

# Quantum Game Analysis on Extrinsic Incentive Mechanisms for P2P Services

Shengling Wang<sup>1</sup>, Member, IEEE, Weiman Sun, Liran Ma<sup>2</sup>, Member, IEEE,  
Weifeng Lv<sup>3</sup>, and Xiuzhen Cheng<sup>4</sup>, Fellow, IEEE

**Abstract**—Peer-to-peer (P2P) services such as mobile P2P transmissions and resource sharing, provide efficient methods to deliver data without the deployment of any central server. Nevertheless, the *free-riding* phenomenon inherent in such services presses a need for incentive mechanisms to stimulate contributions of data transmissions or sharing. As a result, it is imperative to answer the following questions: *whether*, and if so *to what extent*, an incentive mechanism can invoke such contributions? To answer these questions, we employ an  $n$ -player continuous quantum game model to analyze the general extrinsic incentive mechanisms as well as the reputation-based incentive mechanisms, a typical class of extrinsic incentive mechanisms. We focus on studying the extrinsic incentive mechanisms in this paper due to their wide scope of applications stemming from the fact that they promote cooperative behaviors by offering rewards rather than depending on the internal bounds (e.g., social ties) among peers, which may not always exist between any pair of peers. To the best of our knowledge, we are the first to analyze the extrinsic incentive mechanisms for P2P services from a quantum game perspective. Such a perspective is adopted because the extended strategy space in the quantum game broadens the range for searching optimal strategies and the introduction of *entanglement* makes the proposed analytical frameworks more practical due to the consideration of the peers' relationships imposed by the rewards in extrinsic incentive mechanisms. Our quantum game-based analytical framework is generic because it is compatible with classic game-based schemes. The analytical results can provide a straightforward insight on evaluating the potential of the extrinsic incentive mechanisms and can serve as important references for designing new extrinsic incentive mechanisms.

**Index Terms**—P2P services, incentive mechanisms, quantum game

## 1 INTRODUCTION

MOBILE peer-to-peer (P2P) transmissions [1], [2], [3], a method taking advantage of contacts among people carrying mobile devices for disseminating data, can effectively solve communication issues in challenged networks without persisting connectivity. Another similar application is P2P resource sharing, where resources (e.g., files) can be directly shared among peers in a system. Mobile P2P transmissions and P2P resource sharing have a common trait that any peer can act as a service provider as well as a service user. Due to such a common trait, we call mobile P2P transmissions and resource sharing as *P2P services*, which can provide services without the need of any central server.

The success of P2P services relies on the solutions to a long-standing problem that is originated from the classical game model named the *prisoner's dilemma* [4], where two

players of a game can choose to be cooperative or defective. In P2P services, the dilemma stems from the conflict that defection (i.e., not servicing others) is the dominant strategy for each player while cooperation (e.g., downloading resources or relaying data for others) can maximize the overall social welfare.

The prisoner's dilemma in P2P systems gives birth to *free riders* who choose to only enjoy contributions from others while providing no service to others. As rational and intelligent players, free riders adopt the dominant strategy of defection, which is detrimental to the whole system. Hence, incentive mechanisms are typically adopted to constrain free-riding. The state-of-the-art incentive mechanisms can be divided into two categories based on the natures of the motivators: *extrinsic* and *intrinsic*, with the former promotes cooperative behaviors by providing rewards (such as the high precedence in being helped for data transmissions) [5], [6], [7], [8], [9], [10], [11], [12], [13] and the latter encourages reciprocal cooperation by utilizing internal bounds (such as the social ties) among the peers [14], [15], [16], [17], [18].

Given such a variety of incentive mechanisms, we raise the following fundamental questions: *whether*, and if so, *to what extent*, an incentive mechanism can motivate cooperation in P2P services? The answers to these questions can offer an in-depth vision on evaluating the potential of an incentive mechanism. Moreover, these answers can be used as important guidelines for designing new incentive mechanisms. Our unique approach to answering these questions is based on the quantum game theory, which is the marriage

- S. Wang is with the College of Information Science and Technology, Beijing Normal University, Beijing 100875, P.R.China. E-mail: wangshengling@bnu.edu.cn.
- W. Sun is with the Department of Computer Science, Columbia University, New York, NY 10032. E-mail: ws2517@columbia.edu.
- L. Ma is with the Department of Computer Science, Texas Christian University, Fort Worth, TX 76129. E-mail: l.ma@tcu.edu.
- W. Lv is with the School of Computer Science and Engineering, Beihang University, Beijing 100191, China. E-mail: lwf@buaa.edu.cn.
- X. Cheng is with the Department of Computer Science, George Washington University, Washington, DC 20052. E-mail: cheng@gwu.edu.

Manuscript received 10 Oct. 2018; revised 4 July 2019; accepted 23 July 2019.  
Date of publication 5 Aug. 2019; date of current version 18 Dec. 2019.  
(Corresponding author: Shengling Wang.)  
Recommended for acceptance by J. Zhan.  
Digital Object Identifier no. 10.1109/TPDS.2019.2933416

of quantum mechanics and game theory. A quantum game is a powerful tool because it extends the *strategy space* and introduces the concept of *entanglement*. The extended strategy space allows us to search optimal strategies from a wider range, and the concept of entanglement enables us to more accurately depict the relationship among rational players. As a result, the quantum game theory is more suitable for analyzing incentive mechanisms because it can realistically model the impacts of the relationship among players in the decision-making process through the concept of entanglement.

In the conference version [19] of this paper, we proposed two analytical frameworks to study the general extrinsic and intrinsic incentive mechanisms in P2P services based on the quantum game theory. In this paper, we go one step beyond by focusing on the extrinsic incentive mechanisms since most of the time, there exist no internal bounds among peers and extrinsic incentive mechanisms have wider scope of applications. More specifically, in this paper, we analyze a typical and frequently-used class of extrinsic incentive mechanisms, namely the reputation-based incentive mechanisms, which introduce the reputation or quasi-reputation metric for evaluating the contribution of a node and giving the corresponding reward. The reputation-based incentive mechanisms can be further divided into two types: memoryless and memory.

To analyze the extrinsic incentive mechanisms, we adopt an  $n$ -player continuous quantum game model where the state of each player is described by the single-mode electromagnetic field and the reward for a unit amount of transmission is regarded as the degree of entanglement. To the best of our knowledge, we are the first to analyze incentive mechanisms for mobile P2P services from a quantum game perspective. Additionally, our proposed analytical framework is generic enough to be compatible with the classic game-based schemes. Conclusively, we make the following major contributions:

- The influence of the reward strength (i.e., the degree of entanglement) from a general extrinsic incentive mechanism on the optimal strategy and the optimal expected payoff of each player is quantitatively analyzed; further, the mathematical relationship between the relay cost and the critical degree of entanglement that can lead to the emergence of cooperation for general extrinsic incentive mechanisms is deduced.
- The dependence of the cooperation ratio, the optimal strategy, and the optimal utility of each player on the entanglement degree under three reputation distributions in the memory reputation-based extrinsic incentive mechanisms are analyzed; we also study the ranking correlation among the reputation, the optimal strategy, and the optimal utility of each player in different reputation distribution scenarios.
- The impacts of the entanglement degree on the cooperation ratio, the optimal strategy, and the optimal utility of each player are investigated for the memoryless reputation-based extrinsic incentive mechanisms in centralized and distributed settings based on real-world data.

This paper is organized as follows. The related work is presented in Section 2. Section 3 introduces the quantum

game model for extrinsic incentive mechanisms. The general extrinsic incentive mechanisms as well as the reputation-based incentive mechanisms are analyzed respectively from the quantum game perspective in Sections 4 and 5. We conclude this paper in Section 6.

## 2 RELATED WORK

In this section, the state-of-the-art extrinsic incentive mechanisms for P2P services are summarized. Examples of extrinsic incentive mechanisms include [5], [6], [7], [8], [9], [10], [11], [12], [13]. Specifically, Ning et al. [5] introduced the concept of *virtual checks* to avoid the requirement of accurately knowing whom and how many credits an ad provider should pay. With virtual checks, the interactions among nodes are modeled as a two-player cooperative game whose optimal solution is obtained through the Nash Bargaining Theorem. Mobicent [6], a credit-based incentive system for delay tolerant networks (DTNs), employs an underlying routing protocol to find the most efficient routing paths for users, which is also incentive compatible.

A typical kind of extrinsic incentive mechanisms is reputation-based, which introduce reputation or quasi-reputation metrics to evaluate the contribution of a node and give the corresponding reward. In [7], credit and reputation clearance are attached to the data a node forwards for others so as to build a fair and attractive protocol. RELICS [8] adopts a ranking system where a node transmits more messages would be elevated to a higher rank, and a higher-ranked node takes precedence over the lower-ranked ones when its need is being serviced. BuSIS [9] evaluates the contributions of nodes and grades them by their service level while stimulating selfish nodes to earn more credits by providing services to others. In [10], cooperative nodes can receive a certain amount of credits. Multicent [11] models the packet exchanges between a pair of nodes as a game, and the reward credit to a node is determined by the payoff function of the game. By allowing users to be aware of their status and participate in communications more actively, [12] presents a point-based incentive system to activate file sharing communications and prevent free-riding syndrome. Trust relations between different types of peers and trust calculation are carefully studied in [13] to incentivize peers to cooperate in a hybrid P2P system.

The work focuses on analyzing incentive mechanisms for P2P services is limited. As a pioneer work, [20] builds a unified framework based on which different incentive schemes are examined by applying various classical game theoretic models. This paper analyzes how to determine one peer's amount of content produced, the level of sharing and the amount of content that it downloads from others, so as to achieve Pareto efficiency for the social welfare under different cases (i.e., non-cooperative and cooperative peers). This analytical perspective is different from ours.

## 3 QUANTUM GAME FORMULATION

In this section, we quantize P2P services and shed light on the effectiveness of the extrinsic incentive mechanisms. To that aim, we introduce the necessary quantum game basics in advance as follows.

