practically useful class of auctions with far fewer parameters is that of reserve price-based auctions. *Vickrey auctions with bidder-specific reserves* have only n degrees of freedom, the reserve prices $r_1, \ldots, r_n$.[8] Such an auction awards the item to the highest bidder that meets its reserve, and charges the smallest bid that would have won (the winning bidder's reserve price, or the highest bid by a different bidder that clears its reserve, whichever is larger).

Perhaps the most natural choice for bidder i's reserve price r_i is the monopoly price for its distribution F_i (Section 1.2). This choice guarantees a constant fraction of the optimal expected revenue, where the constant is independent of the number of bidders and the valuation distributions.

same record would be one of the favorites to win the World Series. Similarly, is a six-week turnaround for referee reports fast or slow? The answer depends on whether the submission was sent to *Econometrica* or *Science*.

[7] Recall the virtual valuation function is given by $\varphi_i(b_i) = b_i - (1 - F_i(b_i))/f_i(b_i)$.

[8] The informal notion of "degrees of freedom" in an auction class can be made precise using concepts from statistical learning theory, such as the pseudodimension. See Morgenstern and Roughgarden (2015) for further details.

Theorem 4.1 (Simple Versus Optimal Auctions) *For all $n \geq 1$ and regular distributions $F_1, \ldots, F_n$, the expected revenue of an n-bidder single-item Vickrey auction with monopoly reserve prices is at least 50% of that of the optimal auction.*

Thus, knowing a single statistic about each bidder's valuation distribution (its monopoly price) already suffices for approximately optimal expected revenue.[9]

Theorem 4.1 follows from Chawla et al. (2007) and Hartline and Roughgarden (2009). It can also be derived from the "prophet inequality" of Samuel-Cahn (1984); see Chapter 4 of Hartline (2017) or Lecture 6 of Roughgarden (2016b).

The guarantee of 50% can be improved for many distributions. It is tight in the worst case, however, even with only two bidders and arbitrary, not necessarily monopoly, reserve prices (Hartline and Roughgarden, 2009). To improve the guarantee without making additional assumptions, we must add complexity to the auction format. The next section describes a principled way of doing so.

4.2 t-Level Auctions and Simplicity-Optimality Trade-Offs

Virtual welfare maximizers are theoretically optimal but overly complex. Reserve-price-based auctions are reasonably simple but extract only 50% of the optimal expected revenue in the worst case. Can we interpolate between these two extremes? Can we quantify the trade off between simplicity and optimality? It's not clear how to make sense of this question without using the approximately optimal mechanism design paradigm.

Morgenstern and Roughgarden (2015) proposed *t-level single-item auctions* for this purpose. Such an auction has t parameters per bidder, which can be viewed as an increasing sequence of t reserve prices. Given a bid profile, the *level* of a bidder is defined as the number of its reserves that its bid clears. For example, if a bidder has three reserves 5, 7, and 9 and submits a bid of 8, then it has level 2.

The allocation rule of a t-level auction is defined as follows. If every bidder has level 0, then the item remains unallocated. Otherwise, the item is awarded to the bidder with the largest level, with ties broken by bid. That is, the winner is the highest bidder at the top occupied level. Since different bidders can have different reserve prices, the winner need not be the highest bidder overall. As usual, the winning bidder pays the lowest bid at which it would continue to win. 1-level auctions are the same as Vickrey auctions with bidder-specific reserves.

t-level auctions are naturally interpreted as discrete approximations to virtual welfare maximizers. Each level ℓ corresponds to a constraint of the form "If any bidder has level at least ℓ, do not sell to any bidder with level less than ℓ." For every ℓ, we can interpret bidders' ℓth reserve prices as the bidder values that map to some common virtual value. For example, 1-level auctions treat all values below a reserve price as having a negative virtual value, and above the reserve use values as proxies for virtual values. 2-level auctions use the second reserve to refine the virtual value estimates, and so on. With this interpretation, it is intuitively clear that as $t \to \infty$, it is possible to estimate bidders' virtual valuation functions and thus approximate Myerson's optimal auction to arbitrary accuracy. The next theorem is a quantitative version of this intuition; for normalization purposes, it restricts attention to distribution with support $[0, 1]$. The proof idea is to "round" an

[9] Even simpler are the Vickrey auctions with a single *anonymous* reserve price. Anonymous reserve prices also suffice to extract a constant fraction of the optimal expected revenue, although the constant degrades to 37% (Alaei et al., 2015).

optimal auction to a t-level auction without losing much expected revenue, using the reserve prices to approximate each bidder's virtual value.

Theorem 4.2 (Morgenstern and Roughgarden (2015)) *There is a constant $c > 0$ such that, for every number n of bidders, $\epsilon > 0$, and valuation distributions $F_1, \ldots, F_n$ with support $[0, 1]$, there is a $\frac{c}{\epsilon}$-level auction with expected revenue within ϵ of optimal.*

The guarantee in Theorem 4.2 translates to a relative approximation of $1 - \epsilon$ (with a different constant c' in place of c), except in the uninteresting case where the optimal expected revenue is very close to 0.

4.3 Multi-Parameter Problems

The approximation guarantees in Theorems 4.1 and 4.2 hold more generally in most single-parameter environments (Hartline and Roughgarden, 2009; Morgenstern and Roughgarden, 2015), almost at the level of generality of Myerson's optimal auction theory (Myerson, 1981).

Multi-parameter problems like multi-item auctions, however, pose a notorious challenge to optimal auction theory. In most such settings, there is no understanding of the optimal auction, other than being the solution to an astronomically large linear program. For an overview of the solvable special cases, see Daskalakis et al. (2017) and the references therein.

Hart and Reny (2015) suggested studying the seemingly simple case of a single buyer and multiple items, where the buyer has an additive valuation and its values for different items are independent. They documented several troublesome and counterintuitive properties possessed by optimal multi-item auctions, even in this restricted setting.

Hart and Nisan (2017) proposed using approximation to make progress on this class of multi-item auction problems. Passing to approximation can bypass the challenge of characterizing the optimal auction. The reason is that the analyst can instead use an analytically tractable upper bound on the optimal expected revenue, and prove that an auction of interest captures a significant fraction of this upper bound.

