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Quantum Superconductor-Metal Transitions in the Presence of Quenched Disorder

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Abstract InO_x films that are less disordered than those exhibiting direct quantum superconductor-insulator transitions feature quantum superconductor-metal transitions tuned by magnetic field. Resistance data across this superconductor-metal transition obey activated scaling, with critical exponents suggesting that the transition is governed by an infinite randomness critical point in the universality class of the random transverse-field Ising model in two dimensions. The transition is accompanied by quantum Griffiths effects. This unusual behavior is expected for systems with quenched disorder in the presence of Ohmic dissipation. Disorder leads to the formation of large rare regions which are locally ordered superconducting puddles dispersed in a metallic matrix. Their dissipative dynamics causes the activated scaling, as predicted by a renormalization group theory.

Keywords Superconductor-Metal Transitions · Quenched Disorder · Quantum Griffiths Effects

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1 Introduction

The effect of quenched random disorder on quantum superconductor to insulator transitions (SITs) in two dimensional (2D) and quasi-2D systems is usually addressed theoretically by studying how the average disorder strength evolves under coarse graining, i.e., with increasing length scales. If the disorder decreases, then the system will behave as if it were clean at large length scales, and the critical behavior will not be affected by the randomness [1]. If the disorder strength remains finite at large length scales, the universality class of the phase transition differs from that of the corresponding clean transition. If the disorder increases without limit under coarse graining, the transition features exotic infinite-randomness critical behavior [2,3].

On the other hand, it has become clear that rare strong disorder fluctuations and the rare spatial regions that support them can play a dominant role. This is especially important in the case of quantum phase transitions, because quenched disorder is perfectly correlated in the imaginary time direction. Imaginary time acts as an extra dimension at quantum phase transitions. As the extension of this extra dimension, the inverse temperature, diverges as one approaches the quantum critical point, one effectively has an infinitely large disorder fluctuation. Depending on the order parameter symmetry and the character of the quantum dynamics, the rare regions can have no effect on the transition, or they can change its universality class [4,5]. If the rare regions order independently, they can even lead to the destruction of the global sharp phase transition and induce a smeared transition [6].

Quantum SITs and superconductor to metal transitions (SMTs) of 2D or quasi-2D films are studied experimentally by changing an external control parameter of the system such as the disorder strength, a perpendicular (also parallel) magnetic field, or the carrier density. Since zero absolute temperature is experimentally inaccessible, the presence of such a transition is inferred from changes in measurable properties that are influenced by quantum fluctuations present at nonzero temperature. In the case of SITs

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and SMTs, measurements of resistance as a function of the control parameter and the temperature are often analyzed using power-law scaling [7,8]. For magnetic-field tuned transitions, this takes the form:

$$R(\delta, T) = \Phi\left(\delta T^{-1/\nu z}\right) \tag{1}$$

Here $\delta = |B - B_c|/B_c$ is the distance from the critical field B_c and Φ is a scaling function. This scaling form implies that the magnetoresistance isotherms (R vs. B curves at fixed T) all cross at the critical field B_c. Moreover, the magnetoresistance isotherms are expected to collapse into two branches when plotted as function of $\delta T^{-1/\nu z}$ for the correct value of the exponent product νz. Here, ν is the correlation length exponent and z is the dynamical critical exponent. In principle, knowledge of these exponents can be used to identify the universality class of the transition.

The key to the scaling analysis is the fact that the magnetoresistance isotherms cross at a single value of the control parameter, which in the case of the present work is the perpendicular magnetic field. However there have been a number of investigations in which instead of a single value of field, there is a continuum of crossing points, and there is no well-defined crossing field. This was first reported in a detailed fashion by Gantmakher and coworkers some twenty years ago [9]. A potential explanation is that the apparently broadened transition is caused by some sort of macroscopic spatial inhomogeneity in the superconducting coupling strength, resulting in different areas of the film ordering at different values of magnetic field. However, in a number of recent publications describing work on two-dimensional crystalline superconductors, it was realized that there was, within a range of fields, a systematic variation of the crossing fields with temperature [10–13]. An analysis using power law scaling Eq. 1 at selected crossing fields within the range, employing nearby isotherms to determine an exponent product νz, collapsed data around each selected field. This resulted in a systematic variation with temperature of what might be termed an effective exponent product. The values of νz appeared to diverge in the zero temperature limit. This was interpreted as evidence of a quantum Griffiths singularity [14–16] associated with an infinite randomness critical point [2, 3]. This type of behavior has been predicted by a renormalization group calculation for a quantum SMT [17,18]. The universality class to which the transition would then belong would be the random transverse-field Ising model [4,5]. The authors of the above experimental papers then fit the divergence of νz with the predicted expression $C(B - B_c)^{-\nu\psi}$ where ν and ψ are the correlation length and tunneling exponents, respectively, and B_c is the field at which the divergence occurs. The adjustable parameters are C, an arbitrary constant, and $\nu\psi$. The fact that the exponent product determined by the fit agrees with numerical work on the random transverse field Ising model was taken by the authors as evidence of an infinite randomness critical point governed by activated scaling. There was no effort in these works directed at collapsing the data actually employing the activated scaling form.

