

A RISK-CENTRIC MODEL OF DEMAND RECESSIONS AND SPECULATION*

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We provide a continuous-time “risk-centric” representation of the New Keynesian model, which we use to analyze the interactions between asset prices, financial speculation, and macroeconomic outcomes when output is determined by aggregate demand. In principle, interest rate policy is highly effective in dealing with shocks to asset valuations. However, in practice monetary policy faces a wide range of constraints. If these constraints are severe, a decline in risky asset valuations generates a demand recession. This reduces earnings and generates a negative feedback loop between asset prices and aggregate demand. In the recession phase, average beliefs matter because they not only affect asset valuations but also determine the strength of the amplification mechanism. In the *ex ante* boom phase, belief disagreements (or heterogeneous asset valuations) matter because they induce investors to speculate. This speculation exacerbates the crash by reducing high-valuation investors’ wealth when the economy transitions to recession, which depresses (wealth-weighted) average beliefs. Macroprudential policy that restricts speculation in the boom can Pareto improve welfare by increasing asset prices and aggregate demand in the recession. *JEL* Codes: E00, E12, E21, E22, E30, E40, G00, G01, G11.

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I. INTRODUCTION

Prices of risky assets—such as stocks, bonds, and houses—fluctuate considerably without meaningful changes in the underlying payoffs. These fluctuations, which are due to a host of rational and behavioral mechanisms, are generically described as the result of a “time-varying risk premium” (see [Cochrane 2011](#); [Campbell 2014](#); [Shiller 2014](#) for recent reviews). Although fluctuations in risky asset prices affect the macroeconomy in a multitude of ways, a growing empirical literature suggests that aggregate demand plays a central role and therefore interest rate policy can mitigate the macroeconomic impact of asset price shocks. [Pflueger, Siriwardane, and Sunderam \(2020\)](#) show that prices of volatile stocks have high predictive power for interest rates and economic activity, and [Cieslak and Vissing-Jørgensen \(2020\)](#) argue that the Fed pays attention to stock prices and cuts interest rates after stock price declines (the Fed put). However, the ability of interest rate policy to quickly respond to asset price shocks is limited by a host of practical concerns, such as exchange rate volatility, balance sheet fragilities, decision lags, and transmission lags. An important current concern is that, with interest rates close to their effective lower bound in much of the developed world, interest rate policy will be unable to respond to large negative asset price shocks.

This connection between risky asset prices and aggregate demand suggests that speculation—a pervasive feature of financial markets driven by heterogeneous asset valuations—can lead to more severe downturns. There is an old tradition in macroeconomics that emphasizes speculation as a central feature of asset prices in boom-bust cycles (see, e.g., [Minsky 1977](#); [Kindleberger 1978](#)). In recent empirical work, [Mian and Sufi \(2018\)](#) argue that speculation also has played a key role in the U.S. housing cycle. However, speculation and its interaction with aggregate demand are largely missing from the modern macroeconomic theory connecting asset prices with economic activity, which mostly focuses on financial frictions (see [Gertler and Kiyotaki 2010](#) for a review). This omission is especially important in the current low interest rate environment, as monetary policy has even less space than usual to mop up a sharp decline in risky asset prices following a speculative episode.

In this article, we build a risk-centric macroeconomic model—that is, a model in which risky asset prices are at the core of the

analysis—with the two key features highlighted above. First, we explore the role of the aggregate demand channel and interest rate frictions in causing recessions driven by a rise in the “risk premium”—our catchall phrase for shocks to asset valuations. Second, we study the effect of financial speculation on the severity of these recessions and derive the implications for macroprudential policy. To isolate our insights, we remove all financial frictions.

Our analysis relies on the standard aggregate demand mechanism present in the New Keynesian model but formulated in terms of a risk-centric decomposition (as opposed to the usual Euler equation–based approach). Specifically, we decompose the demand block of the equilibrium into two relations: an output–asset price relation that captures the positive association between asset prices and aggregate demand through a wealth effect on consumption (and a marginal-Q effect on investment when we add investment), and a risk balance condition that describes asset prices given risks, risk attitudes, beliefs, and the interest rate. This decomposition isolates the characterization of asset prices from the “macroeconomics” side of the model. Therefore, it facilitates the study of a variety of forces that affect asset prices—including financial speculation—in a macroeconomic environment. Our decomposition highlights that the interest rate policy influences aggregate demand through its impact on financial markets and asset prices (whereas the New Keynesian literature typically emphasizes intertemporal substitution considerations).¹

Our model is set in continuous time with diffusion productivity shocks and Poisson shocks that move the economy between high and low risk premium states. The supply side is a stochastic AK economy with sticky prices (which we extend to an endogenous growth model when we add investment). The demand side has risk-averse consumer-investors who demand goods and risky assets. We focus on interest rate frictions and financial speculation. By “interest rate frictions,” we mean factors that might constrain or delay the adjustment of the risk-free interest rate to shocks. For concreteness, we work with a zero lower bound on the policy interest rate, but our mechanism is applicable with other interest rate constraints, such as a currency union, a fixed exchange rate,

1. Our decomposition (and its implications for the transmission of monetary policy) matches the reasoning in actual central banks’ statements when dealing with the risk-off events that have plagued the world economy over the past few decades (see, e.g., [Cieslak and Vissing-Jørgensen 2020](#)).

or delays in the monetary policy reaction. By “financial speculation,” we mean trading risky financial assets among investors that have heterogeneous valuations for these assets. We capture speculation by allowing investors to have belief disagreements (with respect to the transition probabilities between high and low risk premium states), but our results apply if speculation is driven by other sources of heterogeneous valuations. In particular, optimists in our model can also capture more risk-tolerant investors (e.g., banks or institutional investors), whereas pessimists can capture less risk-tolerant investors (e.g., households or retail investors).

To fix ideas, consider an increase in perceived volatility (equivalently, a decrease in optimism). This is a “risk premium shock” that exerts downward pressure on risky asset prices without a change in current productivity (the supply-determined output level). Consequently, monetary policy responds by reducing the interest rate, which stabilizes asset prices and aggregate demand. However, if the interest rate is constrained, the rise in the risk premium reduces asset prices and generates a demand recession.

Dynamics play a crucial role in this environment, as the recession is exacerbated by feedback mechanisms. When investors expect the higher risk premium to persist, the decline in future demand lowers expected earnings, which exerts further downward pressure on asset prices. With endogenous investment, there is a second mechanism, as the decline in investment lowers the growth of potential output, which further reduces expected earnings and asset prices. In turn, the decline in asset prices feeds back into current consumption and investment, generating scope for severe spirals in asset prices and output. [Figure I](#) illustrates these dynamic mechanisms. The feedbacks are especially powerful when investors are pessimistic and think the higher risk premium will persist. Hence, average beliefs matter in our economy because they not only have a direct impact on asset prices but also determine the strength of the amplification mechanism.

In this environment, belief disagreements (or heterogeneous asset valuations) matter in two important ways. First, asset prices depend on the wealth-weighted average belief among optimists and pessimists. Therefore, in the recession, greater wealth in the hands of optimists increases asset prices as well as aggregate demand and output. This result highlights that wealth distribution matters for aggregate demand not only because of financial frictions (e.g., [Bernanke, Gertler, and Gilchrist 1999](#))

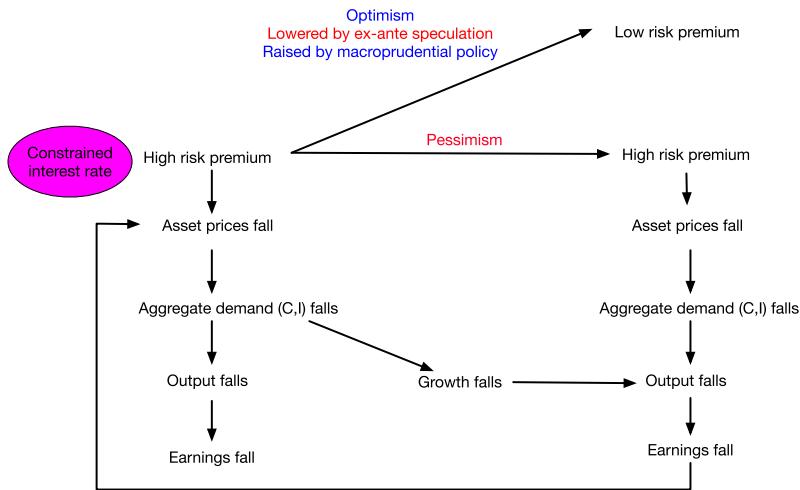


FIGURE I
Output-Asset Price Feedbacks during a Risk-Centric Demand Recession

or heterogeneous marginal propensities to consume (MPC) (e.g., [Auclet 2019](#)), as emphasized by the previous macroeconomics literature, but also because of heterogeneous asset valuations. In fact, in our model there are no financial frictions, and optimists and pessimists have the same MPCs. Increasing optimists' wealth share in the recession raises aggregate spending not because optimists spend more than pessimists but because they raise asset valuations and induce all consumer-investors to spend more (while also increasing aggregate investment when we add investment).

Second, belief disagreements create speculation, which amplifies the fluctuations in asset valuations and aggregate demand. Investors take speculative positions that reflect their beliefs. This speculation makes the wealth-weighted belief extrapolate recent realizations—even though individual investors have fixed beliefs and do not extrapolate. In particular, good realizations vindicate optimists and increase their wealth share, which makes the wealth-weighted belief more optimistic. Conversely, bad realizations increase pessimists' wealth share and make the wealth-weighted belief more pessimistic. Therefore, speculation amplifies the fluctuations in asset prices. When the interest rate is constrained, speculation also amplifies demand-driven boom-bust cycles and worsens macroeconomic outcomes.

Specifically, we find that speculation during the low risk premium phase (boom) exacerbates the recession when there is a transition to the high risk premium phase (bust). During the boom, optimists take on risk by selling insurance contracts to pessimists that enrich optimists if the boom persists but reduce their wealth share if there is a transition to recession. This reallocation of wealth in the recession lowers asset prices and leads to a more severe recession.

These effects motivate macroprudential policy that restricts speculation during the boom. We show that macroprudential policy that makes optimistic investors behave as if they were more pessimistic (implemented via portfolio risk limits) can generate a Pareto improvement in social welfare. This result is not driven by paternalistic concerns (the planner respects investors' own beliefs), and the result does not depend on whether optimists or pessimists are closer to the truth. Rather, the planner improves welfare by internalizing aggregate demand externalities. During the recession, the economy benefits from wealthy optimists (or high-valuation investors) because they raise asset prices and aggregate demand. However, optimists who take on speculative positions during the boom (and pessimists who take the opposite side of those positions) do not internalize the effect of their risk-taking on asset prices and aggregate demand during the recession. This leads to excessive risk-taking by optimists that can be offset by macroprudential policy. Therefore, our model supports a variety of policies used in practice—such as a leverage limit or a risk limit—that preserve optimists' (or high-valuation investors') wealth for the recession state. Moreover, our model supports procyclical macroprudential policy. Whereas macroprudential policy can be useful during the recession, these benefits can be outweighed by its immediate negative effect on asset prices. This adverse price impact is not a concern during the boom, because it is offset by the interest rate policy, but it lowers asset prices and output in the recession since the interest rate is constrained.

Although there is an extensive empirical literature supporting the components of our model (see [Section VII](#) for a brief summary), we present additional empirical evidence consistent with our results. We focus on three implications. First, our model predicts that shocks to asset valuations generate a more severe demand recession when the interest rate is constrained. Second, the recession reduces firms' earnings and leads to a further decline in

asset prices. Third, the recession is more severe when the shock takes place in an environment with more speculation.

To investigate these predictions, we assemble a quarterly panel data set of 20 advanced countries between 1990 and 2017 and divide the panel into countries that are part of the Eurozone or the European Exchange Rate Mechanism (the Euro/ERM sample) and those that have their own currencies (the non-Euro/ERM sample). Countries in the first group have a constrained interest rate with respect to local asset price shocks because they share a common monetary policy. The second group has a less constrained interest rate. We find that a negative house price shock in a non-Euro/ERM country is associated with an initial decline in economic activity, followed by a decline in the policy interest rate and output stabilization. In contrast, a similar shock in a Euro/ERM country is not associated with an interest rate response (compared with other Euro/ERM countries) and is followed by a more persistent and larger decline in economic activity. We also find that the house price shock is followed by a larger decline in earnings and stock prices of publicly traded firms in the Euro/ERM sample (although the standard errors are larger for these results). Finally, we find that past bank credit expansion—which we use as a proxy for speculation on house prices—is associated with more severe outcomes following the house price shock in the Euro/ERM sample (but not in the other sample).

I.A. Literature Review

Our article is related to three main literatures: two in macroeconomics and one in finance. On the macroeconomics side, several recent papers within the New Keynesian literature emphasize demand shocks that might drive business cycles while also affecting asset prices, such as “news shocks” (Beaudry and Portier 2006), “noise shocks” (Lorenzoni 2009; Blanchard, L’Huillier, and Lorenzoni 2013), “confidence shocks” (Ilut and Schneider 2014), “uncertainty shocks” (Basu and Bundick 2017; Fernández-Villaverde et al. 2015), and “disaster shocks” (Isore and Szczerbowicz 2017). Our first contribution to this literature is to reformulate the standard New Keynesian model in terms of a risk-centric decomposition. With this decomposition at hand, we provide an integrated treatment of these demand shocks. We refer to these demand shocks as “risk premium shocks” to emphasize their close connection with asset prices and the finance literature

on time-varying risk premia.² Our second contribution to this literature is to show that heterogeneity in asset valuations matters in these environments. Among other things, heterogeneous valuations lead to speculation that exacerbates demand recessions and provides a distinct rationale for macroprudential regulation.

Another important macroeconomic literature focuses on uncertainty and its role in driving macroeconomic fluctuations (e.g., Bloom 2009; Baker, Bloom, and Davis 2016; Bloom et al. 2018; Baker, Bloom, and Terry 2019). We contribute to this literature by showing how uncertainty affects aggregate activity through asset prices and their effect on aggregate demand. We also illustrate that in our model, uncertainty shocks have stronger effects when monetary policy is constrained, consistent with recent empirical evidence (e.g., Plante, Richter, and Throckmorton 2018). Finally, we show that *ex ante* financial speculation amplifies the damage from uncertainty shocks.

On the finance side, a large literature emphasizes investors' beliefs as a key driver of financial boom-bust cycles (see, e.g., Gennaioli and Shleifer 2018 for the role of beliefs in the recent crisis). A strand of this literature argues that heterogeneity in the degree of optimism combined with short-selling constraints can lead to speculative asset price bubbles that substantially amplify the financial cycle (e.g., Harrison and Kreps 1978; Scheinkman and Xiong 2003; Geanakoplos 2010; Simsek 2013a; Barberis et al. 2018). Related contributions emphasize that disagreements exacerbate asset price fluctuations more broadly—even without short-selling constraints or bubbles—because they create endogenous fluctuations in agents' wealth distribution (e.g., Detemple and Murthy 1994; Zapatero 1998; Basak 2000, 2005; Xiong and Yan 2010; Kubler and Schmedders 2012; Korinek and Nowak 2017; Cao 2017). Our article features similar forces but explores them in an environment where output is not necessarily at its supply-determined level.³

2. See Galí (2020) for an OLG variant of the New Keynesian model with rational bubbles (see also Biswas, Hanson, and Phan *forthcoming*), which also highlights the role of asset prices on aggregate demand. However, his analysis does not focus on the risk-balance condition, which is a key block in our analysis. Also, there is a large body of work that emphasizes the links between asset prices and macroeconomic outcomes through financial frictions (e.g., Kiyotaki and Moore 1997). Our model removes all these financial frictions for clarity.

3. With respect to these papers, we show that speculation during the boom worsens the asset price bust and exacerbates the demand recession. Consequently, and unlike much of this literature, macroprudential policy that restricts

There are five additional connections worth highlighting. In our setting, speculation generates macroeconomic outcomes as if there is a representative agent with extrapolative beliefs. This relates our article to a growing literature that emphasizes extrapolation as a key driver of asset prices and business cycles (see, e.g., [Bordalo, Gennaioli, and Shleifer 2018](#); [Bordalo et al. 2019](#)). Although the two mechanisms are likely to reinforce each other, speculation makes distinct predictions for trading volume and heterogeneity in asset positions (see [Remark 8](#)).

The interactions between heterogeneous valuations, risk premia, and interest rate lower bounds are central themes of the literature on structural safe-asset shortages and safety traps (see, e.g., [Caballero, Farhi, and Gourinchas 2017b](#); [Caballero and Farhi 2018](#)). We contribute to this literature by considering a broader set of factors that can drive the risk premium (in addition to safe-asset scarcity) and, more important, by focusing on dynamics. We analyze the connections between boom and recession phases of recurrent business cycles driven by risk premium shocks. We show that speculation between optimists and pessimists during the boom exacerbates a future risk-centric demand recession, and we derive the implications for macroprudential policy. In contrast, [Caballero and Farhi \(2018\)](#) show how pessimists can create a demand recession in otherwise normal times and derive the implications for fiscal policy and unconventional monetary policy.⁴

At a methodological level, this article belongs to the new continuous-time macrofinance literature started by the work of [Brunnermeier and Sannikov \(2014, 2016a\)](#) and summarized in [Brunnermeier and Sannikov \(2016b\)](#) (see also [Basak and Cuoco 1998](#); [Adrian and Boyarchenko 2012](#); [He and Krishnamurthy 2012, 2013](#); [Di Tella 2017, 2019](#); [Silva 2020](#); [Moreira and Savov 2017](#); [Di Tella and Hall 2019](#)). This literature

speculation can improve welfare even if the planner is not paternalistic and respects investors' (heterogeneous and possibly overoptimistic) beliefs. Adding paternalistic concerns reinforces our normative conclusions (see [Section VI](#)). More broadly, our article is part of a large finance literature that investigates the effect of belief disagreements and speculation on financial markets (e.g., [Lintner 1969](#); [Miller 1977](#); [Varian 1989](#); [Harris and Raviv 1993](#); [Chen, Hong, and Stein 2002](#); [Fostel and Geanakoplos 2008](#); [Simsek 2013b](#); [Iachan, Nenov, and Simsek 2015](#)).

4. This article is also related to an extensive literature on liquidity traps that has exploded since the Great Recession (see, for instance, [Tobin 1975](#); [Krugman 1998](#); [Eggertsson and Woodford 2006](#); [Hall 2011](#); [Christiano, Eichenbaum, and Trabandt 2015](#); [Bacchetta, Benhima, and Kalantzis forthcoming](#); [Midrigan et al. 2016](#); [Guerrieri and Lorenzoni 2017](#); [Rognlie, Shleifer, and Simsek 2018](#)).

highlights the full macroeconomic dynamics induced by financial frictions. While the structure of our economy shares many features with theirs, our model has no financial frictions, and the macroeconomic dynamics stem not from the supply side (relative productivity) but from the aggregate demand side.