Hart and Nisan (2017) focused on two simple mechanisms: selling items separately (one price per item, with the buyer picking a utility-maximizing bundle); and a take-it-or-leave-it offer for the bundle of all items. They proved that, as the number of items grows large, neither mechanism guarantees a constant fraction of the optimal revenue. In a significant advance, Babaioff et al. (2014) proved that, for every distribution over additive valuations with independent item values, one of these two mechanisms extracts a constant fraction of the optimal revenue.[10] Yao (2015) extended this result to multiple buyers, and Rubinstein and Weinberg (2015) to more general valuation distributions.

5 Case Study #2: Low-Dimensional Message Spaces

In this section we switch gears and study the problem of allocating multiple items to bidders with private valuations to maximize the social welfare. We instantiate the approximately optimal

[10] Because the mechanism's expected revenue is compared to an upper bound on the optimal expected revenue, there are two sources of suboptimality: in the auction itself (due to revenue loss relative to an optimal auction), and in the analysis (due to slack between the upper bound and the actual expected revenue of an optimal auction). For this reason, the numerical value of the constant is not particularly satisfying when taken at face value.

mechanism design paradigm to identify conditions on bidders' valuations that are necessary and sufficient for the existence of simple combinatorial auctions with near-optimal welfare at equilibrium. The take-away from this section is that rich bidding spaces are an essential feature of every good combinatorial auction when items are complements, while simple auctions can perform well when bidders' valuations are complement-free.

5.1 Motivating Question

Recall from Section 2.2 the standard setup for allocating multiple items via a combinatorial auction. There are n bidders and m non-identical items. Each bidder has, in principle, a different private valuation $v_i(S)$ for each bundle S of items it might receive. In this section, we assume that the objective is to determine an allocation $S_1, \ldots, S_n$ that maximizes the social welfare $\sum_{i=1}^{n} v_i(S_i)$. The VCG mechanism is dominant-strategy incentive-compatible and welfare-maximizing but, as a direct-revelation mechanism, it requires an exorbitant number of bids (2^m) from each bidder.

In this case study, we apply the approximately optimal mechanism design paradigm to study the following question.

Does a near-optimal combinatorial auction require rich bidding spaces?

Thus, as in the previous case study, we seek conditions under which "simple auctions" can "perform well." This time, our design space of "simple auctions" consists of mechanism formats in which the dimension of every player's bid space is growing polynomially with the number m of items (say m or m^2), rather than exponentially with m as in the VCG mechanism.

"Performing well" means, as usual, achieving objective function value (here, social welfare) close to that of a benchmark. We use the *VCG benchmark*, meaning the welfare obtained by the best arbitrarily complex mechanism (the VCG mechanism), which is simply the maximum-possible social welfare.

This case study contributes to the debate about whether or not package bidding is an important feature of combinatorial auctions, a topic over which much blood and ink has been spilled over the past twenty years. We can identify auctions with no or limited packing bidding with low-dimensional mechanisms, and those that support rich package bidding with high-dimensional mechanisms. With this interpretation, our results make precise the intuition that flexible package bidding is crucial when items are complements, but not otherwise.

5.2 A Simple Auction: Selling Items Separately

Our goal is to understand the power and limitations of the entire design space of low-dimensional mechanisms. To make this goal more concrete, we begin by examining a specific simple auction format.

The simplest way of selling multiple items is by selling each separately. Several specific auction formats implement this general idea. We analyze one such format, simultaneous first-price auctions (Bikhchandani, 1999). In this auction, each bidder submits simultaneously one bid per item—only m bidding parameters, compared with its 2^m private parameters—and each item is sold in parallel using a first-price auction.

When do we expect simultaneous first-price auctions to have reasonable welfare at equilibrium? Not always. With general bidder valuations, and in particular when items are complements, we might expect severe inefficiency due to the "exposure problem" (e.g., Milgrom (2004)). For example,

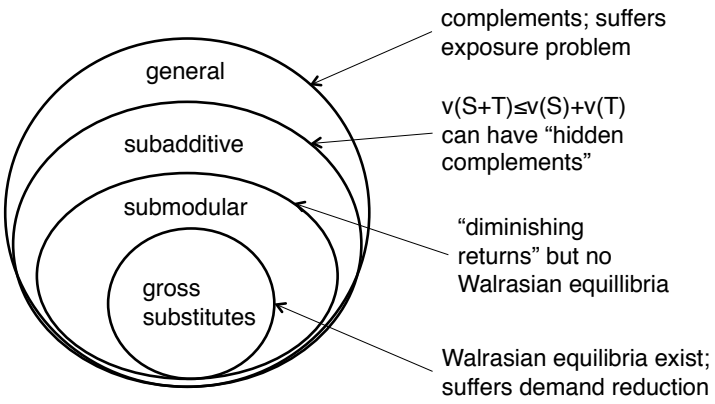

Figure 1: A hierarchy of valuation classes.

consider a bidder in an auction for wireless spectrum licenses that has large value for full coverage of California but no value for partial coverage. When items are sold separately, such a bidder has no vocabulary to articulate its preferences, and runs the risk of obtaining a subset of items for which it has no value, at a significant price.

Even when there are no complementarities amongst the items, we expect inefficiency when items are sold separately (e.g., Krishna (2010)). The first reason is "demand reduction," where a bidder pursues fewer items than it truly wants, in order to obtain them at a cheaper price. Second, if bidders' valuations are drawn independently from different valuation distributions, then even with a single item, Bayes-Nash equilibria are not always fully efficient.

5.3 Valuation Classes

Our discussion so far suggests that simultaneous first-price auctions are unlikely to work well with general valuations, and suffer from some degree of inefficiency even with simple bidder valuations. To parameterize the performance of this auction format, we introduce a hierarchy of bidder valuations (Figure 1); the literature also considers more fine-grained hierarchies (Feldman et al., 2015; Lehmann et al., 2006).