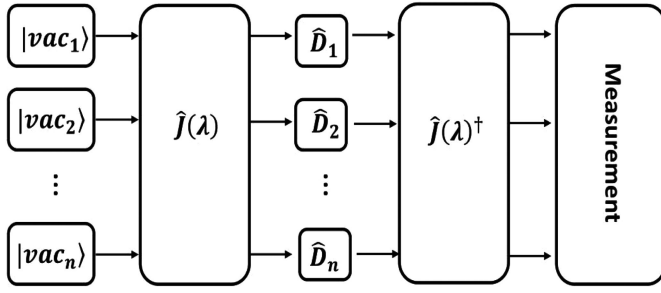


Fig. 1. Quantum model for extrinsic incentive mechanisms.

### 3.1 Quantum Game Basics

Similar to the classical game theory, players in a quantum game act based on a specific set of strategies. In this subsection, we explain a number of important concepts that are associated with quantum game players and their actions.

The state of player  $m$  in a quantum game resides in a Hilbert space, and can be represented as a state vector  $|H_m\rangle$ , where  $|\cdot\rangle$  is known as the Dirac notation. In quantum mechanics, a Hilbert space consists of a set of base vectors. Hence in the quantum game, the base vectors correspond to the player's classical strategies, and all the base vectors compose his quantum state. That is,  $|H_m\rangle = \sum_{\mu=1}^{K_m} p_m^\mu |t_m^\mu\rangle$ , where  $K_m$  is the number of all strategies of player  $m$ ,  $t_m^\mu$  is the  $\mu$ th classical strategy of player  $m$ , and  $|p_m^\mu|^2$  is the probability that the player adopts the classical strategy  $t_m^\mu$ . Note that  $|H_m\rangle$  includes player  $m$ 's basic information, i.e., all his possible strategies and the corresponding probabilities.

When a quantum system contains multiple independent players, the system state is defined as

**Definition 3.1 (System state of multiple independent players).** The state  $|H\rangle$  of a composite system consisting of  $n$  independent players is the direct product of all the states of its players, namely  $|H\rangle = |H_1\rangle \otimes |H_2\rangle \cdots \otimes |H_n\rangle$ , where  $|H_m\rangle$  is the state of player  $m$ ,  $m = 1, 2, \dots, n$ .

Note that if players are dependent, the system state cannot be represented as the direct product of the states of all its players and should be described using entanglement. More specifically, entanglement, the key concept in a quantum game, can be employed to describe the relationship among players. Since players often have more or less correlations with others in real world, which obviously would impact their strategies, the entanglement embedded in the game scheme makes the quantum game approach more realistic than the classical counterparts.

A generic quantum game starts with an initial system state. After each player applies the operation which is defined in the following, one can obtain the results of the quantum game (e.g., payoffs, actual strategies) from the measurements conducted on the final state.

**Definition 3.2 (Operation of a Player).** An operation of a player is defined as a unitary operator.

### 3.2 Quantum Game Model

In this subsection, we introduce the quantum game model that can formulate the extrinsic incentive mechanisms in P2P services. To be specific, in our scenario, the strategy of

each node is the amount of data relayed or shared for each received request. Since the strategy is continuous, we adopt the single-mode electromagnetic field [21] whose quadrature amplitudes have a continuous set of eigenstates as the state of each node  $m$ , denoted as  $|vac_m\rangle$ . We assume that there are  $n$  nodes in the network/system. Thus, the direct product of  $n$  nodes' states  $|vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle$  is the input of the quantum game. We adopt a popular quantum game [22], [23], shown in Fig. 1, to model the behavior of each node. The model includes the following three stages:

- 1) All nodes are entangled through an entangling gate  $\hat{J}(\lambda)$ . The initial quantum state of the system is  $|\psi_{init}\rangle = \hat{J}(\lambda)|vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle$ . Note that  $\lambda$  is the degree of entanglement.
- 2) Each node  $m$  operates on its state using a unitary operator  $\hat{D}_m$  ( $m = 1, 2, \dots, n$ ). A unitary operator  $\hat{D}_m$  has the feature of  $\hat{D}_m \hat{D}_m^\dagger = 1$ , where  $\hat{D}_m^\dagger$  is the adjoint operator of  $\hat{D}_m$ .
- 3) Lastly, a disentangling gate  $\hat{J}(\lambda)^\dagger$  is applied on the system and a measurement is taken on the final state  $|\psi_f\rangle$  of the system, which reads as

$$|\psi_f\rangle = \hat{J}(\lambda)^\dagger (\hat{D}_1 \otimes \hat{D}_2 \otimes \cdots \otimes \hat{D}_n) \hat{J}(\lambda) |vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle. \quad (1)$$

The quantum game model mentioned above contains entanglement operations. The entanglement phenomenon occurs when a composite physical system cannot be described independently upon its sub-systems. Similarly, in a P2P service with an extrinsic incentive mechanism, nodes, like the sub-systems, are associated with each other by serving others and being rewarded. As a result, it is suitable to model the relationship among nodes using the concept of *entanglement*. The degree of entanglement describes the extent to which a node is willing to serve others, which is closely related to the reward strength. Subsequently, we set the entanglement degree  $\lambda$  to be the reward for delivering unit data in an extrinsic incentive mechanism. A high degree of entanglement means that highly cooperative nodes can get more transmission help and resource sharing from others. Nevertheless, when  $\lambda$  is set to 0 and a disentangling operator  $\hat{J}(\lambda)^\dagger$  ( $\hat{J}(\lambda)^\dagger \hat{J}(\lambda) = 1$ ) is involved before measurement, our proposed analytical framework reduces to a classical game-based one.

The entangling gate  $\hat{J}(\lambda)$  in our model has the form similar to that in [21], which is given below:

$$\hat{J}(\lambda) = \exp \left\{ - \sum_{j=1}^{n-1} \sum_{m>j}^n \lambda (\hat{a}_m^\dagger \hat{a}_j^\dagger - \hat{a}_m \hat{a}_j) \right\}. \quad (2)$$

In the above equation,  $\lambda \in [0, \infty]$ ;  $\hat{a}_m$  and  $\hat{a}_m^\dagger$  are the annihilation and creation operator of the  $m$ th electromagnetic field mode, respectively. We use the particle-number representation to eliminate the completeness of calculating the direct product of  $n$  players' state. Since the interaction between two nodes is equivalent to each other,  $\hat{J}(\lambda)$  is symmetric with respect to the interchange of the two field modes. The term  $\lambda(\hat{a}_m^\dagger \hat{a}_j^\dagger - \hat{a}_m \hat{a}_j)$  refers to that node  $m$  is entangled with node  $j$  in terms of the degree of entanglement  $\lambda$ . The summation

in (2) means that any node  $m$  is entangled with the rest of the  $n - 1$  nodes. When  $\lambda \neq 0$ , the states of any node are affected by those of the other nodes in the network. Thus, a selfish node is forced to consider other nodes' strategies before making a decision rather than directly taking free-riding. Therefore, it is possible for selfish nodes to choose to cooperate.

We refer the operation of node  $m$  as  $\hat{D}_m$ . The form of the unitary operator  $\hat{D}_m$  for node  $m$  is similar to that in [21]. Hence, the operation set of node  $m$  is as follows

$$S_m = \{\hat{D}_m(x_m) = \exp(-ix_m\hat{P}_m)|x_m \in [0, \infty]\},$$

where  $i$  is the imaginary unit,  $x_m$  refers to the operation parameter of node  $m$  and  $\hat{P}_m = \frac{i}{\sqrt{2}}(\hat{a}_m^\dagger - \hat{a}_m)$  is its *momentum* operator. As shown in [21],  $\hat{D}_m$  is exactly the quantum operation that includes the classical operation as well.

After the disentangling gate  $\hat{J}(\lambda)^\dagger$ , final measurements are taken on the *position* operator  $\hat{X}_m = \frac{1}{\sqrt{2}}(\hat{a}_m^\dagger + \hat{a}_m)$  of each node  $m$  and the result is  $q_m$  ( $m = 1, 2, \dots, n$ ). If the degree of entanglement  $\lambda = 0$ ,  $q_m = x_m$ , which means that each player adopts the classical strategies  $x_m$ . Otherwise, since the initial state of the composite system is entangled,  $q_m$  is the linear combination of each player's classical strategy. That is,  $q_m = \sum_{i=1}^n c_i x_i$ , where  $c_i$  is the coefficient correlated with the degree of entanglement  $\lambda$ .

## 4 ANALYTICAL FRAMEWORK FOR THE GENERAL EXTRINSIC INCENTIVE MECHANISMS

The evaluation of the extrinsic incentive mechanisms needs to be based on the strategy of each node, which is typically determined by its utility. Hence, we start by formulating the utility in a generic extrinsic incentive scenario. The two words *node* and *player* are used interchangeably hereafter.

### 4.1 Utility Formulation

We model the utility of a node that serves others based on the feature of the extrinsic incentive mechanisms. To be specific, the utility  $\$m$  of a serving node  $m$  is formulated as follows:

$$\$m = -c(q_m) + \mu_1 q_m, \quad (3)$$

where  $c(q_m)$  is the cost of transmitting  $q_m$  units of data, and  $\mu_1$  is the reward for delivering a unit of data. Commonly,  $\mu_1$  is denoted as the reward strength in general extrinsic incentive mechanisms. It is worthy of noting that according to the quantum game model introduced in Section 3.2,  $\mu_1 = \lambda$ , the degree of entanglement. We adopt the mainstream cost form [24] in the market-economy to model the relaying cost  $c(q_m)$ , that is,

$$c(q_m) = c_0 + bq_m + aq_m^2, \quad (4)$$

where  $c_0$  represents the fixed relaying cost,  $bq_m + aq_m^2$  is the variable relay cost depending on the amount of transmitted data  $q_m$ , and  $b$  and  $a$  are corresponding coefficients. Generally speaking, we set  $a$  to be small, making the term  $aq_m^2$  negligible when  $q_m$  is relatively small. However, as  $q_m$  grows larger, it becomes the dominant factor for the total relay cost. Note that the above utility has a generic form. For any given extrinsic incentive mechanism, the utility of each node may

be different. Such a difference, however, does not affect the applicability of our proposed framework.