Our results on macroprudential policy are related to recent work that analyzes the implications of aggregate demand externalities for the optimal regulation of financial markets. For instance, [Korinek and Simsek \(2016\)](#) show that in the run-up to deleveraging episodes that coincide with a zero lower bound on the interest rate, policies targeted at reducing household leverage can improve welfare (see also [Farhi and Werning 2017](#)). In these papers, macroprudential policy reallocates wealth across agents and states so that agents with a higher MPC hold relatively more wealth when the economy is depressed because of deficient demand. The mechanism in our article is different and works through heterogeneous asset valuations (instead of heterogeneous MPCs).⁵

The macroprudential literature beyond aggregate demand externalities is mostly motivated by the presence of pecuniary externalities that make the competitive equilibrium constrained inefficient (e.g., [Caballero and Krishnamurthy 2003](#); [Lorenzoni 2008](#); [Bianchi and Mendoza 2018](#); [Jeanne and Korinek 2019](#)). The friction in this literature is market incompleteness or collateral constraints that depend on asset prices (see [Dávila and Korinek 2018](#) for a detailed exposition). We show that a decline in asset prices is damaging not only for the reasons emphasized in this literature but also because it lowers aggregate demand.

The rest of the article is organized as follows. In [Section II](#) we present an example that illustrates the main mechanism and motivates the rest of our analysis. [Section III](#) presents the general environment and defines the equilibrium. [Section IV](#) characterizes the equilibrium in a benchmark setting with common beliefs. This section shows how risk premium shocks can lower asset prices and induce a demand recession, and how feedback loops between asset prices and aggregate demand exacerbate the recession. [Section V](#) characterizes the equilibrium with belief disagreements and heterogeneous asset valuations. This section illustrates how

5. See [Farhi and Werning \(2016\)](#) for a synthesis of some key mechanisms that justify macroprudential policies in models that exhibit aggregate demand externalities.

a greater optimists' wealth share increases asset valuations and mitigates the recession, and how speculation amplifies asset price fluctuations and worsens the recession. [Section VI](#) shows the aggregate demand externalities associated with optimists' risk-taking and establishes our results on macroprudential policy. [Section VII](#) presents our empirical analysis and summarizes supporting evidence from the related literature. [Section VIII](#) concludes. The [Online Appendix](#) contains the omitted derivations and proofs as well as the details of our empirical analysis.

II. A STEPPING-STONE RISK-CENTRIC EXAMPLE

Here we present a simple, largely static example that serves as a stepping stone to our main dynamic model. We start with a representative-agent setup and characterize the standard aggregate demand mechanism in the New Keynesian model but formulated in terms of our risk-centric decomposition. We use this decomposition to illustrate how risk premium shocks generate a demand recession when the interest rate is constrained. We then consider heterogeneous valuations and illustrate how speculation affects demand recessions.

II.A. A Two-Period Risk-Centric Aggregate Demand Model

Consider an economy with two dates, $t \in \{0, 1\}$, a single consumption good, and a single factor of production—capital. For simplicity, capital is fixed and normalized to 1. Potential output is equal to capital's productivity, z_t , but actual output can be below this level because of a shortage of aggregate demand, $y_t \leq z_t$. For simplicity, we assume output is equal to its potential at the last date, $y_1 = z_1$, and focus on the endogenous determination of output at the previous date, $y_0 \leq z_0$. We assume the productivity at date 1 is uncertain and log-normally distributed,

$$(1) \quad \log y_1 = \log z_1 \sim N\left(g - \frac{\sigma^2}{2}, \sigma^2\right).$$

We also normalize the initial productivity to 1, $z_0 = 1$, so that g captures the (log) expected growth rate of productivity, and σ captures its volatility.

There are two types of assets. There is a market portfolio that represents claims to the output at date 1 (which accrues to production firms as earnings), and a risk-free asset in zero net

supply. We denote the price of the market portfolio with Q , and its log return with

$$(2) \quad r^m(z_1) = \log \frac{z_1}{Q}.$$

We denote the log risk-free interest rate with r^f .

For now, the demand side is characterized by a representative investor, who is endowed with the initial output and the market portfolio. At date 0, she chooses how much to consume, c_0 , and what fraction of her wealth to allocate to the market portfolio, ω^m , with the residual fraction invested in the risk-free asset. When asset markets are in equilibrium, she will allocate all of her wealth to the market portfolio, $\omega^m = 1$, and her portfolio demand will determine the risk premium.

We assume the investor has Epstein-Zin preferences with discount factor $e^{-\rho}$ and relative risk aversion coefficient (RRA) γ . For simplicity, we set the elasticity of intertemporal substitution (EIS) equal to 1. Allowing for a more general EIS leaves our results qualitatively unchanged (see [Remark 2](#)).

The supply side of the economy is described by New Keynesian firms that have fixed nominal prices (see [Remark 1](#) for the role of this assumption and [Online Appendix B.1.3](#) for details). These firms meet the available demand at these prices as long as prices are higher than their marginal cost. These features imply that output is determined by the aggregate demand for goods (consumption) up to the capacity constraint,

$$(3) \quad y_0 = c_0 \leq z_0.$$

Because prices are fully sticky, the real interest rate is equal to the nominal interest rate, which is controlled by the central bank. We assume that the interest rate policy attempts to replicate the supply-determined output level. However, there is a lower bound constraint on the interest rate, $r^f \geq 0$. Thus, the interest rate policy is described by $r^f = \max(r^{f*}, 0)$, where r^{f*} is the natural interest rate that ensures output is at its potential, $y_0 = z_0$.

To characterize the equilibrium, first note that there is a tight relationship between output and asset prices. Specifically, the assumption on the EIS isolates the consumption wealth effect: the investor consumes a fraction of her lifetime income in the first

period,

$$(4) \quad c_0 = \frac{1}{1 + e^{-\rho}}(y_0 + Q).$$

Combining this expression with [equation \(3\)](#), we obtain the following equation:

$$(5) \quad y_0 = e^{\rho} Q.$$

We refer to this equation as the output–asset price relation—generally, it is obtained by combining the consumption function (and when there is investment, also the investment function) with goods market clearing. The condition says that asset prices increase aggregate wealth and consumption, which in turn leads to greater output.

Next note that asset prices must also be consistent with equilibrium in risk markets. In [Online Appendix A.1](#), we show that up to a local approximation, the investor’s optimal weight on the market portfolio is determined by

$$(6) \quad \omega^m \sigma \simeq \frac{1}{\gamma} \frac{E[r^m(z_1)] + \frac{\sigma^2}{2} - r^f}{\sigma}.$$

In words, the optimal portfolio risk (left side) is proportional to “the Sharpe ratio” on the market portfolio (right side). The Sharpe ratio captures the reward per risk, where the reward is determined by the risk premium: the (log) expected return in excess of the (log) risk-free rate. This is the standard risk-taking condition for mean-variance portfolio optimization. It applies approximately in the two-period model, and the approximation becomes exact when there is a representative household and the asset markets are in equilibrium ($\omega^m = 1$).

Substituting the asset market clearing condition, $\omega^m = 1$, and the expected return on the market portfolio from [equations \(1\)](#) and [\(2\)](#), we obtain the following equation:

$$(7) \quad \sigma = \frac{1}{\gamma} \frac{g - \log Q - r^f}{\sigma}.$$

We refer to this equation as the risk balance condition—generally, it is obtained by combining investors’ optimal portfolio allocations with asset market clearing and the equilibrium return on the

market portfolio. The equilibrium level of the Sharpe ratio on the market portfolio (right side) needs to be large enough to convince investors to hold the risk generated by the productive capacity (left side).

Next consider the supply-determined equilibrium in which output is equal to its potential, $y_0 = z_0 = 1$. [Equation \(5\)](#) reveals that this requires the asset price to be at a particular level, $Q^* = e^{-\rho}$. Combining this with [equation \(7\)](#), the interest rate also needs to be at a particular level,

$$(8) \quad r^{f*} = g + \rho - \gamma\sigma^2.$$

Intuitively, the monetary policy needs to set the interest rate low enough to induce sufficiently high asset prices and aggregate demand to clear the goods market.

Now suppose the initial parameters are such that $r^{f*} > 0$, so the equilibrium features Q^* , r^{f*} , and supply-determined output, $y_0 = z_0 = 1$. Consider a “risk premium shock” that raises the volatility, σ , or risk aversion, γ . The immediate effect of this shock is to create an imbalance in the risk balance condition [\(7\)](#). The economy produces too much risk (left side) relative to what investors are willing to absorb (right side). In response, the monetary policy lowers the risk-free interest rate (captured by the decline in r^{f*}), which increases the risk premium and equilibrates the risk balance condition [\(7\)](#). Intuitively, the central bank lowers the opportunity cost of risky investment and induces investors to absorb more risk.

Next suppose the shock is large enough that the natural interest rate becomes negative, $r^{f*} < 0$, and the actual interest rate becomes constrained, $r^f = 0$. In this case, the risk balance condition is reestablished via a decline in the price of the market portfolio, Q . This decline in asset prices increases the expected return on risky investment, which induces investors to absorb risk. However, the decline in Q reduces aggregate wealth and induces a demand-driven recession. Formally, we combine [equations \(5\)](#) and [\(7\)](#) to obtain

$$(9) \quad \log y_0 = \rho + \log Q, \quad \text{where } \log Q = g - \gamma\sigma^2.$$

Note that in the constrained region, asset prices and output are sensitive to beliefs about future prospects. For instance, a decrease in the expected growth rate g (pessimism)—rational or

otherwise—decreases asset prices and worsens the recession. In fact, while we considered shocks that raise σ or γ , [equations \(8\)](#) and [\(9\)](#) reveal that shocks that lower g lead to the same effects. The key point for our risk-centric perspective is that g still operates by reducing the expected return on risky assets and hence creating an imbalance in risk markets.

II.B. Heterogeneous Valuations and Speculation

We consider heterogeneous asset valuations and investigate how speculation affects demand recessions. We capture heterogeneous valuations with belief disagreements about productivity growth. Specifically, there are two types of investors, optimists and pessimists, who believe $\log z_1$ is distributed according to, respectively, $N(g^o - \frac{\sigma^2}{2}, \sigma^2)$ and $N(g^p - \frac{\sigma^2}{2}, \sigma^2)$. We assume $g^o > g^p$ so that optimists perceive greater growth. Beliefs are dogmatic—that is, investors know each others' beliefs and they agree to disagree. Optimists are endowed with a fraction α of the market portfolio and of date 0 output (and pessimists are endowed with the remaining fraction). Hence, α denotes the wealth share of optimists. The rest of the model is unchanged.

Following similar steps to those in the baseline case, we solve for “rstar” as (see [Online Appendix A.3](#)),

$$(10) \quad r^{f*} \simeq \alpha g^o + (1 - \alpha) g^p + \rho - \gamma \sigma^2.$$

When $r^{f*} < 0$, the interest rate is constrained and $r^f = 0$, so we have a demand recession with

$$(11) \quad \log y_0 = \rho + \log Q, \text{ where } \log Q \simeq \alpha g^o + (1 - \alpha) g^p - \gamma \sigma^2.$$

Hence equilibrium prices and output depend on optimists' wealth share, α . During the recession, increasing α improves outcomes because optimists increase asset prices, which increases aggregate wealth and everyone's spending. In our dynamic model, α is endogenous because investors (ex ante) speculate on their different beliefs. Moreover, speculation reduces α during the recession because optimists think the risk premium shock is unlikely. This exacerbates the recession and motivates macroprudential policy.

REMARK 1 (ROLE OF NOMINAL PRICE RIGIDITY). In our model (as well as in other New Keynesian models), nominal price rigidity plays two roles. First, and most important, it creates a real interest rate rigidity. To see this, consider an alternative

economy in which prices are fully flexible and the nominal interest rate is at a lower bound. How would this economy react to a risk premium shock that requires a decline in the real interest rate? By definition, the real rate is equal to the nominal interest rate minus expected inflation in nominal prices. Because the nominal interest rate is constrained, the economy must generate expected inflation: either the current nominal prices must decline or the future nominal prices must increase (or a combination of the two). Nominal price rigidity hinders such an adjustment and translates into real interest rate rigidity. As our analysis illustrates, this rigidity reduces real asset prices in financial markets, which in turn reduces aggregate demand in goods markets. Nominal price rigidity plays another role by making firms respond to the decline in aggregate demand by cutting production—instead of cutting their nominal price to increase the demand for their goods (see [Online Appendix B.1.3](#) for a formalization and [Remark 3](#) for a discussion of how partial price flexibility affects our results).

REMARK 2 (MORE GENERAL EIS). In [Online Appendix A.2](#), we extend the baseline two-period model (without disagreements) to cases where the EIS is different from 1. In these cases, a risk premium shock affects output through two channels. As before, it exerts a downward influence on asset prices and consumption through a wealth effect. But it also reduces the attractiveness of investment opportunities, which further affects consumption depending on the balance of income and substitution effects. When the EIS is greater than 1, the second channel works against the wealth effect because investors substitute toward consumption. When the EIS is less than 1, the second channel reinforces the wealth effect. Importantly, we show that the wealth effect dominates regardless of the EIS. When the interest rate is constrained, a risk premium shock reduces equilibrium output and the asset price. When the EIS is greater than 1, the substitution effect dampens these declines but it does not overturn them.

III. DYNAMIC ENVIRONMENT AND EQUILIBRIUM

In this section we introduce our general dynamic environment and define the equilibrium. We then partially characterize

the equilibrium. In subsequent sections we further characterize this equilibrium in various special cases of interest. Throughout, we simplify the analysis by abstracting away from investment. In [Online Appendix D](#), we extend the environment to introduce investment and endogenous growth. We discuss additional results related to investment at the end of [Section IV](#).

1. Potential Output and Risk Premium Shocks. The economy is set in infinite continuous time, $t \in [0, \infty)$, with a single consumption good and a single factor of production, capital. Let $k_{t,s}$ denote the capital stock at time t and in the aggregate state $s \in S$. Suppose that when fully utilized, $k_{t,s}$ units of capital produce $Ak_{t,s}$ units of the consumption good. Hence, $Ak_{t,s}$ denotes the potential output in this economy. Capital follows the process

$$(12) \quad \frac{dk_{t,s}}{k_{t,s}} = gdt + \sigma_s dZ_t.$$

Here, g denotes the expected productivity growth, which is an exogenous parameter in the main text (we endogenize it in [Online Appendix D](#)). The term dZ_t denotes the standard Brownian motion, which captures “aggregate productivity shocks.”⁶

The states, $s \in S$, differ only in terms of the volatility of aggregate productivity, σ_s . For simplicity, there are only two states, $s \in \{1, 2\}$, with $\sigma_1 < \sigma_2$. State $s = 1$ corresponds to a low-volatility state, whereas state $s = 2$ corresponds to a high-volatility state. At each instant, the economy in state s transitions into the other state $s' \neq s$ according to a Poisson process. We use these volatility shocks to capture the time variation in the risk premium due to various unmodeled factors (see [Section II](#) for an illustration of how risk, risk aversion, and beliefs play a similar role in our analysis).

2. Investor Types. There is a finite number of investor types denoted by $i \in I$. Investor types are identical in all respects except for their beliefs about state transitions. Each type consists of a continuum of identical investors with mass normalized to 1. We

6. Note that fluctuations in $k_{t,s}$ generate fluctuations in potential output, $Ak_{t,s}$. We introduce Brownian shocks to capital, $k_{t,s}$, as opposed to total factor productivity, A , because this leads to a slightly more tractable analysis when we extend the model to include investment (see [Online Appendix D](#)). In the main text, we could equivalently introduce shocks to A and conduct the analysis by normalizing all relevant variables with $A_{t,s}$ as opposed to $k_{t,s}$.

focus on symmetric equilibria in which investors within a type choose identical allocations.

3. Transition Probabilities and Belief Disagreements. We let $\lambda_s^i > 0$ denote the perceived Poisson transition probability in state s (into the other state) according to type i investors. These probabilities capture the degree of investors' (relative) optimism or pessimism. For instance, greater λ_2^i corresponds to greater optimism because investors expect the high risk premium conditions to end relatively soon. Likewise, smaller λ_1^i corresponds to greater optimism because investors expect the low risk premium conditions to persist longer. We first analyze the special case with common beliefs (Section IV) and then investigate belief disagreements and speculation (Section V). When investors disagree, they know each others' beliefs and they agree to disagree.

4. Menu of Financial Assets. There are three types of financial assets. First, there is a market portfolio that represents a claim on all output. We let $Q_{t,s}k_{t,s}$ denote the price of the market portfolio, so $Q_{t,s}$ denotes the price per unit of capital. We let $r_{t,s}^m$ denote the instantaneous expected return on the market portfolio conditional on no transition. Second, there is a risk-free asset in zero net supply. We denote its instantaneous return by $r_{t,s}^f$. Third, in each state s , there is a contingent Arrow-Debreu security that trades at the (endogenous) price $p_{t,s}^{s'}$ and pays 1 unit of the consumption good if the economy transitions into the other state $s' \neq s$. This security is also in zero net supply and it ensures that financial markets are dynamically complete.

5. Price and Return of the Market Portfolio. Absent transitions, the price of the market portfolio per unit of capital follows an endogenous diffusion process,

$$(13) \quad \frac{dQ_{t,s}}{Q_{t,s}} = \mu_{t,s}^Q dt + \sigma_{t,s}^Q dZ_t \quad \text{for } s \in \{1, 2\}.$$

Combining equations (12) and (13), the price of the market portfolio (absent transition) follows

$$(14) \quad \frac{d(Q_{t,s}k_{t,s})}{Q_{t,s}k_{t,s}} = \left(g + \mu_{t,s}^Q + \sigma_s \sigma_{t,s}^Q \right) dt + \left(\sigma_s + \sigma_{t,s}^Q \right) dZ_t.$$

The expected return and the volatility of the market portfolio (absent transition) are then given by

$$(15) \quad r_{t,s}^m = \frac{y_{t,s}}{Q_{t,s} k_{t,s}} + g + \mu_{t,s}^Q + \sigma_s \sigma_{t,s}^Q \quad \text{and} \quad \sigma_{t,s}^m = \sigma_s + \sigma_{t,s}^Q.$$

Here, $y_{t,s}$ denotes the endogenous level of output at time t . The first term in $r_{t,s}^m$ captures the “dividend yield” component of return. The remaining terms capture the (expected) capital gain conditional on no transition, which reflects the expected growth of capital, of the price per unit of capital, and of their stochastic interaction.

Equations (13)–(15) describe the prices and returns conditional on no state transition. If there is a transition at time t from state s into state $s' \neq s$, then the price per unit of capital jumps from $Q_{t,s}$ to a potentially different level, $Q_{t,s'}$. Therefore, investors that hold the market portfolio experience instantaneous capital gains or losses.

6. Consumption and Portfolio Choice. Investors continuously make consumption and portfolio allocation decisions. Specifically, at any time t and state s , each type i investor has some financial wealth denoted by $a_{t,s}^i$. She chooses her consumption rate, $c_{t,s}^i$; the fraction of her wealth to allocate to the market portfolio, $\omega_{t,s}^{m,i}$; and the fraction of her wealth to allocate to the contingent security, $\omega_{t,s}^{s',i}$. The residual fraction, $1 - \omega_{t,s}^{m,i} - \omega_{t,s}^{s',i}$, is invested in the risk-free asset. For analytical tractability, we assume the investor has log utility. In particular, we set the RRA and the EIS equal to 1 (see Remark 6 for a discussion of how a more general RRA affects our results). The investor then solves a standard portfolio problem that we formally state in Online Appendix B.1.1.

