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# Multi-objective MILP model for PMU allocation considering enhanced gross error detection: A weighted goal programming framework

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ABSTRACT

This paper proposes a multi-objective mixed-integer linear programming (MILP) model for the optimal phasor measurement unit (PMU) placement (OPP) problem. A majority of the solutions presented for the OPP problem focus on minimizing cost while guaranteeing system observability. While this is a good approach to the OPP problem, the effect of PMU allocation on various energy management system applications should also be considered. This paper addresses the OPP problem considering PMU installation costs, system observability, and gross error detection. In order to allocate PMUs considering gross error detection, the Vulnerability Index (VI) is used to quantify the vulnerability of each element in the system. A weighted goal programming (WGP) framework, which allows for the optimization of contradictory goals, which is a characteristic of the goals considered in this paper, is presented. The goals and weights of the framework are user inputs to the program besides a hard budget restriction. The results of this paper show how the WGP framework allows the priorities of the decision maker to influence the final PMU allocation presented by the MILP, making this model a valuable framework for utilities planning to install PMUs on their network.

## 1. Introduction

It is well known that phasor measurement units (PMUs) have multiple benefits to offer power systems monitoring and control by frequently providing precise wide-area measurements of both the magnitude and phase of voltages and currents [1]. Besides providing more accurate measurements, PMUs can further increase measurement redundancy on power systems [2]. Ideally, PMUs could be installed at every bus so that redundancy is maximized and every bus has precise and synchronous measurements. Unfortunately, this is not a feasible strategy due to the high cost of PMUs. This cost is composed of procurement of the devices, installation, ongoing maintenance, accessories such as software and additional measurements, and the development of a wide-area measurement system (WAMS) [3]. Because of this, a major topic of recent research has been developing strategies to use the minimal amount of PMUs on a system while maximizing the positive impact on the monitoring and control of the power system. This research topic is commonly known as the optimal PMU placement (OPP) problem and begs the question of what qualifies as a positive impact.

The first and still popular idea to solve the OPP was to allocate PMUs such that the system can be observable. This would allow PMU measurements to be used in state estimation (SE) either in conjunction with SCADA measurements or on their own [4]. This idea remains a crucial one for PMU allocation due to the interest in the use of linear state estimators, as shown in a survey conducted by the North American Synchrophasor Initiative (NASPI) [5] as well as in a report by the North American Electric Reliabilty Corporation (NERC) [6]. In order to optimize the allocation of PMUs with observability in mind, a variety of optimization techniques were studied. These include genetic algorithm, dual search methods, greedy algorithm, and linear integer programming, among others [7–10]. Manousakis et al. [11] reviews further heuristic methods used to solve the OPP problem. More recently, more realistic cost scenarios are considered with a genetic algorithm to focus on cost minimization specifically [12]. The NERC report [6] points out that there are two strategies to allocating PMUs for observability: (1) minimize the number of PMUs to achieve observability and (2) maximize the observability for a given number of PMUs. One of the goals of this paper is to present a strategy that can take on either of these strategies or even a hybrid of them based on the user preference and inputs.

The initial works of OPP strategies for observability only considered allocation of PMUs at buses, but did not focus on which branches to

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measure currents. These strategies assumed that a PMU was installed with all possible current measurements and that all of the PMUs would have the same installation cost. In reality, the addition of current measurements will augment the cost of PMU installation. In the authors' previous work [13], these current measurement costs are taken into account through an updated cost function modeled through a mixed integer linear programming (MILP) model that considers not only which buses to install a PMU, but also which branches to install current measurements. This helped reduce the cost of PMU installation while maintaining observability.

Beyond allocating PMUs for observability goals, research has been done considering other specific aspects of power systems real-time monitoring. [14] allocates PMUs with the goal of improving the accuracy of the SE, but does not consider the potential for gross errors. [15] attempts to enhance the detection of parameter errors through strategically placed PMUs, but this is done without considering system observability. Madani et al. [16] takes a more holistic view for power systems monitoring by looking at various application and infrastructure concerns which could be enhanced through integrating more focused strategies. Chen and Abur [17] aims to enable better bad data detection through PMU measurements, but does not guarantee observability or consider the selection of specific branch measurements. Aghaei et al. [18] considers a multi-objective approach, minimizing the number of PMUs while maximizing the system's expected redundancy. However, simply maximizing redundancy doesn't guarantee that the utility of the new PMU measurements is optimized. Adding redundancy to a secure area of the system is not as useful as adding redundancy to a vulnerable area. Li et al. [19] takes an information theory approach and does not directly consider observability or cost. Another multiobjective approach is taken in [20], where the objectives are maximizing reliability with the minimum number of PMUs. Khiabani et al. [20], however, uses a heuristic approach that does not guarantee optimality and does not provide a way to directly consider financial costs. In [21], the authors present a strategy for enhancing bad data detection by allocating PMUs while still guaranteeing observability. This model, however, does not explicitly include the installation costs of PMUs. The objective function in [21] is formulated such that a minimum amount of PMUs will be allocated, which means the improvement in bad data detection is limited. In this paper, the authors expand on this formulation by including both installation costs as well as enhancing bad data detection explicitly. In general, previous strategies for the OPP problem provide solutions without user input. Any weighting or metrics are calculated or based on equipment specifications. Since there are many potential solutions to the OPP problem, this paper presents a framework that allows meaningful user input and can be easily expanded to include previously presented indices and metrics.

This paper presents a multi-objective MILP model and WGP framework for the OPP problem. The specific contributions of this work towards the state of the art are threefold:

1. An explicit mathematical model of the interdependencies between the installation costs of PMUs and gross error detection.
2. A weighted goal programming framework applied to the OPP problem which allows for customization based on the user's desired approach and the addition of new goals.
3. An improved topological observability constraint which considers both the PMU voltage measurements as well as the current measurements.

The remainder of this paper is organized as follows. Section 2 presents key technical aspects of the OPP and goal programming. The multi objective MILP model and WGP framework are presented and discussed in Section 3. Section 4 presents a case study. Section 5 provides concluding remarks of this work.