### 4.2 Optimal Strategies

With the above quantum game model for P2P services, we now move forward to derive the optimal strategy for each node. Specifically, we have

**Theorem 4.1.**

$$\begin{aligned} \hat{J}(\lambda)^\dagger \hat{P}_m \hat{J}(\lambda) &= \frac{\hat{P}_m}{n} \left( e^{(n-1)\lambda} + (n-1)e^{-\lambda} \right) \\ &+ \sum_{j=1, j \neq m}^n \frac{\hat{P}_j}{n} \left( e^{(n-1)\lambda} - e^{-\lambda} \right) \end{aligned}$$

**Proof.** We use the mathematical induction to prove Theorem 4.1 as follows:

- a) When  $n = 2$ ,  $\hat{J}(\lambda) = \exp\{-\lambda(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2)\}$ . Because  $\hat{J}(\lambda)^\dagger = \hat{J}(\lambda)^{-1}$ ,  $\hat{J}(\lambda)^\dagger = \exp\{\lambda(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2)\}$ . Let  $\hat{A} = \lambda(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2)$ . According to the Baker-Campbell-Hausdorff formula, we have

$$\begin{aligned} \hat{J}(\lambda)^\dagger \hat{a}_1 \hat{J}(\lambda) &= \sum_{j=0}^{\infty} \frac{1}{j!} [\hat{A}^{(j)}, \hat{a}_1] \\ &= \hat{a}_1^\dagger \sum_{j=0}^{\infty} \frac{1}{(2j)!} \lambda^{2j} - \hat{a}_2 \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \lambda^{2j+1}. \end{aligned} \quad (5)$$

Eq. (5) is obtained by employing the commutation relations  $[\hat{A}^{(0)}, \hat{a}_1] = \hat{a}_1$  and  $[\hat{A}^{(j)}, \hat{a}_1] = [\hat{A}, [\hat{A}^{(j-1)}, \hat{a}_1]]$  for  $\forall j \geq 1$ . Similarly,

$$\hat{J}(\lambda)^\dagger \hat{a}_1^\dagger \hat{J}(\lambda) = \hat{a}_1^\dagger \sum_{j=0}^{\infty} \frac{1}{(2j)!} \lambda^{2j} - \hat{a}_2 \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \lambda^{2j+1}.$$

Due to  $\hat{P}_1 = \frac{i}{\sqrt{2}}(\hat{a}_1^\dagger - \hat{a}_1)$ ,

$$\begin{aligned} \hat{J}(\lambda)^\dagger \hat{P}_1 \hat{J}(\lambda) &= \frac{i}{\sqrt{2}} \hat{J}(\lambda)^\dagger (\hat{a}_1^\dagger - \hat{a}_1) \hat{J}(\lambda) \\ &= \hat{P}_1 \sum_{j=0}^{\infty} \frac{1}{(2j)!} \lambda^{2j} - \hat{P}_2 \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \lambda^{2j+1} \\ &= \hat{P}_1 \left( \frac{e^\lambda + e^{-\lambda}}{2} \right) + \hat{P}_2 \left( \frac{e^\lambda - e^{-\lambda}}{2} \right), \end{aligned}$$

and because of the symmetric property of  $\hat{J}(\lambda)$ , we have

$$\hat{J}(\lambda)^\dagger \hat{P}_2 \hat{J}(\lambda) = \hat{P}_2 \left( \frac{e^\lambda + e^{-\lambda}}{2} \right) + \hat{P}_1 \left( \frac{e^\lambda - e^{-\lambda}}{2} \right).$$

- b) Suppose when  $n = k$ , for any arbitrary positive integer  $m < k$ , we have

$$\begin{aligned} \hat{J}(\lambda)^\dagger \hat{P}_m \hat{J}(\lambda) &= \frac{\hat{P}_m}{k} (e^{(k-1)\lambda} + (k-1)e^{-\lambda}) \\ &+ \sum_{j=1, j \neq m}^k \frac{\hat{P}_j}{k} (e^{(k-1)\lambda} - e^{-\lambda}), \end{aligned}$$

where  $\hat{J}(\lambda)_k = \exp\{-\sum_{m < j=1, m \neq j}^k \lambda(\hat{a}_m^\dagger \hat{a}_j^\dagger - \hat{a}_m \hat{a}_j)\}$  and



$$\begin{aligned}\hat{J}(\lambda)_k^\dagger \hat{a}_m \hat{J}(\lambda)_k &= \sum_{j=0}^{\infty} \frac{1}{j!} [\hat{G}^{(j)}, \hat{a}_m] \\ &= \hat{a}_m \zeta_1 + \nu_1 \sum_{j=1, j \neq m}^k \hat{a}_j - \hat{a}_m^\dagger \zeta_2 + \nu_2 \sum_{j=1, j \neq m}^k \hat{a}_j^\dagger.\end{aligned}\quad (6)$$

In (6),  $\hat{G} = \sum_{m < j=1, m \neq j}^k \lambda(\hat{a}_m^\dagger \hat{a}_j^\dagger - \hat{a}_m \hat{a}_j)$ ;  $\zeta_1$  and  $\zeta_2$  are respectively the even-power and the odd-power terms in the Taylor expand series of  $e^{(k-1)\lambda} + (k-1)e^{-\lambda}$ ; similarly,  $\nu_1$  and  $\nu_2$  are respectively the even-power and the odd-power terms in the Taylor expand series of  $e^{(k-1)\lambda} - e^{-\lambda}$ .

- (c) When  $n = k + 1$ ,  $\hat{J}(\lambda)_{k+1} = \exp\{-\sum_{m=1}^k \lambda(\hat{a}_m^\dagger \hat{a}_{k+1}^\dagger - \hat{a}_m \hat{a}_{k+1})\} \hat{J}(\lambda)_k$ . Let  $\hat{E} = \hat{G} + \hat{C}$ , where  $\hat{C} = \sum_{m=1}^k \lambda(\hat{a}_m^\dagger \hat{a}_{k+1}^\dagger - \hat{a}_m \hat{a}_{k+1})$ . Taking advantage of the Baker-Campbell-Hausdorff formula, we can obtain

$$\begin{aligned}\hat{J}(\lambda)_{k+1}^\dagger \hat{a}_1 \hat{J}(\lambda)_{k+1} &= \sum_{g=0}^{\infty} \frac{1}{g!} [\hat{E}^{(g)}, \hat{a}_1] \\ &= \hat{a}_1 \frac{k}{k+1} \sum_{g=0}^{\infty} \left[ \frac{1}{(2g)!} \lambda^{2g} (k^{2g-1} + 1) \right] \\ &\quad - \hat{a}_1^\dagger \frac{k}{k+1} \sum_{g=0}^{\infty} \left[ \frac{1}{(2g+1)!} \lambda^{2g+1} (k^{2g} - 1) \right] \\ &\quad + \sum_{j=1, j \neq 1}^{k+1} \hat{a}_j \frac{1}{k+1} \sum_{g=0}^{\infty} \left[ \frac{1}{(2g)!} \lambda^{2g} (k^{2g} - 1) \right] \\ &\quad + \sum_{j=1, j \neq 1}^{k+1} \hat{a}_j^\dagger \frac{1}{k+1} \sum_{g=0}^{\infty} \left[ \frac{1}{(2g+1)!} \lambda^{2g+1} (k^{2g+1} - 1) \right].\end{aligned}\quad (7)$$

Here (7) is calculated by using the commutation relations  $[\hat{E}^{(0)}, \hat{a}_1] = \hat{a}_1$ ,  $[\hat{E}, \hat{a}_1] = [\hat{G} + \hat{C}, \hat{a}_1] = [\hat{G}, \hat{a}_1] + [\hat{C}, \hat{a}_1]$  and  $[\hat{E}^{(g)}, \hat{a}_1] = [\hat{E}, [\hat{E}^{(g-1)}, \hat{a}_1]]$  for  $\forall g > 1$ .

We then can get  $\hat{J}(\lambda)_{k+1}^\dagger \hat{a}_1^\dagger \hat{J}(\lambda)_{k+1}$  by a similar way. As  $\hat{P}_1 = \frac{i}{\sqrt{2}}(\hat{a}_1^\dagger - \hat{a}_1)$ , we have

$$\begin{aligned}\hat{J}(\lambda)_{k+1}^\dagger \hat{P}_1 \hat{J}(\lambda)_{k+1} &= \hat{P}_1 \frac{1}{k+1} \left[ \sum_{g=0}^{\infty} \frac{1}{g!} (k\lambda)^g + k \sum_{g=0}^{\infty} \frac{1}{g!} (-\lambda)^g \right] \\ &\quad + \sum_{j=1, j \neq 1}^{k+1} \hat{P}_j \frac{1}{k+1} \left[ \sum_{g=0}^{\infty} \frac{1}{g!} (k\lambda)^g - \sum_{g=0}^{\infty} \frac{1}{g!} (-\lambda)^g \right] \\ &= \hat{P}_1 \frac{1}{k+1} (e^{k\lambda} + k e^{-\lambda}) + \sum_{j=1, j \neq 1}^{k+1} \hat{P}_j \frac{1}{k+1} (e^{k\lambda} - e^{-\lambda}).\end{aligned}$$

Considering the symmetric form of  $J(\hat{\lambda})_{k+1}$ , we can easily get  $J(\hat{\lambda})_{k+1}^\dagger \hat{P}_m J(\hat{\lambda})_{k+1}$  for any arbitrary  $m$  by the same method. Hence, Theorem 4.1 is proved.  $\square$

#### Theorem 4.2.

$$\begin{aligned}\hat{J}(\lambda)^\dagger \hat{D}_m \hat{J}(\lambda) &= \exp\{-ix_m \left[ \hat{P}_m \frac{1}{n} (e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \right. \\ &\quad \left. + \sum_{j=1, j \neq m}^n \hat{P}_j \frac{1}{n} (e^{(n-1)\lambda} - e^{-\lambda}) \right]\}.\end{aligned}$$