7. Equilibrium in Asset Markets. Asset markets clear when the total wealth held by investors is equal to the value of the market portfolio before and after the portfolio allocation decisions,

$$(16) \quad \sum_i a_{t,s}^i = Q_{t,s} k_{t,s} \quad \text{and} \quad \sum_i \omega_{t,s}^{m,i} a_{t,s}^i = Q_{t,s} k_{t,s}.$$

Contingent securities are in zero net supply, which implies

$$(17) \quad \sum_i \omega_{t,s}^{s',i} a_{t,s}^i = 0.$$

The market-clearing condition for the risk-free asset (which is also in zero net supply) holds when conditions (16) and (17) are satisfied.

8. Nominal Rigidities and the Equilibrium in Goods Markets. The supply side of our model features nominal rigidities similar to the standard New Keynesian model. We relegate the details to [Online Appendix B.1.3](#). There is a continuum of monopolistically competitive production firms that own the capital stock and produce intermediate goods (which are then converted into the final good). For simplicity, these production firms have preset nominal prices that never change (see [Remark 3](#) for a discussion of the case with partial price flexibility). The firms choose their capital utilization rate, $\eta_{t,s} \in [0, 1]$, which leads to output $y_{t,s} = \eta_{t,s} A k_{t,s}$. We assume firms can increase factor utilization for free until $\eta_{t,s} = 1$ and they cannot increase it beyond this level.

As we show in the [Online Appendix](#), these features imply that output is determined by aggregate demand for goods up to the capacity constraint. Combining this with market clearing in goods, output is determined by aggregate consumption (up to the capacity constraint),

$$(18) \quad y_{t,s} = \eta_{t,s} A k_{t,s} = \sum_i c_{t,s}^i, \quad \text{where } \eta_{t,s} \in [0, 1].$$

Moreover, all output accrues to production firms in the form of earnings.⁷ Hence, the market portfolio can be thought of as a claim on all production firms.

9. Interest Rate Rigidity and Monetary Policy. Our assumption that production firms do not change their prices implies that the aggregate nominal price level is fixed. The real risk-free interest rate, then, is equal to the nominal risk-free interest rate, which is determined by the interest rate policy of the central bank. We assume there is a lower bound on the nominal interest rate, which we set at 0 for convenience,

$$(19) \quad r_{t,s}^f \geq 0.$$

7. In this model, firms own the capital, so the division of earnings between returns to capital and monopoly profits is indeterminate. Because there is no investment, this division is inconsequential. When we introduce investment in [Online Appendix D](#), we make additional assumptions to determine how earnings are divided.

The zero lower bound is motivated by the presence of cash in circulation (which we leave unmodeled for simplicity).

We assume that the interest rate policy aims to replicate the level of output that would obtain without nominal rigidities subject to the constraint in inequality (19). Without nominal rigidities, capital is fully utilized, $\eta_{t,s} = 1$ (see [Online Appendix B.1.3](#)). Thus, we assume that the interest rate policy follows the rule

$$(20) \quad r_{t,s}^f = \max(0, r_{t,s}^{f*}) \quad \text{for each } t \geq 0 \text{ and } s \in S.$$

Here, $r_{t,s}^{f*}$ is recursively defined as the instantaneous natural interest rate that obtains when $\eta_{t,s} = 1$ and monetary policy follows the rule in [equation \(20\)](#) at all future times and states.

DEFINITION 1. The equilibrium is a collection of processes for allocations, prices, and returns such that capital and its price evolve according to [equations \(12\)](#) and [\(13\)](#), the instantaneous return and the volatility of the market portfolio are given by [equation \(15\)](#), investors maximize expected utility (see [Online Appendix B.1.1](#)), asset markets clear (see [equations \(16\)](#) and [\(17\)](#)), production firms maximize earnings (cf. [Online Appendix B.1.3](#)), goods markets clear (see [equation \(18\)](#)), and the interest rate policy follows the rule in [equation \(20\)](#).

REMARK 3 (PARTIAL PRICE FLEXIBILITY). Our assumption of fixed nominal prices is extreme. However, allowing some nominal price flexibility does not necessarily circumvent the lower bound in inequality (19). In fact, if monetary policy follows an inflation-targeting policy regime, partial price flexibility leads to expected price deflation during a demand recession—the opposite of what the economy needs to circumvent the lower bound on the nominal interest rate (see [Remark 1](#)). Intuitively, individual firms respond to the recession by cutting their individual nominal prices, which increases their individual demand given aggregate demand. However, the decline in nominal prices does not necessarily stabilize aggregate demand—whether or not this happens depends on monetary policy. In an inflation-targeting regime, nominal prices decline during the recession and get stabilized at a lower level once the economy exits the recession. This creates expected deflation that strengthens the bound in inequality (19) and exacerbates the recession (see [Werning 2012](#);

[Korinek and Simsek 2016](#); [Caballero and Farhi 2018](#) for further discussion).

In the rest of this section, we provide a partial characterization of the equilibrium. In subsequent sections, we use this characterization to describe the equilibrium for various specifications of investors' beliefs.

III.A. Equilibrium in the Goods Market

First consider the goods market. The following result establishes that there is a tight relationship between output and asset prices as in the two-period model.

LEMMA 1 (OUTPUT-ASSET PRICE RELATION). The equilibrium level of output (per unit of capital) satisfies

$$(21) \quad \frac{y_{t,s}}{k_{t,s}} = A\eta_{t,s} = \rho Q_{t,s}.$$

The equilibrium return and the volatility of the market portfolio (absent transition) are given by

$$(22) \quad r_{t,s}^m = \rho + g + \mu_{t,s}^Q + \sigma_s \sigma_{t,s}^Q \quad \text{and} \quad \sigma_{t,s}^m = \sigma_s + \sigma_{t,s}^Q.$$

As before, the output–asset price relation in [equation \(21\)](#) follows from the wealth effect. In view of log utility, each investor optimally consumes a constant fraction of her wealth (see [Online Appendix B.1.1.](#)),

$$(23) \quad c_{t,s}^i = \rho a_{t,s}^i.$$

This implies that aggregate consumption is a constant fraction of aggregate wealth (see [equation \(16\)](#)),

$$(24) \quad \sum_i c_{t,s}^i = \rho Q_{t,s} k_{t,s}.$$

Combining this with [equation \(18\)](#), we obtain the relation in [\(21\)](#). Substituting this into [equation \(15\)](#), we further obtain [equation \(22\)](#). In view of the output–asset price relation, the dividend yield on the market portfolio is equal to the consumption rate ρ .

As before, the output–asset price relation implies that full factor utilization, $\eta_{t,s} = 1$, obtains only if the price per unit of

capital is at a particular level $Q^* \equiv \frac{A}{\rho}$. This is the efficient price level that ensures that the implied consumption clears the goods market. Likewise, the economy features a demand recession, $\eta_{t,s} < 1$, only if the price per unit of capital is strictly below Q^* . Combining these observations with the interest rate policy in [equation \(20\)](#), we also summarize the goods market with

$$(25) \quad Q_{t,s} \leq Q^*, r_{t,s}^f \geq 0, \quad \text{where at least one condition is an equality.}$$

The equilibrium at any time and state takes one of two forms. If the natural interest rate is nonnegative, then the interest rate policy ensures that the price per unit of capital is at the efficient level, $Q_{t,s} = Q^*$, capital is fully utilized, $\eta_{t,s} = 1$, and output is equal to its potential, $y_{t,s} = Ak_{t,s}$. Otherwise, the interest rate is constrained, $r_{t,s}^f = 0$, the price is lower, $Q_{t,s} < Q^*$, and output is determined by aggregate demand according to [equation \(21\)](#).

III.B. Equilibrium in Asset Markets

Next consider asset markets. The equilibrium in these markets depends on investors' relative wealth. We define type i investors' wealth share as

$$(26) \quad \alpha_{t,s}^i = \frac{a_{t,s}^i}{k_{t,s} Q_{t,s}}.$$

By definition, the wealth shares sum to 1, $\sum_i \alpha_{t,s}^i = 1$ (see [equation \(16\)](#)). These wealth shares matter because they determine the wealth-weighted average belief for the transition probability, defined as

$$(27) \quad \bar{\lambda}_{t,s} \equiv \sum_i \alpha_{t,s}^i \lambda_s^i.$$

We will show that asset prices are determined as if there is a representative investor that has the wealth-weighted average belief. However, the wealth-weighted average belief is not constant over time because investors have speculative portfolios that affect their wealth shares. Therefore, we start by characterizing investors' optimal portfolios and the resulting wealth dynamics. We use the notation \dot{x} to denote the time derivative of variable x , that is, $\dot{\alpha}_{t,s}^i = \frac{d\alpha_{t,s}^i}{dt}$ denotes the drift in type i investors' wealth share.

LEMMA 2 (WEALTH-SHARE DYNAMICS). Investors hold identical positions on the market portfolio,

$$(28) \quad \omega_{t,s}^{m,i} = 1 \quad \text{for each } i.$$

They hold possibly heterogeneous positions on the contingent security given by

$$(29) \quad \omega_{t,s}^{s',i} = \lambda_s^i - \bar{\lambda}_{t,s}.$$

Type i investors' wealth share evolves according to

$$(30) \quad \begin{aligned} \frac{\dot{\alpha}_{t,s}^i}{\alpha_{t,s}^i} &= -\omega_{t,s}^{s',i} = \bar{\lambda}_{t,s} - \lambda_s^i, & \text{if there is no state change,} \\ \frac{\alpha_{t,s'}^i}{\alpha_{t,s}^i} &= \frac{\lambda_s^i}{\bar{\lambda}_{t,s}}, & \text{if there is a change to } s'. \end{aligned}$$

[Equation \(28\)](#) says that investors' belief disagreements do not affect their positions on the market portfolio. In contrast, [equation \(29\)](#) shows that belief disagreements do affect investors' positions on the contingent security, and [equation \(30\)](#) describes the resulting wealth dynamics. When type i investors assign a relatively large probability to transition, $\lambda_s^i > \bar{\lambda}_{t,s}$, they purchase the contingent security that pays if there is a transition, $\omega_{t,s}^{s',i} > 0$. As long as the economy remains in the same state, their wealth share drifts downward, $\dot{\alpha}_{t,s}^i < 0$. However, if there is a transition to the other state, their wealth share makes an upward jump, $\frac{\alpha_{t,s'}^i}{\alpha_{t,s}^i} > 1$. Conversely, when type i investors assign a relatively small probability to transition, they sell the contingent security. This ensures that their wealth share drifts upward if the economy remains in the same state, and it makes a downward jump if there is a transition. These dynamics are important for our main result (see [Section V](#)).

We provide a sketch proof of [Lemma 2](#), which is useful for developing further intuition and obtaining additional results. We derive investors' portfolio optimality conditions in [Online Appendix B.1.1](#). A type i investor's portfolio weight on the market portfolio is determined by

$$(31) \quad \omega_{t,s}^{m,i} \sigma_{t,s}^m = \frac{1}{\sigma_{t,s}^m} \left(r_{t,s}^m - r_{t,s}^f + \lambda_s^i \frac{a_{t,s}^i}{a_{t,s'}^i} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}} \right).$$

That is, she invests in the market portfolio until the risk of her portfolio (left side) is equal to the Sharpe ratio of the market portfolio (right side). This is similar to the optimality condition in the two-period model (see [equation \(6\)](#)), but the dynamic model also features state transitions. Our notion of the Sharpe ratio accounts for potential revaluation gains or losses from transitions (the term $\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}$) and the adjustment of marginal utility in case there is a transition (the term $\frac{a_{t,s}^i}{a_{t,s'}^i}$).⁸

Likewise, the investor's optimal portfolio allocation to the contingent securities implies

$$(32) \quad \frac{p_{t,s}^{s'}}{\lambda_s^i} = \frac{a_{t,s}^i}{a_{t,s'}^i}.$$

The portfolio weight, $\omega_{t,s}^{s',i}$, is implicitly determined as the level that ensures this equality. The investor buys contingent securities until the perceived price-to-probability ratio of a state (or the state price) is equal to the investor's relative marginal utility in that state.

Substituting [equation \(32\)](#) into [equation \(31\)](#) shows that different investor types allocate identical portfolio weights to the market portfolio, $\omega_{t,s}^{m,i} = \omega_{t,s}^m$. Together with market clearing (see [equation \(16\)](#)), this implies [equation \(28\)](#).

To establish the remaining results in the lemma, we rewrite [equation \(32\)](#) in terms of wealth shares to obtain (see [equation \(26\)](#))

$$(33) \quad \alpha_{t,s'}^i = \alpha_{t,s}^i \lambda_s^i \frac{1}{\kappa_{t,s}}, \quad \text{where } \kappa_{t,s} = p_{t,s}^{s'} \frac{Q_{t,s'}}{Q_{t,s}}.$$

Here, $\kappa_{t,s}$ is a variable that depends on asset prices but that is common across investor types. Aggregating this equation across all investor types, and using $\sum_i \alpha_{t,s'}^i = 1$, we obtain the second part of [equation \(30\)](#). In [Online Appendix B.1.2](#), we further derive [equation \(29\)](#) and the first part of [equation \(30\)](#) by combining [equation \(33\)](#) with investors' budget constraints.

8. The presence of state transitions makes the Sharpe ratio in our model slightly different from its common definition, which corresponds to the expected return in excess of the risk-free rate normalized by volatility.

REMARK 4 (DETERMINISTIC WEALTH DYNAMICS WITHIN A STATE). Lemma 2 shows that investors' wealth shares follow deterministic dynamics absent state transitions. This property is driven by [equation \(28\)](#), which ensures that investors' relative wealth shares are not influenced by dZ_t . As our proof illustrates, [equation \(28\)](#) is driven by complete markets and constant RRA preferences. Complete markets ensure disagreements per se do not induce investors to trade the market portfolio. Intuitively, since investors disagree about transition risk, they settle these disagreements by trading the contingent security for the corresponding risk instead of the market portfolio. Constant (and common) RRA preferences ensure risk-sharing considerations do not generate trade on the market portfolio either.

REMARK 5 (SEPARABILITY OF WEALTH DYNAMICS AND ASSET PRICES). Lemma 2 also implies that investors' wealth shares follow the same dynamics regardless of asset prices or monetary policy. This separability property comes from assuming complete markets and log utility. As we discuss in the previous remark, these assumptions imply that investors do not trade the market portfolio—such trade would make asset prices relevant for wealth shares (see [Caballero and Simsek 2019](#)). Log utility (which features RRA equal to 1) is also necessary because, as captured by [equation \(33\)](#), it ensures that investors' marginal utility depends only on their wealth. In particular, investors do not have dynamic hedging motives—these motives would make asset prices relevant for wealth shares (see [Remark 6](#) and [Online Appendix E](#) for further discussion of the case with a more general RRA).

The separability property is convenient because it breaks the analysis with belief disagreements into two steps. We first use Lemma 2 to characterize investors' wealth dynamics and the evolution of the wealth-weighted average belief in [equation \(27\)](#). We then characterize equilibrium asset prices. The following lemma facilitates the second step by establishing the equilibrium conditions for asset prices given the wealth-weighted average belief. The proof follows from [equations \(31\)–\(33\)](#) and is relegated to [Online Appendix B.1.2](#).

LEMMA 3 (RISK BALANCE CONDITION). The equilibrium price of the market portfolio satisfies

$$(34) \quad \sigma_{t,s}^m = \frac{1}{\sigma_{t,s}^m} \left(r_{t,s}^m - r_{t,s}^f + \bar{\lambda}_{t,s} \left(1 - \frac{Q_{t,s}}{Q_{t,s'}} \right) \right),$$

where $r_{t,s}^m = \rho + g + \mu_{t,s}^Q + \sigma_s \sigma_{t,s}^Q$ and $\sigma_{t,s}^m = \sigma_s + \sigma_{t,s}^Q$

[see Lemma 1].

The equilibrium price of the contingent security satisfies

$$(35) \quad p_{t,s}^{s'} = \bar{\lambda}_{t,s} \frac{Q_{t,s}}{Q_{t,s'}}.$$

[Equations \(34\)](#) and [\(35\)](#) are identical to their counterparts in an alternative economy in which a representative investor has the wealth-weighted average belief $\bar{\lambda}_{t,s}$.

Lemma 3 shows that asset prices are determined as if there is a representative investor that has the wealth-weighted average belief. [Equation \(34\)](#) is the risk balance condition: the dynamic counterpart to [equation \(7\)](#) in the two-period model. In each state, the total risk in the economy (the left side) is equal to the Sharpe ratio according to the wealth-weighted average belief (the right side). Note that the Sharpe ratio accounts for the fact that the aggregate wealth and (aggregate) marginal utility will change if there is a state transition.⁹ Likewise, [equation \(35\)](#) shows that the equilibrium price of the contingent security reflects the wealth-weighted average belief and the change in (aggregate) marginal utility after transition.

1. *First-Best Equilibrium.* For future reference, we derive the first-best equilibrium without interest rate rigidities. In this case, there is no lower bound constraint on the interest rate, so the price per unit of capital is at its efficient level at all times and states, $Q_{t,s} = Q^*$. Combining this with [equation \(34\)](#), we solve for

9. To see this, observe that the term $\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s'}}$ is actually equal to $\frac{Q_{t,s}}{Q_{t,s'}} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}$. Here, $\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}$ denotes the capital gains and $\frac{Q_{t,s}}{Q_{t,s'}}$ denotes the aggregate marginal utility adjustment.

“rstar” as

$$(36) \quad r_s^{f*} = \rho + g - \sigma_s^2 \quad \text{for each } s \in \{1, 2\}.$$

Hence, in the first-best equilibrium the risk premium shocks are fully absorbed by the interest rate. Note also that by Lemma 2, investors’ wealth shares do fluctuate when there are belief disagreements. In the first-best equilibrium, these wealth-share fluctuations affect the equilibrium price of the contingent security (see [equation \(35\)](#)) but not the equilibrium price of capital, aggregate demand, or the interest rate. Next, we characterize the equilibrium with interest rate rigidities.

IV. COMMON BELIEF BENCHMARK AND AMPLIFICATION

In this section, we analyze the equilibrium in a benchmark case with a single investor type with belief denoted by $\lambda_s \equiv \lambda_s^i$. We use this benchmark to illustrate how the spirals between asset prices and output exacerbate the recession and how pessimism amplifies these spirals.

Because the model is linear, we focus on equilibria in which the price per unit of capital and the interest rate remain constant within states, $Q_{t,s} = Q_s$ and $r_{t,s}^f = r_s^f$. In particular, there is no price drift or volatility within a state, $\mu_{t,s}^Q = \sigma_{t,s}^Q = 0$. Combining this with [equation \(34\)](#), we obtain the risk balance conditions

$$(37) \quad \sigma_s = \frac{\rho + g + \lambda_s \left(1 - \frac{Q_s}{Q_{s'}}\right) - r_s^f}{\sigma_s} \quad \text{for each } s \in \{1, 2\}.$$

The equilibrium is then characterized by finding four unknowns, (Q_1, r_1^f, Q_2, r_2^f) , that solve the two conditions in [equation \(37\)](#) together with the two goods market equilibrium conditions in [equation \(25\)](#). We solve these equations under the following parametric restriction.