## 2. Background

### 2.1. Vulnerability Index

The Vulnerability Index (VI), known as UI in [21], quantifies the vulnerability of a bus to undetectable gross errors in measurements associated with that bus, considering the classical weighted least squares state estimation (WLS SE) model [22]. In this paper, the VI concept is extended to system branch measurements. This is most important because the addition of branch current measurements will help with gross error detection, not just add cost. The VI is developed based on the Innovation concept presented in [23–25]. In these works, a geometrical view of the measurement error based on the classical WLS SE model is presented. In the WLS SE, (1) is solved to minimize the weighted norm of the residuals, where $z \in \mathbb{R}^m$ is the measurement vector and $x \in \mathbb{R}^N$ is the state variables vector. Also, $h: \mathbb{R}^N \to \mathbb{R}^m$ is a continuously nonlinear differentiable function, $r \in \mathbb{R}^m$ is the measurement residual vector assumed to have zero mean, standard deviation $\sigma$ and Gaussian probability distribution, and $N = 2n - 1$ is the number of unknown state variables to be estimated ($n$ is the number of buses of the power system):

$$z = h(x) + r \tag{1}$$

Let $\hat{x}$ be the solution of this minimization problem thus, the estimated measurements vector is given by $\hat{z} = h(\hat{x})$ and the residuals vector is defined as the difference between $z$ and $\hat{z}$, i.e. $r = z - \hat{z}$. The linearization of (1), at a certain operating point $x^*$, implies:

$$\Delta z = H\Delta x + r \tag{2}$$

where $H$ is the Jacobian matrix of the measurement model.

If the system represented by (2) is observable, then the vector space $\mathbb{R}^m$ of the measurements can be decomposed in a direct sum of two vector sub-spaces, in the following way:

$$\mathbb{R}^m = \mathbb{R}(H) \oplus (\mathbb{R}(H))^{\perp} \tag{3}$$

so, the range space of $H$, given by $\mathbb{R}(H)$, is an $N$-dimensional vector subspace that belongs to $\mathbb{R}^m$ and $(\mathbb{R}(H))^{\perp}$ is its orthogonal complement, i.e. if $u \in \mathbb{R}(H)$ and $v \in (\mathbb{R}(H))^{\perp}$, then $\langle u, v\rangle = u^T R^{-1} v = 0$. Then, it is possible to decompose the measurements error vector $e$ into two components: undetectable ($e_U$) and detectable ($e_D$).

The detectable component is the same as the measurement residual, $r$ from (1), and is commonly used for error detection through the Chi-squared test. The undetectable component of the error can be estimated using the projection matrix of the WLS SE solution. For that purpose, the Innovation Index (II) is defined as the ratio between these two components of the error, $e$:

$$\mathrm{II}_i = \frac{||e_{\mathrm{D}i}||_W}{||e_{\mathrm{U}i}||_W} \tag{4}$$

where the weight matrix $W$ is the inverse of the diagonal measurement covariance matrix. Using the II, the composed measurement error (CME) can be easily calculated from the residual, $r$:

$$\mathrm{CME}_i = r_i\sqrt{1 + \frac{1}{\mathrm{II}_i^2}} \tag{5}$$

From there, sensitivity analysis of both the residual and the CME is done to compute the VI metric. In classical WLS SE, the sensitivity of the residuals is defined as the change in residuals due to arbitrary perturbation introduced in the measurement vector. Given the solution of the linear estimation model, this sensitivity matrix can be expressed as $r = S_r e$. $S_r$ can be easily calculated based on the Jacobian matrix and $W$ used in the WLS SE method [22]. Applying the same logic to the CME, the sensitivity of the CME can be expressed as $CME = S_{CME} e$ and $S_{CME}$ calculated using (5):

$$\mathrm{VI}_i = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\sum_{j=1}^{J}(S_{\mathrm{CME}}(k,j) - S_r(k,j))}^2 \tag{6}$$

where $K$ is the set of measurements associated with bus or branch $i$, $J$ is the set of all measurements, and $S_{CME}$ and $S_r$ are the sensitivities of the CME and residual, respectively, with respect to gross errors in measurements. For a bus, the set of measurements associated with that bus is all measurements taken directly on that bus or any branch connected to that bus. For a branch, this set is any measurement taken directly on that branch or on the two buses that the branch connects. This means that a single measurement will impact multiple VI values on the system. However, the impact will be different for each bus and branch a measurement is associated with, so it should be taken into account multiple times. The VI of a bus or branch quantifies the vulnerability of that bus or branch measurement to gross errors with undetectable components. The higher the VI, the more vulnerable that system component is to undetected errors considering the classical WLS SE model. Therefore, the objective of the optimization problem should be to allocate PMUs at buses and branches with the highest VI. Innovation based metrics and analysis has been used to enhance the detection of gross errors before [24,26–29,21], but not while mathematically modeling the interdependency between gross error detection and PMU installation costs. By modeling this interdependency, it will be possible to determine if the utility of a certain PMU or specific PMU measurement outweighs the cost of installation.

### 2.2. PMU cost model

In [30], the US Department of Energy (DOE) presented a study on the real-life costs of PMU installation. This was done by interviewing nine utility companies that went through a PMU installation project. It shows that the actual PMU devices make up a small percentage of the total cost, normally in the 5–10% range. The majority of costs in PMU installation projects come from the necessary communication infrastructure to synchronize the PMUs and send the measurement data. This can vary widely from company to company. This cost depends on the existing communication infrastructure; is it sufficient for the addition of PMUs, does it need to be upgraded, or does a brand new communication infrastructure need to be installed alongside the PMUs? Other factors for cost include security and labor costs. Ultimately, the DOE found that the average cost per PMU installed was anywhere from $40,000 to $180,000.

In the OPP frameworks presented in [31,15,13,21], the costs of PMUs was broken down to a base cost that includes voltages measurements and additional costs for optional current measurements. The framework presented in this paper requires the same type of cost model. Since taking any measurements with PMUs requires a communication system, this will be much larger than the additional current measurement costs. In this paper, the base cost of a PMU installation is considered as $50,000 while the addition of current measurements is assumed to cost $5000 per channel. The ratio between these comes from the 5–10% range of device costs plus a small amount of labor required. These values are in line with the DOE study [30] and costs used in other research [31,15,13,21]. This work considers the costs will be the same regardless of which bus a PMU is installed on.