The activated scaling form of the resistance predicted for a quantum SMT governed by an infinite randomness fixed point differs from Eq. 1. It reads [19]

$$R\left(\delta, \ln\frac{T_0}{T}\right) = \Phi\left[\delta\left(\ln\frac{T_0}{T}\right)^{1/\nu\psi}\right], \tag{2}$$

where once again $\delta = |B - B_c|/B_c$ is the distance from the critical field and T_0 is a microscopic temperature scale, which acts as an additional fitting parameter. Equation 2 predicts a single crossing point in magnetic field. To account for the range of crossing fields reported in recent papers, corrections to the leading scaling behavior need to be included. These corrections are required because the inverse disorder strength acts as an irrelevant variable in the scaling analysis. The corrections become less important as the temperature is decreased towards absolute zero. This is shown in Lewellyn *et al.* [20]. If a system is governed by activated scaling with corrections to scaling, then $R(\delta, T)$ curves at different temperatures do not cross at $\delta = 0$. The crossing shifts with temperature and approaches $\delta = 0$ in the limit $T \to 0$. Furthermore, the effective value of νz obtained from a power-law scaling analysis involving isotherms near the chosen crossing points at different temperatures is given by

$$\left(\frac{1}{\nu z}\right)_{\text{eff}} = \left(\frac{1}{\nu\psi}\right)_{\text{eff}} \frac{1}{\ln(T_0/T)}. \tag{3}$$

where $(\nu\psi)_{\text{eff}}$ is the exponent product for the universality class of the quantum phase transition exhibiting activated scaling. Here again, ν is the correlation length exponent of the transition and ψ is the tunneling exponent. In the zero temperature limit $\left(\frac{1}{\nu z}\right)$ will vanish, or $(\nu z)_{\text{eff}}$ will diverge. Thus the conclusion that a divergence of $(\nu z)_{\text{eff}}$ is consistent with an infinite randomness critical point is justified.

In this paper, we will first describe studies of amorphous InO_x, which lead to smeared crossing points similar to those reported recently. We will then go beyond the considerations of these earlier works by presenting an analysis which leads to a collapse of the magnetoresistance isotherms into two branches governed by activated scaling. This analysis enables us to determine the critical exponents of the transition, and it provides stronger evidence for the quantum SMT being governed by an infinite-randomness fixed point in the universality class of the random transverse-field Ising model. The present work is an elaboration of findings we reported earlier [20].

2 Experiment

The InO_x films which were studied were about 30 nm thick and were grown by electron beam evaporation of In_2O_3.

During deposition an O_2 partial pressure between 2×10^{-5} and 9×10^{-4} mbar was maintained in the chamber by bleeding gas through a needle valve, while continuing to pump [21]. Amorphous films were produced when the substrate temperature was kept below about 40°C. After removal from the deposition system, the films were annealed in air for many hours at temperatures between 40°C and 60°C. The annealing process does not alter the carrier concentration, but reduces the disorder. Usually such films, when cooled down to low temperatures and subjected to magnetic fields perpendicular to the plane, exhibit SITs. However, the films reported on in the present paper were not used for about three years. Subsequent transport measurements, which were carried out initially using a Quantum Design physical property measurement system and then with an Oxford Kelvinox 25 dilution refrigerator, exhibited superconductor-metal transitions. This suggests that during the long-term storage further annealing towards metallic behavior occurred.