The biggest set corresponds to general valuations, which can encode complementarities among items. The other three sets denote different notions of "complement-free" valuations. In this survey, we focus on the most permissive of these, *subadditive valuations*. Such a valuation v_i is monotone ($v_i(T) \subseteq v_i(S)$ whenever $T \subseteq S$) and satisfies $v_i(S \cup T) \leq v_i(S) + v_i(T)$ for every pair S, T of bundles. This class is significantly larger than the well-studied classes of gross substitutes and submodular valuations.[11] In particular, subadditive valuations can have "hidden complements," with two items becoming complementary once a third item is acquired, while submodular valuations cannot (Lehmann et al., 2006).

[11]Submodularity is the set-theoretic analog of "diminishing returns:" $v_i(S \cup \{j\}) - v_i(S) \leq v_i(T \cup \{j\}) - v_i(T)$ whenever $T \subseteq S$ and $j \notin S$. The gross substitutes condition—which states that a bidder's demand for an item only increases as the prices of other items rise—is strictly stronger and guarantees the existence of Walrasian equilibria (Kelso and Crawford, 1982; Gul and Stacchetti, 1999).

5.4 When Do Simultaneous First-Price Auctions Work Well?

Our intuition about the performance of simultaneous first-price auctions translates nicely into rigorous statements. First, for general valuations, selling items separately can be a disaster.

Theorem 5.1 (Hassidim et al. (2011)) *With general bidder valuations, simultaneous first-price auctions can have mixed-strategy Nash equilibria with expected welfare arbitrarily smaller than the VCG benchmark.*

For example, equilibria of simultaneous first-price auctions need not obtain even 1% of the maximum-possible welfare when there are complementarities between many items.

On the positive side, even for the most permissive notion of complement-free valuations—subadditive valuations—simultaneous first-price auctions suffer only bounded welfare loss.

Theorem 5.2 (Feldman et al. (2013)) *If every bidder's valuation is drawn independently from a distribution over subadditive valuations, then the expected welfare obtained at every Bayes-Nash equilibrium of simultaneous first-price auctions is at least 50% of the expected VCG benchmark value.*

In Theorem 5.2, the valuation distributions of different bidders do not have to be identical, just independent. The guarantee improves to roughly 63% for the special case of submodular bidder valuations (Syrgkanis and Tardos, 2013).

Taken together, Theorems 5.1 and 5.2 suggest that simultaneous first-price auctions should work reasonably well if and only if there are no complementarities among items.

5.5 Negative Results

We now return to the question of when simple mechanisms, meaning mechanisms with low-dimensional bid spaces, can achieve non-trivial welfare guarantees. Section 5.4 considered the special case of simultaneous first-price auctions; here we consider the full design space.

First, the poor performance of simultaneous first-price auctions with general bidder valuations is not an artifact of the specific format: *every* simple mechanism is vulnerable to arbitrarily large welfare loss when there are complementarities among items. This impossibility result argues forcefully for a rich bidding language, such as flexible package bidding, in such environments.

Theorem 5.3 (Roughgarden (2014b)) *With general bidder valuations, no family of simple mechanisms guarantees equilibrium welfare at least a constant fraction of the VCG benchmark.*

In Theorem 5.3, the mechanism family is parameterized by the number of items m; "simple" means that the number of dimensions in each bidder's bid space is bounded above by some polynomial function of m. The theorem states that for every such family and constant $c > 0$, for all sufficiently large m, there is a valuation profile and a full-information mixed Nash equilibrium of the mechanism with expected welfare less than c times the maximum possible.[12] The proof of Theorem 5.3 builds on techniques from the field of complexity theory, specifically communication complexity (Kushilevitz and Nisan, 1996; Roughgarden, 2016a).

[12]Technically, Theorem 5.3 proves this statement for an ϵ-approximate Nash equilibrium—meaning every player mixes only over strategies with expected utility within ϵ of a best response—where $\epsilon > 0$ can be made arbitrarily small. The same comment applies to Theorem 5.4.

We already know from Theorem 5.2 that, in contrast, simple auctions can have non-trivial welfare guarantees with complement-free bidder valuations. Our final result states that no simple mechanism outperforms simultaneous first-price auctions with these bidder valuations.

Theorem 5.4 (Roughgarden (2014b)) *With subadditive bidder valuations, no family of simple mechanisms guarantees equilibrium welfare more than 50% of the VCG benchmark.*

6 Case Study #3: Algorithmic Mechanism Design

In this section we address a third possible failure of optimal mechanisms: excessive computation. As in the previous section, we study the problem of allocating resources to players with private valuations to maximize the social welfare.

Section 6.1 introduces the theory of computational complexity, which studies the amount of computational resources necessary and sufficient to solve different computational problems. Section 6.2 interprets this theory in the context of the recent FCC Incentive Auction. Section 6.3 states our design goals. Sections 6.4 and 6.5 instantiate the approximately optimal mechanism design paradigm in single- and multi-parameter settings, respectively. Our single-parameter example concerns allocating a limited-capacity shared resource, and we'll see that "greedy" mechanisms often perform well. Our main multi-parameter example is multi-unit auctions, and we'll see how to modify the VCG mechanism, with bounded loss of social welfare, so that it becomes computationally tractable.

6.1 Computational Complexity

The field of *computational complexity* analyzes the amount of computational resources, such as the amount of time, required to solve a computational problem. Examples of computational problems include sorting a given set of numbers, sequencing a given set of tasks, computing a shortest path between a given origin and destination in a network, and so on. A positive result in this field takes the form of a computationally efficient algorithm—an algorithm that solves every instance of a problem in a reasonable amount of time. The most common definition of "reasonable" is as a polynomial-time algorithm: the running time (i.e., number of elementary steps) performed by the algorithm grows as a polynomial function of the size of the instance (e.g., the number of tasks to be sequenced, or the number of vertices and edges in the given network). Equivalently, the input sizes that the algorithm can solve in a fixed amount of time scales multiplicatively with increasing computational power. An example of an inefficient algorithm is one that exhaustively searches through an exponential number of possible solutions (cf., Section 2.3). A standard textbook treatment of computational complexity is Sipser (2006); see also Roughgarden (2010) for a survey aimed at economists.