### 2.3. Goal programming

GP is a multi-objective programming technique in which one tries to minimize the distance of the solution to previously defined goals [32]. The GP framework is designed to optimize objectives that are contradictory in nature. The most common example of this is using a cost based objective against a disutility based objective. Added costs will minimize disutility and vice versa. A GP framework can mathematically model the interdependencies between problem objectives. It will be impossible to achieve all of the objectives in the problem, so optimization is extremely valuable in this scenario. GP has two major subsets: weighted GP (WGP) and lexicographic GP (LGP). This work uses a WGP framework to mathematically model the two goals of the problem. WGP is used because it allows for direct trade-offs to be mathematically modeled between the different goals in the problem. LGP uses preemptive priority levels for the goals of the problem where each priority level is optimized individually. Higher priority levels are considered infinitely more important than lower levels [33], which is not useful for the two goals considered in the OPP problem presented here.

In WGP, the objective function is defined by the weighted normalized distance between the predefined goal for one objective and the explicit mathematical model which estimates the latter. The values of the various goals in the WGP objective function are chosen by the user. Since the objectives in WGP are contradictory, the goals set by the user should not be simultaneously achievable. If they are, the solution will be relatively trivial. The easiest goals to set are the extreme values for each goal. These are often trivial or expected solutions based on the optimization of just that objective. The advantage of using WGP is the ability to find a middle ground between these individual optimizations. To complete the formulation of the objective function, each differential variable is multiplied by a weight and these terms are then added. An important aspect of creating a fair model is the normalization of the differential variables. If the objectives are not normalized properly, objectives with larger magnitudes will attract the solution towards that objective.

There have been many proposed techniques of normalization for GP models. In this paper, the zero-one normalization technique is used because it normalizes each differential variable individually rather than the objective function as a whole [32]. This technique requires a closed feasible set for each objective, which is not an issue with the OPP problem [34]. In this technique, the normalization factor for each objective is the difference between the goal and the worst possible value for that objective. Hence the differential value will be between zero and one. Using this normalization technique, the extreme goal of an objective to be minimized would be 0 while the extreme goal of an objective to be maximized would be 1.

The weights in WGP can have two functions: normalization of the goals and to indicate preference of the decision makers. Since normalization has already been taken care of, the model presented in this paper uses the weights to indicate preference. This will be discussed further in the presented formulation in Section 3.

## 3. OPP using goal programming approach formulation

### 3.1. WGP framework

The WGP framework for the OPP problem is illustrated in Fig. 1. In order to actually use this framework, existing software can be utilized with a supplementary post processing step. Topology processing and state estimation are standard procedures that have been done by utilities for many years. The VI analysis consists of some simple calculations that are done using the projection matrix of the WLS SE solution and the residuals of the solution. These calculations are discussed in Section 2.1 where the final VI metric is shown in (6). One may further use the extended WLS SE model [35]. The PMU cost model was described in Section 2.2, but as long as a cost model provides a final cost for the base PMU and additional current measurements, the proposed model can easily handle different cost models. The goals, weights, and budget are decided and set by the user of the framework. The inputs for this framework are described in detail later, but are easily changed to provide the user various options for their final optimal PMU allocation.

### 3.2. Goal programming formulation

As discussed, minimizing the number of PMUs is good for cost

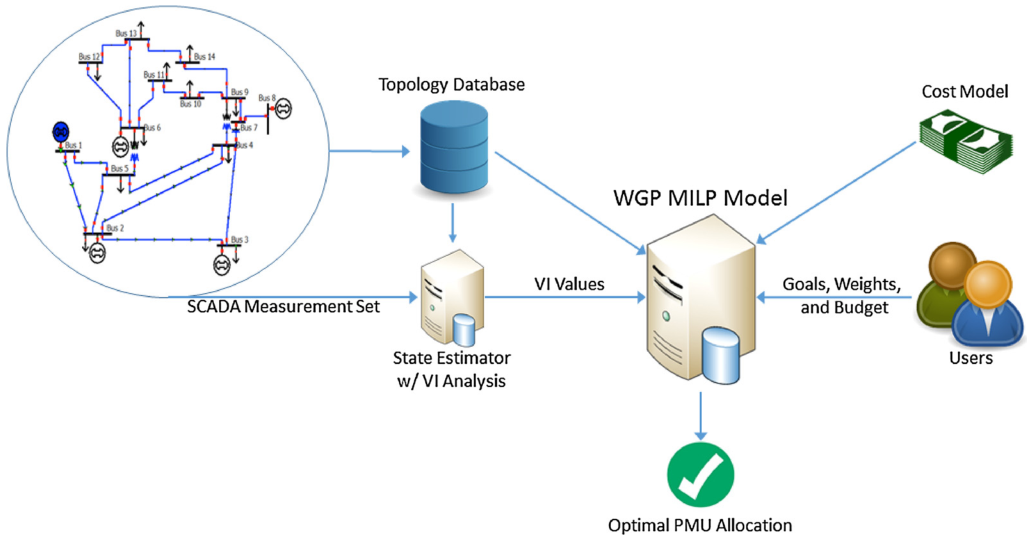


**Fig. 1.** Presented weighted goal programming framework.

considerations, however, this minimal number does not need to be strictly enforced. In reality, utilities may have flexibility with their budget that allows them to allocate more PMUs than necessary [30]. This will add additional redundancy to the system as well, consequently improving the accuracy of state estimation and gross error analysis. Considering this possibility, this paper presents a multi-objective MILP model using a WGP structure that can be easily used as a framework by utilities to develop a PMU allocation strategy that fits their needs and goals. The two objectives considered in this paper are financial cost and gross error detection. Since these are contradictory goals, a WGP framework is ideal for creating an explicit mathematical model of these interdependent conflicting objective values.