The temperature range over which transport measurements are reliable is limited by factors such as electromagnetic noise, self-heating due to the measurement current, and the limitations of the cooling power of the dilution refrigerator [22]. The electrical leads to the sample and the thermometers were filtered only at room temperature, so that some noise at 300 K was delivered to the low temperature environment. Measurements of resistance were confined to currents and temperatures at which the I-V characteristics were linear, eliminating the possibility of heating due to the measurement current.

The InO_x films studied exhibited zero-field transition temperatures of approximately $2.8K$. Curves of resistance R vs. temperature T of one of the films at various perpendicular magnetic fields B are shown in Fig. 1.

At a field, $B \approx 7$T, the temperature derivative of resistance dR/dT changes sign. This change occurs at a resistance that is much lower than the quantum resistance $h/4e^2$ for Cooper pairs, which is the typical value found for direct quantum SITs. The films exhibited metallic behavior at fields in excess of 8 T as exemplified by the linear dependencies of their conductances on the natural logarithm of temperature. This is shown in Fig. 2

In addition, there was what might be termed an anomalous metallic regime at magnetic fields intermediate between those in which the films were obviously superconducting, and those which were clearly metallic. In this regime the values of dR/dT were positive, suggesting the onset of superconductivity at lower temperatures. However, their resistances did not fall to zero at the lowest temperatures deemed to yield reliable data. In subsequent analysis we will assume that a film in this field range would at sufficiently lower temperatures become superconducting, but it could be what is referred to as a failed superconductor [23].

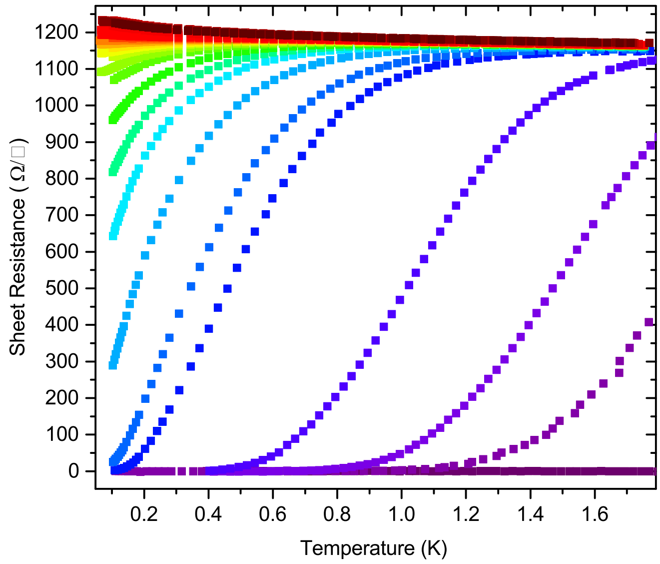

Fig. 1 Resistance versus temperature at magnetic fields of 0.0, 3.0, 4.0, 5.0, 6.0, 6.2, 6.5, 6.7, 6.8, 6.9, 7.0, 7.050, 7.125, 7.150, 7.175, 7.225, 7.250, 7.275, 7.300, 7.325, 7.4, 8.0, 9.0, 10.0, 11.0, and 12.0 T

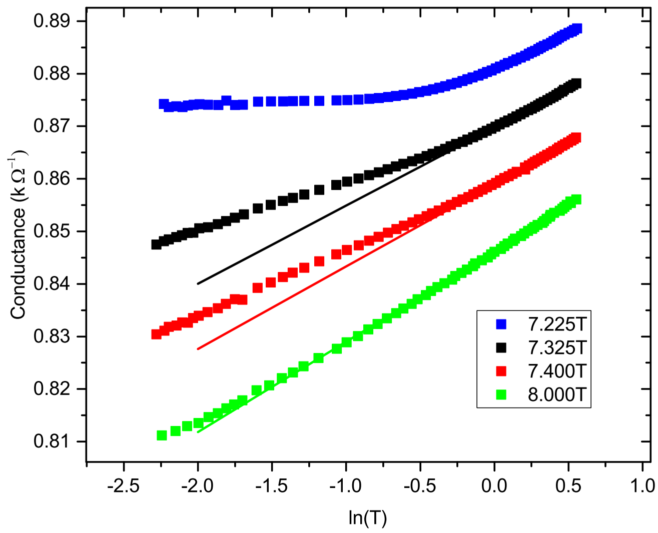

Fig. 2 Conductance versus the natural logarithm of the temperature at magnetic fields of 7.225, 7.325, 7.400, and 8.000 T (top to bottom) with an offset of 0.01 kΩ^{-1}. As the applied magnetic field increases the linear fits to the data hold to lower temperatures indicating a crossover to a quantum corrected quasi-2D metal