Unfortunately, many computational problems, including many that arise in economics, are "NP-hard." A formal definition of this term is outside the scope of this survey, but the bottom line is that NP-hard problems do not admit computationally efficient algorithms under widely believed mathematical assumptions (specifically, the "$P \neq NP$" conjecture).

While the NP-hardness of a problem rules out any always-fast, always-correct algorithm for the problem (assuming $P \neq NP$), it is not a death sentence. In some (but not all) applications, the instances of an NP-hard problem relevant to practice are relatively easy and can be solved in a reasonable amount of time.

6.2 Computational Complexity in Practice: The FCC Incentive Auction

The lessons of computational complexity show up frequently in the real world. For an example germane to this survey, consider the US FCC Incentive Auction of 2016–17. This auction consisted of two phases: (i) freeing up a designated band of spectrum by buying it back from TV broadcasters; and (ii) reselling the cleared spectrum to interested companies (Milgrom, 2017). Two computational problems are closely associated with the buying-back phase. The first is the problem of checking whether a given set of broadcasters can stay on-air, i.e., can be feasibly repacked into the band of spectrum not designated for sale to companies. In the second computational problem, given every broadcaster's value for remaining on-air, the goal is to find the subset of broadcasters with maximum total value (welfare) that can be feasibly repacked.

While both computational problems are NP-hard, the problem of checking feasibility can be reformulated as a *satisfiability (SAT)* problem, for which effective SAT-solver software exists (Newman et al., 2017).[13] For the problem of welfare maximization, however, no effective heuristic is known.[14] This demonstrates that computational complexity can be a true hurdle for mechanism design, forcing the designer to embrace an approximation approach, as was done in the FCC Incentive Auction.

6.3 Design Goals

For the rest of this section, our goal is to design a mechanism that: (i) is dominant-strategy incentive-compatible, or DSIC (an important requirement in our motivating example—the Incentive Auction should be simple for broadcasters to participate in); (ii) is welfare-maximizing, subject to feasibility; and (iii) runs in polynomial time. When the welfare maximization requirement involves solving an NP-hard problem, properties (ii) and (iii) are incompatible (even ignoring the DSIC requirement) and one of them must be relaxed. We consider relaxing (ii) and settling for approximate welfare-maximization. A fundamental question in algorithmic game theory, first posed by Nisan and Ronen (2001), is whether the DSIC requirement leads to further loss in the approximation factor. In other words, is *mechanism* design fundamentally more difficult than *algorithm* design?[15]

6.4 Approximation in Single-Parameter Settings

We introduce a single-parameter abstraction of the packing scenario described in our motivating example (Section 6.2). There are n players (broadcasters) with single-parameter values $v_1, \ldots, v_n$ for being chosen by the mechanism (staying on-air). There is a feasibility constraint $\mathcal{F} \subseteq 2^{[n]}$ over player sets, where $A \in \mathcal{F}$ if and only if the player set A can be feasibly chosen (repacked).[16] The auction outcome is the chosen (on-air) player set $A^* \in \mathcal{F}$, and its welfare is $\sum_{i \in A^*} v_i$.

[13] An instance of the satisfiability problem is a logical formula in a specific format with a number of free Boolean variables; the question is whether it's possible to assign values to the free variables so that the formula is satisfied (i.e., is true).

[14] Milgrom (2017), Section 4.3: "In the actual auction, Vickrey outcomes [...] cannot be computed at all."

[15] We focus on the case of DSIC mechanism design, where in general the answer is yes (Papadimitriou et al., 2008; Dobzinski, 2011; Dughmi and Vondrák, 2015; Dobzinski and Vondrák, 2016; Daniely et al., 2018). The same question makes sense for Bayesian incentive-compatible (BIC) mechanisms, and for this version of the question, recent research has produced strong and general positive results (Hartline and Lucier, 2015; Hartline et al., 2015; Bei and Huang, 2011; Dughmi et al., 2017).

[16] We assume that $\mathcal{F}$ is downward-closed, i.e., if A is feasible and $A' \subseteq A$ then A' is also feasible.

Myerson (1981) characterizes the range of DSIC mechanisms in such single-parameter settings as *monotone* allocation rules coupled with *critical bid* payments (analogous to the second-price payment). The approximate mechanism design question therefore reduces in these settings to approximation algorithm design, subject to the extra condition of monotonicity.

6.4.1 Example: Knapsack

A knapsack constraint corresponds to a shared resource with limited capacity W. Every player has a publicly-known size $w_i \leq W$ (e.g., the size of bandwidth it needs to stay on-air), and $A \in \mathcal{F}$ if and only if the set of players A fits within the knapsack ($\sum_{i \in A} w_i \leq W$). The computational problem of finding a welfare-maximizing player set among the sets that fit within the knapsack is NP-hard; assuming $P \neq NP$, there is no polynomial-time algorithm that solves the problem in general.

The classic algorithm of Ibarra and Kim (1975) achieves the next best thing: it guarantees 99% of the optimal welfare in polynomial time.[17] This algorithm is not monotone, but Briest et al. (2011) show how to tweak it to get a monotone allocation rule without compromising on the approximation factor. Together with Myerson's critical bid pricing, this gives a mechanism that satisfies all three desiderata (i)–(iii) above (up to a tiny loss in welfare).

6.4.2 A Simple Greedy-Based Mechanism

Next we consider a mechanism for the knapsack problem that is based on the "greedy" approach, which is remarkably simple to describe and analyze.

DSIC Greedy-Based Mechanism for the Knapsack Problem

1. Solicit bids $b_1, \ldots, b_n$, and re-index the players so that $\frac{b_1}{w_1} \geq \cdots \geq \frac{b_n}{w_n}$.
2. Choose the biggest prefix $\{1, 2, \ldots, i\}$ of players with total size at most W.
3. Return either this greedy solution or the highest bidder, whichever has a higher sum of bids.
4. Charge Myerson's critical bid prices.