The presented WGP formulation is as follows:

$$\min_{x_s, x_r} \quad \omega_{\mathrm{VI}}\delta_{\mathrm{VI}} + \omega_{\mathrm{cost}}\delta_{\mathrm{cost}} \tag{7}$$

$$\text{where} \quad \delta_{\mathrm{VI}} = \mathrm{VI}_{\mathrm{goal}} - ([\mathrm{VI}_{\mathrm{bs},1}\mathrm{VI}_{\mathrm{bs},2}...\mathrm{VI}_{\mathrm{bs},n}]x_s + [\mathrm{VI}_{\mathrm{br},1}\mathrm{VI}_{\mathrm{br},2}...\mathrm{VI}_{\mathrm{br},2m}]x_r) \tag{8}$$

$$\delta_{\mathrm{cost}} = (c_s x_s + c_r x_r) - c_{\mathrm{goal}} \tag{9}$$

$$\text{s. t.} \quad x_s + A_r \mathrm{Q} x_r \geq 1 \tag{10}$$

$$\mathrm{P} x_s \geq x_r \tag{11}$$

$$c_s x_s + c_r x_r \leq b \tag{12}$$

$$\delta_{\mathrm{VI}}, \delta_{\mathrm{cost}} \geq 0 \tag{13}$$

where

$$x_{s,i} = \begin{cases} 1 & \text{if PMU at bus } i \\ 0 & \text{if no PMU at bus } i \end{cases} \tag{14}$$

$$x_{r,2j-1} = \begin{cases} 1 & \text{if current phasor measured at sending end of branch } j \\ 0 & \text{otherwise} \end{cases}$$
$$x_{r,2j} = \begin{cases} 1 & \text{if current phasor measured at receiving end of branch } j \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$c_{s,i} = \text{cost of installing a PMU at bus } i \tag{16}$$

$$c_{r,j} = \text{cost of measuring current at this location} \tag{17}$$

$$A_{r,\mathrm{ij}} = \begin{cases} 1 & \text{if branch } j \text{ is connected to bus } i \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$Q_{j,2j-1} = 1$$
$$Q_{j,2j} = 1 \tag{19}$$

$$P_{2j-1,i} = \begin{cases} 1 & \text{if sending end of branch } j \text{ is connected to bus } i \\ 0 & \text{otherwise} \end{cases}$$
$$P_{2j,i} = \begin{cases} 1 & \text{if receiving end of branch } j \text{ is connected to bus } i \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

The objective function (7) for this framework follows the standard WGP formulation. Each goal has its own term in the objective function which consists of the weight of that objective and the difference variable for that goal. Because this problem has two goals; enhanced error detection and cost; there are two terms in the (7). The difference factor for each goal is defined in (8) and (9). This is also where the actual decision vectors $x_s$ and $x_r$ influence the objective function value. $x_s$, as described in (14), has one element per bus to decide if a PMU is to be installed at that bus. $x_r$, as described in (15), has two elements per branch so that a current measurement can be taken from a PMU at either end of the branch or both.

In (8), $VI_{bs,i}$ represents the VI of bus $i$ while $VI_{br,j}$ represents the VI of branch location $j$. Both of these are calculated based on Eq. (6). Since the framework aims to allocate PMUs at locations with the highest VI values, the VI goal should be a high value that the program attempts to match. Therefore, the difference is the goal subtracted by the total normalized VI of buses and branches where PMU measurements are allocated. Cost, on the other hand, is to be minimized, so the difference factor is the total cost subtracted by the goal. The specific values for $c_s$ and $c_r$, described in (16) and (17), may vary from utility to utility, but provided a final value for the base PMU cost and additional current cost, this framework will handle different costs easily. As discussed in 2.2, this paper uses a cost model where each bus has the same costs, but that is not a requirement for this framework. Overall, the objective function minimizes the weighted sum of these difference factors.

The first constraint (10) guarantees observability of the system with PMU measurements. The matrix $A_r$, as described in (18) in this constraint is built based on the topology of the system. Each element represents whether a certain branch is connected to a certain bus. Naturally, each column will have two elements with value 1, while the rest will be 0. The $Q$ matrix in (19) simply expands $A_r$ to line up with the decision vector $x_r$. As discussed earlier, it is important to include this constraint due to the interest in using PMU measurements for linear state estimator purposes [5,6]. This constraint is very similar to the observability constraint in [13], but has the added variable $x_s$. In previous works, it was not necessary to include this variable because the

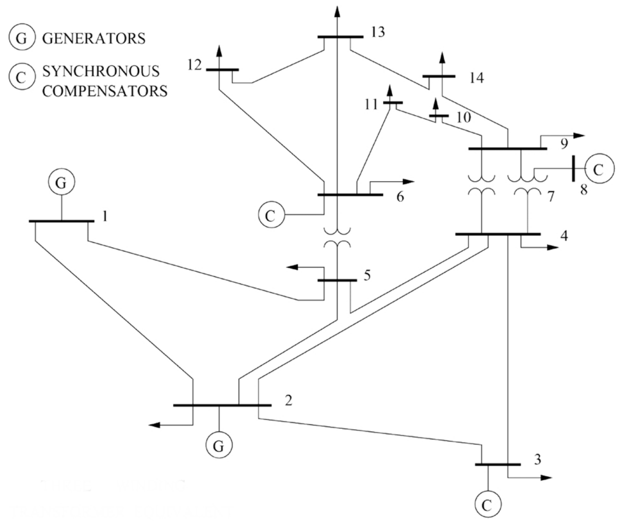


**Fig. 2.** 14-Bus IEEE System.

optimization problem was always looking to allocate the minimum number of PMUs. Therefore, many branch currents were necessary to meet the observability constraint anyway. It's possible for the WGP framework to be configured to allocate PMUs at every bus. In this case, no branch measurements should be necessary to meet the observability constraint since each bus voltage is being measured directly. Without the $x_s$ variable in (10), however, $\frac{n}{2}$ branch measurements would be required to meet the constraint, where $n$ is the number of buses on the system. The addition of $x_s$ makes this is a more complete observability constraint.

It is important to note that constraint (10) only guarantees topological observability since it is a topology based equation. As discussed in [36], in order to use a set of measurements for SE purposes, these measurements must make the system numerically observable as well, meaning the Jacobian matrix is full rank. With this in mind, the authors included a separate numerical observability check by setting up a Jacobian matrix for only PMU measurements. None of the solutions found by the WGP presented a numerical observability problem.

Constraint (11) acts as a physical constraint to ensure that any potential current measurements are being taken from a bus that has a PMU allocated to it. The matrix $P$, as described in (20), is built based on the topology of the system, similar to $A_r$. The difference is that each branch is split into two columns so that the sending and receiving end of each branch has its own column. Constraint (12) is a budget constraint where the total cost of the PMU installation and added current measurements is below a budget specified by the user.

The constraints in (13) are important to understand when setting the goals in the problem. These are non-negativity constraints on the differential variables for each goal. This means that when a goal is set by the user, the final solution is limited such that a goal cannot be surpassed (making the difference a negative value). For example, the IEEE 14-bus system requires four PMUs to guarantee observability. However, if a utility has the budget to allocate five or more PMUs and the goal is set to use that money, the model will allocate at least five PMUs even if a potentially better solution only uses four PMUs. More specific examples of this type of scenario will be discussed in Section 4.