The minimum temperature at which data are considered to be reliable was determined from the behavior in the high-field metallic regime at fields well above the quantum SMT. The conductance in this regime corresponds to that of a two-dimensional quantum corrected metal and should be a linear function of the natural logarithm of temperature down to very-low temperatures [24,25]. The deviation of the data in this regime was taken as the minimum temperature at which reliable measurements and analysis could be carried out. A plot of the conductance vs. temperature in high magnetic fields is shown in Fig. 2 .

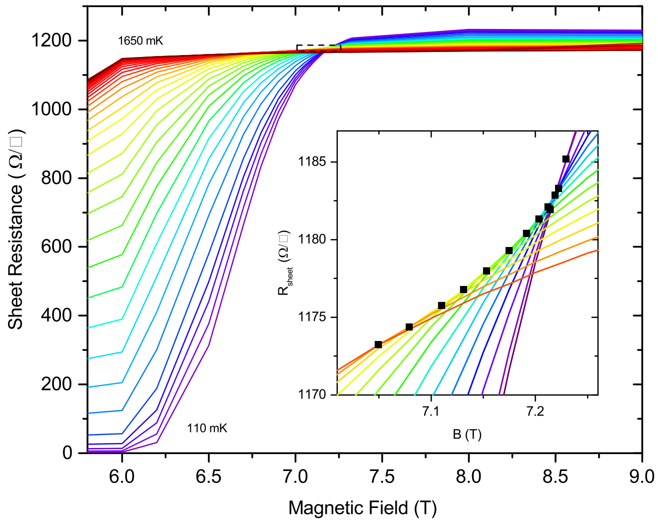

Fig. 3 Resistance versus magnetic field isotherms at temperatures ranging from 110 mK to 1650 mK. The crossover region indicated by the dashed box is shown in more detail in the inset. Crossings between neighboring isotherms are indicated with a black square

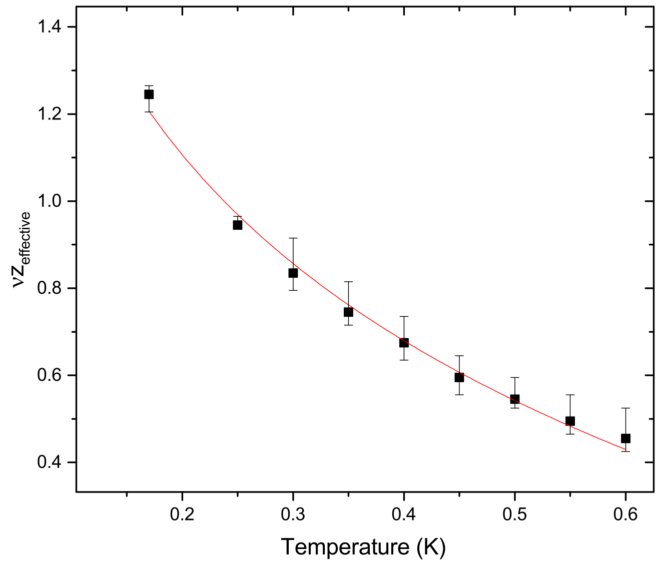

Fig. 4 Effective exponent $(\nu z)_{\text{eff}}$ vs. temperature. The solid line is a two-parameter fit to the data of Eq. 3 with $(\nu\psi)_{\text{eff}} = 0.62$ and $T_0 = 1.21$ K