ASSUMPTION 1. $\sigma_2^2 > \rho + g > \sigma_1^2$.

In view of this restriction, the low risk premium state 1 features positive interest rates, efficient asset prices, and full factor utilization, $r_1^f > 0$, $Q_1 = Q^*$, and $\eta_1 = 1$, whereas the high risk premium state 2 features zero interest rates, lower asset prices, and imperfect factor utilization, $r_2^f = 0$, $Q_2 < Q^*$, and $\eta_2 < 1$. In particular, the analysis with common beliefs reduces to finding

two unknowns, (Q_2, r_1^f) , that solve the two risk balance conditions in [equation \(37\)](#) (after substituting $Q_1 = Q^*$ and $r_2^f = 0$).

IV.A. Equilibrium in the High Risk Premium State

After substituting $r_2^f = 0$, the risk balance [equation \(37\)](#) for the high risk premium state $s = 2$ can be written as

$$(38) \quad \sigma_2 = \frac{\rho + g + \lambda_2 \left(1 - \frac{Q_2}{Q}\right)}{\sigma_2}.$$

If the price were at its efficient level, $Q_2 = Q^*$, the risk (the left side) would exceed the Sharpe ratio (the right side). As in the two-period model, the economy generates more risk than investors are willing to absorb at the constrained interest rate. As before, the price per unit of capital, Q_2 , must fall to equilibrate the risk markets. Rearranging the expression, we obtain a closed-form solution,

$$(39) \quad Q_2 = Q^* \left(1 - \frac{\sigma_2^2 - (\rho + g)}{\lambda_2}\right).$$

As this expression illustrates, we require a minimum degree of optimism to ensure an equilibrium with positive price and output.

ASSUMPTION 2. $\lambda_2 > \sigma_2^2 - (\rho + g)$.

This requirement is a manifestation of an amplification mechanism that we describe next.

IV.B. Amplification from Endogenous Output and Earnings

In the two-period model of [Section II](#), the future payoff from the market portfolio is exogenous (z_1). Therefore, a decline in the price of capital (Q) increases the dividend yield and the market return, $r^m(z_1) = \frac{z_1}{Q}$ (see [equation \(2\)](#)). In contrast, in the current model the instantaneous payoff from the market portfolio is endogenous and given by $y_{t,2} = \rho Q_2 k_{t,2}$. Therefore, a decline in the price of the market portfolio does not affect the dividend yield ($\frac{y_{t,2}}{Q_2 k_{t,2}} = \rho$) and leaves the market return absent transitions unchanged, $r_{t,2}^m = \rho + g$ (see [equation \(22\)](#)). Unlike in the two-period model, a decline in asset prices does not increase the market return (aside from state transitions). The intuition is that a lower

price reduces output and economic activity, which reduces firms' earnings and leaves the dividend yield constant. Thus, asset price declines no longer play a stabilizing role through the dividend yield, leaving the economy susceptible to a spiraling decline.

In view of this amplification mechanism, one might wonder how the risk market ever reaches equilibrium once the price, Q_2 , falls below its efficient level, Q^* . The stabilizing force is captured by the last term in [equation \(38\)](#), $\lambda_2(1 - \frac{Q_2}{Q^*})$. A decline in the price increases the expected capital gain from transition into the recovery state $s = 1$, which increases the expected return to capital and the Sharpe ratio. The stabilizing force is stronger when investors are more optimistic and perceive a higher transition probability into the recovery state, λ_2 . Assumption 2 ensures that the stabilizing force is sufficiently strong to counter the impact of the risk premium shock. If this assumption were violated, a risk premium shock would trigger a downward price spiral that would lead to an equilibrium with zero asset prices and zero output.

Finally, consider the comparative statics of the equilibrium price with respect to the exogenous shifter of the risk premium, σ_2^2 (see [equation \(36\)](#)). Using [equation \(39\)](#), we obtain $\frac{d(\frac{Q_2}{Q^*})}{d\sigma_2^2} = -\frac{1}{\lambda_2}$. Hence, risk premium shocks reduce asset prices and output by a greater magnitude when investors are more pessimistic about recovery (lower λ_2). These observations illustrate that average beliefs matter in this environment not only because they have a direct effect on asset prices but also because they determine the strength of the amplification mechanism.

IV.C. Equilibrium in the Low Risk Premium State

Following similar steps for the low risk premium state $s = 1$, we also obtain a closed-form solution for the interest rate in this state,

$$(40) \quad r_1^f = \rho + g - \sigma_1^2 - \lambda_1 \left(\frac{Q^*}{Q_2} - 1 \right).$$

Intuitively, given the expected return on capital, the interest rate adjusts to ensure that the risk balance condition is satisfied at the efficient price level, $Q_1 = Q^*$. For our conjectured equilibrium, we also assume an upper bound on λ_1 , which ensures that the implied interest rate is positive.

ASSUMPTION 3. $\lambda_1 < \frac{\rho+g-\sigma_1^2}{\frac{Q^*}{Q_2}-1}$, where $\frac{Q^*}{Q_2}$ is given by [equation \(39\)](#).

Note that [equation \(40\)](#) implies r_1^f is decreasing in the transition probability, λ_1 , as well as in the asset price drop conditional on transition, $\frac{Q^*}{Q_2}$. Interest rates are kept relatively low because investors fear a recession triggered by an increase in the risk premium when the interest rate is constrained. The following result summarizes the equilibrium characterization in this section.

PROPOSITION 1. Consider the model with a single belief type and Assumptions 1–3. There is an equilibrium in which the price per unit of capital and the interest rate are constant within each state, $Q_{t,s} = Q_s$ and $r_{t,s}^f = r_s^f$. The low risk premium state 1 features a positive interest rate, efficient asset prices, and full factor utilization, $r_1^f > 0$, $Q_1 = Q^*$, and $\eta_1 = 1$. The high risk premium state 2 features a zero interest rate, lower asset prices, and a demand-driven recession, $r_2^f = 0$, $Q_2 < Q^*$, and $\eta_2 < 1$, as well as lower consumption and output, $\frac{c_{t,2}}{k_{t,2}} = \frac{y_{t,2}}{k_{t,2}} = \rho Q_2$. The price in state 2 and the interest rate in state 1 are given by [equations \(39\)](#) and [\(40\)](#).

IV.D. Equilibrium with Investment and Endogenous Growth

In [Online Appendix D](#), we extend the baseline environment to incorporate investment. This leads to two main changes. First, the growth rate in [equation \(12\)](#) becomes endogenous, $g_{t,s} = \varphi(\iota_{t,s}) - \delta$, where $\iota_{t,s} = \frac{i_{t,s}}{k_{t,s}}$ denotes the investment rate per unit of capital, $\varphi(\cdot)$ denotes a neoclassical production technology for capital, and δ denotes the depreciation rate. Second, under the simplifying assumption that output accrues to agents as returns to capital (i.e., no monopoly profits), optimal investment is an increasing function of the price per unit of capital, $Q_{t,s}$.¹⁰ Moreover, using a convenient functional form for $\varphi(\cdot)$, we obtain a linear relation between the investment rate and the price, $\iota(Q_{t,s}) = \psi(Q_{t,s} - 1)$ for some $\psi > 0$.

10. Without this assumption, investment would be a function of $\tilde{Q}_{t,s} \leq Q_{t,s}$, which represents a claim on the rental rate of capital in future periods (excluding monopoly profits). The difference, $Q_{t,s} - \tilde{Q}_{t,s}$, captures the price of a claim on monopoly profits. Hence, allowing for profits would have a quantitative impact on investment, though we believe it would leave our qualitative results unchanged. We leave an investigation of this issue for future research.

In this setting, aggregate demand is the sum of consumption and investment. Using the expression for optimal investment, we generalize the output–asset price relation (21) to

$$(41) \quad A\eta_{t,s} = \rho Q_{t,s} + \psi(Q_{t,s} - 1).$$

Hence, output is increasing in asset prices because asset prices generate a wealth effect on consumption and increase investment through a marginal-Q channel. Substituting optimal investment into the endogenous growth expression, we further obtain

$$(42) \quad g_{t,s} = \psi q_{t,s} - \delta, \quad \text{where} \quad q_{t,s} = \log Q_{t,s}.$$

Hence, this setting also features a growth-asset price relation: lower asset prices reduce investment, which translates into lower endogenous growth and lower potential output in future periods. The rest of the model is unchanged (see [Online Appendix D](#) for details).

In the [Online Appendix](#), we characterize the equilibrium in this extended environment and generalize [Proposition 1](#). We find that risk premium shocks—captured by a transition to state 2—generate a decline in investment and endogenous growth, as well as in consumption and output as in the baseline model. The decline in investment and endogenous growth generates a second amplification mechanism that reinforces the mechanism we described earlier. Specifically, the recession lowers asset prices further, not only by reducing output and earnings but also by reducing growth in potential output and earnings. [Figure I](#) illustrates the two amplification mechanisms. Henceforth, we return to the baseline model without investment.

REMARK 6 (MORE GENERAL RRA). In [Online Appendix E](#), we extend the baseline dynamic model to cases where the RRA is different from one while keeping the EIS equal to one (see [Remark 2](#) for the role of the EIS). This introduces dynamic hedging motives: investors' marginal utility for future states reflects not only their wealth but also the attractiveness of investment opportunities in the corresponding state. These motives affect our analysis in two ways. First, investors' wealth-share dynamics are influenced by their relative dynamic hedging motives in addition to belief disagreements. These effects complicate the analysis with belief disagreements (in particular, the separability property from [Section III.B](#)

no longer applies), but they are largely orthogonal to the effects of speculation that we discuss subsequently. Second, with common beliefs (in which case the analysis is tractable), dynamic hedging motives further amplify the effect of risk premium shocks on asset prices and output in the empirically relevant case of RRA greater than one (see [Di Tella 2017](#)). Specifically, an increase in the (relative) risk premium in the high risk premium state makes the investment opportunities in this state (relatively) less attractive, which increases investors' willingness to hedge the high risk premium state. In equilibrium, this translates into a lower price and output in the high risk premium state (and a lower interest rate in the low risk premium state).

V. BELIEF DISAGREEMENTS AND SPECULATION

We now turn to the main case with belief disagreements. We show that a greater wealth share for optimists raises asset valuations and mitigates the recession. We also establish that speculation induced by belief disagreements exacerbates asset price fluctuations and worsens the recession.

We restrict attention to two investor types, optimists and pessimists, with beliefs denoted by $\{(\lambda_1^i, \lambda_2^i)\}_{i \in \{o, p\}}$. Beliefs satisfy the following assumption.

ASSUMPTION 4. $\lambda_2^o > \lambda_2^p$ and $\lambda_1^o \leq \lambda_1^p$.

When the economy is in the high risk premium state, optimists find the transition into the low risk premium state relatively likely ($\lambda_2^o > \lambda_2^p$); when the economy is in the low risk premium state, optimists find the transition into the high risk premium state relatively unlikely ($\lambda_1^o \leq \lambda_1^p$). Hence, optimism and pessimism are relative: an optimist is someone who is optimistic relative to a pessimist. In fact, we do not need to specify the “objective distribution” for our theoretical results (including the welfare results). We do, however, need relative optimism and pessimism to be persistent across the two risk premium states (see [Remark 7](#) at the end of this section).

As our analysis in [Section III.B](#) suggests, the equilibrium with belief disagreements depends on investors' wealth shares. In the two-type context, we simplify the notation and denote optimists'

wealth share without a superscript (see [equation \(26\)](#)):

$$\alpha_{t,s} \equiv \frac{a_{t,s}^o}{k_{t,s} Q_{t,s}}.$$

Pessimists' wealth share is the residual, $1 - \alpha_{t,s} = \frac{a_{t,s}^p}{k_{t,s} Q_{t,s}}$. Optimists' wealth share, $\alpha_{t,s}$, is the relevant state variable for this economy. Specifically, we establish an equilibrium in which all variables can be written as a function of $\alpha_{t,s}$. To this end, we also write the wealth-weighted average belief as a function of optimists' wealth share (see [equation \(27\)](#)):

$$(43) \quad \bar{\lambda}_{t,s} = \bar{\lambda}_s(\alpha_{t,s}) \equiv \alpha_{t,s} \lambda_s^o + (1 - \alpha_{t,s}) \lambda_s^p.$$

We present our main result in this section, which characterizes the equilibrium with belief disagreements. The result requires Assumptions 1–3 to hold for both beliefs. These assumptions ensure that there exists an equilibrium in which the low risk premium state 1 always features a positive interest rate, efficient price level, and full factor utilization, $r_{t,1}^f > 0$, $Q_{t,1} = Q^*$, and $\eta_{t,1} = 1$, whereas the high risk premium state 2 always features a zero interest rate, a lower price level, and insufficient factor utilization, $r_{t,2}^f = 0$, $Q_{t,2} < Q^*$, and $\eta_{t,2} < 1$.

PROPOSITION 2. Consider the model with two belief types. Suppose Assumptions 1–3 hold for each belief, and that beliefs are ranked according to Assumption 4. Then, there is an equilibrium in which the log price and interest rate can be written as functions of optimists' wealth share, $q_{t,s} = q_s(\alpha_{t,s})$, $r_{t,s}^f = r_s^f(\alpha_{t,s})$, where optimists' wealth share evolves according to

$$(44) \quad \begin{aligned} \frac{\dot{\alpha}_{t,s}}{\alpha_{t,s}} &= -\omega_{t,s}^{s',o} = (\lambda_s^p - \lambda_s^o)(1 - \alpha_{t,s}) && \text{if there is no state change,} \\ \frac{\alpha_{t,s'}}{\alpha_{t,s}} &= \frac{\lambda_s^o}{\bar{\lambda}_s(\alpha_{t,s})} && \text{if there is a change to } s'. \end{aligned}$$

In the high risk premium state 2, the interest rate is 0, $r_2^f(\alpha) = 0$, and the log price per unit of capital is below the efficient level, $q_2(\alpha) < q^*$. The price function is the solution to the differential equation

$$(45) \quad q_2'(\alpha)(\lambda_2^o - \lambda_2^p)\alpha(1 - \alpha) = \rho + g + \bar{\lambda}_2(\alpha)\left(1 - \frac{\exp(q_2(\alpha))}{Q^*}\right) - \sigma_2^2,$$

with boundary conditions $q_2(0) = q_2^p$ and $q_2(1) = q_2^o$. The solution, $q_2(\alpha)$, is strictly increasing in α . In particular, greater optimists' wealth share in the high risk premium state, $\alpha_{t,2}$, increases the price per unit of capital, $Q_{t,2}$, as well as consumption and output, $\frac{c_{t,2}}{k_{t,2}} = \frac{y_{t,2}}{k_{t,2}} = \rho Q_{t,2}$.

In the low risk premium state 1, the log price is at its efficient level, $q_1(\alpha) = q^*$, and the interest rate is strictly positive, $r_1^f(\alpha) > 0$. The interest rate function is given by

$$(46) \quad r_1^f(\alpha) = \rho + g - \bar{\lambda}_1(\alpha) \left(\frac{Q^*}{\exp(q_2(\alpha'))} - 1 \right) - \sigma_1^2 \quad \text{where } \alpha' = \alpha \frac{\lambda_1^o}{\bar{\lambda}_1(\alpha)}.$$

The function $r_1^f(\alpha)$ is strictly increasing in α . In particular, a greater optimists' wealth share in the low risk premium state, $\alpha_{t,1}$, increases the interest rate, $r_{t,1}^f$.

The characterization of optimists' wealth dynamics follows from Lemma 3. When the economy is in the recession state, optimists purchase the upside contingent security from pessimists (because $\lambda_2^o > \lambda_2^p$). As long as the economy remains in the recession state, optimists' wealth share drifts downward. However, if there is a transition to the boom state, then optimists' wealth share makes an upward jump. Conversely, when the economy is in the boom state, optimists sell the downside contingent security to pessimists (since $\lambda_1^o \leq \lambda_1^p$). This ensures that optimists' wealth share drifts upward when the economy remains in the boom state, but it makes a downward jump if there is a transition to the recession state. Figure II illustrates these dynamics for a particular parameterization (described subsequently).

The rest of the proposition characterizes the equilibrium price in the high-risk state and the interest rate in the low-risk state as functions of optimists' wealth share. Equations (45) and (46) are similar to their counterparts with common beliefs (see equations (38) and (40)). The main difference is that the asset price and the interest rate depend on the wealth-weighted average belief in the corresponding state, $\bar{\lambda}_2(\alpha)$ and $\bar{\lambda}_1(\alpha)$. Because increasing optimists' wealth share, α , makes the wealth-weighted average belief more optimistic (in either state), these equations suggest that greater α should increase the asset price and the interest rate. The proposition verifies this intuition. Figure III illustrates the equilibrium asset price and interest rate functions for a particular parameterization.

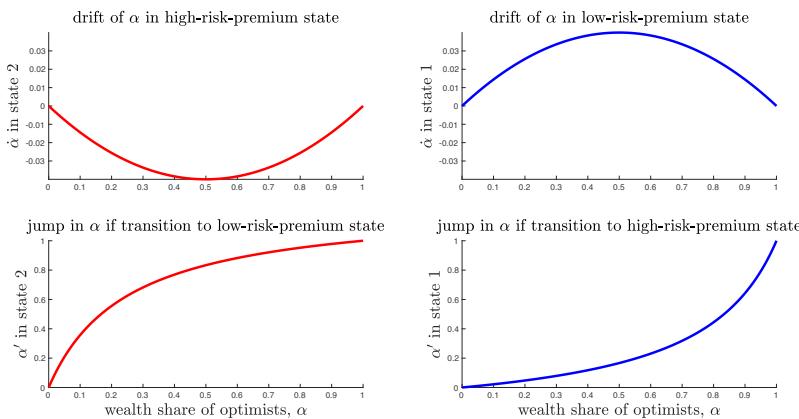


FIGURE II

Equilibrium Dynamics of Investors' Wealth Shares with Heterogeneous Beliefs

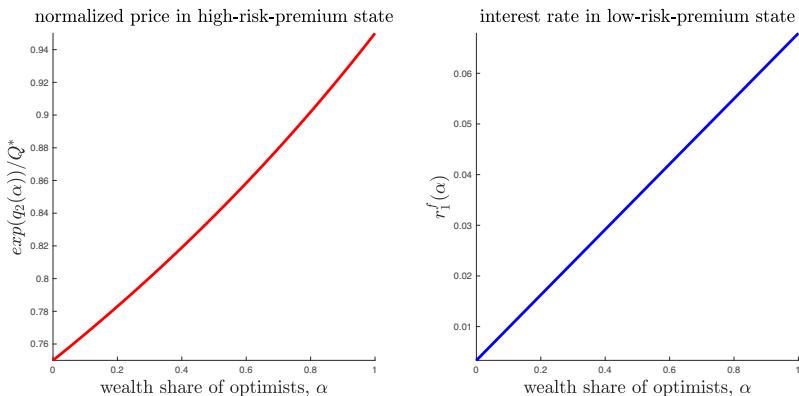


FIGURE III

Equilibrium Price and Interest Rate Functions with Heterogeneous Beliefs

Proposition 2 has two important implications. First, in the recession state, a greater wealth share for optimists increases not only asset prices but also consumption and output (in view of the output–asset price relation). This observation motivates policies that redistribute wealth to optimists in the recession state—including macroprudential policy, which we analyze in the next section. The result is reminiscent of the recent macroeconomic literature that emphasizes the importance of wealth distribution for aggregate spending in environments with heterogeneous

MPCs. However, the mechanism here is different and works through general equilibrium effects driven by heterogeneous asset valuations. To see this, consider the effect of a wealth transfer from pessimists to optimists in partial equilibrium—keeping asset prices unchanged, and in general equilibrium—allowing asset prices to adjust. In partial equilibrium, this transfer would not stimulate aggregate spending because pessimists and optimists have the same MPC (equal to ρ). As optimists increase their spending, pessimists reduce their spending by the same amount. In general equilibrium, the transfer increases asset prices and aggregate spending. In fact, relative to the partial equilibrium benchmark, optimists and pessimists both increase their spending.