### 3.3. User inputs and flexibility

In the presented approach for the OPP problem, there are five variables that can be controlled by the user to make sure the framework produces a solution that fits their needs:

1. VI goal
2. Cost goal
3. VI objective weight
4. Cost objective weight
5. Budget

Since the VI metric (6) represents the vulnerability of a bus or branch to undetected gross errors in measurements, the VI goal is essentially how much of the overall vulnerability of the system should be addressed through adding PMU measurements on the system. The extreme case of this would be covering the whole system, or allocating a PMU at every bus on the system and measuring every branch current. In this case, the numerical value for the $VI_{goal}$ would be simply the normalized sum of all bus and branch $VI$ values. Because the model uses zero-one normalization, this case is equivalent to a goal of 1. Therefore for the user, it is easiest to simply define a value between 0 and 1, the normalized range for the VI goal. The cost goal is the desired amount of money that will be spent on PMU installation, while the budget is a hard limit on how much money is allowed to be spent. While the normalized cost values are also between 0 and 1, the framework can take in a real dollar amount and calculate the normalized value. This is for the user's convenience. The extreme limiting case for the cost goal would be spending no money, i.e. $c_{goal} = 0$.

The two weight variables can be used to determine the priority of the goals in the problem. It is simplest to keep these values between 0 and 1 because the ratio of the weights is more important than the actual values. For example, weighting each goal with the same value means the optimization will be done based simply on the difference factors (8) and (9). The weights can also be used to temporarily ignore a certain goal if its weight is set to 0. The effects of changing these five variables is explored and presented in the case study discussed in Section 4.

## 4. Case studies

### 4.1. PMU allocation decisions

In order to validate the multi-objective MILP model and WGP framework, the IEEE 14-bus system is used, illustrated in Fig. 2, along with the IEEE 118-bus system. The framework is programmed and solved in MATLAB using the recently updated Optimization Toolbox 8.1, which uses the common branch and bound method to solve the MILP [37]. The initial SCADA measurement set used to calculate the bus VI values consisted of real and reactive power flows, real and reactive power injections, and voltage magnitudes. As discussed in Section 2, the cost model is such that all $c_s$ values are \$50,000, while all $c_r$ values are \$5,000. This means the maximum cost for the 14 bus system is \$900,000, while the maximum cost for the 118 bus system is \$7,690,000. The various tests are done by changing the five variables discussed in Section 3. The first set of tests is done with constant goals for the two objectives while varying the goal weights. The constant goals used are the extreme cases: $c_{goal} = 0$ and $VI_{goal} = 1$. The objective of this test case is to investigate the model's results when varying the VI and cost weights. The results of these tests are shown in Tables 1 and 2. In these tables, the **VI** column shows the value for the second term in the right side of Eq. (8), which is the sum of VI values for the buses and branches covered in the PMU allocation solution.

In the first result (hereby known as the WGP Base case), the weights are both set to 1, meaning each goal is weighted equally. As one might expect, this results in PMUs being allocated to about half of the buses on each system, eight PMUs on the 14 bus system and 55 PMUs on the 118 bus system. It is interesting to note that significantly more than half of the available branch measurements are allocated in this scenario. Given the fact that branch measurements are cheaper to install based on the cost model, it is logical that more branches are being used relative to the number available. The next two results examine the extreme cases of completely eliminating a goal. These both give trivial solutions. If the cost goal weight is set to zero, PMUs are allocated at every bus on the

**Table 1**
14 bus PMU allocation with constant goals at extremes.

| $\omega_{VI}$ | $\omega_{cost}$ | PMU buses | Branches | Cost ($1k) | VI |
|---|---|---|---|---|---|
| 1 | 1 | 8 | 28 | $540 | 0.68234 |
| 1 | 0 | 14 | 40 | $900 | 1.0000 |
| 0 | 1 | 4 | 10 | $250 | 0.24778 |
| 0.25 | 1 | 4 | 10 | $250 | 0.26128 |
| 0.5 | 1 | 4 | 12 | $260 | 0.29028 |
| 0.75 | 1 | 5 | 19 | $345 | 0.42624 |
| 1 | 0.25 | 14 | 40 | $900 | 1.0000 |
| 1 | 0.5 | 13 | 39 | $845 | 0.97162 |
| 1 | 0.75 | 12 | 37 | $785 | 0.92631 |

**Table 2**
118 bus PMU allocation with constant goals at extremes.

| $\omega_{VI}$ | $\omega_{cost}$ | PMU buses | Branches | Cost ($1M) | VI |
|---|---|---|---|---|---|
| 1 | 1 | 55 | 220 | $3.85 | 0.67581 |
| 1 | 0 | 118 | 358 | $7.69 | 1.0000 |
| 0 | 1 | 32 | 86 | $2.03 | 0.24188 |
| 0.25 | 1 | 33 | 90 | $2.10 | 0.30712 |
| 0.5 | 1 | 38 | 130 | $2.55 | 0.45082 |
| 0.75 | 1 | 43 | 176 | $3.03 | 0.55288 |
| 1 | 0.25 | 117 | 357 | $7.635 | 0.99834 |
| 1 | 0.5 | 108 | 343 | $7.115 | 0.97256 |
| 1 | 0.75 | 74 | 268 | $5.04 | 0.80483 |

system and all possible branch measurements are taken as well. If the VI goal weight is set to zero, the minimum number of PMUs and branch measurements are allocated to achieve observability, which are the results of past work in PMU allocation [7–10,13,19,20].

The next set of results explores lowering the VI goal weight, so that the cost goal has more impact on the objective function. As expected, these results show PMU allocation decisions in between the even weights result and the minimum allocation result. As the VI goal weight decreases, fewer PMUs are being allocated and the cost goes down as well. One important point to note is that in the 14 bus result for $\omega_{VI} = 0.25$, the allocation is actually slightly different than the minimum allocation result even though they both have four PMUs and ten branches covered. The difference can be seen by the different VI values for each solution. This is because the VI values are considered enough to differentiate between two different allocation results that have the same cost. As expected, the VI value for the 0.25 weight solution is slightly higher. In the last set of tests in Tables 1 and 2, the cost goal weight is reduced and the expected pattern emerges in the allocation results. As the cost goal weight decreases, the number of PMUs allocated increases, increasing the cost and VI values as well.