**Proof.** By using a Taylor series of  $\hat{D}_m(x_m)$ ,  $\hat{J}(\lambda)^\dagger \hat{D}_m \hat{J}(\lambda)$  can be expressed as

$$\hat{J}(\lambda)^\dagger \hat{D}_m \hat{J}(\lambda) = \hat{J}(\lambda)^\dagger \sum_{h=0}^{\infty} \frac{1}{h!} (-ix_m \hat{P}_m)^h \hat{J}(\lambda). \quad (8)$$

We have  $\hat{J}(\lambda)^\dagger (-ix_m \hat{P}_m)^h \hat{J}(\lambda) = [(-ix_m) \hat{J}(\lambda)^\dagger \hat{P}_m \hat{J}(\lambda)]^h$  for every term in (8) due to  $\hat{J}^\dagger \hat{J} = 1$ . Hence,  $\hat{J}(\lambda)^\dagger \hat{D}_m \hat{J}(\lambda)$  can be easily calculated since we know the exact result of  $(-ix_m) \hat{J}(\lambda)^\dagger \hat{P}_m \hat{J}(\lambda)$  according to Theorem 4.1.  $\square$

Based on Theorem 4.2, the final state of the quantum game model  $|\psi_f\rangle$  in (1) can be written as

$$\begin{aligned}|\psi_f\rangle &= [(\hat{J}(\lambda)^\dagger \hat{D}_1 \hat{J}(\lambda)) \otimes (\hat{J}(\lambda)^\dagger \hat{D}_2 \hat{J}(\lambda)) \otimes \cdots \otimes \\ &\quad (\hat{J}(\lambda)^\dagger \hat{D}_n \hat{J}(\lambda))] |vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle \\ &= \exp\{-i[x_1 \frac{1}{n} (e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \\ &\quad + \sum_{j=1, j \neq 1}^n x_j \frac{1}{n} (e^{(n-1)\lambda} - e^{-\lambda})] \hat{P}_1\} |vac_1\rangle \otimes \\ &\quad \exp\{-i[x_2 \frac{1}{n} (e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \\ &\quad + \sum_{j=1, j \neq 2}^n x_j \frac{1}{n} (e^{(n-1)\lambda} - e^{-\lambda})] \hat{P}_2\} |vac_2\rangle \otimes \cdots \otimes \\ &\quad \exp\{-i[x_n \frac{1}{n} (e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \\ &\quad + \sum_{j=1, j \neq n}^n x_j \frac{1}{n} (e^{(n-1)\lambda} - e^{-\lambda})] \hat{P}_n\} |vac_n\rangle.\end{aligned}$$

After forwarding  $|\psi_f\rangle$  to measurement, node  $m$  can measure the data it relays to other nodes as follows:

$$q_m = \frac{x_m}{n} (e^{(n-1)\lambda} + (n-1)e^{-\lambda}) + \sum_{j=1, j \neq m}^n \frac{x_j}{n} (e^{(n-1)\lambda} - e^{-\lambda}).$$

In light of the conditions of the Nash Equilibrium, that is,  $\frac{\partial \mathcal{S}_1}{\partial x_1} = \frac{\partial \mathcal{S}_2}{\partial x_2} = \cdots = \frac{\partial \mathcal{S}_n}{\partial x_n} = 0$  and  $\frac{\partial^2 \mathcal{S}_m}{\partial x_m^2} < 0$  ( $m = 1, 2, \dots, n$ ), we can obtain the optimal value of  $x_m$ , i.e.,  $x_m^*$ , as follows:

$$x_1^* = x_2^* = \cdots = x_n^* = \frac{(\lambda - b)(e^{(n-1)\lambda} + (n-1)e^{-\lambda})}{2ae^{2\lambda(n-1)} + (n-1)2ae^{(n-2)\lambda}}.$$

Subsequently, the optimal strategy of each node is:

$$\begin{aligned}q_1^*(x_1^*, x_2^*, \dots, x_n^*) &= q_2^*(x_1^*, x_2^*, \dots, x_n^*) = \cdots \\ &= q_n^*(x_1^*, x_2^*, \dots, x_n^*) \\ &= \frac{e^{(n-1)\lambda}(\lambda - b)(e^{(n-1)\lambda} + (n-1)e^{-\lambda})}{2ae^{2\lambda(n-1)} + (n-1)2ae^{(n-2)\lambda}}.\end{aligned}\quad (9)$$

Thus, the optimal utility  $\mathcal{S}_m^*$  ( $m = 1, 2, \dots, n$ ) of each player can be calculated according to (3).

#### 4.3 Numerical Analysis

In this subsection, we investigate whether nodes decide to serve others and to what extent they are willing to help in general extrinsic incentive mechanisms via numerical analysis. To that aim, we first give the following definition:

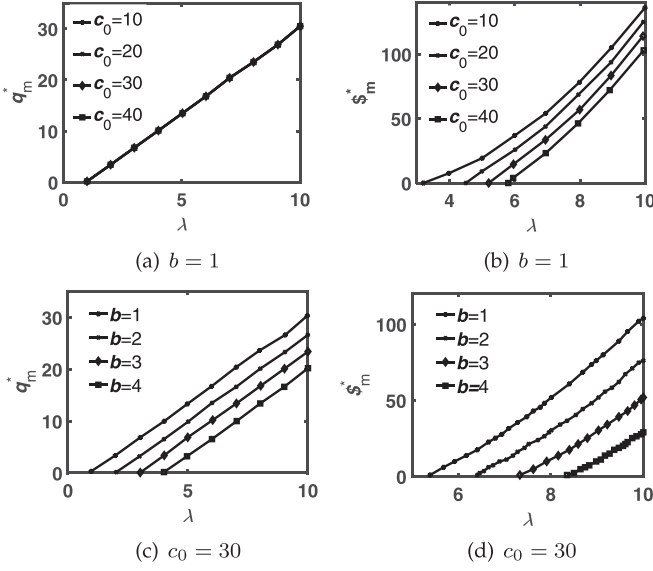


Fig. 2. Evolution of  $q_m^*$  and  $s_m^*$  under different settings.

**Definition 4.1 (Critical degree of entanglement).** The critical degree of entanglement  $\lambda_c$  is a value which leads to the emergence of cooperation under an extrinsic incentive mechanism.

In other words, under our extrinsic incentive mechanism, a player cooperates when  $\lambda \geq \lambda_c$ , but defects otherwise.

Next, we utilize Matlab R2013b on a laptop with Intel Core i5 Processor (2.3 GHz) and 8 GB memory to conduct the numerical analysis, which is the default experimental

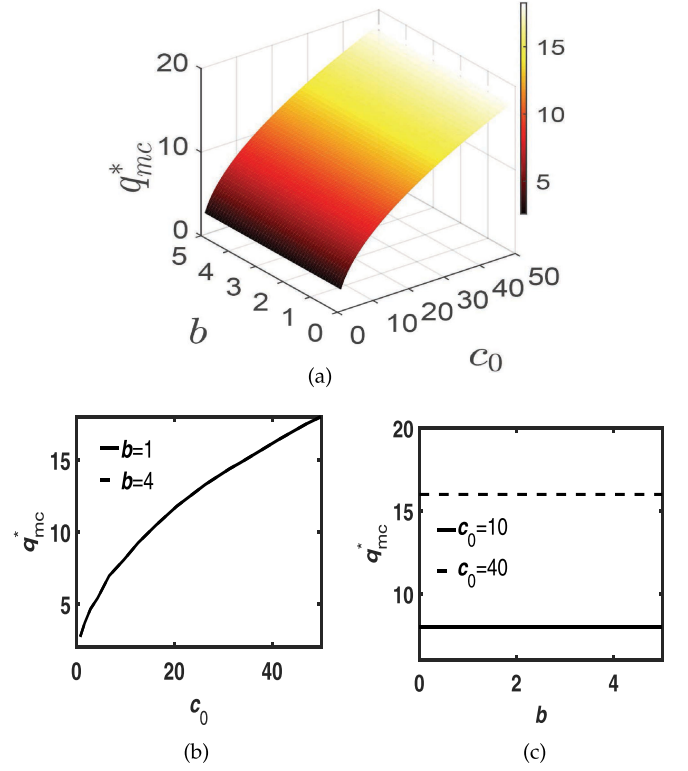


Fig. 4. Critical values of  $q_m^*$  with different  $c_0$  and  $b$ .

environment throughout the whole paper. Fig. 2a, 2b plot node  $m$ 's optimal strategy  $q_m^*$  and its corresponding utility  $s_m^*$  when  $\lambda$  and  $c_0$  vary but  $b=1$ . We also give  $q_m^*$  and  $s_m^*$  with different  $\lambda$  and  $b$  when  $c_0=30$  in Fig. 2c, 2d. It is worth noting that we have done extensive experiments and found out that the trends of  $q_m^*$  and  $s_m^*$  are consistent with those in Fig. 2 for different sets of  $b$  and  $c_0$  values.

In Fig. 2a, one can observe that the lines depicting  $q_m^*$  under different values of  $c_0$  overlap, which indicates that node  $m$ 's best strategy has nothing to do with  $c_0$  (i.e., the fixed transmission cost). Additionally, Fig. 2a, 2c demonstrate that when  $\lambda$  increases to a certain level,  $q_m^*$  approaches positive. However, this does not mean that node  $m$  would necessarily relay or share  $q_m^*$  data because his utility  $s_m^*$  may be less than zero as shown in Fig. 2b, 2d. Nodes adopt their calculated optimal strategies only when  $s_m^* > 0$ .

In Fig. 3, we illustrate  $\lambda_c$  with different values of  $c_0$  and  $b$ . One can see that  $\lambda_c$  goes up with the increase of either  $c_0$  or  $b$ . This is because  $\lambda_c$  needs to be adaptive to offset the negative impacts (i.e., the increase of the cost of relaying or sharing data) on the cooperativeness of a node. Our results indicate that the emergence of cooperation does exist in P2P services, which depends upon the critical degree of entanglement  $\lambda_c$ .