In effect, this mechanism greedily considers players one at a time, ordered according to their "bang-per-buck." (The second solution is needed only to handle the case where there is a single bidder that is both very big and also has a very high valuation.) Holding all other bids fixed, by bidding higher a player can only go from being unchosen to being chosen by the mechanism. This monotonicity coupled with the pricing rule ensures that the mechanism is DSIC. It also has a non-trivial approximation guarantee:

Theorem 6.1 (Folklore) *The greedy-based mechanism for the knapsack problem runs in polynomial time and, assuming truthful bids, achieves at least* 50% *of the optimal welfare.*

[17] In fact, the guarantee is $(100 - c)\%$ of the welfare, where c can be an arbitrarily small constant. Formally, for any precision parameter ϵ, the algorithm guarantees $(1 - \epsilon) \cdot 100\%$ of the optimal welfare and runs in time polynomial in n and $\frac{1}{\epsilon}$. Similar comments apply to uses of "99%" later in this survey.

To see why, first imagine that the greedy prefix $\{1, 2, \ldots, i\}$ of players filled up the knapsack exactly, with no space left over. In this case, this prefix is an optimal solution—the player ordering ensures that every unit of space in the knapsack is used in the most-valuable possible way. The only issue is when the greedy prefix leaves some of the knapsack unfilled, because the sum of the sizes of the first i players is less than W and that of the first $i+1$ players is $W' > W$. The prefix $\{1, 2, \ldots, i+1\}$ would be optimal for a knapsack with capacity W', and hence is only better-than-optimal for the smaller knapsack capacity W. Each of the sets $\{1, 2, \ldots, i\}$ and $\{i+1\}$ is a feasible solution with the original capacity W, so one of them captures at least 50% of the optimal welfare. The greedy-based mechanism does at least as well.[18]

6.4.3 Performance of the Greedy Approach in Practice

The FCC Incentive Auction is more complicated than a knapsack setting, because its feasibility constraint $\mathcal{F}$ must take into account not only sizes but also potential interferences among geographically-close broadcasters. This results in a more difficult welfare-maximization problem, and greedy approaches cannot guarantee any constant fraction of the optimal welfare. However, when a greedy approach is applied in simulations, its empirical performance is excellent, achieving 95% of the optimum on average (Newman et al., 2017).[19]

What characteristics of "typical instances" make them easier to approximate that arbitrary instances? Approximation helps us identify relevant parameters that govern the difficulty of welfare maximization. Milgrom (2017) points to the distance of $\mathcal{F}$ from a matroid (see, e.g., Oxley (1992)) as one such parameter. Back in knapsack settings, item sizes are a crucial parameter: if the size of every item is at most α% of the knapsack capacity (say 5% or 10%), then the greedy-based approach guarantees welfare within $(100 - \alpha)$% of optimal, even in the worst case (see, e.g., Dütting et al. (2017)). These examples illustrate how stronger worst-case guarantees are often possible under stronger assumptions about the instances of interest.

6.5 Approximation in Multi-Parameter Settings

As we have seen, in single-parameter settings there is a successful paradigm for designing computationally efficient mechanisms with good approximation guarantees: (i) Characterize the design space of implementable algorithms (i.e., monotone allocation rules); (ii) Optimize over this design space (i.e., find the best computationally efficient monotone algorithm), or use a simple algorithm from the design space (like a greedy algorithm). This paradigm has had limited success in multi-parameter settings. The reason is that the characterization of implementable multi-parameter allocation rules (the "cyclic monotonicity" condition of Rochet (1987)) is quite inconvenient to work with.

An alternative idea focuses on the VCG mechanism, which can be seen as an ingenious way to transform a welfare-maximizing algorithm into a DSIC mechanism. Can this method be extended to *approximately* welfare-maximizing algorithms? Unfortunately, plugging an arbitrary approximation algorithm into the VCG mechanism does not generally preserve incentive-compatibility (Nisan and

[18] The guarantee of 50% is tight in the worst case. Let $\epsilon > 0$ be arbitrarily small. If there are 3 players with sizes $\frac{W}{2} + \epsilon$, $\frac{W}{2}$, $\frac{W}{2}$ and valuations $\frac{W}{2} + 2\epsilon$, $\frac{W}{2}$, $\frac{W}{2}$, then the greedy-based mechanism chooses player 1 and achieves welfare $\frac{W}{2} + 2\epsilon$, while the optimal solution (players 2 and 3) has welfare W.

[19] The problem instances considered by Newman et al. (2017) were kept small enough that the exponential-time computation of the benchmark (the optimal welfare) could be carried out in a reasonable amount of time.

Ronen, 2007). A more specific approach is to first commit to a restricted range of outcomes (prior to receiving players' bids), and then, given players' bids, to run the VCG mechanism with respect to the restricted range. The hope is to define a range small or well-structured enough to enable computationally-efficient welfare maximization over it, yet large enough to contain a near-optimal outcome for every valuation profile. The resulting DSIC mechanisms are called *maximum-in-range (MIR)* mechanisms.

For example, consider a multi-unit auction setting with n bidders and m homogeneous items (called units). We do not assume that bidders have decreasing marginal values. Assume that m is much bigger than n ($m = 2^n$, say), and that the goal is to compute an approximately welfare-maximizing allocation in time polynomial in n and $\log_2 m$.[20] Such a computation cannot even take the time to examine a bidder's valuation for each of the m possible allocations to it.

One simple maximal-in-range solution is to commit to selling units only in multiples of m/n^2—equivalently, to bundle the units into n^2 blocks of equal size—and then implement the VCG mechanism for this restricted range using dynamic programming. The resulting mechanism uses computation polynomial in n and $\log_2 m$, and is guaranteed to produce an allocation with near-optimal welfare.

Theorem 6.2 (Dobzinski and Nisan (2010)) *The maximum-in-range multi-unit auction above runs in time polynomial in n and $\log m$ and, assuming truthful bids, achieves at least 50% of the optimal welfare.*

Can we do better? Not with maximum-in-range mechanisms: Dobzinski and Nisan (2010) prove that no such mechanism can guarantee more than 50% of the optimal welfare. What about with a more general class of mechanisms?