As mentioned in Section 3.3, the ratio of the weights is more important than the actual values. In order to confirm this idea, tests were run to determine if the same weight ratio produces the same result from the WGP framework. For example, weights of 2 and 1 were used instead of 1 and 0.5. A few examples of these tests on the 14 bus system are shown in Table 3. It is up to the user to determine how they want to change the weights of each goal, but it is important to understand that the ratio of these weights is what will determine the final solution of the WGP framework.

**Table 3**
Weight ratio tests on the 14 bus system.

| $\omega_{VI}$ | $\omega_{cost}$ | PMU buses | Branches | Cost ($1k) | VI |
|---|---|---|---|---|---|
| 1 | 4 | 4 | 10 | $250 | 0.26128 |
| 1 | 2 | 4 | 12 | $260 | 0.29028 |
| 1.5 | 2 | 5 | 19 | $345 | 0.42624 |
| 4 | 1 | 14 | 40 | $900 | 1.0000 |
| 2 | 1 | 13 | 39 | $845 | 0.97162 |
| 2 | 1.5 | 12 | 37 | $785 | 0.92631 |

**Table 4**
14 bus PMU allocation with constant weights at 1.

| $VI_{goal}$ | $c_{goal}$ ($1k) | PMU buses | Branches | Cost ($1k) | VI |
|---|---|---|---|---|---|
| 0.25 | $0 | 4 | 10 | $250 | 0.24972 |
| 0.5 | $0 | 6 | 20 | $400 | 0.49887 |
| 0.75 | $0 | 8 | 28 | $540 | 0.68234 |
| 1 | $250 | 8 | 28 | $540 | 0.26128 |
| 1 | $750 | 12 | 37 | $785 | 0.92631 |

**Table 5**
118 bus PMU allocation with constant weights at 1.

| $VI_{goal}$ | $c_{goal}$ ($1M) | PMU buses | Branches | Cost ($1M) | VI |
|---|---|---|---|---|---|
| 0.25 | $0 | 32 | 86 | $2.03 | 0.25000 |
| 0.5 | $0 | 40 | 152 | $2.76 | 0.50000 |
| 0.75 | $0 | 55 | 220 | $3.85 | 0.67581 |
| 1 | $3.00 | 55 | 220 | $3.85 | 0.67581 |
| 1 | $5.00 | 73 | 270 | $5.00 | 0.79999 |

Tables 4 and 5 explore the effects of varying the goals in the problem rather than the weights. For these tests, both weights are held constant at one. When changing the goals in the WGP framework, it is important to understand the effect of the non-negativity constraints in (13). These constraints ensure that the difference factor for each goal is not a negative value. Negative difference values would render the goals set by the user useless since making those differences as negative as possible would minimize the overall objective function. This is demonstrated well in the test results shown in Tables 4 and 5. The first set of tests lower the VI goal. By setting this goal to 0.5, for example, the user is telling the WGP framework that they want PMUs to cover buses and branches whose normalized VI values sum to no more than 0.5. This limits the number of PMUs and branch measurements allocated by the optimization problem, but the framework will still optimize within that constraint. Using the example of 0.5, it is easy to see that the VI value for the 14 bus solution is very close to 0.5 and is exactly 0.5 for the 118 bus solution. This gives the user a way to limit the cost of the PMU allocation solution without have specific cost goals in mind, but rather considering how much of the vulnerability on the system do they want to address via PMU measurements. One important observation about using the VI goal is that while zero-one normalization is used for the goals, setting the VI goal to zero will not have a feasible solution. This is due to the observability constraint (10). The minimum VI value for a feasible solution is provided by looking at the scenario where the VI goal is ignored by giving it a weight of 0. For the 14 bus system, this minimum is 0.24778. For the 118 bus system, the minimum is 0.24188.

The final test of Tables 4 and 5 show how the cost goal can be used to set a minimum cost on the solution. This is again because of the non-negativity constraints. The first test shows that setting a cost goal that is below what the solution would have been anyway does not affect the final solution. The last test shows that a minimum cost can be set easily and increases the number of PMUs and branch measurements allocation by the WGP framework. This would be used in the (admittedly unlikely) scenario where the user must use a certain amount of funding on their PMU allocation project. Similar to the minimum VI goal, the cost goal has a maximum value for feasibility, which would the cost in the case of all possible PMU measurements being allocated in the solution.

As mentioned earlier, the feasibility limits discussed for each goal are due to the non-negativity constraints (13). Let us say that both weights are set to 1, the cost goal to the maximum possible cost, and the VI goal to 0 (these are the opposite extremes as in the tests in Tables 1 and 2). If the constraints (13) are used, the problem is infeasible, which makes sense since the goals are not achievable. If the constraints (13) are not used, the framework produces the same solution as the WGP Base case. In fact, any set of goals in this situation would produce the

**Table 6**
14 bus PMU allocation with varying budgets.

| Budget ($1k) | PMU buses | Branches | Cost ($1k) | VI |
|---|---|---|---|---|
| $300 | 5 | 10 | $300 | 0.32552 |
| $500 | 7 | 26 | $480 | 0.60879 |
| $750 | 8 | 28 | $540 | 0.68234 |

**Table 7**
118 bus PMU allocation with varying budgets.

| Budget ($1M) | PMU buses | Branches | Cost ($1M) | VI |
|---|---|---|---|---|
| $3.00 | 43 | 170 | $3.00 | 0.54709 |
| $3.50 | 49 | 204 | $3.47 | 0.60879 |
| $7.50 | 55 | 220 | $3.85 | 0.67851 |

WGP Base solution. It is important to use the non-negativity constraints such that the goals can be used as meaningful inputs if the user so chooses.

Finally, Tables 6 and 7 show how using the budget constraint impacts the PMU allocation solution. This works intuitively as a normal budget constrain, which is basically the opposite of the cost goal. If a budget is used, the cost of the allocation cannot exceed that value. The tests shown use the weights and goals of the WGP Base case, meaning the only input that is being changed is the budget. From Tables 1 and 2, we know that that the base case solutions for the 14 and 118 system cost $540,000 and $3,850,000, respectively. The first two tests have budgets below these values and therefore show a solution with fewer PMUs. The second test shows that even if the budget restricts the solution, the WGP framework is smart enough not to blindly use all of the budget. This can be caused by the physical constraint (11) or simply that of the available branch measurements, not all of them will reduce the objective function. The final budget test simply shows that if the optimal solution based on the weights and goals has a cost lower than the budget, then the budget will not have an impact on the solution.