We denote the optimal strategy  $q_m^*$  of node  $m$  when  $\lambda = \lambda_c$  as the critical optimal strategy  $q_{mc}^*$ . Fig. 4 demonstrates that  $q_{mc}^*$  changes with different  $c_0$  and  $b$  when employing the same parameter set as in Fig. 3. We observe that  $q_{mc}^*$  rises up with the increase of  $c_0$ , while stays the same when  $b$  varies. The rationales behind these results are: i)  $\lambda_c$  increases with  $c_0$ , which leads to the growth of  $q_{mc}^*$ ; and ii) the increase of  $b$  is offset by the reward according to the mechanism, and thus makes  $q_{mc}^*$  unchanged.

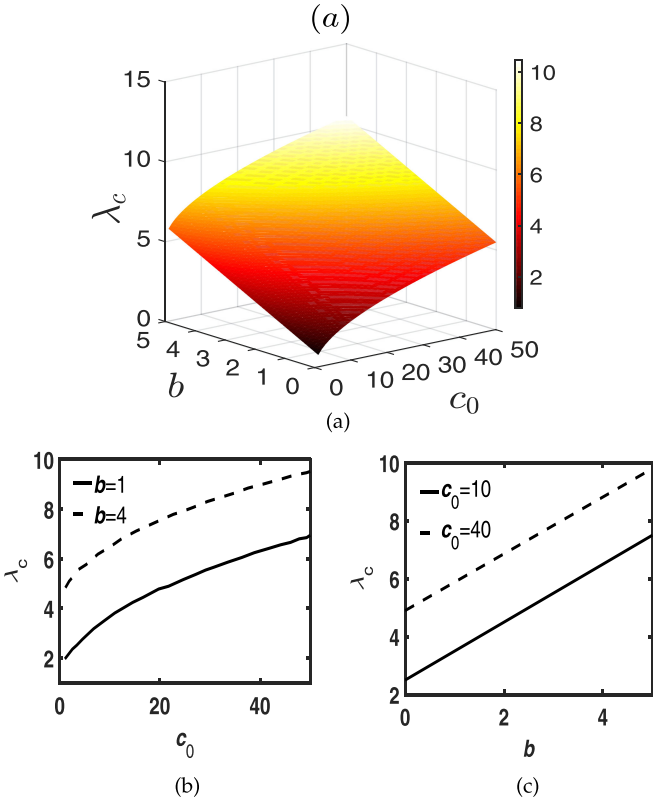


Fig. 3. Critical value of  $\lambda$  with different  $c_0$  and  $b$ .

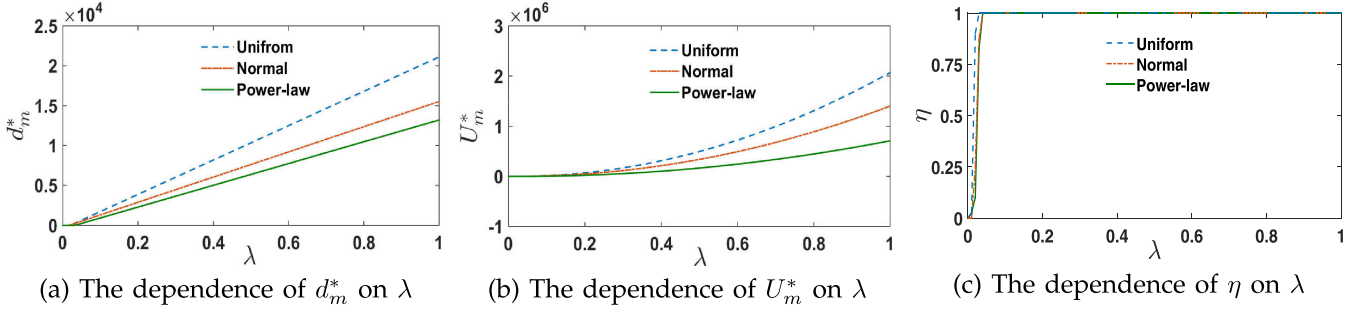


Fig. 5. The dependence of  $d_m^*$ ,  $U_m^*$ , and  $\eta$  on  $\lambda$  under different initial reputation distributions ( $t = 0$ ).

## 5 ANALYTICAL FRAMEWORK FOR REPUTATION-BASED INCENTIVE MECHANISMS

As a typical class of extrinsic incentive mechanisms, reputation-based ones employ the reputation value to evaluate the contribution of a node, which is also a metric for reacting to the service requests of the node. Reputation-based incentive mechanisms can be further categorized into two categories: memory and memoryless ones. In the following, we first introduce our analytical methods; then we detail the analytical results for memory reputation-based incentive mechanisms as well as their memoryless counterparts.

### 5.1 Analytical Methods

The utility of each player is the basis of its behavior motivation and should be formulated in advance. Similarly, the utility ( $U_m$ ) of a node  $m$  in a reputation-based incentive mechanism includes the cost of delivering data and the corresponding reward. However, the reward is not only related to the amount of data  $d_m$  the node helps to serve but also its reputation  $r_m$ . Obviously, the bigger the  $d_m$  and  $r_m$ , the more reward the node can obtain. Hence, the utility of a node  $m$  in a reputation-based incentive mechanism can be written as

$$U_m = -c(d_m) + \mu_2 r_m d_m, \quad (10)$$

where  $c(d_m)$  is the cost of relaying data and  $\mu_2 r_m$  is the reward for a unit amount of the delivered data, which is proportional to the reputation ( $r_m$ ) of node  $m$  and  $\mu_2$  is the reward strength in the reputation-based scheme.

Based on the above utility, we can also employ the quantum model to depict reputation-based incentive mechanisms, where the entanglement degree  $\lambda = \mu_2$ , i.e., the reward strength. Then the corresponding optimal strategy and the optimal amount of relayed data  $d_m^*$  for each player can be calculated using the similar method in Section 4, and thus  $d_m^* = \frac{e^{(n-1)r_m\lambda}(r_m\lambda-b)(e^{(n-1)r_m\lambda}+(n-1)e^{-r_m\lambda})}{2ae^{2r_m\lambda(n-1)}+(n-1)2ae^{(n-2)r_m\lambda}}$  based on which we can obtain the utility of any node in light of (10).

### 5.2 Analytical Results for Memory Reputation-Based Incentive Mechanisms

In a memory reputation-based incentive mechanism, the reputation of each node is time-varying and is associated with its amount of data transmitted at the current period and the previous reputation which may decrease with time. Hence, nodes can improve their reputation by transmitting or sharing data at the current period; and if they do not transmit

data at the following periods, their reputation may decrease to 0 quickly. This kind of system has the advantage of taking into account nodes' history information. Such a reputation system is employed by many incentive mechanisms such as [7]. Let  $r_m^t$  be the node  $m$ 's reputation at time  $t$ ; then its reputation at time  $t + 1$  can be written as

$$r_m^{t+1} = e^{-\rho\Delta t} r_m^t + \tilde{d}_m, \quad (11)$$

where  $\rho$  is the decrease rate of reputation,  $\Delta t$  is the time interval, and  $\tilde{d}_m$  is the normalized amount of data transmitted or shared by node  $m$ .

Using (11) to compute the reputation  $r_m$  in (10), one can obtain the optimal strategy for each player via the method described in the previous subsection.<sup>1</sup>

In the following we report our simulation results. In our simulations, we consider three initial reputation distributions, namely normal, power-law, and uniform distributions. Then we investigate the system evolution under these three different initial settings.

We first analyze the characteristics before the system begins to evolve, i.e.,  $t = 0$ . We depict the dependence of the optimal amount of relayed or shared data  $d_m^*$ , the best utility  $U_m^*$ , and the cooperation ratio  $\eta^2$  on the entanglement degree  $\lambda$  for different initial reputation distributions when  $t = 0$  in Fig. 5. Note that the  $d_m^*$  and  $U_m^*$  ( $m = 1, 2, \dots, n$ ) of all players show similar tendency; thus we randomly select a player's  $d_m^*$  and  $U_m^*$  to plot.

From Fig. 5, one can learn that  $d_m^*$ ,  $U_m^*$ , and  $\eta$  increase with  $\lambda$  no matter what the initial reputation distribution is. Thus in a memory reputation-based incentive mechanism, players tend to transmit more data, gain more benefit, and are more willing to cooperate as the reward provided by the system increases. Additionally, when the initial reputation is uniformly distributed, the critical degree of entanglement defined in Section 4, i.e.,  $\lambda_c$ , is about 0.04, and when the initial reputation follows a normal or a power-law distribution,  $\lambda_c$  is roughly 0.05. In other words, under our parameter settings, the memory reputation-based incentive mechanisms can facilitate a system to obtain a high level of cooperation before the system starts to evolve even if its reward strength  $\lambda_c$  is quite small.

1. Note that the above utility has a generic form. For any given extrinsic incentive mechanism, the utility of each node may be different. Such a difference, however, does not affect the applicability of our proposed framework.

2. The ratio of the number of cooperative players to the number of total players.

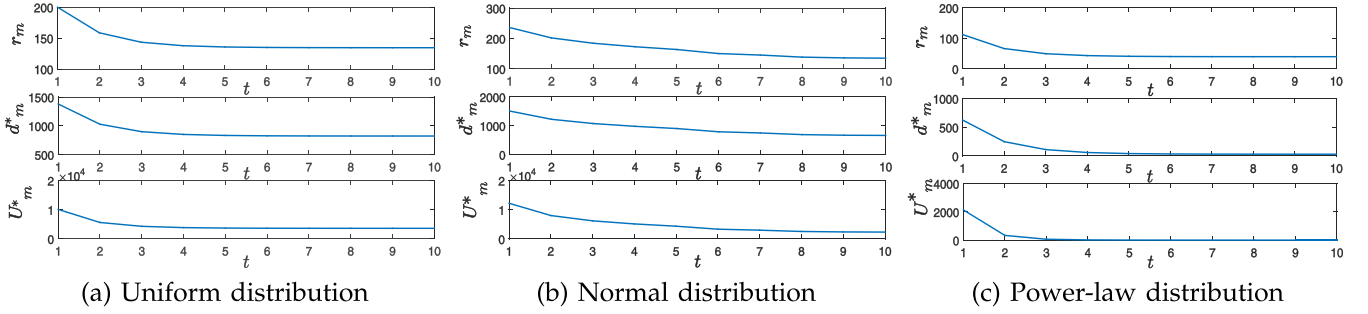


Fig. 6. The evolution of  $r_m$ ,  $d_m^*$ , and  $U_m^*$  under different initial reputation distributions.