Second, the proposition implies that financial markets are effectively extrapolative even though investors' beliefs are fixed. Good realizations increase optimists' wealth share, which raises effective optimism and increases the asset price (in the high risk premium state) or the interest rate (in the low risk premium state). Conversely, bad realizations reduce optimists' wealth share, which reduces effective optimism and decreases the asset price or the interest rate. These results also imply that speculation exacerbates asset price boom-bust cycles and leads to more severe recessions (see [Remark 8](#) at the end of this section).

Next we provide a sketch proof for [Proposition 2](#), which provides further intuition. We then present a simulation that illustrates how speculation exacerbates boom-bust cycles and demand recessions.

V.A. Sketch Proof of [Proposition 2](#)

The wealth dynamics in [equation \(44\)](#) follow from [Lemma 2](#). Because we search for an equilibrium that satisfies $q_{t,s} = q_s(\alpha_{t,s})$, and $\alpha_{t,s}$ follows a deterministic path absent transition, $q_{t,s}$ also follows a deterministic path absent transition. Therefore, $\sigma_{t,s}^Q = 0$ (see [Remark 4](#)).

To characterize the rest of the equilibrium, consider [Lemma 3](#), which describes the risk balance condition. Applying [equation \(34\)](#) for the high risk premium state $s = 2$ and substituting $r_{t,2}^f = 0$ and $\sigma_{t,2}^Q = 0$, $\mu_{t,2}^Q = \frac{dQ_{t,2}}{dt} \frac{1}{Q_{t,2}} = \dot{q}_{t,2}$, we obtain

$$(47) \quad \sigma_2 = \frac{1}{\sigma_2} \left(\rho + g + \dot{q}_{t,2} + \bar{\lambda}_{t,2} \left(1 - \frac{Q_2}{Q^*} \right) \right).$$

This condition is similar to its counterpart with common beliefs, with two differences (see [equation \(38\)](#)). First, the transition probability effectively depends on the wealth-weighted average belief, $\bar{\lambda}_{t,2}$. Second, the expected return to the market portfolio features the price drift term $\dot{q}_{t,2}$ (see [equation \(22\)](#)), which is not necessarily 0 because optimists' wealth share changes over time.

Combining [equations \(44\)](#) and [\(47\)](#), we obtain a differential equation system that describes the joint dynamics of the log price and optimists' wealth share, $(q_{t,2}, \alpha_{t,2})$, conditional on no transition. In [Online Appendix B.3](#), we show that this system is saddle path stable: for any initial wealth share, $\alpha_{t,2} \in (0, 1)$, there exists a unique equilibrium price level, $q_{t,2} \in [q_2^p, q_2^o]$, such that the solution satisfies $\lim_{t \rightarrow \infty} \alpha_{t,2} = 0$ and $\lim_{t \rightarrow \infty} q_{t,2} = q_2^p$. We further show that the saddle path is strictly increasing in $\alpha_{t,2}$. These observations imply that the equilibrium price is an increasing function of optimists' wealth share, $q_2(\alpha)$ (which corresponds to the saddle path). Substituting this function into [equation \(47\)](#) and using [equation \(44\)](#), we also obtain the differential [equation \(45\)](#) that characterizes this function in the α -domain.

Finally, we obtain [equation \(46\)](#) by applying the risk balance condition [\(34\)](#) for the low risk premium state $s = 1$. The term α' denotes optimists' wealth share after an immediate transition into the high risk premium state (see [equation \(44\)](#)). The interest rate is increasing in α both because the wealth-weighted transition probability to the high risk premium state, $\bar{\lambda}_1(\alpha)$, is decreasing in α ; and because the price that would obtain after transition, $q_2(\alpha')$, is increasing in α .

V.B. Numerical Illustration

We illustrate the equilibrium characterized in [Proposition 2](#) using a simple parameterization (see [Online Appendix B.4](#) for details). For the baseline parameters, we set $g = 5\%$, $\rho = 4\%$, $\sigma_1^2 = 2\%$, $\sigma_2^2 = 10\%$. For investors' beliefs about transition probabilities, we set $\lambda_1^o = \frac{1}{25}$, $\lambda_1^p = \frac{1}{5}$ for the boom state, and $\lambda_2^o = \frac{1}{5}$, $\lambda_2^p = \frac{1}{25}$ for the recession state.

[Figure II](#) illustrates the corresponding dynamics for optimists' wealth share. [Figure III](#) illustrates the corresponding equilibrium prices. The left panel of [Figure III](#) illustrates the price of capital in the recession (normalized by the efficient price level) as a function of optimists' wealth share. When pessimists dominate the economy, the price of capital and output decline by 25%. In

contrast, when optimists dominate, they decline by only 5%. The right panel of [Figure III](#) illustrates the interest rate in the boom as a function of optimists' wealth share. The risk-free rate during the boom is close to 7% when optimists dominate the economy but it is close to 0% when pessimists dominate.

V.C. Amplification from Speculation

We next use our numerical example to illustrate how speculation amplifies the business cycle driven by risk premium shocks. We fix investors' beliefs and simulate the equilibrium for a particular realization of uncertainty over a 40-year horizon. We choose the objective simulation belief to be in the "middle" of optimists' and pessimists' beliefs in terms of the relative entropy distance.¹¹ We also focus on the "average path"—the path in which the length of each boom or bust is exactly equal to its mean value implied by the simulation belief. [Figure IV](#) illustrates the dynamics of equilibrium variables along this path. For comparison, the dashed red line plots the equilibrium that would obtain in the common-beliefs benchmark if all investors shared the "middle" simulation belief, and the dot-dashed blue line plots the first-best equilibrium that would obtain without interest rate rigidities (color versions online).

The figure illustrates two points. First, as we establish in [Section IV](#), the price per unit of capital is more volatile and the interest rate is more compressed in the common-belief benchmark than in the first-best equilibrium. In the high risk premium state, the interest rate cannot decline enough to equilibrate the risk balance condition, which leads to a drop in asset prices and a demand recession. Moreover, asset prices and output decline substantially even though the interest rate is above the first-best level by only 1 percentage point—illustrating the amplification mechanism. In

11. Formally, given two probability distributions $(p(\tilde{s}))_{\tilde{s} \in S}$ and $(q(\tilde{s}))_{\tilde{s} \in S}$, the relative entropy of p with respect to q is defined as $\sum_{\tilde{s}} p(\tilde{s}) \log \left(\frac{p(\tilde{s})}{q(\tilde{s})} \right)$. In a setting similar to ours, [Blume and Easley \(2006\)](#) show that investors whose belief is closer to the objective distribution in terms of the relative entropy distance dominate the economy in the long run. We choose the simulation belief (in both the boom and the recession state) to be in the "middle" of optimists' and pessimists' beliefs to prevent this outcome and ensure that there is a nondegenerate long-run wealth distribution. This helps illustrate the destabilizing effects of speculation without taking a stand on whether optimists or pessimists are "correct." Our welfare results in the next section do not require this assumption because we evaluate investors' expected utilities according to their own beliefs.

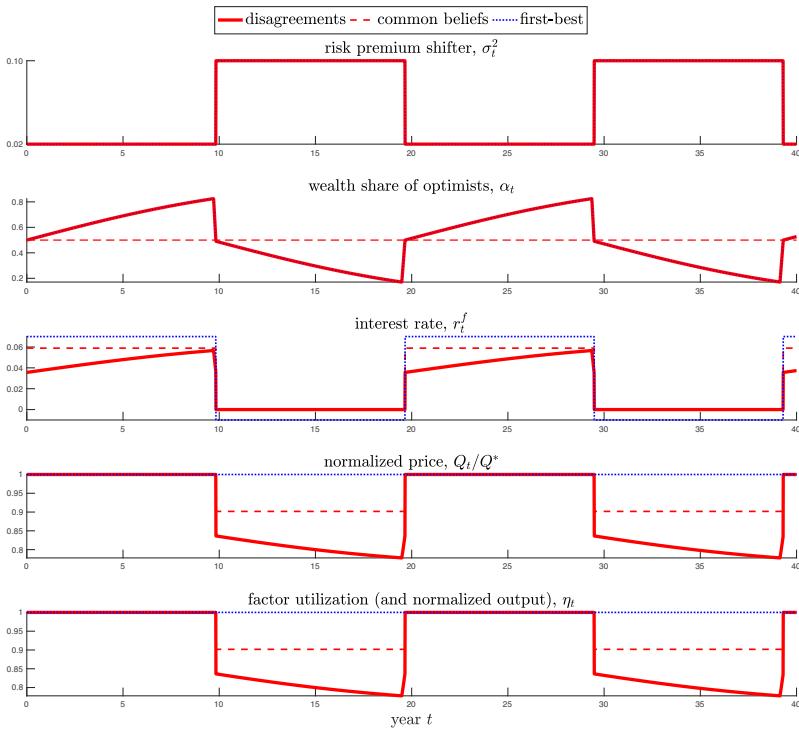


FIGURE IV
A Simulation of the Equilibrium Variables over Time

The solid red lines illustrate the equilibrium with belief disagreements. The dashed red lines (resp. the dotted blue lines) illustrate the common belief benchmark (resp. the first-best benchmark).

the low risk premium state, the fear of transition into the recessionary high risk premium state keeps the interest rate lower than in the first-best benchmark.

Second, risk-centric recessions are more severe when investors have belief disagreements (and this also compresses interest rates). The intuition follows from Proposition 2. Speculation in the low risk premium state decreases optimists' wealth share once the economy transitions into the high risk premium state, as illustrated by the second-to-top panel of Figure IV. This translates into lower asset prices and a more severe demand recession, as illustrated by the bottom panels of Figure IV. Speculation also increases optimists' wealth share if the boom continues, but this

effect does not translate into higher asset prices or output because it is (optimally) neutralized by the interest rate response. The adverse effects of speculation on demand recessions motivate the analysis of macroprudential policy, which we turn to in the next section.

REMARK 7 (INTERPRETATION OF BELIEF DISAGREEMENTS). As this discussion suggests, what matters for our results in this section is persistent heterogeneous valuations for risky assets that ensure: (i) during the recession, a greater wealth share for high-valuation investors increases the (relative) price of risky assets, and (ii) during the boom, high-valuation investors absorb relatively more of the recession risks. Belief disagreements generate these features naturally, under the mild assumption that optimists and pessimists do not flip roles between booms and recessions, but other sources of heterogeneous valuations would lead to similar results.¹² For example, with heterogeneous risk aversion, more risk-tolerant agents take on more aggregate risk (i.e., they insure less risk-tolerant agents), which reduces their wealth share and the (relative) price of risky assets following negative shocks to fundamentals (see, for instance, [Garleanu and Pedersen 2011](#); [Longstaff and Wang 2012](#)). From this perspective, belief disagreements can also capture institutional reasons for heterogeneous valuations such as capacity or mandates for handling risk. Investment banks, for example, have far more capacity to handle and lever risky positions than do pensioners and money market funds.

REMARK 8 (OTHER SOURCES OF EXTRAPOLATIVE DYNAMICS). In our model, asset prices are determined as if there is a representative investor with the wealth-weighted average belief, $\bar{\lambda}_{t,s}$ (see Lemma 3). Our analysis in this section shows that speculation generates extrapolative dynamics for the wealth-weighted average belief. Therefore, an alternative model in which agents are homogeneous but have individually extrapolative beliefs would generate identical price dynamics

12. The no-flipping assumption is supported by recent survey evidence that shows belief heterogeneity is largely explained by persistent individual heterogeneity ([Giglio et al. 2019](#)). The assumption is also consistent with an extensive psychology literature that documents the prevalence of optimism, as well as its heterogeneity and persistence, since it is largely a personal trait (see [Carver, Scheier, and Segerstrom 2010](#) for a review).

(as long as the extrapolation mechanism generates identical paths for $\bar{\lambda}_{t,s}$). Unlike our model, such an alternative model would feature homogeneous asset positions. Thus, it would not generate trading volume, which is a distinctive feature of speculative episodes in practice (see, e.g., [Hong and Stein 2007](#); [DeFusco, Nathanson, and Zwick 2017](#)). More broadly, extrapolation and speculation are both likely to be at play and reinforce each other during speculative episodes.

VI. WELFARE ANALYSIS AND MACROPRUDENTIAL POLICY

Because our model features constrained monetary policy, most of the aggregate demand boosting policies that have been discussed in the New Keynesian literature are effective in our environment. We skip a discussion of these policies (our results still apply as long as these policies are imperfect). Instead, we focus on macroprudential policy interventions that impose restrictions on risk market participants with the objective of obtaining macroeconomic benefits. In practice, most macroprudential policies restrict risk-taking by banks—especially large ones. Interpreting banks as high-valuation investors (see [Remark 7](#)) or as lenders to such investors (see [Section VII](#)), we capture these policies in reduced form by imposing portfolio risk limits on optimists.

Our model features heterogeneous beliefs, which makes the welfare analysis challenging. We mainly focus on the standard Pareto criterion in which the planner evaluates investors' expected utility according to the investors' own beliefs. This criterion ensures that our results are not driven by paternalistic concerns. Instead, the planner improves welfare by internalizing aggregate demand externalities. The standard criterion is also appropriate if we interpret belief disagreements as a modeling device that captures heterogeneous valuations due to other factors (see [Remark 7](#)). However, if we interpret belief disagreements literally, then a paternalistic criterion such as the belief-neutral welfare criterion developed by [Brunnermeier, Simsek, and Xiong \(2014\)](#) could be more appropriate. In [Section VI.C](#), we illustrate how to use a belief-neutral criterion in our context.

We provide a decomposition of investors' value functions that simplifies the theoretical analysis. Specifically, because our model features complete markets and no frictions other than interest rate rigidities, aggregate demand externalities are the only source of inefficiency. Therefore, the first-best benchmark that corrects

for these inefficiencies is Pareto efficient. Motivated by this observation, we decompose investors' value functions into two components: a first-best value function that would obtain if there were no demand recessions and a gap value function that captures the loss of value due to demand recessions. Introducing the gap value function simplifies the analysis considerably because, up to a first order, macroprudential policies affect social welfare only through their effect on investors' gap values.

Using the model with two belief types from the previous section, we first characterize investors' value functions in equilibrium according to their own beliefs. We also define the gap value functions and illustrate the aggregate demand externalities. We consider macroprudential policy that induces optimists to act more pessimistically (by imposing appropriate portfolio risk limits). We show that this policy can lead to a Pareto improvement of social welfare. We focus on macroprudential policy in the boom (low risk premium) state and briefly discuss macroprudential policy in the recession (high risk premium) state.

VI.A. Equilibrium Value Functions

Because the model is linear, a type i investor's expected utility can be written as (see [Online Appendix B.1.1](#))

$$(48) \quad V_{t,s}^i(a_{t,s}^i) = \frac{1}{\rho} \log \left(\frac{a_{t,s}^i}{Q_{t,s}} \right) + v_{t,s}^i.$$

Here, $v_{t,s}^i$ denotes the normalized value function per unit of capital stock. Consider the equilibrium characterized in [Proposition 2](#). In [Online Appendix C.1](#), we characterize the normalized value corresponding to this equilibrium as the solution to the differential equation system,

$$(49) \quad \rho v_{t,s}^i - \frac{\partial v_{t,s}^i}{\partial t} = \log \rho + q_{t,s} + \frac{1}{\rho} \left(-(\lambda_s^i - \bar{\lambda}_{t,s}) + \lambda_s^i \log \left(\frac{\lambda_s^i}{\lambda_{t,s}} \right) \right) + \lambda_s^i (v_{t,s'}^i - v_{t,s}^i).$$

The equilibrium price, $q_{t,s}$, affects the investor's welfare because it determines output and consumption (see [equations \(24\)](#) and [\(21\)](#)). Consumption growth, g , and volatility, σ_s^2 , also affect welfare. Finally, speculation affects the investor's (perceived) welfare. This

is captured by the term $-(\lambda_s^i - \bar{\lambda}_{t,s}) + \lambda_s^i \log(\frac{\lambda_s^i}{\bar{\lambda}_{t,s}})$, which is 0 with common beliefs and strictly positive with disagreements.

1. Gap Value Function. To facilitate the policy analysis, we break down the normalized value function into two components,

$$(50) \quad v_{t,s}^i = v_{t,s}^{i,*} + w_{t,s}^i.$$

Here, $v_{t,s}^{i,*}$ denotes the first-best value function that would obtain if there were no interest rate rigidities. It is characterized by solving [equation \(49\)](#) with the efficient price level, $q_{t,s} = q^*$, for each t, s . The residual, $w_{t,s}^i = v_{t,s}^i - v_{t,s}^{i,*}$, denotes the gap value function, which captures the loss due to interest rate rigidities and demand recessions. The first-order impact of macroprudential policy on social welfare depends only on the gap value function. Using [equation \(49\)](#), we characterize the gap value function as the solution to the following system,

$$(51) \quad \rho w_{t,s}^i = q_{t,s} - q^* + \frac{\partial w_{t,s}^i}{\partial t} + \lambda_s^i (w_{t,s'}^i - w_{t,s}^i).$$

The gap value function corresponds to the investor's present discounted value of utility losses from output gaps relative to the efficient level. In view of the output–asset price relation [\(21\)](#), the function accounts for the output gaps in terms of the asset price gaps. Recall that the equilibrium features $q_{t,1} = q^*$ and $q_{t,2} < q^*$. Thus, the key objective of policy interventions is to increase the asset price in the high risk premium state, in order to mitigate the demand recession.

VI.B. Aggregate Demand Externalities

In [Online Appendix C.1](#) we show that the gap value can be written as a function of optimists' wealth share, $w_s^i(\alpha)$. Combining [equations \(51\)](#) and [\(44\)](#), we characterize this function as the solution to the following system in the α -domain,

$$(52) \quad \begin{aligned} \rho w_s^i(\alpha) &= q_s(\alpha) - q^* - (\lambda_s^o - \lambda_s^p) \alpha (1 - \alpha) \frac{\partial w_s^i(\alpha)}{\partial \alpha} \\ &+ \lambda_s^i (w_{s'}^i(\alpha') - w_s^i(\alpha)), \end{aligned}$$

where $\alpha' = \alpha \frac{\lambda_s^o}{\lambda_s(\alpha)}$. Recall that the price function in the high risk premium state, $q_2(\alpha)$, is increasing in optimists' wealth share (see [Figure III](#)). This leads to the following result.