The results presented show a small number of possible input configurations for this WGP framework in order to validate that the inputs change the PMU allocation decisions in the way they are meant to. All of the tests done by the authors passed the numerical observability check by producing Jacobian matrices of full rank, confirming that this is not a concern when using the WGP framework. The tests presented in this paper hone in on each different type of input (weights, goals, and budget) in order to directly show they work as intended in the WGP framework. If a user chooses to, they can change all five inputs at once. This will depend on the needs and goals of the user and it is up to the user to determine which solution works best for their situation.

### 4.2. Gross error detection tests

In order to verify that the WGP model with the VI metric does improve the detection of gross errors in measurements, error analysis tests were done on some of the PMU allocation solutions. [21,29] show that the VI metric can be used to enhance gross error detection, so these tests are just confirmation that the WGP uses the VI metric in an intelligent way. These tests consist of 1000 measurement set samples where in 25% of the samples, errors of a random size between 5 and 25 standard deviations are added to one of the measurements at random. While the detection of gross errors in measurements is important for state estimation, it is also important to avoid false positives, which can lead unnecessary changes to system operation just like an error could. Since it is more common to have an error free measurement set, a majority of the samples are error free. The gross error detection in this paper is done through a CME based Chi-squared test [26]. These tests are done for solutions provided by the WGP framework and the solutions from [20]. In [20], a multi-objective framework is used to solve the OPP

**Table 8**
14 bus gross error analysis results.

| Measurement set | Cost ($1k) | Correct decision % |
|---|---|---|
| SCADA only | $0 | 85.1 |
| WGP Base | $540 | 94.7 |
| 0.95 Reliability | $545 | 92.9 |
| WGP $\omega_{VI}$ = 0.75 | $345 | 94.1 |
| [20] 0.99 Reliability | $340 | 93.6 |
| WGP $\omega_{VI}$ = 0.25 | $250 | 92.8 |
| [20] 0.9983 Reliability | $275 | 92.2 |

**Table 9**
118 bus gross error analysis results.

| Measurement set | Cost ($1M) | Correct decision % |
|---|---|---|
| SCADA only | $0 | 83.0 |
| WGP $\omega_{cost}$ = 0.75 | $5.04 | 89.1 |
| 0.95 Reliability | $5.455 | 88.0 |
| WGP Base | $3.85 | 90.3 |
| [20] 0.99 Reliability | $4.075 | 88.8 |
| WGP $\omega_{VI}$ = 0.25 | $2.10 | 85.1 |
| [20] 0.9983 Reliability | $2.545 | 84.6 |

problem with minimal PMUs while maximizing reliability. Various solutions are provided based on differing PMU reliability levels. Similar solutions based on cost are chosen for gross error detection comparisons. The results of these tests are shown in Tables 8 and 9.

As shown in the results, the VI metric continues to help improve gross error detection when used in the WGP framework. In general, more PMU measurements leads to better gross error detection, regardless of the allocation method. The one exception to that is that most expensive solutions to the 118 bus system. Basically, once the PMU allocation goes beyond the WGP Base solution, gross error detection results seem to drop a little bit. This isn't entirely unexpected since it has been shown that for a Chi-squared test, the rate of Type I errors (false positives) can increase with a large number of measurements [38]. The WGP solutions, shown in bold, provide a higher rate of correct decisions on if there is an error or not for all comparable solutions, even when WGP provides the cheaper solution. This trend holds true for both the 14 and 118 bus system.

### 4.3. Optimization performance analysis

When formulating any type of optimization problem, it is important to consider and verify that the method being used does in fact converge to optimal solutions and does so a timely manner. For multi-objective optimization problems, a good way to analyze the performance is through Pareto optimality. A solution is said to be Pareto optimal if none of the objectives can be improved upon without deteriorating at least one other objective [39]. In this paper, the WGP MILP formulation only has two objectives, so a solution is Pareto optimal if lowering the cost must lower the VI or if raising the VI must raise the cost. With two objectives, this can be visualized relatively easily by plotting the Pareto frontier alongside the solutions found by the WGP framework. These results are shown in Figs. 3 and 4 for the 14 and 118 bus systems, respectively. The Pareto frontiers in orange were generated by fixing the cost of the PMU allocation and solving the optimization problem such that the optimal VI value for that cost is found. The *x*-axis of the plots are the cost in dollars, while the *y*-axis is the distance between the normalized VI value of the solution and 1, the max normalized VI value for the problem.

In the 14 bus Pareto frontier, it is interesting to point out the spikes that seem to occur. These come from the physical constraint of the problem where branch measurements must come from an allocated PMU. Since branch measurements are cheaper than the actual PMU, the