In Fig. 6, we depict the evolution of  $r_m$ ,  $d_m^*$ , and  $U_m^*$  for different initial reputation distributions. One can see that in all the three settings, once the reputation  $r_m$  decreases,  $d_m^*$  and  $U_m^*$  decrease at the same time. The evolution can finally reach a stable state.

We demonstrate the cooperation ratio  $\eta$  on different  $t$  and  $\lambda$  under the three different initial reputation settings in Figs. 7, 8, and 9, respectively. One can see that when  $\lambda$  increases,  $\eta$  increases as well at all time, which means that the system reward strength  $\lambda$  has a significant influence on the cooperative ratio. When  $\lambda$  is small,  $\eta$  decreases as time goes by, while  $\eta$  stays at 1 for large  $\lambda$  values. In other words, the number of cooperative players decreases as time goes if the system reward strength is not large enough, while the players always cooperate when the system reward is large. Moreover,  $\lambda_c$  increases as time goes, indicating that the

system needs to provide more reward to achieve cooperation as the system evolves.

We also conduct the Kendall correlation analysis [25], [26] between the ranking by reputation  $r$  and that by the optimally transmitted or shared data  $d^*$ , and between the ranking by  $r$  and that by the best utility a player obtains  $U^*$ . The results are shown in Figs. 10, 11, and 12.

One can see that under all the three distributions, as time goes by, the correlation between the ranking by  $r$  and that by  $d^*$  and the correlation between the ranking by  $r$  and that by  $U^*$  increase, which means that a player with a high reputation tends to transmit more data and get more payoff as time goes. With the increase of  $\lambda$ , under the uniform and power-law distributions, the correlation between the ranking by  $r$  and that by  $d^*$  and the correlation between the ranking by  $r$  and that by  $U^*$  increase; while under the normal distribution, the correlation between the ranking by  $r$  and that by  $d^*$  decreases and the correlation between the ranking by  $r$  and that by  $U^*$  increases. These phenomenon can be explained as follows: under the initial uniform and power-law distributions of the reputation, when the system provides more reward, a player with a high reputation tends to transmit more data and get more payoff; while under the initial normal distribution of the reputation, a player that has a high reputation tends to transmit less data but can get more payoff when the system reward increases.

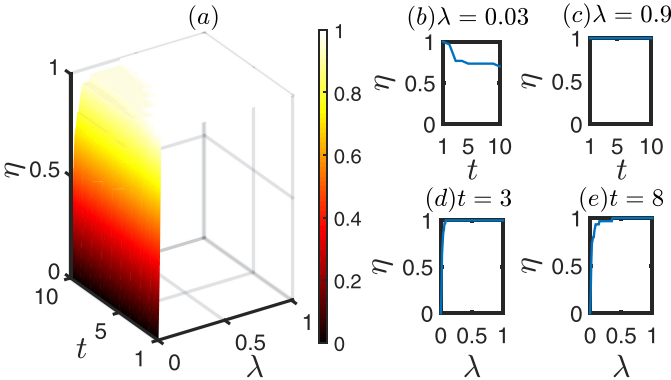


Fig. 7. Cooperation ratio  $\eta$  on different  $t$  and  $\lambda$  under the initial uniform reputation distribution.

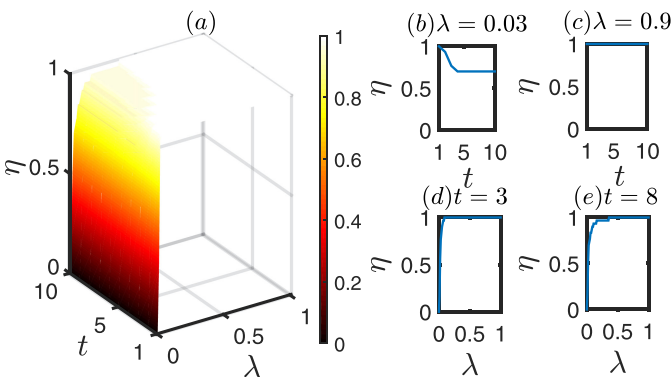


Fig. 8. Cooperation ratio  $\eta$  on different  $t$  and  $\lambda$  under the initial normal reputation distribution.

### 5.3 Analytical Results for Memoryless Reputation-Based Incentive Mechanisms

The reputation in memoryless mechanisms is equivalent to counting the contributions of the nodes in the present

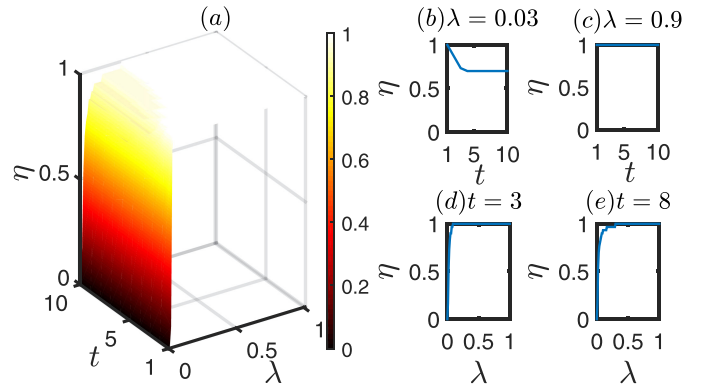


Fig. 9. Cooperation ratio  $\eta$  on different  $t$  and  $\lambda$  under the initial power-law reputation distribution.



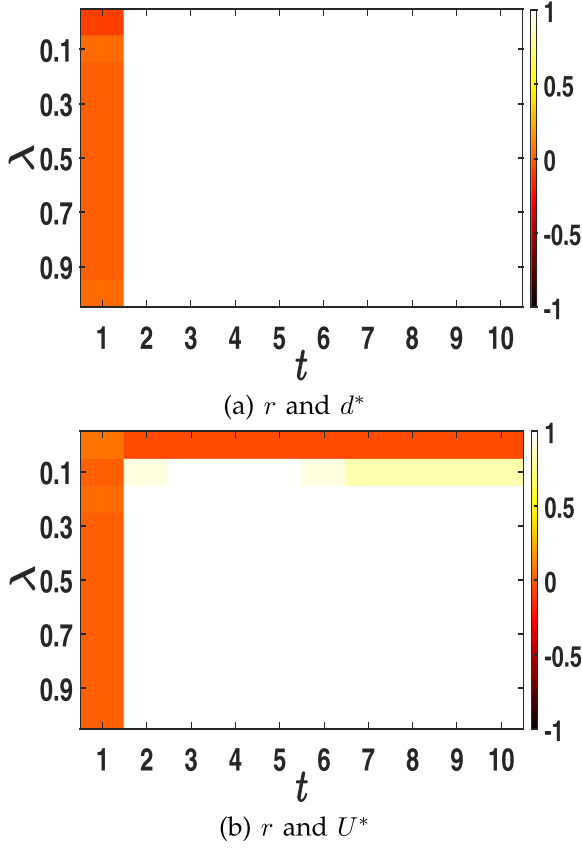


Fig. 10. The correlation analysis on rankings under the uniform distribution.

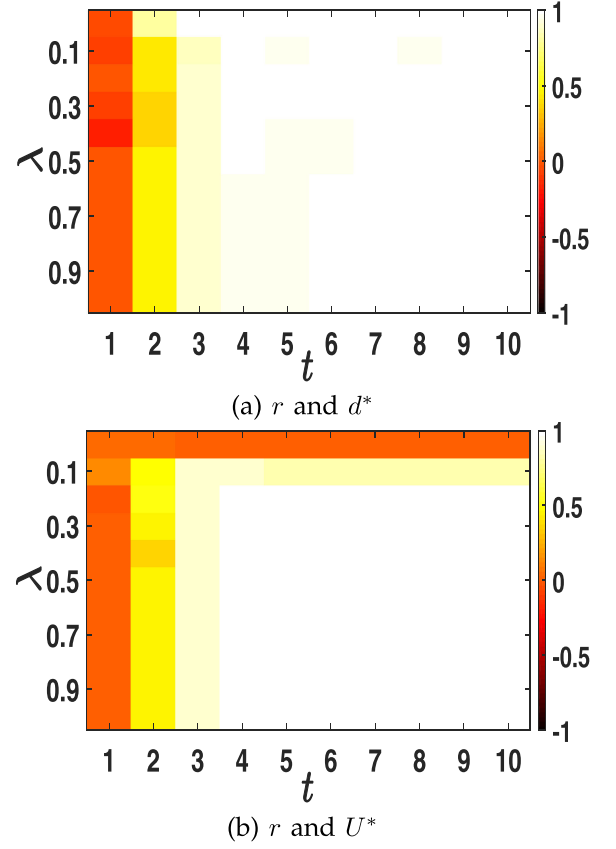


Fig. 12. The correlation analysis on rankings under the power-law distribution.