A *randomized* mechanism outputs a distribution over allocations and payments. It is *truthful (in expectation)* if for any valuation reports of the other players, the expected utility of a player is maximized by reporting truthfully, where the expectation is over the randomization internal to the mechanism. The maximum-in-range approach can be immediately generalized to randomized mechanisms: A *maximum-in-distributional-range (MIDR)* mechanism pre-decides on a range of distributions over allocations; based on the reported valuations, it chooses the distribution from the range that induces the maximum expected welfare. MIDR mechanims are sufficiently powerful to obtain almost optimal welfare in multi-unit auctions:

Theorem 6.3 (Dobzinski and Dughmi (2013)) *There exists a maximum-in-distributional-range mechanism for homogeneous items that runs in time polynomial in n and $\log m$ and achieves in expectation at least 99% of the optimal welfare.*

The maximum-in-distributional-range approach is useful also in multi-item auctions with non-identical items. Circling back to FCC spectrum auctions, a reasonable model for how potential spectrum buyers value sets of channels is the class of *coverage* valuations, which assign value to every channel set according to the population it covers. There is a computationally efficient MIDR mechanism based on convex programming that achieves roughly 63% of the optimal welfare for bidders with coverage valuations (Dughmi et al., 2016).

[20] Exact welfare-maximization is provably impossible under this time constraint, even ignoring incentive-compatibility. If the running time need only be polynomial in m rather than $\log_2 m$, then the VCG mechanism can be implemented efficiently using an algorithmic technique called dynamic programming.

7 Discussion and Alternatives to Approximation

7.1 A Second Look at Approximation

Fundamentally, the goal of the approximately optimal mechanism design framework is to make comparisons between competing mechanisms for a problem and identify a "best-in-class" mechanism. Even with a given objective function, it is not always obvious how to compare two different mechanisms: the first will generally perform better in some cases (e.g., for some valuation profiles, or some prior distributions) and the second in other cases. This issue does not come up in classical welfare-maximization settings: because the VCG mechanism maximizes the social welfare ex post, it is better than every competing mechanism (with respect to the social welfare objective) irrespective of the valuation profile. It also does not arise in the traditional optimal auction setting: once the prior distribution is fixed, competing auctions can be unequivocally ranked according to expected revenue.

In many of the canonical applications of approximately optimal mechanism design, including the ones described in this survey, the theoretically optimal mechanism serves only as a benchmark. The performance of a mechanism of interest (like a "simple" mechanism) is assessed through a relative comparison to this benchmark. A given mechanism generally approximates the benchmark better in some cases than others, and two different mechanisms generally have incomparable performance, which each one superior to the other in some cases. In approximately optimal mechanism design, the performance of a mechanism is usually summarized with a single number, by taking the worst-case (i.e., minimum) performance guarantee achieved in a setting of interest. For example, the 50% guarantee in Theorem 4.1 holds in the worst case over all single-item auction settings with regular valuation distributions, the 50% guarantee in Theorem 5.2 holds in the worst case over all subadditive valuation distributions, and the 50% guarantee in Theorem 6.1 holds in the worst case over all valuation profiles.[21] With this single number attached to every mechanism, there is an obvious way to compare them, according to their worst-case approximation guarantees. This approach seems to enable results that would be hard or impossible to establish by other means, such as the quantification of simplicity-optimality trade-offs in Theorem 4.2 and the optimality of simultaneous first-price auctions among low-dimensional mechanisms (Theorems 5.2 and 5.4).[22]

Instead of taking the worst case over all settings of interest, why not take a Bayesian approach and impose a prior distribution over these settings? For example, in the single-item context of Theorem 4.1, we could assume that the seller has a "higher-order belief" in the form of a distribution over valuation distributions. But as pointed out by Segal (2003), "the dependence on the seller's prior is simply pushed to a higher level." Complexity and detail-dependence—the very disadvantages we were trying to avoid—creep back in, rendering this approach ineffective for our purposes.

In the remainder of this section, we discuss other approaches from the literature that aim to reveal useful mechanism design techniques or understand simplicity-optimality tradeoffs. Each of these approaches has both merits and downsides, and each is an important tool in the market

[21] All three results are also tight in a worst-case sense, meaning there exists a setting of interest in which the mechanism's approximation guarantee achieves the worst-case bound.

[22] Making the right modeling choices can be key to obtaining meaningful qualitative insights. One choice is the settings of interest—for example, the restriction to regular distributions in Theorem 4.1 or subadditive valuations in Theorem 5.2. The choice of the benchmark can also be important. For example, should the performance of a simple DSIC mechanism be compared to that of the best arbitrarily complex DSIC mechanism, or the best BIC mechanism? Of course, the more permissive the benchmark, the harder it is to approximate it.

designer's toolbox.

7.2 Alternative I: Robust Max-Min Optimality

Robust mechanism design is a rising paradigm in economics, surveyed in this issue by Carroll (2018). It has roots in operations research and robust optimization, where uncertainty about parameters is represented by deterministic "uncertainty sets," and optimization is in the max-min sense over these sets.[23] In recent years, robust mechanism design has offered justifications for the ubiquity of several popular auction and contract formats, often capturing the common wisdom about what makes them so widely embraced in practice.

Robust mechanism design shares some of the "worst-case" or "max-min" flavor of approximately optimal mechanism design, but it strives for max-min optimality rather than the optimal approximation of a benchmark. For concreteness, consider robustness against detailed knowledge of an exponential-sized joint distribution, representing the distribution of values for bundles of goods. Following Carroll (2017), assume instead knowledge only of the *marginal* distributions of values of individual goods. The paradigm of robust mechanism design replaces the original model, in which every market instance corresponds to a joint distribution, with a new model, in which every market instance corresponds to a set of marginals. The objective of maximizing the expected revenue for each instance is then replaced with a max-min objective: we measure a mechanism's performance on a "partial" instance (i.e., marginals) by its performance on the worst-case "full" instance (i.e., joint distribution) that is compatible with the partial instance. The result is a new problem formulation with new instances (partial) and objective (max-min) to maximize. In this new model, the optimal mechanism is well defined—it is the mechanism that maximizes the max-min objective for every partial instance. A successful outcome of this modeling approach could be the novel justification of a widely-used mechanism format as the robustly optimal solution, or the identification of a new and potentially useful robustly optimal mechanism.