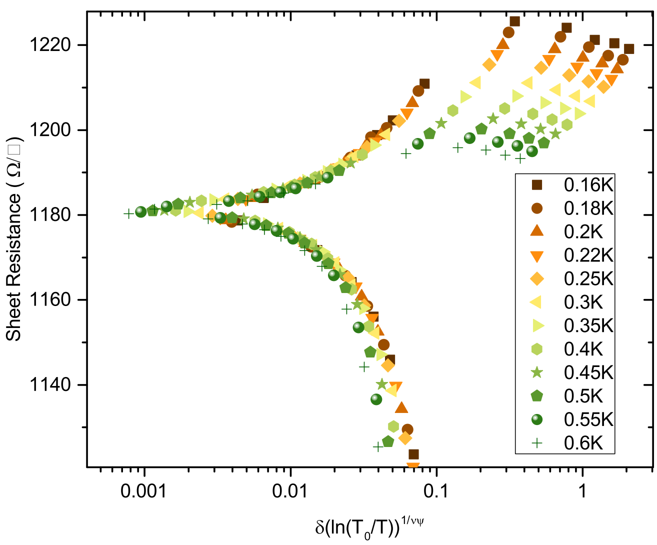

Fig. 5 Collapse of the magnetoresistance data using the activated scaling form described in Eq. 2. The data collapse around critical field B_c = 7.21 T, with $\nu\psi = 0.62$ and $T_0 = 1.21$ K which were determined from the fit in Fig. 4

3 Analysis

Magnetoresistance isotherms were generated using the measured $R(T,B)$ curves by carrying out a matrix inversion of the temperature-swept data. Initially, it seemed that there was a single crossing point, which would be typical of a direct quantum phase transition. See Fig. 3 .

However, a detailed examination of the crossing region revealed a range of temperatures and magnetic fields over which isotherms crossed as shown in the inset of Fig. 3 . The crossing fields were increasing with decreasing temperature and appeared to saturate in the limit of zero temperature. This unusual behavior is not compatible with the standard scaling analysis in which there is a single crossing point. In the earlier experimental papers mentioned above, similar behavior was found, and the smeared crossing and its field and temperature dependence were interpreted as evidence of the quantum Griffiths effect.

To analyze our data, we first follow the approach of Xing et al. in which power-law scaling was applied to arbitrarily chosen crossings of magnetoresistance isotherms within the smeared crossing regime [10]. We then successfully apply activated scaling to our data, providing much stronger evidence than that presented in earlier work for the existence of the quantum Griffiths effect in quenched disordered superconductors.

To find the effective values of νz vs. T, one considers a sequence crossing points over very narrow temperature intervals such that the magnetoresistance isotherms within each interval have a well-defined crossing field $B_x(T)$. Then for each interval, a power law scaling analysis is performed, collapsing the isotherms around each crossing field according to Eq. (1). The result of this analysis is an effective, temperature dependent νz which is shown in Fig. 4. In a conventional quantum phase transition analyzed in this way there would be a single crossing point and νz would be a constant (or, at least, it would approach a constant with decreasing temperature). In the present case νz strongly increases as the temperature is lowered, which suggests highly unconventional behavior.

According to the renormalization group theory, the temperature dependence of $(\nu z)_{\text{eff}}$ is given by Eq. 3. The expression on the right-hand side of Eq. 3 vanishes in the zero-

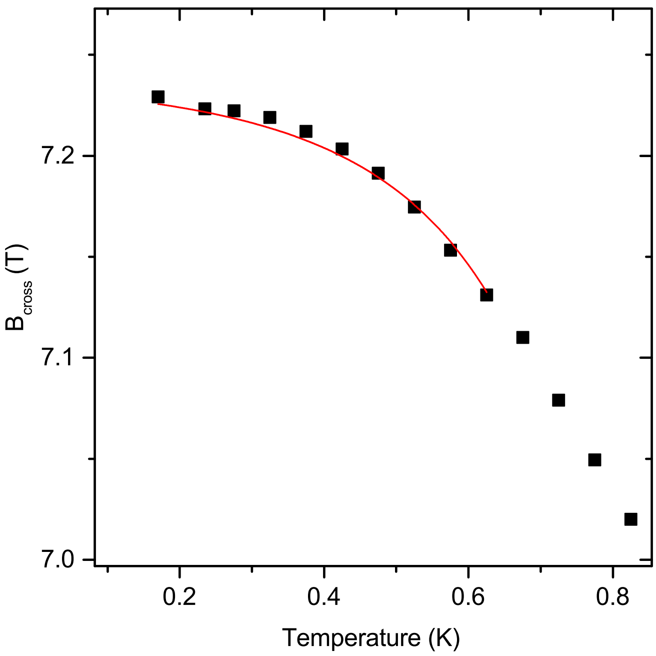

Fig. 6 Crossing field of neighboring isotherms versus temperature. The solid line is a fit of Eq. 4 to the data with $\nu\psi$ and T_0 determined from the fit in Fig. 4