LEMMA 4. The gap value function satisfies $\frac{dw_s^i(\alpha)}{d\alpha} > 0$ for each s, i , and $\alpha \in (0, 1)$.

Optimists' wealth share is a scarce resource that brings asset prices and output in the recession state closer to their first-best level. Thus, the gap value function in the recession state is increasing in optimists' wealth share. The gap value function in the boom state is also increasing, because the economy can always transition into the recession state, where optimists' wealth is useful (see [Lemma 5](#) for a ranking of the marginal value of optimists' wealth across the two states).

This result also illustrates the aggregate demand externality. Optimists' wealth share is an endogenous variable that fluctuates due to investors' portfolio decisions (see [Figure IV](#)). Individual optimists who take positions in contingent markets—and pessimists who take the other side of these positions—do not take into account the effect of their decisions on asset prices and social welfare. This leads to inefficiencies that can be corrected by macroprudential policy.

VI.C. Macroprudential Policy

To evaluate the direction of the inefficiency, we consider a constrained policy exercise where the planner can induce optimists to choose allocations as if they have less optimistic beliefs.¹³ Specifically, optimists are constrained to choose allocations as if they have the beliefs $\lambda^{o,pl} \equiv (\lambda_1^{o,pl}, \lambda_2^{o,pl})$, which satisfy $\lambda_1^{o,pl} \geq \lambda_1^o$ and $\lambda_2^{o,pl} \leq \lambda_2^o$. Pessimists continue to choose allocations according to their own beliefs. Throughout, we use $\lambda_s^{i,pl}$ to denote investors' as-if beliefs and λ_s^i to denote their actual beliefs. For pessimists, the two beliefs coincide. We also use $\bar{\lambda}_s^{pl}(\alpha) = \alpha \lambda_s^{o,pl} + (1 - \alpha) \lambda_s^p$ to denote the weighted average as-if belief.

In [Online Appendix C.2](#), we show that the planner can implement this policy by imposing inequality restrictions on optimists' portfolio weights, while allowing them to make unconstrained

13. For simplicity, we restrict attention to time-invariant policies. The planner commits to a policy at time 0, $(\lambda_1^{o,pl}, \lambda_2^{o,pl})$, and implements it throughout.

consumption-savings decisions. To understand this implementation, note that optimists' position on the contingent security is given by (see [equation \(44\)](#))

$$(53) \quad \omega_{t,s}^{s',o,pl} = (\lambda_s^{o,pl} - \lambda_s^p) (1 - \alpha_{t,s}).$$

With their own beliefs, optimists sell the contingent security in the boom state, $\omega_{t,1}^{2,o} \leq 0$ (since $\lambda_1^o \leq \lambda_2^p$), and purchase it in the recession state, $\omega_{t,2}^{1,o} > 0$ (since $\lambda_2^o > \lambda_2^p$). Therefore, [equation \(53\)](#) implies that in the boom state, the planner prevents optimists from selling too much of the contingent security, $\omega_{t,1}^{2,o} \geq \underline{\omega}_{t,1}^{2,o}$. In the recession state, the policy prevents optimists from purchasing too much of the contingent security, $\omega_{t,2}^{1,o} \leq \bar{\omega}_{t,2}^{1,o}$. In either state, the policy constrains optimists' weight on the market portfolio not to exceed the market average, $\omega_{t,s}^{m,o} \leq 1$. In the absence of this constraint, optimists would speculate by increasing their exposure to the market portfolio.

The characterization of the equilibrium with macroprudential policy is the same as in [Section V](#). In particular, [equations \(44\)](#) and [\(47\)](#) still hold, and the only difference is that investors' beliefs are replaced by as-if beliefs, $\lambda_s^{i,pl}$. We denote the resulting price functions with $q_s^{pl}(\alpha)$ to emphasize that they are determined by as-if beliefs. The equation system that characterizes the gap value function is given by the following analogue of [equation \(52\)](#),

$$(54) \quad \rho w_s^i(\alpha) = q_s^{pl}(\alpha) - q^* - (\lambda_s^{o,pl} - \lambda_s^p) \alpha (1 - \alpha) \frac{\partial w_s^i(\alpha)}{\partial \alpha} + \lambda_s^i (w_{s'}^i(\alpha')^{pl} - w_s^i(\alpha)),$$

where $\alpha'^{pl} = \alpha \frac{\lambda_s^{o,pl}}{\lambda_s^{i,pl}(\alpha)}$. This system illustrates that macroprudential policy can affect the gap value via two channels. First, it might affect equilibrium asset prices (captured by the term $q_s^{pl}(\alpha)$). Second, the policy affects the dynamics of optimists' wealth share, which influence the gap value. For example, in the boom state $s = 1$, the policy increases $\lambda_1^{o,pl}$, which increases optimists' wealth share after a transition to the recession (captured by the term α'^{pl}) at the expense of reducing optimists' wealth share if there is no transition (captured by the term $-(\lambda_s^{o,pl} - \lambda_s^p)$).

1. Planner's Pareto Problem. To trace the Pareto frontier, we allow the planner to make a one-time wealth transfer among the investors at time 0. In [Online Appendix C.2](#), we show that the

planner's Pareto problem can be reduced to

$$(55) \quad \max_{\lambda^{o,pl}} v_{0,s}^{pl} = \alpha_{0,s} v_{0,s}^o + (1 - \alpha_{0,s}) v_{0,s}^p.$$

Hence, the planner maximizes a wealth-weighted average of investors' normalized values. The wealth shares (chosen by the planner via the one-time transfer) correspond to the planner's Pareto weights on optimists and pessimists. We decompose the planner's value function into first-best and gap value components, $v_{0,s}^{pl} = v_{0,s}^{pl,*} + w_{0,s}^{pl}$.

Because the first-best benchmark does not feature any frictions, it is Pareto efficient (due to the first welfare theorem). This in turn implies that the marginal impact of the policy on the planner's first-best value function is 0, $\frac{\partial v_{0,s}^{pl,*}}{\partial \lambda^{o,pl}}|_{\lambda^{o,pl}=\lambda^o} = 0$. Consequently, the first-order impact of the policy is characterized by its impact on the planner's gap value function,

$$(56) \quad w_{0,s}^{pl} = \alpha_{0,s} w_{0,s}^o + (1 - \alpha_{0,s}) w_{0,s}^p.$$

2. Macroprudential Policy in the Boom State. Now suppose the economy is in the boom state $s = 1$. The planner can use macroprudential policy in the current state, $\lambda_1^{o,pl} \geq \lambda_1^o$ (she can induce optimists to act as if transition into the recession is more likely), but not in the other state $\lambda_2^{o,pl} = \lambda_2^o$ (she cannot influence optimists' actions in the recession state). Effectively, this policy induces optimists to sell less of the contingent security that pays in case there is a transition to the recession, while also preventing optimists from increasing their position in the market portfolio. We present our main result of this section showing that this policy improves welfare. The result requires a technical assumption (no disagreement in state $s = 1$) that we discuss subsequently.

PROPOSITION 3. Consider the equilibrium with two belief types characterized in Proposition 2. Suppose agents' beliefs satisfy $\lambda_1^o = \lambda_1^p$ and consider the macroprudential policy in the boom state, $\lambda_1^{o,pl} \geq \lambda_1^o$ (and suppose $\lambda_2^{o,pl} = \lambda_2^o$). The policy increases the gap value according to each belief,

$$\left. \frac{\partial w_1^i(\alpha)}{\partial \lambda_1^{o,pl}} \right|_{\lambda_1^{o,pl}=\lambda_1^o} > 0 \text{ for each } i \in \{o, p\} \text{ and } \alpha \in (0, 1).$$

The policy also increases the planner's value, $\frac{\partial v_1^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}|_{\lambda_1^o} = \frac{\partial w_1^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}|_{\lambda_1^o} > 0$. In particular, regardless of the planner's Pareto weights, there exists a Pareto-improving macroprudential policy.

We present a heuristic derivation of this result, which facilitates the intuition. For small changes, macroprudential policy in the boom state does not affect current asset prices, which remain equal to the efficient level, $q_1^{pl}(\alpha) = q^*$ (since the interest rate in the boom state is not constrained). Hence, the policy affects the gap value only through its effect on optimists' wealth dynamics and the associated aggregate demand externalities. Differentiating [equation \(54\)](#) for $s = 1$ with respect to optimists' as-if beliefs and evaluating at the no-policy benchmark, $\lambda_1^{o,pl} = \lambda_1^o$, we obtain

$$(57) \quad (\rho + \lambda_1^i) \frac{\partial w_1^i(\alpha)}{\partial \lambda_1^{o,pl}} = \alpha(1 - \alpha) \left[\frac{\partial w_2^i(\alpha')}{\partial \alpha} - \frac{\partial w_1^i(\alpha)}{\partial \alpha} \right] + \lambda_1^i \frac{\partial w_2^i(\alpha')}{\partial \lambda_1^{o,pl}}.$$

Here, $\alpha' = \alpha \frac{\lambda_1^o}{\lambda_1(\alpha)} = \alpha$ given the assumption $\lambda_1^o = \lambda_1^p$.

The two terms inside brackets capture the direct impact of the policy on welfare through aggregate demand externalities. The first term illustrates that the policy generates positive aggregate demand externalities—because it increases optimists' wealth share if there is a transition into the recession state. On the other hand, the second term illustrates that the policy also generates negative aggregate demand externalities—because it reduces optimists' wealth share if there is no transition. In a dynamic setting, macroprudential policy in the boom state is associated with some costs as well as benefits. The costs emerge because the policy prevents optimists from accumulating wealth that could be useful in a future recession. However, intuition suggests the benefits should outweigh the costs as long as future recessions are not too different from an imminent recession. The following lemma verifies this under the assumption $\lambda_1^o = \lambda_1^p$.

LEMMA 5. When $\lambda_1^o = \lambda_1^p$, the gap value function satisfies $\frac{dw_2^i(\alpha')}{d\alpha} > \frac{dw_1^i(\alpha)}{d\alpha}$ for each i and $\alpha \in (0, 1)$.

As expected, optimists are more useful if there is an immediate transition to the recession state, in which case their benefits materialize immediately. Any delay in this transition reduces

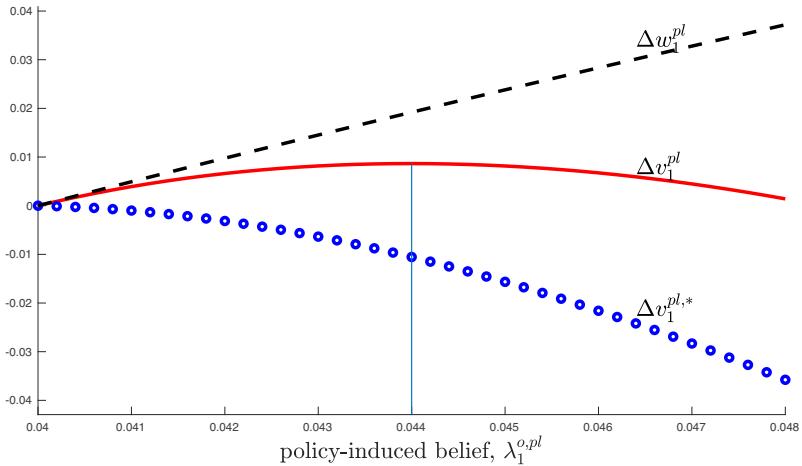


FIGURE V

Welfare Effect of Macroprudential Policy in the Boom State According to the Pareto Criterion

The solid red line illustrates the change in the planner's total value. The dashed black line (resp. the dotted blue line) illustrates the change in the planner's gap value (resp. the first-best value).

the benefits by postponing them. Combining this lemma with [equation \(57\)](#) provides a heuristic derivation of [Proposition 3](#) (see [Online Appendix C.2](#) for the proof).

What happens when we relax the assumption $\lambda_1^o = \lambda_1^p$? This is largely a technical assumption. Our numerical simulations (e.g., [Figure V](#)) suggest that a version of [Proposition 3](#) also holds when $\lambda_1^o < \lambda_1^p$ under appropriate technical assumptions, but we are unable to provide a proof.¹⁴

14. There are two distinct challenges. First, we cannot generalize [Lemma 5](#), although the ranking is intuitive and should hold unless there are strong nonlinearities in the gap value function. Specifically, the proof of [Lemma 5](#) establishes

$$\frac{\partial w_1^i(b_{0,1})}{\partial b} = \frac{\lambda_1^i}{\lambda_1^i + \rho} \int_0^\infty e^{-(\rho + \lambda_1^i)t} (\rho + \lambda_1^i) \frac{\partial w_2^i(b_{t,2})}{\partial b} dt,$$

where $b_{0,1}$ denotes a transformed version of α at the initial state, and $b_{t,2}$ denotes the same variable after a transition into the recession state after a period of length t . When $\lambda_1^o = \lambda_1^p$, we also have $b_{t,2} = b_{0,1}$ (since there is no speculation in the boom state), which yields $\frac{\partial w_1^i(b_{0,1})}{\partial b} = \frac{\lambda_1^i}{\lambda_1^i + \rho} \frac{\partial w_2^i(b_{0,1})}{\partial b} < \frac{\partial w_2^i(b_{0,1})}{\partial b}$. When $\lambda_1^o < \lambda_1^p$, the same result holds and the ranking is unchanged if the value function is linear in the transformed variable b . Hence, the ranking can fail only if there are sufficiently large nonlinearities in the gap value function.

[Figure V](#) illustrates the result for our earlier parameterization, which features $\lambda_0^o < \lambda_1^p$. We fix optimists' wealth share ($\alpha = \frac{1}{2}$) and calculate the effect of macroprudential policy on the planner's value function and on its components. The policy reduces the planner's first-best value function because it distorts investors' allocations according to their own beliefs. However, for small policy changes, this decline is small (due to the first welfare theorem). The policy also increases the planner's gap value function. This increase is large enough that the policy increases the actual value function and generates a Pareto improvement. As the policy becomes larger, the gap value continues to increase whereas the first-best value decreases. Moreover, the decline in the first-best value is negligible for small policy changes but it becomes sizeable for large policy changes. Thus, the constrained-optimal macroprudential policy obtains at an intermediate level.

3. Macroprudential Policy According to A Belief-Neutral Gap-Value Criterion. When we interpret belief disagreements literally (see [Remark 7](#)), it is questionable whether the utility from speculation should count toward social welfare. A recent literature argues that the Pareto criterion is not the appropriate notion of welfare for environments with belief disagreements. If investors' beliefs are different due to mistakes (e.g., in Bayesian updating), it is arguably more appropriate to evaluate their utility according to a belief that is common across investors. This approach removes the speculative utility from welfare calculations, and it motivates larger policy interventions. In this case, macroprudential policy not only improves macroeconomic outcomes, it also mitigates the microeconomic costs associated with speculation (see, e.g., [Simsek 2013b](#); [Dávila 2020](#); [Heimer and Simsek 2019](#)).

In our context, when investors have belief disagreements, it might be natural for a planner to focus exclusively on increasing investors' gap value, $w_s^i(\alpha)$, as opposed to their total value, $v_s^i(\alpha)$. This sidesteps difficult questions about whether speculation increases or reduces welfare. It also accords well with the goals of macroprudential policy in practice: the planner exclusively focuses on minimizing output gaps relative to a frictionless benchmark. Although reasonable, this approach still faces a major

Second, in the more general case, pessimists and optimists disagree about the benefits of macroprudential policy. The planner takes a weighted average of these perceptions, which complicates the analysis.

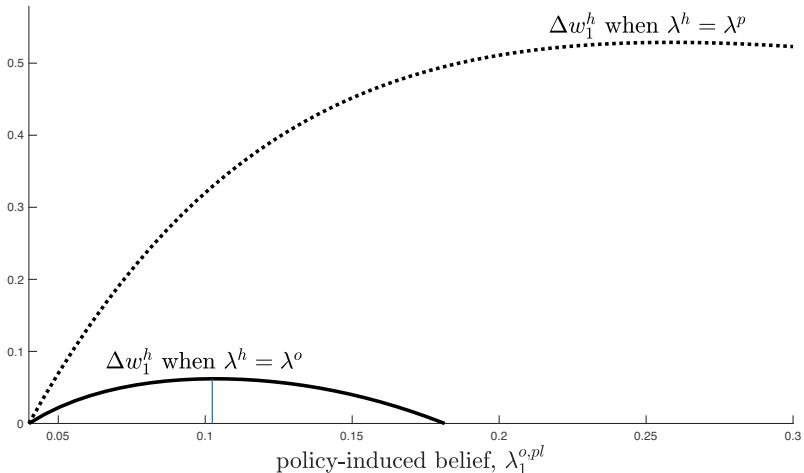


FIGURE VI

Welfare Effect of Macroprudential Policy in the Boom State According to the Belief-Neutral Gap Value Criterion

The solid line (resp. the dotted line) illustrates the change in the gap value evaluated under the optimists' belief (resp. the pessimists' belief).

challenge: given that the planner thinks investors' beliefs might be wrong, what belief should she use to evaluate the gap value?

In recent work, [Brunnermeier, Simsek, and Xiong \(2014\)](#) offer a belief-neutral welfare criterion that circumvents this problem. The basic idea is to require the planner to evaluate welfare according to a single belief and make the welfare comparisons robust to the choice of the single belief. Specifically, their baseline criterion says that an allocation is belief-neutral superior to another allocation if it increases welfare under every belief in the convex hull of investors' beliefs. Proposition 3 suggests their criterion can be useful in this context since macroprudential policy increases the gap value according to each belief.

For a numerical illustration, fix some $h \in [0, 1]$ and let $w_s^h(\alpha; \lambda_1^{o,pl})$ denote the value function for an investor when the planner implements policy $\lambda_1^{o,pl}$ and evaluates utility under the beliefs $\lambda_s^h = \lambda_s^p + h(\lambda_s^o - \lambda_s^p)$ for each s .¹⁵ [Figure VI](#) illustrates the effect of macroprudential policy on the gap value according

15. This value function solves the differential equation system (54) after replacing the actual belief, λ_s^i , with the belief λ_s^h .

to the optimists' belief (solid line) and the pessimists' belief (dotted line). As the figure suggests, tightening the policy toward $\lambda_1^{o,pl} = 0.104$ generates a belief-neutral improvement in the gap value. Beyond this level, tightening the policy improves the gap value according to pessimists but not according to optimists—who perceive smaller benefits from macroprudential policy because they find the transition into the recession state unlikely. In this example, the belief-neutral optimal policy represents a substantial intervention: it induces optimists to act as if $\lambda_1^{o,pl} = 0.104$ (roughly a 10% chance of transition to recession in a year), whereas optimists' own belief is $\lambda_1^o = 0.04$ (roughly a 4% chance of transition) and pessimists' belief is $\lambda_1^p = 0.2$ (roughly a 20% chance of transition). Note that the belief-neutral gap-value criterion supports a much larger policy intervention than the Pareto criterion (see [Figure V](#)).