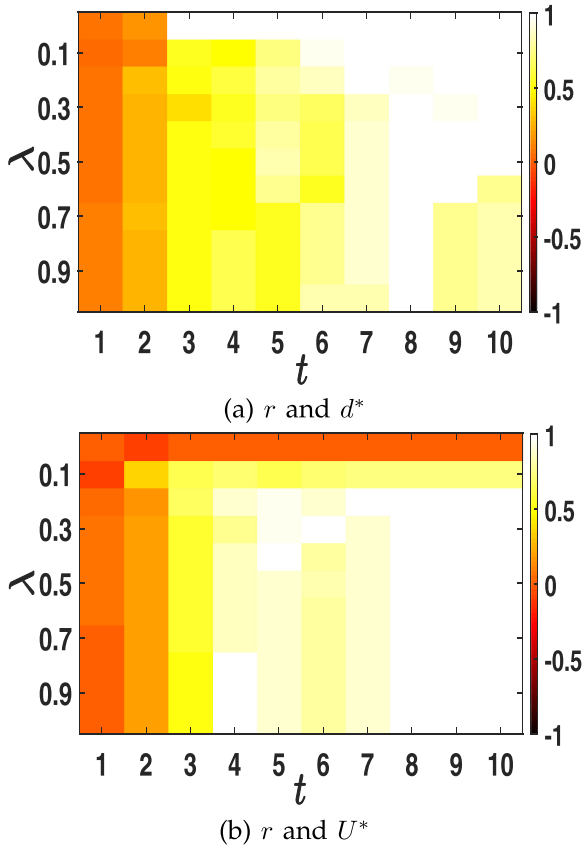


Fig. 11. The correlation analysis on rankings under the normal distribution.

period. The memoryless reputation-based incentive mechanisms can be implemented in two systems: centralized and distributed.

### 5.3.1 Centralized System

In a centralized system, there exists a central server who knows all the nodes' relaying or sharing information in the present period and helps the system to calculate and store the reputation of each node [12]. The memoryless reputation-based incentive mechanisms in a centralized system are quite fair and can avoid malicious players to some extent. In the most general form, the memoryless reputation  $s_m$  of node  $m$  can be calculated as follows,

$$s_m = X_m - Y_m, \quad (12)$$

where  $X_m$  and  $Y_m$  are respectively the numbers of times that node  $m$  relays or shares data for other nodes and gets data from other nodes.

### 5.3.2 Distributed System

In a distributed environment, each node needs to store the local trust values for the nodes it contacts, and a distributed reputation calculation algorithm is implemented to compute the global trust value of each node. Tian et al. [13] proposed an incentive mechanism in a distributed peer-to-peer environment. Cost-effective algorithms to obtain the global trust values for each node were presented in [27], [28]. Since the reputation was well formulated from the mathematical viewpoint in [27], we utilize its methods in our numerical

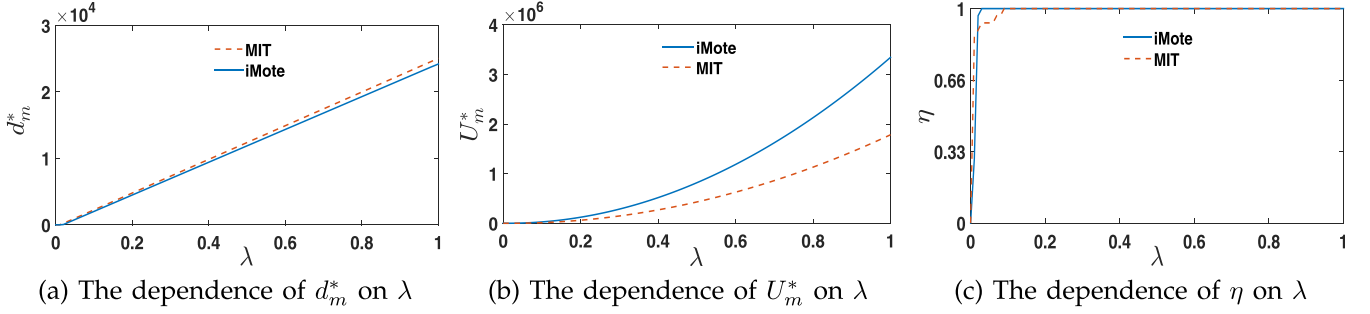


Fig. 13. Analysis on the memoryless reputation-based incentive mechanism for the iMote and MIT data in a centralized system.

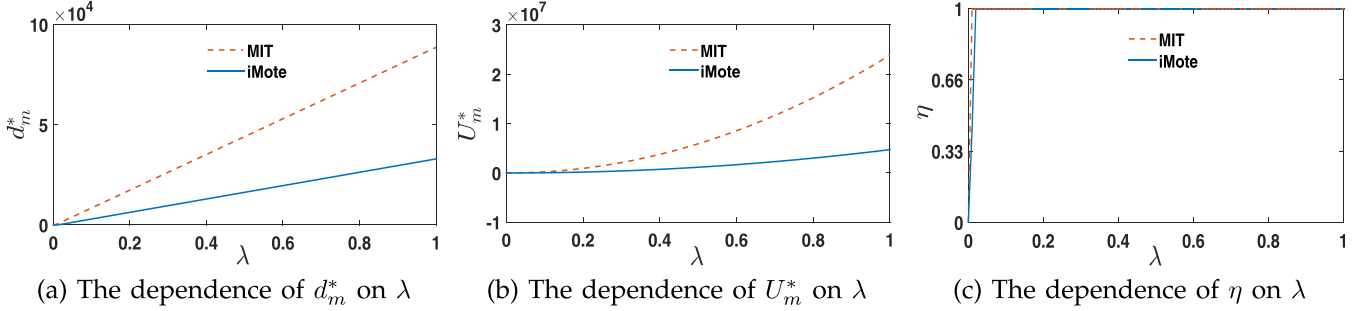


Fig. 14. Analysis on the memoryless reputation-based incentive mechanism for the iMote and MIT data in the distributed system.

analysis. Suppose there are  $n$  players in the system; for each player  $i$ , its reputation can be represented by

$$r_i^{(k+1)} = (1-a)(c_{i1}r_1^{(k)} + c_{i2}r_2^{(k)} + \dots + c_{in}r_n^{(k)}) + ap_i, \quad (13)$$

where  $c_{ni}$  is the local trust value of player  $i$  from the perspective of player  $n$ ,  $p_i$  is the pre-reputation of player  $i$ ,  $k$  is the iteration number, and  $a$  is a weight coefficient. By continuously calculating (13), if  $k$  is large,  $r_i^{(k+1)}$  can converge to a value, which corresponds to the global trust value of player  $i$ .

### 5.3.3 Simulation Analysis

We utilize two real world data sets, namely MIT and iMote, two popular data sets to study P2P transmission [29], [30], [31], to investigate two kinds of the memoryless reputation-based incentive mechanisms. The MIT data set has captured communication, proximity, location, and activity information from 100 subjects at MIT over the course of the 2004 – 2005 academic year [32]. It includes 897921 records, with each offering the following information: the ID of the person that records the phone call (person\_oid), the direction of the call (direction), and the phone number on the other end of the call (phonenumber\_oid). The iMote data includes a number of traces of Bluetooth sightings by groups of users carrying small devices (iMotes) for a number of days in office, conference, and city environments [33]. The three documents (contacts\*.dat, table\*.dat, MAC3Btable\*.dat) in this data set give information about the devices' ID in sightings among Bluetooth devices and iMote devices and the length of the contact time.

In each real-world data-based simulation, we assume every node  $m$  has its own data. Once meeting other nodes, the node needs to require whether others are willing to help relaying, and at the same time, it should compute  $d_m^*$  to

determine the best amount of data it needs to relay for others.<sup>3</sup> Hence, the reputation of node  $m$  can be updated which affects its best utility  $U_m^*$ .

In Fig. 13, we depict the dependence of  $d_m^*$ ,  $U_m^*$ , and  $\eta$  on  $\lambda$  in a centralized system. One can see that  $d_m^*$ ,  $U_m^*$ , and  $\eta$  increase with  $\lambda$ , and the system exhibits the emergence of cooperation when  $\lambda$  is small for both data sets. Similarly, the dependence of  $d_m^*$ ,  $U_m^*$ , and  $\eta$  on  $\lambda$  in the distributed environment are shown in Fig. 14, which demonstrates that  $d_m^*$ ,  $U_m^*$ , and  $\eta$  also increase with  $\lambda$ , and a quite small  $\lambda$  can lead to the emergence of cooperation in the system for both data sets. Conclusively, for a memoryless reputation-based incentive mechanism, as the reward provided by the mechanism increases, players tend to transmit more data and gain more payoffs correspondingly, and the system can guarantee a high degree of cooperation.

## 6 CONCLUSION

In this paper, we adopt an  $n$ -player continuous quantum game model to analyze the general extrinsic incentive mechanisms as well as the memoryless and memory reputation-based incentive mechanisms. More specifically, the impact of the reward strength from a general extrinsic incentive mechanism on the optimal strategy and the optimal expected payoff of each player is quantitatively analyzed and when the cooperation for a general extrinsic incentive mechanism emerges is deduced. The influence of the entanglement degree under three reputation distributions in memory reputation-based incentive mechanisms are analyzed, followed by the study on the ranking correlations among the reputation, the optimal strategy, and the optimal utility of each player under different reputation distribution scenarios. We

3. if  $d_m^* = 0$ , it will not help relaying.

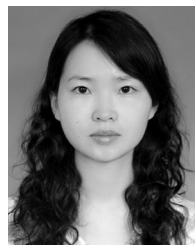
also study the power of the entanglement degree under the memoryless reputation-based incentive mechanisms based on real-world data. To the best of our knowledge, we are the first to analyze extrinsic incentive mechanisms for P2P services from a quantum game perspective. Our proposed quantum game-based analytical framework is generic; thus it is suitable for various scenarios because they are compatible with the classic game-based schemes.

## ACKNOWLEDGMENTS

This work has been supported by the National Science Foundation of China No. 61772080 and No. U1811463, and partially supported by the National Science Foundation of the US under grants OAC-1829553, CNS-1912755, IIS- 1741279 and CNS-1704397.