temperature limit so that $(\nu z)_{\text{eff}}$ is expected to diverge. Equation 3 can then be used to determine the parameter $(\nu\psi)_{\text{eff}}$ of the activated scaling form. The solid line in Fig. 4 is the result of a two-parameter fit of Eq. 3 to the data. The parameters are $(\nu\psi)_{\text{eff}}$ and T_0 . The resultant $(\nu\psi)_{\text{eff}} = 0.62$ is in good agreement with numerical predictions for a two-dimensional infinite-randomness critical point in the random transverse field Ising universality class. The range of temperatures spanned in Fig. 4 does not extend to low enough values to unequivocally determine that $(\nu z)_{\text{eff}}$ diverges in the zero-temperature limit, but fits by a curve that diverges support its likely divergence.

To make the case even stronger that the quantum SMT belongs to a universality class governed by activated scaling, we scaled the full set of magnetoresistance isotherms using Eq. 2. Having determined $(\nu\psi)_{\text{eff}}$ and T_0 from power law scaling at the selected crossing points, the only unknown parameter is the critical field of the quantum phase transition. Determining B_c can be accomplished by employing a numerical method used by Skinner, Ruhman and Nahum in which the variance of the magnetoresistance isotherms plotted against the scaling parameter is minimized [26]. A value of $B_c = 7.21$ T, was found to produce the best collapse. The scaling is shown in Fig. 5 . Similar results were found in another sample that was studied.

This method produced a well-defined best value for B_c, but the variance as a function of $\nu\psi$ and T_0 did not have a well-defined minimum. Instead the variance was roughly minimized over an extended region of values. The values of these quantities determined from the fit by Eq. 3 as shown in Fig.4, fell within this region and when used in the numerical analysis of the magnetoresistance isotherms yielded a reasonable scaling collapse. It is not a surprise that a unique value of $\nu\psi$ could not be determined employing this method. Equation 2, the activated scaling function assumes $\nu\psi$ to be a constant. However, corrections to scaling result in a weak temperature dependence, which was not considered.

The use of Eq. 2 to collapse the data ignores the corrections to scaling which are essential to the temperature dependence of the crossing fields. These corrections vanish in the $T \to 0$, limit, with the crossing field converging to a fixed value. The temperature dependence of the crossing fields can be used as a further check on the consistency of the analysis leading to Fig. 5. In Ref. [20] we showed that the shift in crossing fields is given by

$$\delta_x(T) \sim u \left(\ln \frac{T_0}{T} \right)^{-\frac{1}{\nu\psi} - \frac{\omega}{\psi}}, \tag{4}$$

where u is the leading irrelevant variable responsible for the corrections, and ω is the associated exponent. Here $\delta_x(T) = (B_c - B_x(T))/B_c$ which is the shift in the crossing fields with temperature. The crossing fields shown in the inset of Fig. 3 are plotted as a function of temperature in Fig. 6.

The line is a fit of Eq. 4 to the data. As $T \to 0$, $\delta_x \to 0$ and the crossing fields approach B_c. The extrapolated zero-temperature limit of the crossing fields in Fig. 6 is slightly higher than, but within 0.3%, of the B_c used for best collapse of the data shown in Fig. 5. Examining Fig. 5, we note that the scaling fails at large values of the scaling parameter for both the upper and lower branches. The disconnect regions in the upper branch correspond to fields of 8, 9, 10, 11, and 12 T. There is a similar breakdown in the lower branch at 6.7 T. We believe that the range of magnetic fields between 6.7 and 8 T corresponds to the regime of quantum criticality governed by quantum fluctuations, where scaling should work. At low fields and at sufficiently low temperatures the film is in a superconducting state not influenced by quantum fluctuations of the order parameter. Correspondingly, at high fields at low temperatures it is in a quantum-corrected metallic state that is also not influenced by quantum fluctuations of the order parameter. The breakdown of scaling at high fields in the upper branch corresponds to the magnetic field at which the conductance becomes a linear function of the logarithm of the temperature, which is the signature of a quantum corrected metal.