4. Dynamics of Equilibrium with Macroprudential Policy. We consider how macroprudential policy in the boom state affects the dynamics of equilibrium variables. [Figure VII](#) illustrates the evolution of equilibrium over a 40-year horizon when the planner implements the belief-neutral optimal policy, $\lambda_1^{o,pl} = 0.104$. For comparison, the figure also replicates the evolution of the equilibrium variables without macroprudential policy from [Figure IV](#). Macroprudential policy in the boom state ensures that optimists' wealth share drops less when there is a transition into the recession state. This leads to higher asset prices and output when the economy transitions to the recession. However, the policy is not without its drawbacks. As the periods before year 10 illustrate, macroprudential policy in the boom state slows down the growth of optimists' wealth share when the economy remains in the boom state.

5. Macroprudential Policy in the Recession State. Our analysis so far concerns macroprudential policy in the boom state and maintains the assumption that $\lambda_2^{o,pl} = \lambda_2^o$. In [Online Appendix C.2](#), we analyze the opposite case, when the economy is currently in the recession state $s = 2$ and the planner can apply macroprudential policy in this state, $\lambda_2^{o,pl} \leq \lambda_2^o$ (she can induce optimists to act as if the recovery is less likely), but not in the other state, $\lambda_1^{o,pl} = \lambda_1^o$. Proposition 4 in the [Online Appendix](#) shows that in contrast to Proposition 3, this policy can reduce social welfare. Consider the two countervailing forces.

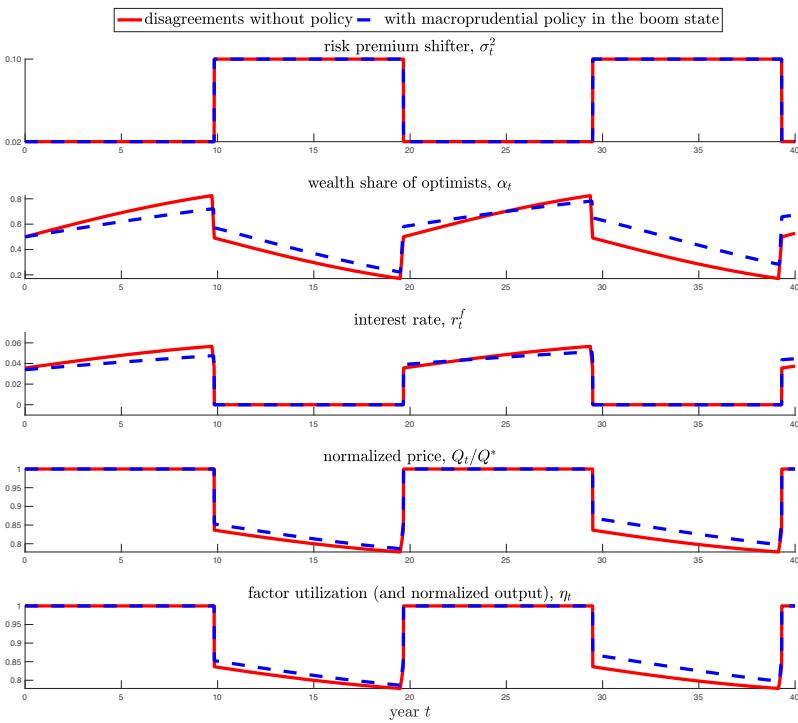


FIGURE VII

A Simulation of the Equilibrium Variables over Time with Macroprudential Policy

The dashed blue line (resp. the solid red line) illustrates the equilibrium with disagreements and with macroprudential policy (resp. without macroprudential policy).

As before, macroprudential policy increases the gap value by increasing optimists' wealth share if the economy stays in the recession state. However, unlike before, macroprudential policy also reduces current asset prices because the price is below the efficient level, $q_2^{pl}(\alpha) < q^*$, and it is increasing in optimists' as-if optimism, $\lambda_2^{\alpha,pl}$ (see [equation \(39\)](#)). This channel reduces the gap value (see [equation \(54\)](#)). When optimists' wealth share is large ($\alpha \rightarrow 1$), the latter channel dominates and macroprudential policy reduces the gap value and social welfare. Even when the current asset price channel does not dominate, macroprudential policy in the recession state is less useful than in the boom state. We verify this intuition in numerical simulations.

It is important to emphasize that macroprudential policy in the boom state does not lower asset prices due to the monetary policy response. Specifically, recall that the policy increases optimists' as-if pessimism in the boom state, $\lambda_1^{o,pl}$. Although this does not affect the asset price in the boom state, $q_1^{pl}(\alpha) = q^*$, it reduces the interest rate for a given level of optimists' wealth share, $r_1^f(\alpha)$ (see [equation \(40\)](#)). As macroprudential policy reduces the demand for risky assets, monetary policy lowers the interest rate to dampen its effect on asset prices and aggregate demand.

Our analysis in this section provides support for procyclical macroprudential policy. In states where output is not demand constrained (in our model, the boom state $s = 1$), macroprudential policy that restricts high-valuation investors' risk-taking is desirable. This policy improves welfare by ensuring that high-valuation investors bring more wealth to the demand-constrained states, which increases asset prices and output. In states where output is demand constrained (in our model, the recession state $s = 2$), macroprudential policy is less useful because it has an immediate negative impact on asset prices and aggregate demand.

VII. EMPIRICAL EVIDENCE

Our empirical analysis focuses on three predictions. First, our model predicts that risk premium shocks generate an interest rate reduction when the interest rate is not constrained and a more severe demand recession when the interest rate is constrained. Second, the recession reduces firms' earnings and leads to a further reduction in asset prices. Third, the recession is more severe when the shock takes place in an environment with more speculation. To investigate these predictions, we compare the response to house price shocks in Eurozone countries (which have constrained interest rates with respect to national shocks) to the response in non-Eurozone developed countries (which have less constrained interest rates). At the end of the section, we discuss empirical evidence from the recent literature suggesting that similar results apply for price shocks to other asset classes, such as stocks, and for other interest rate constraints, such as the zero lower bound.

Although our model relies on the zero lower bound constraint, the mechanisms are more general, and we find it more convenient to work with the currency-union constraint in our empirical

analysis. The zero lower bound has only recently become a practical constraint, limiting data availability, and it calls for an asymmetric specification that requires separate responses to positive and negative price shocks (since the monetary policy can raise the interest rate in response to positive shocks). In contrast, individual Eurozone countries have had constrained interest rates (with respect to national shocks) for much longer, and the constraint has been symmetric.

A major challenge in this exercise is identifying the risk premium shock that drives asset prices. As we clarify in [Section II](#), the exact source of the shock is not important for our mechanisms (e.g., risk, risk aversion, and beliefs have similar effects). Therefore, our strategy is to control for factors that do not act as a risk premium shock according to our model. In particular, we control for supply shocks and demand shocks that are not specific to house prices—including monetary policy shocks—and we interpret the residual change in house prices as a plausibly exogenous risk premium shock. Specifically, our risk premium shock is a surprise change in house prices in a country after controlling for contemporaneous and recent changes in output and the policy interest rate.¹⁶

Our model has a single type of capital, which can be interpreted as a value-weighted average of housing, stocks, and other assets in positive net supply. We focus on house prices for two reasons. First, housing wealth is large and its size (relative to output) is comparable between Eurozone and non-Eurozone developed countries (see Table F.3 in [Online Appendix F](#)). In contrast, stock markets in Eurozone countries are typically much smaller than in non-Eurozone developed countries, which makes stocks less suitable for our empirical strategy (see Table F.4 in the [Online Appendix](#)). Second, house prices are less volatile and seem to react to monetary policy shocks with some delay (see Figure F.6 in the [Online Appendix](#)). This feature enables us to control for monetary policy shocks by including contemporaneous and past

16. Although our controls are imperfect, we also report the differential effects of these shocks in Eurozone countries compared with their effects outside the Eurozone, which provides additional robustness. For example, our model illustrates that permanent supply shocks (e.g., an increase in A) shift asset prices and output regardless of whether the interest rate is constrained (see [Sections III](#) and [IV](#)). This suggests that common omitted supply shocks would lead to a similar bias inside and outside the Eurozone, and this bias is mitigated by focusing on the differential responses.

realizations of policy interest rates. We also interpret the future changes in interest rates as the monetary policy response to the risk premium shock, which enables us to test a key prediction of our model. This strategy works less well for stocks, because stock prices react to monetary policy shocks quickly, which might create a correlation between stock prices and interest rates with the opposite sign (since stock price declines driven by monetary policy shocks are typically followed by interest rate hikes—the opposite of risk premium shocks).¹⁷

VII.A. Data Sources

We assemble a quarterly cross-country panel data set of financial and economic variables for advanced economies. We obtain data on house price indices from the quarterly data set described in [Mack, Martínez-García, and Grossman \(2019\)](#). We obtain data on macroeconomic activity such as GDP, investment, and consumption from the OECD. We also obtain financial market data such as the policy interest rate, stock price indices, and earnings (of publicly traded firms) from Global Financial Data (GFD) and the Bank for International Settlements (BIS). [Online Appendix F](#) describes the data sources and variable construction.

VII.B. Sample Selection

Our sample covers 20 advanced economies from the first quarter of 1990 until the last quarter of 2017. Our selection of countries is driven by the availability of consistent house price data. We start the sample in 1990 because monetary policy in most advanced economies had shifted from focusing on stabilizing inflation to stabilizing output by this time, as in our model. Our results are robust to alternative sample selections.¹⁸

17. Formally, we assume house prices react to monetary policy shocks with a delay of at least one quarter. Figure F.6 in the [Online Appendix](#) plots impulse responses to shocks to the policy interest rate and provides support for this assumption. Specifically, a surprise increase in the policy interest rate is followed by a decline in house prices, but the response starts after the first quarter and takes several quarters to complete. The same figure shows that the assumption is clearly violated for stock prices. A surprise increase in the policy interest rate also reduces stock prices, but all of the response takes place in the same quarter as the shock.

18. Figures F.4 and F.5 in the [Online Appendix](#) show that starting the sample in 1980 leaves our results (except for the effect on inflation) qualitatively unchanged.

To capture interest rate constraints, we divide the data into two categories. The first category, which we refer to as the Euro/ERM sample, consists of country-quarters in which the country was a member of the Euro area or the European Exchange Rate Mechanism (ERM) for most of the calendar year. The ERM system, which preceded the euro, required member countries to keep their exchange rates within a narrow band of a central currency. This system constrained countries' relative policy interest rates (albeit imperfectly), and most member countries eventually adopted the euro. The countries in the euro area share the same policy interest rate (determined by the European Central Bank). The second category, which we refer to as the non-Euro/ERM sample, consists of the remaining country-quarters. Table F.1 in the [Online Appendix](#) describes Euro/ERM status by country and year.

VII.C. *Empirical Specification*

To describe how the economy behaves after house price shocks, we follow the local projection method developed by [Jordà \(2005\)](#). In particular, we regress several outcome variables at various horizons after time t on (residual) house price changes at time t . Specifically, we estimate equations of the type

$$(58) \quad Y_{j,t+h}^h - Y_{j,t-1}^h = \alpha_j^h + \gamma_t^h + \beta^{p,h} (-\Delta \log P_{j,t}) + \beta^{c,h} \text{controls}_{j,t} + \varepsilon_{j,t}^h,$$

where j denotes the country, t denotes the quarter, h denotes the horizon, Y denotes an outcome variable, P denotes the (real) house price index, and $\Delta \log P_{j,t} = \log P_{j,t} - \log P_{j,t-1}$ denotes its quarterly log change. We include time and country fixed effects, so our "house price shock" is a decline in house prices in a quarter, after accounting for the average price decline in the sample countries and various other controls within the country. Our control variables include the contemporaneous value and 12 lags of the first difference of log GDP—to control for supply shocks and demand shocks that are not specific to house prices. Likewise, we include the contemporaneous value and 12 lags of the policy interest rate—to control for monetary policy shocks. We also include 12 lags of the first difference of log house prices—to capture the momentum in house prices—and 12 lags of the first difference of the outcome variable—to control for other dynamics that might influence the outcomes. We weight each regression by countries' relative GDP, and estimate [equation \(58\)](#) for horizons 0 to 12.

To evaluate the responses within and outside the Eurozone, we include indicator variables for Euro/ERM and non-Euro/ERM status, and we interact all right-hand-side variables (including the fixed effects) with these indicators. We let $\beta_{euro}^{p,h}$ and $\beta_{non}^{p,h}$ denote the coefficient on the interaction of the price shock with the corresponding indicator. Our specification is equivalent to running the regressions separately within the Euro/ERM and non-Euro/ERM samples.¹⁹ We report the sequence of coefficients, $\{\beta_{euro}^{p,h}\}_{h=0}^{12}$ and $\{\beta_{non}^{p,h}\}_{h=0}^{12}$, which provide an estimate of the impulse response functions for the respective samples. We also report 95% confidence intervals calculated according to [Newey and West \(1987\)](#) standard errors with a bandwidth of 20 quarters.

We look at outcome variables for which our model makes a clear prediction, such as the policy interest rate, the unemployment rate (a proxy for factor underutilization), and the logs of GDP, investment, and consumption. We also include the log (core) CPI. Even though it is constant in our model (by assumption), variants of our model predict that it should decline in a demand recession. We analyze public firms' earnings and log stock prices to investigate spillover and amplification effects, as well as log house prices to investigate the price dynamics following the initial shock. All relevant variables except for the policy interest rate are adjusted for inflation to focus on real effects, as in our model. For earnings, we use the ratio of earnings to the initial stock price level as our dependent variable.²⁰

Table F.2 in the [Online Appendix](#) describes the summary statistics by Euro/ERM status for the variables that enter our regression analysis. The Euro/ERM sample has 760 country-quarters and the non-Euro/ERM sample has 1,130

19. The point estimates from our regression are identical to those obtained from running separate regressions within each sample. However, because our standard errors account for autocorrelation of the residuals, the joint regression will have slightly different standard errors (for example, the joint regression will account for the fact that residuals are correlated from before and after a country joined the ERM). The joint regression is preferable because it uses more data and thus gives more precise standard errors.

20. Earnings sometimes take a negative value, which makes a log transformation problematic. Instead, we change the specification in [equation \(58\)](#) slightly so that the dependent variable is $\frac{\text{earnings}_{t+h} - \text{earnings}_{t-1}}{\text{stock price}_{t-1}}$. Likewise, we adjust the control variables that feature earnings by dividing them by the stock price at quarter $t - 1$.

country-quarters.²¹ Both samples are unbalanced because a few countries have imperfect data coverage in earlier years (and because a few countries transition between samples). The two samples are comparable except that the non-Euro/ERM sample experienced slightly faster growth over the sample period.

VII.D. House Price Shocks and Demand Recessions

[Figure VIII](#) plots the estimated sequences of coefficients by Euro/ERM status (see Figure F.1 in the [Online Appendix](#) for the differenced coefficients). The panels in the top two rows illustrate our main empirical findings. The top left panel shows that in the non-Euro/ERM sample (dashed blue line; color version available online), a decline in house prices is followed by a sizable and persistent decline in the policy interest rate. By contrast, in the Euro/ERM sample (solid red line), a decline in house prices does not lead to an additional decline in the country's interest rate relative to other Euro/ERM countries, illustrating the interest rate constraint.²² The remaining panels in the top two rows illustrate that the shock is followed by a more severe demand recession in a Euro/ERM country than in a non-Euro/ERM country. In fact, the panels on GDP, investment, and consumption suggest that the shock initially leads to similar effects in both samples but is followed by milder outcomes in the non-Euro/ERM sample.

These results are consistent with our prediction that risk premium shocks lead to a more severe demand recession when the interest rate is constrained. From the lens of our model, the interest rate policy mitigates a demand recession driven by a

21. These are the sample sizes for our baseline regression in which the outcome variable is the policy interest rate and the horizon is 0 (see [equation \(58\)](#)). For some regressions, the sample size is slightly smaller, because we estimate outcomes at future horizons (which removes some data from the end of the sample period) and because some variables do not have complete coverage.

22. For the Euro era, the Euro/ERM-wide policy interest rate response is common to all countries and is captured by our time fixed effects. During the ERM era, there were severe cross-country monetary policy constraints. Figure F.3 in the [Online Appendix](#) illustrates the results from the same regression without time fixed effects. The figure shows that a negative house price shock in the Euro/ERM sample leads to a decline in the Euro/ERM-wide policy interest rate, but the magnitude of this decline is smaller than in the other sample. This is because house price shocks have a national (or idiosyncratic) component, and the Euro/ERM-wide policy interest rate arguably responds only to the Euro/ERM-wide (or systematic) component of these shocks.

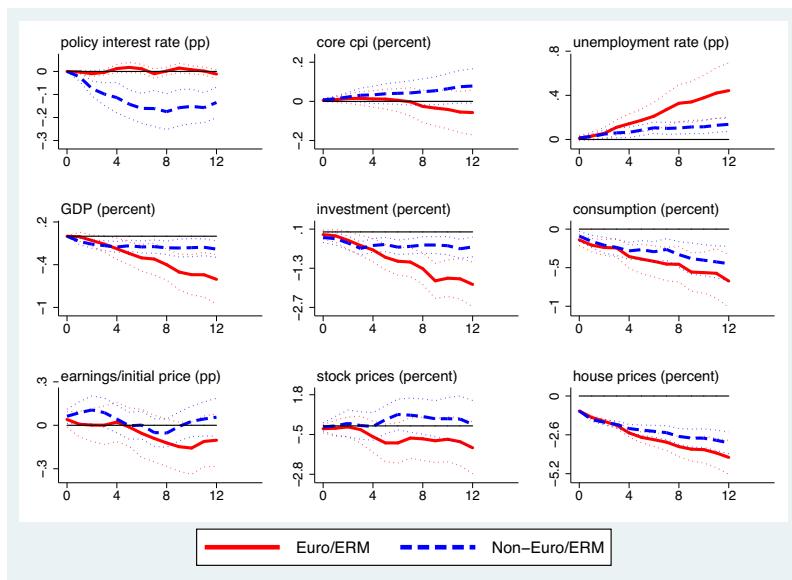


FIGURE VIII
Impulse Responses to a 1% Decrease in Real House Prices

The panels illustrate the results from the regression specification in equation (58) with the addition of the indicator variables for Euro/ERM and non-Euro/ERM status as well as the interaction of all right-hand-side variables with these indicators. The solid red (resp. dashed blue) lines plot the coefficients corresponding to the the negative log house price variable when Euro/ERM status is equal to 1 (resp. 0). For the units, “percent” corresponds to 0.01 log units (i.e., it is approximate) and “pp” corresponds to percentage points. All regressions include time and country fixed effects; contemporaneous value and 12 lags of the first difference of log GDP; contemporaneous value and 12 lags of the policy interest rate; 12 lags of the first difference of log house prices; and 12 lags of the first difference of the outcome variable. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries’ PPP-adjusted GDP in 1990. Data are an unbalanced quarterly panel that spans 1990Q1–2017Q4. All variables except for those in the top panel are adjusted for inflation. Earnings are normalized by the stock price in the quarter before the shock (see note 20). We describe our sources and variable definitions in [Online Appendix F](#).

local risk premium shock outside the Eurozone but not within the Eurozone.²³

23. In our model, risk premium shocks generate a less severe recession in unconstrained countries because the interest rate policy response leads to a smaller decline in asset prices. This suggests that asset price changes might provide an inaccurate measure of the underlying shock. We believe our analysis is robust to

VII.E. Spillover Effects and Amplification

The panels at the bottom row of [Figure VIII](#) illustrate the effect of the house price shock on asset markets. The panels on earnings and stock prices establish that there are spillover effects on the stock market: specifically, earnings and stock prices decline more in the Euro/ERM sample than in the other sample (although the estimates are imprecise due to the high volatility of earnings and prices). The remaining panel illustrates that after the initial shock, house prices decline more persistently and by a larger amount in the Euro/ERM sample.