## REFERENCES

- [1] H. Zheng and J. Wu, "Nsfa: Nested scale-free architecture for scalable publish/subscribe over p2p networks," in *Proc. IEEE 24th Int. Conf. Netw. Protocols*, 2016, pp. 1–10.
- [2] K. Lu, S. Wang, G. Cui, M. Li, and H. Bin-Liaqat, "Multi-reciprocity policies co-evolution based incentive evaluating framework for mobile p2p systems," *IEEE Access*, vol. 5, pp. 3313–3321, 2017.
- [3] C.-M. Liu and C.-C. Lai, "A group-based data-driven approach for data synchronization in unstructured mobile p2p systems," *Wireless Netw.*, vol. 24, no. 7, pp. 2465–2482, 2018.
- [4] B. Alberto, Y.-K. Liu, and A. Vahdat, "Designing incentives for peer-to-peer routing," in *Proc. 24th Annu. Joint Conf. IEEE Comput. Commun. Societies*, 2005, vol. 1, pp. 374–385.
- [5] T. Ning, Z. Yang, H. Wu, and Z. Han, "Self-interest-driven incentives for ad dissemination in autonomous mobile social networks," in *Proc. IEEE INFOCOM*, 2013, pp. 2310–2318.
- [6] B. B. Chen and M. C. Chan, "Mobicent: A credit-based incentive system for disruption tolerant network," in *Proc. IEEE INFOCOM*, 2010, pp. 1–9.
- [7] R. Lu, X. Lin, H. Zhu, X. Shen, and B. Preiss, "Pi: A practical incentive protocol for delay tolerant networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1483–1493, Apr. 2010.
- [8] M. Y. S. Uddin, B. Godfrey, and T. Abdelzaher, "Relics: In-network realization of incentives to combat selfishness in dtms," in *Proc. 18th IEEE Int. Conf. Netw. Protocols*, 2010, pp. 203–212.
- [9] Y. Hu, M. Feng, L. N. Bhuyan, and V. Kalogeraki, "Budget-based self-optimized incentive search in unstructured p2p networks," in *Proc. IEEE INFOCOM*, 2009, pp. 352–360.
- [10] M. E. Mahmoud and X. Shen, "Stimulating cooperation in multi-hop wireless networks using cheating detection system," in *Proc. IEEE INFOCOM*, 2010, pp. 1–9.
- [11] K. Chen and H. Shen, "Multicent: A multifunctional incentive scheme adaptive to diverse performance objectives for dtn routing," in *Proc. IEEE Int. Conf. Sens. Commun. Netw.*, 2013, pp. 532–540.
- [12] J. Moon and Y. Cho, "A point-based incentive system to prevent free-riding on p2p network environments," in *Proc. Int. Conf. Comput. Sci. Appl. - Vol. Part IV*, 2011, pp. 462–471.
- [13] C. Tian, B. Yang, J. Zhong, and X. Liu, "Trust-based incentive mechanism to motivate cooperation in hybrid p2p networks," *Comput. Netw.*, vol. 73, no. C, pp. 244–255, 2014.
- [14] T.-Y. Wu, W.-T. Lee, N. Guizani, and T.-M. Wang, "Incentive mechanism for p2p file sharing based on social network and game theory," *J. Netw. Comput. Appl.*, vol. 41, pp. 47–55, 2014.
- [15] Q. Li, S. Zhu, and G. Cao, "Routing in socially selfish delay tolerant networks," in *Proc. IEEE INFOCOM*, 2010, pp. 857–865.
- [16] M. Pitkanen, M. Conti, S. Giordano, F. Legendre, M. May, and T. Spyropoulos, "Scampi: Service platform for social aware mobile and pervasive computing," *ACM SIGCOMM Comput. Commun. Rev.*, vol. 42, pp. 503–508, Aug. 2012.
- [17] X. Zhuo, Q. Li, G. Cao, Y. Dai, B. Szymanski, and T. La Porta, "Social-based cooperative caching in dtms: A contact duration aware approach," in *Proc. IEEE 11th Int. Conf. Mobile Ad-Hoc Sensor Syst.*, 2011, pp. 92–101.
- [18] Z. Ning, L. Liu, F. Xia, B. Jedari, I. Lee, and W. Zhang, "Cais: A copy adjustable incentive scheme in community-based socially-aware networking," *IEEE Trans. Veh. Technol.*, vol. 66, no. 4, pp. 3406–3419, Apr. 2015.
- [19] W. Sun and S. Wang, "General analysis of incentive mechanisms for peer-to-peer transmissions: A quantum game perspective," in *Proc. IEEE 37th Int. Conf. Distrib. Comput. Syst.*, 2017, pp. 517–526.
- [20] J. Park and M. Van der Schaar, "A game theoretic analysis of incentives in content production and sharing over peer-to-peer networks," *IEEE J. Select. Top. Signal Process.*, vol. 4, no. 4, pp. 704–717, Aug. 2010.
- [21] H. Li, J. Du, and S. Massar, "Continuous-variable quantum games," *Phys. Lett. A*, vol. 306, no. 2, pp. 73–78, 2002.
- [22] J. Eisert, M. Wilkens, and M. Lewenstein, "Quantum games and quantum strategies," *Phys. Rev. Lett.*, vol. 83, no. 15, p. 3077, 1999.
- [23] S. C. Benjamin and P. M. Hayden, "Multiplayer quantum games," *Phys. Rev. A*, vol. 64, no. 3, 2001, Art. no. 030301.
- [24] M. Martin, "A quadratic cost function," 2010. [Online]. Available: <http://math.jccc.net:8180/webMathematica/JSP/mmartin/qcost.jsp>
- [25] H. Abdi, "The kendall rank correlation coefficient," in *Encyclopedia of Measurement and Statistics*. Thousand Oaks, CA, USA: Sage, 2007, pp. 508–510.
- [26] M. G. Kendall, "Rank correlation methods," 1955.
- [27] S. D. Kamvar, M. T. Schlosser, and H. Garcia-Molina, "The eigen-trust algorithm for reputation management in p2p networks," in *Proc. Int. Conf. World Wide Web*, 2003, pp. 640–651.
- [28] R. Zhou and H. Kai, "Powertrust: A robust and scalable reputation system for trusted peer-to-peer computing," *IEEE Trans. Parallel Distrib. Syst.*, vol. 18, no. 4, pp. 460–473, Apr. 2007.
- [29] R. Chen, F. Bao, M. Chang, and J.-H. Cho, "Dynamic trust management for delay tolerant networks and its application to secure routing," *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 5, pp. 1200–1210, May 2014.
- [30] J. Fan, J. Chen, Y. Du, W. Gao, J. Wu, and Y. Sun, "Geocommunity-based broadcasting for data dissemination in mobile social networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 24, no. 4, pp. 734–743, Apr. 2013.
- [31] K. Zhu, W. Li, and X. Fu, "Smart: A social-and mobile-aware routing strategy for disruption-tolerant networks," *IEEE Trans. Veh. Technol.*, vol. 63, no. 7, pp. 3423–3434, Sep. 2014.
- [32] N. Eagle and A. S. Pentland, "CRAWDAD dataset mit/reality (v. 2005-07-01)," Jul. 2005. [Online]. Available: <http://crawdad.org/mit/reality/20050701/blueaware, traceset: blueaware>
- [33] J. Scott, R. Gass, J. Crowcroft, P. Hui, C. Diot, and A. Chaintreau, "CRAWDAD dataset cambridge/haggle (v. 2009-05-29)," May 2009. [Online]. Available: <http://crawdad.org/cambridge/haggle/20090529>



**Shengling Wang** received the PhD degree from Xi'an Jiaotong University, in 2008. After that, she did the postdoctoral research in the Department of Computer Science and Technology, Tsinghua University. Then she worked as an assistant and associate professor, respectively, from 2010 to 2013 with the Institute of Computing Technology, Chinese Academy of Sciences. She is currently a professor with the College of Information Science and Technology, Beijing Normal University. Her research interests include edge computing, cloud computing, mobile/wireless networks, and game theory. She is a member of the IEEE.



**Weiman Sun** received the bachelor's degree from Beijing Normal University, Beijing, China, in 2017. She is currently working toward the master's degree at Columbia University, New York. Her major is computer software and theory. Her current research interests include data mining and edge computing.



**Liran Ma** received the DSc degree in computer science from the George Washington University. He is currently an associate professor with the Department of Computer Science, Texas Christian University. His current research interests include wireless, mobile, and embedded systems, including security and privacy, smartphones, smart health, mobile computing, data analytics, internet of things, and systems security education. It involves building and simulating prototype systems and conducting real experiments and measurements. He is a member of the IEEE.



**Weifeng Lv** received the PhD degree in computer science from Beihang University. He is a professor of computer science, the dean of the School of Computer Science and Engineering, and the vice director of the State Key Laboratory of Software Development Environment, Beihang University. He also serves as the secretary-general of the China Software Industry Association and the director of National Engineering Research Center for Science and Technology Resources Sharing Service. His research interests include massive

information system, urban cognitive computing, swarm intelligence, and smart cities. He received multiple internationally renowned awards, including the second prize of the 2016 China National Science and Technology Invention Award and the first prize of the 2010 Beijing Science and Technology Award.



**Xiuzhen Cheng** received the MS and PhD degrees in computer science from the University of Minnesota-Twin Cities, in 2000 and 2002, respectively. She is a full professor with the Department of Computer Science, George Washington University, Washington, DC. Her current research interests include privacy-aware computing, wireless and mobile security, dynamic spectrum access, mobile handset networking systems (mobile health and safety), cognitive radio networks, and algorithm design and analysis. She has served on the

Editorial Boards of several technical publications (e.g., the *IEEE Transactions on Parallel and Distributed Systems*, the *IEEE Wireless Communications*), and the Technical Program Committees of various professional conferences/workshops (e.g., IEEE INFOCOM, IEEE ICDCS, IEEE ICC, IEEE/ACM IWQoS). She has also chaired several international conferences (e.g., IEEE CNS, WASA). She worked as a program director for the U.S. National Science Foundation (NSF) from April to October 2006 (full time), and from April 2008 to May 2010 (part time). She published more than 170 peer-reviewed papers. She is a fellow of the IEEE.

▷ **For more information on this or any other computing topic, please visit our Digital Library at [www.computer.org/csdl](http://www.computer.org/csdl).**