4 Discussion

The properties of InO_x films depend upon the interplay between carrier concentration and quenched disorder. The former is set by the film deposition process. Annealing can reduce the level of disorder transforming a film from a highly disordered, as prepared state, to a somewhat less disordered more metallic state. Sufficiently disordered, low mobility InO_x films are known to exhibit direct quantum SITs. In contrast, annealed films that are still disordered but have higher mobilities, have been shown in this work to exhibit quantum SMTs governed by an infinite randomness fixed point. The difference between these behaviors lies in the dynamics of rare, locally superconducting regions, close to the quantum critical point. These rare regions are in effect superconducting puddles immersed in an insulating matrix for the low mobility films, and in a metallic matrix for the higher mobility films. According to a classification put forward in Refs. [27,28], the rare region dimensionality needs to be at the lower critical dimension d_c^- of the problem to yield quantum Griffiths singularities. Rare superconducting regions immersed in an insulating matrix are below d_c^- and thus produce only exponentially small corrections to conventional critical behavior. On the other hand, rare regions imbedded in a metallic matrix are right at d_c^- because the coupling to gapless electronic excitations causes Ohmic dissipation that slows down their dynamics. As a consequence, a superconductor-metal transition of a disordered system is expected to exhibit effects due to Griffiths singularities.

It is essential to these arguments that the electrons which cause dissipation penetrate the entire superconducting puddle. Spivak and co-workers [29,23] pointed out that in the limit of large rare-region sizes that the dissipation will scale with the surface of the rare region rather than its volume. This would cut off the quantum Griffiths physics at the lowest temperatures. However, because of the exponential dependence of the rare-region energy scales with its size, this cutoff temperature is expected to be extremely low, leaving a wide temperature range governed by quantum Griffiths physics [30,27].

One must also address the conditions under which quantum Griffiths singularities lead to activated scaling. The answer depends upon whether the Harris criterion is satisfied or not. If the transition in the absence of disorder fulfills the Harris criterion, $d\nu > 2$, then even if Griffiths singularities are present, activated scaling would not be expected. In the case of a clean superconductor-metal transition tuned by magnetic field, $\nu = \frac{1}{2}$ and $d = 2$. As a consequence, the Harris criterion is violated and with the introduction of disorder, activated scaling would be expected [28,31].

These scaling arguments have been confirmed by explicit model calculations. Hoyos et al. studied the effects of dissipation on a disordered quantum phase transition with O(N) order parameter symmetry through the use of a strong disorder renormalization-group theory applied to the Landau-Ginzburg-Wilson field theory appropriate to the problem [17, 18]. With Ohmic dissipation the quantum phase transition was found to be controlled by an infinite randomness fixed point in the universality class of the random transverse field Ising model. The dynamical scaling between the characteristic length scale ξ and the corresponding time scale ξ_τ is not of power-law type, $\xi_\tau \sim \xi^z$, but activated, $\xi_\tau \sim \exp(const \times \xi^\psi)$, leading to Eq. 2.

The films used in this work, which exhibit magnetic field tuned SMTs have higher mobilities than those that exhibit direct quantum-SITs. When the mobility increases, the high-field state becomes more metallic. Ohmic dissipation increases and the quantum critical point changes from that of a conventional SIT to that of a SMT, which is an infinite randomness critical point.

There has been considerable attention paid to the quantum or Bose metal or the so-called failed superconductors [23] in the various discussions of SMTs and SITs. One might ask where this behavior would be found in this data. Films in magnetic fields for which $dR/dT > 0$, with their resistances never falling to zero over the accessible temperature range have been assumed to be superconducting. They could be the regime of failed superconductors or Bose metals. Additional measurements down to lower temperatures could resolve the uncertainty.

In summary, the magnetic-field-tuned quantum SMTs of InO_x films which are less disordered than those exhibiting direct quantum SITs, exhibit quantum Griffiths effects which lead to an infinite-randomness quantum critical point. This is expected for systems with quenched disorder in the presence of Ohmic dissipation and is caused by the formation of large rare regions which are locally ordered superconducting puddles.