These results are consistent with our prediction that the demand recession reduces firms' earnings and leads to a further decline in asset prices. From the lens of the model, stock prices (resp. house prices) decline less in the non-Euro/ERM sample due to the interest rate response, which not only increases the price to earnings ratio (resp. price to rent ratio) but also mitigates the recession and supports earnings (resp. rents).²⁴

VII.F. Speculation and Further Amplification

We need a proxy for speculation to test the final prediction of our model. We choose a measure of bank credit, which is a major catalyst of speculation in housing markets. First, banks can be thought of as high-valuation investors (optimists) because they have a greater capacity to handle risk than do noninstitutional investors, and they have real estate exposures through mortgage loans. Under this interpretation, bank credit provides a measure of banks' exposure to the housing market. Second, banks lend to other high-valuation investors in housing markets, such as

this concern for three reasons. First, to the extent that this concern is relevant, it biases the empirical analysis against finding support for our mechanisms because it implies that an equivalent magnitude of asset price decline corresponds to a larger underlying shock if the country has an unconstrained interest rate. Second, the concern is less relevant in practice than in our model because the interest rate policy affects all assets, which implies that risk-driven price declines in one asset class (such as housing) are partially absorbed by price increases in other asset classes. Third, the concern is less relevant for house prices because they seem to react to interest rate changes with some delay (see [Figure F.6](#) in the [Online Appendix](#)). In fact, the panel of [Figure VIII](#) on house prices suggests that the interest rate response only partially stabilizes risk-driven house price changes, and only with some delay.

24. We cannot test the predictions on rents because we do not have reliable data.

optimistic homebuyers who use bank credit to purchase larger homes or second homes. When bank credit is easily available, perhaps because of banks' optimism about house prices, these high-valuation investors wield a greater influence in the housing market (see [Simsek 2013a](#) for a formalization). Thus, bank credit provides a broad proxy for speculation in the housing market.

Our specific measure of bank credit comes from [Baron and Xiong \(2017\)](#), who construct a variable, credit expansion, defined as the change in the bank-credit-to-GDP ratio in the past three years. They standardize this variable by its mean and standard deviation in each country so that the measure is high when bank credit expansion in a country has been high in recent years relative to its historical trends. They show that their standardized measure predicts the likelihood of a large decline in bank equity prices, and despite the elevated risk it also predicts lower average returns on bank equity. Their preferred interpretation is that bank equity investors are excessively optimistic or neglect crash risk, which in our framework would translate into greater speculation (by banks or their borrowers).

We use BIS data on bank credit to households and nonfinancial firms to construct a close analogue of Baron and Xiong's standardized credit expansion variable (see [Online Appendix F](#) for details). We then run the same regressions as in [equation \(58\)](#), but we include the interaction of the price shock with standardized credit expansion. That is, we estimate

$$(59) \quad Y_{j,t+h}^h - Y_{j,t-1}^h = \alpha_j^h + \gamma_t^h + \left[\begin{array}{l} \beta^{p,h} (-\Delta \log P_{j,t}) \\ + \beta^{pc,h} (-\Delta \log P_{j,t}) \times \text{credit expansion-std} \end{array} \right] \\ + \beta^{c,h} \text{controls}_{j,t} + \varepsilon_{j,t}.$$

In addition to the earlier controls, we include 12 lags of standardized credit expansion to capture its direct impact. As before, we interact all right-hand-side variables with the Euro/ERM and the non-Euro/ERM status indicators. We let $\beta_{euro}^{pc,h}$ and $\beta_{non}^{pc,h}$ denote the coefficient on the interaction of the shock and credit with these indicators. The sequence of coefficients, $\{\beta_{euro}^{pc,h}\}_{h=0}^{12}$ and $\{\beta_{non}^{pc,h}\}_{h=0}^{12}$, provide an estimate of the additional effect of the shock when credit expansion has been one standard deviation above average (relative to its baseline effect with average credit).

[Figure IX](#) plots these sequences and illustrates our findings (see Figure F.2 in the [Online Appendix](#) for the differenced coefficients). The panels in the first two rows show that in the

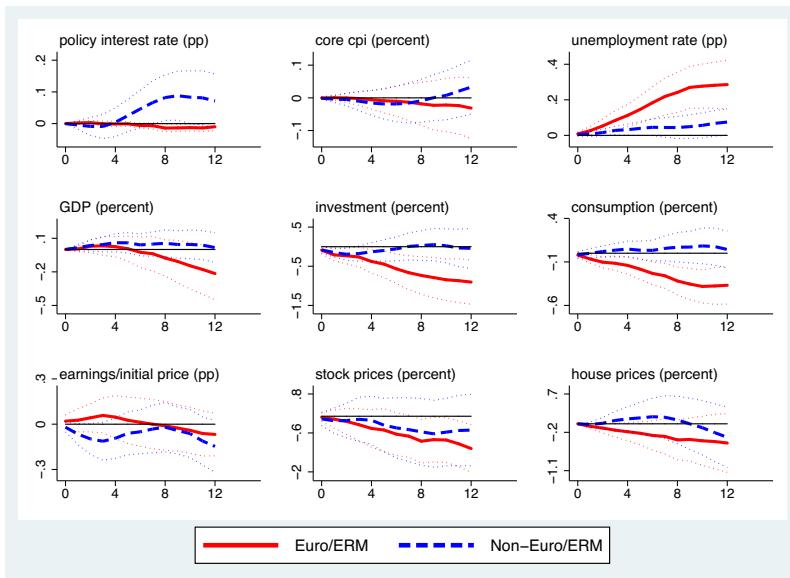


FIGURE IX

Additional Impulse Responses to a 1% Decrease in Real House Prices When Credit Expansion Has Been One Standard Deviation above Average

The panels illustrate the results from the regression specification in equation (59) with the addition of the indicator variables for Euro/ERM and non-Euro/ERM status as well as the interaction of all right-hand-side variables with these indicators. The solid red (resp. dashed blue) lines plot the coefficients corresponding to the interaction of the negative log house price and the standardized credit expansion variables when Euro/ERM status is equal to 1 (resp. 0). For the units, “percent” corresponds to 0.01 log units (i.e., it is approximate) and “pp” corresponds to percentage points. All regressions include time and country fixed effects; contemporaneous value and 12 lags of the first difference of log GDP; contemporaneous value and 12 lags of the policy interest rate; 12 lags of the first difference of log house prices; 12 lags of the first difference of the outcome variable; and 12 lags of standardized credit expansion. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries’ PPP-adjusted GDP in 1990. Data are an unbalanced quarterly panel that spans 1990Q1–2017Q4. All variables except for those in the top panel are adjusted for inflation. Earnings are normalized by the stock price in the quarter before the shock (see note 20). We describe our sources and variable definitions in [Online Appendix F](#).

Euro/ERM sample, house price shocks lead to a greater decline in economic activity when credit expansion has been high in recent years. In contrast, credit expansion does not change the effect of the house price shock in the non-Euro/ERM sample. These results support our prediction that risk premium shocks lead to a

more severe demand recession (in constrained economies) when they take place in an environment with elevated speculation.

On the other hand, the bottom row of [Figure IX](#) presents inconclusive results that neither support nor refute our predictions. We do not find meaningful differences for the additional effect of house price shocks on earnings or house prices when credit expansion has been high (in either sample). We do find a negative effect on stock prices for the Euro/ERM sample, but the effect is not statistically significantly different from the other sample. That said, because standard errors are large, we cannot rule out sizable effects either. Although we tentatively conclude that speculation proxied by credit expansion is associated with deeper risk-centric demand recessions, further empirical research should verify the robustness of this conclusion and the precise channels through which speculation affects the recession.

VII.G. Other Supporting Evidence

Our empirical analysis is related to [Mian and Sufi \(2014, 2018\)](#), who use regional data within the United States to provide evidence for the central role played by the house price cycle and housing speculation in the Great Recession.

[Mian and Sufi \(2014\)](#) argue that house price declines explain much of the job losses between 2007 and 2009. Our results for the Euro/ERM sample suggest that similar results hold in cross-country data, whereas the non-Euro/ERM sample suggests that monetary policy can mitigate the adverse effects of house price shocks. Moreover, while [Mian and Sufi \(2014\)](#) emphasize household deleveraging as the key channel by which house price declines cause damage, some of our empirical results (e.g., the investment response) suggest there are other mechanisms as well. As our model demonstrates, house price declines could lower aggregate demand even without household deleveraging or other financial frictions—although these additional ingredients would amplify the effects.

[Mian and Sufi \(2018\)](#) argue that housing speculation amplified the house price cycle and led to a more severe downturn. As in our empirical exercise, they emphasize bank credit expansion as a major catalyst of speculation. They find that the U.S. areas more exposed to credit expansion in the early 2000s featured greater speculative trading activity (measured from detailed transaction data) and greater belief disagreements (measured from survey

data). They argue that the same areas experienced a larger housing boom but also a much greater bust, so they ended the housing cycle with lower house prices and economic activity. Our empirical results on speculation (although less detailed) suggest that similar results hold in cross-country data. Our model illustrates how greater speculation during the boom naturally leads to lower prices and economic activity once the economy transitions to recession.

In recent work, [Pflueger, Siriwardane, and Sunderam \(2020\)](#) present evidence that suggests risk premium shocks in the stock market also affect aggregate demand and interest rates. Specifically, they construct a measure of risk appetite for the United States as the price of high-volatility (idiosyncratic) stocks relative to low-volatility stocks. They show that a decrease in their measure of risk appetite is followed by a slowdown in economic activity and a decline in the risk-free rate—similar to our results for the non-Euro/ERM sample. [Pflueger, Siriwardane, and Sunderam \(2020\)](#) argue that their risk appetite measure explains almost half of the variation of the one-year risk-free rate in the United States since 1970. This suggests that the time-varying risk premium is an important driver of the risk-free rate in practice. [Chodorow-Reich, Nenov, and Simsek \(2019\)](#) provide further support for the link between the stock market and aggregate demand. Using regional data in the United States, they find that a decline in local stock wealth (driven by aggregate stock prices) decreases local payroll and employment. They also find stronger effects in nontradable industries but no effects for tradeable industries, consistent with a consumption wealth effect as in our model.

Focusing on a value-weighted average of house and stock prices, [Jordà et al. \(2019\)](#) argue that low-frequency fluctuations in the risk premium in developed economies have been associated with a collapse of safe asset returns (as opposed to a spike in risky asset returns). In particular, when the risk premium rises, the risk-free rate tends to fall and the value-weighted average risky asset returns remain relatively stable, as in our model. Looking at more recent years, [Del Negro et al. \(2017\)](#) provide a comprehensive empirical evaluation of the different mechanisms that have put downward pressure on interest rates and show that risk and liquidity considerations played a central role (see also [Caballero, Farhi, and Gourinchas 2017a](#)).

Finally, our mechanisms are supported by a literature that investigates the macroeconomic impact of “uncertainty shocks.”

Using vector autoregressions (VARs), Bloom (2009) shows that an increase in the volatility index in the United States is followed by a slowdown in economic activity. Moreover, although his model does not emphasize monetary policy, his empirical analysis shows that the shock is followed by a decline in the federal funds rate. This response suggests the effects could be more severe if the interest rate were constrained. Recent empirical work verifies this intuition and shows that uncertainty shocks in the United States are associated with a greater decline in economic activity when the federal funds rate is close to 0, arguably because of the zero lower bound constraint on the interest rate (see, for instance, Caggiano, Castelnuovo, and Pellegrino 2017; Plante, Richter, and Throckmorton 2018).

VIII. FINAL REMARKS

We develop a risk-centric macroeconomic model to focus on the role of the aggregate demand channel in causing recessions driven by risky asset price fluctuations and to study the effect of financial speculation on the severity of these recessions. Our analysis reformulates the standard New Keynesian model in terms of a risk-centric decomposition that puts asset prices at the center of the analysis. When the interest rate is constrained, a rise in the risk premium lowers asset prices and triggers a demand recession, which further drives down asset prices. The feedbacks are especially powerful when investors are pessimistic and think the higher risk premium will persist. Hence, average beliefs play a central role in the recession phase not only because they affect asset valuations but also because they determine the strength of the amplification mechanism. In the *ex ante* boom phase, belief disagreements (and more broadly heterogeneous valuations) matter because they induce investors to speculate. This speculation exacerbates the recession because it depletes high-valuation investors' wealth when the risk premium rises, which leads to a greater decline in asset prices and economic activity. Macroprudential policy in the boom improves outcomes by restricting speculation and preserving high-valuation investors' wealth during the recession. This policy intervention leads to a Pareto improvement because it internalizes the aggregate demand externalities that result from speculation.

Our analysis supports "the Fed put": the tendency of central banks to cut interest rates in response to asset price declines

driven by risk premium shocks. In our model, as long as the central bank can cut interest rates without constraints, the economy remains in the first best. The lower-bound constraint on the interest rate can be thought of as the inability to provide a put beyond a certain point. Moreover, our amplification mechanism implies that even small constraints on the interest rate can generate large recessions (see Figure IV for an illustration). This highlights that the Fed put is important not only because of its direct effect on aggregate demand but also because it stabilizes expected future outcomes and prevents a large decline in current asset prices.

In addition, our approach highlights that the interest rate policy affects aggregate demand through its effect on financial markets and asset prices. From this perspective, our analysis supports unconventional monetary policies—such as central bank asset purchases—that attempt to stimulate aggregate demand through their effect on asset prices. More broadly, any policy that reduces perceived market volatility and prevents sudden asset price drops should have a similar effect, providing support for various policies implemented after the subprime and European crises.

In our model, we use a lower-bound constraint as the interest rate friction, but our mechanisms apply if the interest rate is constrained for other reasons. When the interest rate has both an upper bound and a lower bound (such as in a currency union or fixed exchange rate regime), our results often become stronger. In this setting, speculation causes damage not only by lowering asset prices during the recession but also by raising asset prices during the boom, when aggregate demand is stretched above its natural level, which exacerbates the inefficiency. Moreover, in this case macroprudential policy during the boom is beneficial not only because it preserves high-valuation investors' wealth for a future recession but also because it immediately contains the excessive rise in asset prices.

Our results with belief disagreements do not depend on whether optimists or pessimists are right about the transition probabilities. In fact, since the equilibrium is a function of investors' subjective beliefs, the objective belief does not enter the equilibrium characterization.²⁵ Moreover, the objective belief is

25. The objective belief matters if one is interested in understanding the evolution of investors' wealth shares along the objective path (that would be realized in practice). The market selection hypothesis, formulated by Friedman (1953),

largely irrelevant for our policy analysis because we mainly focus on the Pareto criterion and evaluate investors' welfare under their subjective beliefs. For example, we could think of optimists as rational agents and pessimists as Knightian agents (see, e.g., Caballero and Krishnamurthy 2008; Caballero and Simsek 2013). Absent any direct mechanism for alleviating Knightian behavior during severe recessions, the key point that reducing optimists' risk-taking during the boom leads to Pareto improvements survives this alternative motivation. More generally, our results are driven by persistent heterogeneous valuations, so similar results would also apply if investors share the same beliefs but value risky assets heterogeneously for other reasons such as differences in risk tolerance (see [Remark 7](#)).

Our model illustrates that the wealth distribution matters for aggregate demand, not only because of financial frictions or heterogeneous MPCs as emphasized in the previous macroeconomics literature but also because of heterogeneous asset valuations. In the recession, a greater wealth share for high-valuation investors improves asset prices and raises everyone's spending. From this perspective, our analysis supports not only *ex ante* macroprudential policy but also a variety of *ex post* policies that transfer wealth to high-valuation investors in recessions. In recent work, [Kekre and Lenel \(2019\)](#) show that heterogeneous asset valuations (which they refer to as a heterogeneous marginal propensity to take risk) also strengthen the transmission of monetary policy.

As we noted in the introduction, we removed all financial frictions to isolate the aggregate demand mechanism and its interactions with speculation. However, if we were to introduce these realistic frictions in our setting, many of the themes in that literature would reemerge and be exacerbated by aggregate demand feedbacks. For example, asset price declines would depress aggregate demand directly and indirectly by tightening collateral constraints. This mechanism would amplify the feedback loops between asset prices and aggregate demand. Although our analysis shares many similarities with the financial-frictions

posit that investors with incorrect beliefs should be driven out of the market because they consistently lose money. Our model features a version of this hypothesis in the long run (see note 11). However, recent research has identified many reasons this hypothesis is unlikely to apply in practice (see, e.g., [Cao 2017](#); [Borovička 2020](#)). We view our model as capturing the short-run given belief disagreements (or more broadly, heterogeneous valuations) that result from various unmodeled factors.

literature, we have a different focus. Financial frictions highlight the importance of constrained firms or commercial banks that lend to such firms, whereas our heterogeneous valuations approach highlights the importance of institutional investors or financial intermediaries that lend to such investors, for example hedge funds, active mutual funds, investment banks, and broker-dealers. Consistent with our main mechanism, a growing empirical literature suggests that financial intermediaries' balance sheets have a large effect on asset prices (see, e.g., [Adrian, Etula, and Muir 2014](#); [He, Kelly, and Manela 2017](#)).

Relatedly, we assumed complete markets so investors speculate only via contingent Arrow-Debreu securities. When these securities are not available, investors engage in proxy-speculation with leverage. In [Caballero and Simsek \(2019\)](#), we analyze this modified environment and show that our main results still hold: that is, contingent securities do not play an important role beyond providing analytical tractability. However, speculation via leverage breaks the separability property that we derive in [Section III.B](#): asset prices and the interest rate become relevant for the dynamics of investors' wealth shares. Once the separability property breaks down, monetary policy can be an effective prudential tool. In particular, raising interest rates during the low risk premium state can reduce speculation. We explore the role of prudential monetary policy in [Caballero and Simsek \(2019\)](#).

Although this is mostly an applied theory article, we surveyed some of the extensive empirical evidence supporting our analysis and provided our own evidence by contrasting the local response to risk premium shocks—captured by surprise house price changes—in constrained Euro/ERM countries with the local response in less constrained non-Euro/ERM countries. Our evidence suggests that risk premium shocks lead to more severe recessions when the interest rate is constrained, as in our model. The evidence also supports our model's prediction that recessions reduce firms' earnings and lead to a further reduction in asset prices. When we proxy speculation with the size of the bank credit expansion before the shock, we also find some evidence that high speculation increases the severity of recessions driven by risk premium shocks.

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SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online. Data and code replicating tables and figures in this article can be found in [Caballero and Simsek \(2020\)](#), in the Harvard Dataverse, doi: 10.7910/DVN/GABKJE.

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