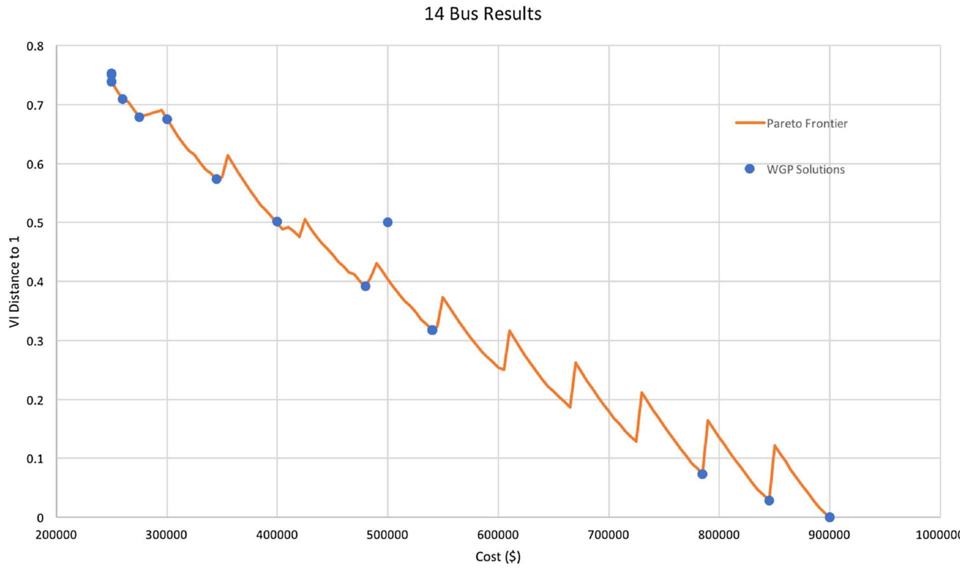


**Fig. 3.** 14 bus results of WGP framework along with Pareto frontier.

solution would ideally have many branch measurements. However, there are only so many branches off of each bus. Each spike represents when a new PMU must be installed to achieve a specific cost. In order to achieve that exact cost, a PMU is added and branch measurements are taken away, driving the $\delta_{VI}$ value up rapidly. As you can see in Fig. 3, the WGP framework avoids these spikes when looking for a solution when the cost of the solution is not set at a specific value. In Fig. 4, the spikes are much smaller and often not even noticeable. This is due to the fact that there is much more flexibility in the solutions due to there being many more buses and branches.

For the both systems, it is clear that the WGP framework almost always delivers a solution that is on the Pareto frontier, meaning they are Pareto optimal solutions. For each system, there is one example of a non-Pareto optimal solution. These solutions were developed deliberately by setting goals that, frankly, don't make sense to set. In order to get these solutions, the VI goal was set very low while the cost goal was set very high. In the 14 bus system, the VI goal was 0.5 while the cost goal was $500,000. Both of these goals were achieved simultaneously, which does minimize the objective function. However, it does not make sense to set goals like this when using WGP. Similarly in the 118 bus system, the VI goal was set to 0.75 while the cost goal was set to $5,000,000. Again, both goals are achieved. When both goals are achieved in the WGP framework, the solution will not be Pareto optimal. Based on this analysis, it is important for a user of this framework to have an understanding of their system when setting goals. They should know the goal limits for their system: the minimum and maximum costs of PMU installation for minimal and full observability, and the normalized VI values of the minimum observability case. These values can be obtained through using the framework with extreme weight values, as done in Tables 1 and 2.

As a final note on the performance of the WGP framework, the authors point out that a vast majority of the solutions for the 118 bus system were found within a few seconds, with the occasional run that took a few minutes. Since the OPP problem is a planning problem (not real time), this is not really an issue for the WGP framework presented.

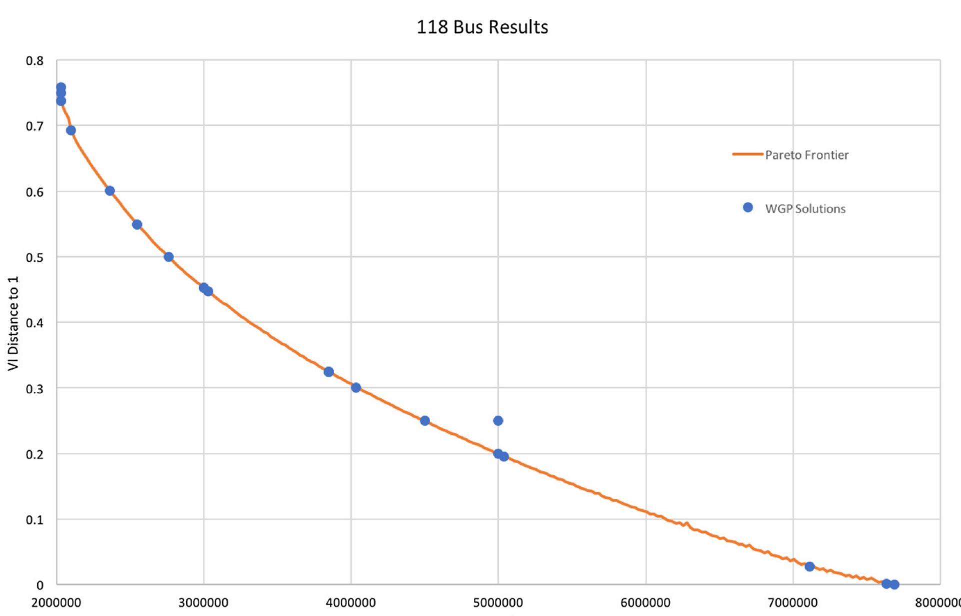


**Fig. 4.** 118 bus results of WGP framework along with Pareto frontier.

## 5. Conclusion

This paper presents a multi-objective MILP model to solve the OPP problem that allows the user to have an input in the final solution. The model is built on a WGP framework. By creating a multi-objective MILP model, this framework can be used by any off-the-shelf optimization solver, making it easily implementable. This paper focuses on two goals for PMU allocation: enhanced gross error detection and financial cost. These are naturally interdependent contradictory goals, so a WGP framework is ideal for mathematically modeling the value of each goal and optimizing the overall solution. Using this framework, the user has control over five different variables within the program. The user can set specific goal values for each goal, prioritize the goals by varying the weights, and set a budget. When setting goal values, it is important for the user to understand the limits of those values for their system. They must also be careful not to set goals that are simultaneously achievable, as this may lead to a non-Pareto optimal solution.

The model was tested on the IEEE 14 and 118 bus systems and the results of the tests show that the model works as expected, varying the number of PMUs as well as the location of the PMUs depending on the values given for the five controllable variables. This model can easily provide solutions from the minimum amount of PMUs required to allocating a PMU at every bus. Given logical input values from the user, the WGP framework will always find the Pareto optimal solution for those inputs. Gross error detection tests show that the WGP framework use the VI metric well, such that gross error detection is improved more by allocating PMUs with this framework than other multi-objective solutions. The intermediate solutions provided by the WGP framework depend on the goals and preferences of the user, making this framework ideal for utilities. The model provides the user a wide variety of options if desired but always finds a solution that fits the goals and priorities provided by the user.

## Authors' contribution

Cody Ruben: conceptualization, methodology, software, validation, formal analysis, investigation, writing – original draft, review and editing, visualization.

Surya C. Dhulipala: methodology, software, writing – original draft, review and editing.

Arturo S. Bretas: conceptualization, resources, writing – review and editing, supervision, project administration, funding acquisition.

Yongpei Guan: conceptualization, methodology, formal analysis, writing – review and editing.

Newton G. Bretas: conceptualization, writing – review and editing.

## Conflict of interest

None declared.

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