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Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. A.B. Harris, J. Phys. C Solid State Physics **7**, 1671 (1974)
2. D.S. Fisher, Phys. Rev. B **51**, 6411 (1995)
3. O. Motrunich, S.C. Mau, D.A. Huse, D.S. Fisher, Phys. Rev. B **61**, 1160 (2000)
4. T. Vojta, J. Phys. A **39**, R143 (2006)
5. T. Vojta, J. Low Temp. Phys. **164**, 299 (2010)

6. T. Vojta, Phys. Rev. Lett. **90**, 107202 (2003)
7. M.P.A. Fisher, Phys. Rev. Lett. **65**, 923 (1990)
8. S.L. Sondhi, S.M. Girvin, J.P. Carini, D. Shahar, Rev. Mod. Phys. **69**, 315 (1997)
9. V.F. Gantmakher, M.V. Golubkov, V.T. Dolgopolov, G.E. Tsydynzhapov, A.A. Shashkin, JETP Lett. **71**, 160 (2000)
10. Y. Xing, H.M. Zhang, H.L. Fu, H. Liu, Y. Sun, J.P. Peng, F. Wang, X. Lin, X.C. Ma, Q.K. Xue, J. Wang, X.C. Xie, Science **350**, 542 (2015)
11. S. Shen, Y. Xing, P. Wang, H. Liu, H.L. Fu, Y. Zhang, L. He, X. Xie, X. C.and Lin, J. Nie, J. Wang, Phys. Rev. B **94**, 144517 (2016)
12. Y. Saito, T. Nojima, Y. Iwasa, Nat. Comm. **9**, 778 (2018)
13. Y. Xing, K. Zhao, P. Shan, F. Zheng, Y. Zhang, H. Fu, Y. Liu, M. Tian, C. Xi, H. Liu, J. Feng, X. Lin, S. Ji, X. Chen, Q.K. Xue, J. Wang, Nano Lett. **17**, 6802 (2017)
14. R.B. Griffiths, Phys. Rev. Lett. **23**, 17 (1969)
15. M. Thill, D.A. Huse, Physica A **214**, 321 (1995)
16. A.P. Young, H. Rieger, Phys. Rev. B **53**, 8486 (1996)
17. J.A. Hoyos, C. Kotabage, T. Vojta, Phys. Rev. Lett **99**, 260601 (2007)
18. T. Vojta, C. Kotabage, J.A. Hoyos, Phys. Rev. B **79**, 024401 (2009)
19. A. Del Maestro, B. Rosenow, J.A. Hoyos, T. Vojta, Phys. Rev. Lett. **105**, 145702 (2010)
20. N.A. Lewellyn, I.L. Percher, J. Nelson, J. G.-Barriocanal, I. Volotsenko, A. Frydman, T. Vojta, A.M. Goldman, Phys. Rev. B **99**, 054515 (2019)
21. Z. Ovadyahu, J. Phys. C: Solid State Physics **19**, 5187 (1986)
22. I. Tamir, A. Benyamini, E.J. Telford, F. Gorniaczyk, A. Doron, T. Levinson, D. Wang, F. Gay, B. Sacépé, J. Hone, K. Watanabe, T. Taniguchi, C.R. Dean, A.N. Pasupathy, D. Shahar, arXiv:1804.04648v1 [cond-mat.supr-con] (2018)
23. A. Kapitulnik, S.A. Kivelson, B. Spivak, Rev. Mod. Phys **91**, 011002 (2019)
24. B.I. Altshuler, A.G. Aronov, D.E. Khmelnitskii, A.I. Larkin, *Quantum Theory of Solids*. Physics Series (MIR Publishers, Moscow, 1982)
25. B. Kramer, A. MacKinnon, Rep. Prog. Phys. **56**, 1469 (1993)
26. B. Skinner, J. Ruhman, A. Nahum, arXiv:1808.05953v3 [cond-mat.stat-mech]
27. T. Vojta, J. Schmalian, Phys. Rev. B **72**, 045438 (2005)
28. T. Vojta, J.A. Hoyos, Phys. Rev. Lett. **112**, 075702 (2014)
29. B. Spivak, P. Oreto, S.A. Kivelson, Phys. Rev. B **77**, 214523 (2008)
30. A.J. Millis, D.K. Morr, J. Schmalian, Phys. Rev. B **66**, 174433 (2002)
31. S. Sachdev, P. Werner, M. Troyer, Phys. Rev. Lett. **92**, 237003 (2004)