Robust mechanism design makes weaker informational assumptions than its classical counterpart, thus tackling head-on the problem of excessive detail-dependence. When the primary criticism of the optimal mechanism stems from sources like communication or computational complexity rather than detail-dependence, however, it is not immediately obvious how to apply to robust mechanism design perspective. In this case, simple mechanisms are desirable due to their practical implementability, not their robustness to details of the environment per se.

7.3 Alternative II: Asymptotic Optimality (Large Market) Results

In asymptotic analysis, the performance of a simple mechanism is measured as the size of the market (number of players) goes to infinity. The hope is to establish that the mechanism becomes optimal in the limit, despite its simplicity. Such a result would imply that as long as the market is sufficiently "large," we can combine the best of both worlds (simplicity and almost-optimality). When successful, the asymptotic approach gives a new sense in which simple mechanisms can be very close to optimal.

The intuition behind "large market results" is as follows: As more players participate in a resource-allocation mechanism, the actions of a single player have an *increasingly negligible* effect on the prices and outcome of the mechanism (under certain conditions).[24] This makes the players

[23] See the work of Bandi and Bertsimas (2014) for an application of robust optimization to auctions.

[24] See Roberts and Postlewaite (1976) for an early example of quantifying this intuition.

more homogeneous from the mechanism's perspective, and their behavior easier to incentivize; it then becomes possible even for simple mechanisms to optimize economic objectives such as revenue and welfare. Put differently, worst-case bounds on inefficiency can be overly pessimistic when determined by pathological "small market" examples, and this issue goes away in larger markets.

A central example of the asymptotic optimality approach is the work of Swinkels (2001), who studies simple auctions for asymmetric players competing over a single good (with multiple units). Whereas in small markets such auctions can have very inefficient equilibria, Swinkels (2001) shows that under certain conditions of "noisy" demand or supply, the equilibria become arbitrarily close to welfare-maximizing as the markets grow large. Roughly, the role of the noisy demand/supply assumption is to provide enough randomness to rule out pathological examples with inefficient equilibria that persist even in large markets. Feldman et al. (2016) extend these results to multiple different goods, as well as to additional simple auction formats. They also combine the large market approach with the approximation paradigm; in one of the settings they study, equilibrium inefficiency decreases but does not vanish in the limit. Segal (2003) establishes a large market result for revenue rather than welfare, showing how the revenue performance of detail-free auctions can converge quickly to that of optimal detail-dependent ones; see also Baliga and Vohra (2003), Goldberg et al. (2006), and Neeman (2003) for related results. Other large market results exist in the context of resource allocation without money, some showing that simple mechanisms increase in efficiency as the market grows large (e.g., Che and Kojima (2010)), others that they gain good properties like truthfulness or stability (e.g., Che et al. (2018)).

There are two main downsides to the asymptotic large market approach: (i) no guarantees for small- to medium-sized markets, which are common in many modern applications (e.g., many keyword advertising auctions); and (ii) in some cases, reliance on sufficient randomness in the market.

7.4 Alternative III: Resource Augmentation

Bulow and Klemperer (1996) first introduced the approach of resource augmentation as an alternative to complex optimal mechanisms. In their seminal auctions vs. negotiations result, they show that in a symmetric single-item auction setting with a common regular valuation distribution, running a standard Vickrey auction with one extra (i.i.d.) bidder earns at least as much expected revenue as a (distribution-dependent) optimal auction without the extra bidder. We can view the players as "resources," in the sense that their competition is what drives up prices and generates high revenue. Adding a player can be viewed as an "augmentation" of these resources.[25] While the intuition of more competition leading to more revenue is clear, it is not a priori apparent by how much the competition needs to be increased for a simple mechanism to outperform the optimal one (and whether this is at all possible for a particular simple mechanism).

Roughgarden et al. (2017b) generalize the result of Bulow and Klemperer (1996) to multi-item auctions, and also make two connections to approximation: first, by combining resource augmentation with approximation to limit the required amount of extra resources; and second, by establishing a framework for transforming a resource augmentation guarantee into an approximation one. Related approximation guarantees were established by Devanur et al. (2011). The work of Eden et al. (2017) further generalizes the approach of augmenting competition by applying it to

[25]The phrase "resource augmentation" was originally coined by Kalyanasundaram and Pruhs (2000) in the context of algorithm analysis, inspired in part by Sleator and Tarjan (1985).

more restrictive benchmarks and challenging revenue maximization settings. Feldman et al. (2017) combine resource augmentation with an approximation guarantee of 99%, and Liu and Psomas (2018) apply this approach to dynamic mechanisms.

One downside of resource augmentation is that comparing the augmented mechanism to the optimal one with no augmentation is in some sense "unfair," like comparing apples to oranges. On the other hand, this approach enables a direct comparison between the cost of resource augmentation and the cost of mechanism complexity, which is important for making an informed choice between a simple mechanism and its complex counterpart. Another limitation of the competition enhancement approach is that it typically assumes that players are i.i.d. and regular; for possible solutions to this issue, see Hartline and Roughgarden (2009) and Sivan and Syrgkanis (2013). Finally, resource augmentation has been applied so far mainly to revenue-maximization problems, although it has recently been adopted for other domains as well (Akbarpour et al., 2018).

8 Conclusion

The main message of this survey is that approximation is useful for achieving *qualitative insights* on mechanism design in *complex settings*. Section 8.1 summarizes briefly our supporting evidence for this statement, and the takeaways from our three case studies. Complexity is quickly becoming the norm, and even the defining feature, in many important economic settings (Nisan, 2017). Many modern transactions take place within complex market environments—ridesharing platforms, crowdsourcing marketplaces, and so on. This suggests that, like other techniques for dealing with complexity, the approximation paradigm will only increase in utility in the coming years.

At present, however, approximation is possibly the most polarizing topic in debate among computer scientists and economists working on mechanism design. Economic theorists have largely ignored this paradigm, passing on the opportunity to add to their arsenal a well-developed and deep mathematical toolbox.[26] For their part, computer scientists have arguably been guilty of devoting disproportional effort to small improvements in approximation factors, often at the expense of useful qualitative insights. They have also been accused of viewing every problem as a nail to which the approximation hammer should be applied.

We postulate that many if not all of these issues are caused by an overly literal interpretation of approximation factors, as detailed in Section 3. In this sense, the debate on approximation has greatly advanced the research community's understanding of the meaning behind approximation guarantees in mechanism design. It goes without saying that a guarantee of (say) 50% does not in itself justify the use of a mechanism; but in the context of a challenging area of mechanism design, in which there is no useful characterization of the optimal mechanism and no explanation as to why certain mechanisms are observed in practice, the same guarantee of a 50%-approximation can become meaningful and enlightening.

As Carroll (2018) notes, the culture of economic theory is becoming gradually more pluralistic; we believe this is an excellent time for economists to take another look at approximation—at the very least, as a useful complement to the widely-accepted approaches in Section 7. In the best-case scenario, approximation could become a leading example of the kind of gains that stem from interdisciplinary research.

[26] See Arnosti et al. (2016) for a recent exception.

8.1 Summary of Case Studies

We briefly summarize the benefits of applying the approximation paradigm to address the following three complexity barriers in mechanism design: (1) opaque and detail-dependent mechanisms; (2) unreasonable communication requirements from participants; and (3) prohibitive computational complexity.

Section 4 considered barrier (1). In single-parameter environments, the pursuit of approximation guarantees guided us to a parameterized family of mechanisms that runs the gamut from complex optimal mechanisms to simple mechanisms with constant-factor approximation guarantees. In multi-parameter environments, the approximation approach led to a relatively simple mechanism—selling items separately or as a single bundle, whichever is better—that provably extracts a constant fraction of the optimal expected revenue in certain multi-item auction settings.

Section 5 focused on barrier (2). Here, the approximation perspective confirms that the simple common method of selling items separately has excellent welfare-performance guarantees, as long as bidders' preferences do not include strong complementarities between items. This mirrors the conventional wisdom among both theoreticians and practitioners in multi-item auction design.

Section 6 addressed barrier (3), and showed that a natural greedy approach to sharing a limited-capacity resource is near-optimal in theory, and significantly exceeds expectations in practice. Meanwhile, in certain multi-item auctions, generalizations of the VCG mechanism achieve near-optimal welfare.

In all three applications, traditional economic tools targeted at characterizing exact and optimal solutions appear inadequate to achieve similar results.

8.2 Directions for Future Research

We highlight three directions for further research on approximation and mechanism design: new applications for the approximation paradigm; improved understanding of the relationships between different notions of complexity (and the corresponding approximation guarantees); and narrowing the gaps between worst-case analysis and the "typical cases" relevant to practice. We believe that more research in these directions is necessary to expand the reach of the theory and improve its coherence and applicability. Many additional questions arise in relation to existing application areas of the approximation paradigm, like those in our case studies.

1. **New frontiers for approximation.** Cutting-edge economic theory and new applications can inspire new ways for the approximation paradigm to contribute. One source of new frontiers is market design in practice. For example, as part of the design of the FCC Incentive Auction, Milgrom and Segal (2017) developed a *reverse greedy heuristic* and analyzed its strong incentive properties. The approximation paradigm can be used to study formally its welfare guarantees (Dütting et al., 2017; Gkatzelis et al., 2017).

   Another source for new opportunities is classical areas of economics in which complexity matters, and perhaps has been studied using one the approaches in Section 7, but to which the approximation paradigm has not yet been applied. For example, *contract design* is a major success story of robust mechanism design (Carroll, 2015), and only very recently has the approximation lens been applied to it (Dütting et al., 2018). Similarly, the large market approach (Section 7.3) has been successful in achieving fair allocations in conjunction with

efficiency and strategyproofness (Che and Kojima, 2010). To relax the large market requirement, researchers are beginning to explore different notions of *approximate* fairness (Budish, 2011; Caragiannis et al., 2018).

Ideas can also flow in the opposite direction. For example, we used the approximation paradigm in Section 4 to formalize simplicity-optimality trade-offs. Could such trade-offs also be formulated using one of the alternative approaches in Section 7? For example, are there settings where the robustly optimal mechanism (in the sense of Section 7.2) becomes gradually more complex as more details of the environment are revealed?

2. **Relations among different complexity measures.** We have demonstrated how the approximation paradigm helps tackle different types of complexity. These have largely been studied in isolation, but researchers are now aspiring to a more holistic and comprehensive understanding of mechanism design complexity. For example, in Section 4 we discussed revenue approximation guarantees for classes of mechanisms with low information requirements. Do any of our conclusions change dramatically if we also enforce computational tractability? See Gonczarowski and Nisan (2017) for a recent example of work in this direction.

   Similarly, Section 5 discussed equilibrium welfare guarantees for auctions with low communication requirements. What if we relax the assumption of convergence to equilibrium, perhaps assuming instead some form of natural dynamics that requires less computation? See Roughgarden et al. (2017a) for a more detailed discussion of this point.

   A final example is the recent work of Dobzinski (2016), who proved a formal relationship between different measures of mechanism complexity—some measures related to the mechanism's format (when viewed as a menu of prices, e.g., the number of such prices), and some to its required resources (communication or computation). Extending such results to additional complexity measures would contribute to a more unified theory of mechanism complexity, and thus also of approximately optimal mechanisms.

3. **Realistic (rather than pessimistic) models of complexity.** In Section 6.4.3 we saw an example of the gap between worst-case approximation guarantees for greedy-based mechanisms and their (much better) performance in practice. The "beyond worst-case analysis" research agenda in computer science advocates sharper analysis of algorithms to capture their true behavior, usually by focusing attention on a subset of the "most relevant" inputs. The same agenda is relevant for the analysis of mechanisms—we wish to develop models that explain when and why the empirical performance of simple mechanisms significantly exceeds their worst-case approximation guarantees. See Psomas et al. (2018) for a recent effort in this direction.

   Echenique et al. (2011) and Barman et al. (2014) approached this issue from the perspective of revealed preference and showed that, in certain settings, every rationalizable set of choice data is in fact consistent with an easy optimization problem. It would be interesting to extend this approach to other settings, such as quasi-linear markets.

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