

Reactions leading to the first excited states of ${}^7\text{Li}$ and ${}^7\text{Be}$ and isospin-mixed states in ${}^8\text{Be}$

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Background: Ratios of cross sections for mirror reactions sometimes deviate from the values expected on the basis of charge symmetry. These deviations are attributed to smoothly varying charge-dependent effects (proton charge, different Q values) and to the effects of isospin mixed states in the compound nucleus. The effects are large and well known for certain positive-parity states in ${}^8\text{Be}$, but information is lacking for negative-parity states.

Purpose: We measure the excitation functions of angle integrated cross sections for two pairs of reactions that involve ${}^8\text{Be}$ as an intermediate state: (1) ${}^6\text{Li}(d, p'){}^7\text{Li}$ (0.478 MeV) and ${}^6\text{Li}(d, n'){}^7\text{Be}$ (0.429 MeV) and (2) ${}^7\text{Li}(p, p'){}^7\text{Li}$ (0.478 MeV) and ${}^7\text{Li}(p, n'){}^7\text{Be}$ (0.429 MeV), measure the ratios of the neutron-emitting and proton-emitting reactions, and examine the implications for the structure of ${}^8\text{Be}$.

Method: The ratios were determined by observing the isotropically emitted γ rays from the decay of ${}^7\text{Li}$ (0.478 MeV) and ${}^7\text{Be}$ (0.429 MeV). Shell model calculations were performed for both positive- and negative-parity states. Results were compared to existing information.

Results: Ratios, usually with an accuracy of $\pm 2\%$, were obtained for deuteron energies from 0.15 to 7.2 MeV and proton energies from 3.0 to 10.0 MeV. There were relatively strong deviations from expectations based on charge symmetry at the lowest deuteron energies and smaller deviations between $E_d = 2$ and 4 MeV. There were very strong deviations for proton energies near 3 and 5.5 MeV, with strong neutron strength near 3 MeV and strong proton strength near 5 MeV. The shell model calculations were generally in good agreement for the positive-parity states and, with some exceptions, for negative-parity states. There is reasonable evidence for the lowest lying $2^- T = 1$ state at $E_x = 20.2$ MeV and for two new isospin mixed pairs, one for 2^+ states near 22 MeV and another for 2^- states near 24 MeV.

Conclusions: The results will constrain future calculations for isospin mixed states in ${}^8\text{Be}$.

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I. INTRODUCTION

The ${}^8\text{Be}$ nucleus has many attractions for theoretical study. It is light enough that many theoretical approaches are possible and, although it consists of only eight nucleons, it exhibits striking phenomena. It has, for example, strong rotational structures and strongly isospin-mixed states. There have been many experimental studies of ${}^8\text{Be}$, but its structure is still not well known, partly because the levels are broad and overlapping and partly because isospin does not seem to be a good quantum number for many of these states.

The present experiments were intended to clarify the prevalence of isospin mixed states in ${}^8\text{Be}$ [1].

For these measurements, we use the systems shown in Fig. 1. Wilkinson [2] noted that the observation of γ -ray emission from ${}^8\text{Be}$, following nucleon decay, was uniquely advantageous for searches for violations of charge independence of nuclear forces and for isospin mixed states. The first excited states, ${}^7\text{Be}(0.429 \text{ MeV})$ and ${}^7\text{Li}(0.478 \text{ MeV})$, both have spin $1/2^-$ so their γ -decay angular distributions are isotropic. A measurement of their intensity at any angle is then a measurement of their total cross section. In the remainder of the paper, we denote these total cross sections ${}^6\text{Li}(d, n')$, ${}^6\text{Li}(d, p')$, ${}^7\text{Li}(p, n')$, and ${}^7\text{Li}(p, p')$ by (d, n') , (d, p') , (p, n') , and (p, p') , respectively, and their ratios by $(d, n')/(d, p')$ and $(p, n')/(p, p')$. Since there are no other stable states, these ratios are not significantly affected by γ -ray cascades. Finally, the Coulomb barriers are low enough that one does not expect Coulomb and phase-space-dependent effects to greatly change the ratios from their values assuming charge symmetry of nuclear forces.

Measurements of $(d, n')/(d, p')$ by Cecil *et al.* [3] and Czerski *et al.* [4] found significant deviations from unity but

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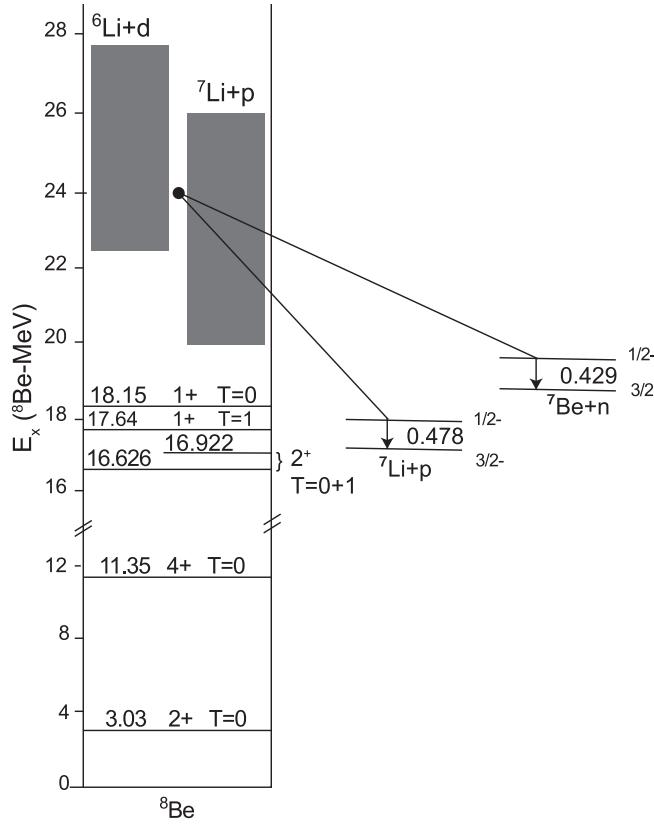


FIG. 1. Energetics of the ${}^6\text{Li} + d$ and ${}^7\text{Li} + p$ reactions leading to the first excited states of ${}^7\text{Li}$ and ${}^7\text{Be}$. Only the lower lying ${}^8\text{Be}$ levels are shown. The ranges of E_x accessed by the two reactions are shown by the shaded regions.

covered a very limited energy range: 0.06–0.18 MeV. An extensive set of measurements comparing the ${}^7\text{Li}(p, p'){}^7\text{Li}$, ${}^7\text{Li}(n, n'){}^7\text{Li}$, and ${}^7\text{Li}(p, n'){}^7\text{Be}$ reactions was carried out by Presser and Bass [5].

This paper describes measurements of $(d, n')/(d, p')$ and $(p, n')/(p, p')$, each over a larger energy range than was previously available, and of the excitation functions of the individual reactions. The data reported here were taken and analyzed [6,7] in the late 1960s but never published.¹ In Secs. II and III, we describe the experimental procedures and the analysis of the data. In Sec. IV, we discuss the variations in the observed ratios and their possible correlation with levels known in the literature and, in Sec. V, with shell model calculations.

II. DEUTERON-INDUCED REACTIONS

Metallic targets of ${}^6\text{Li}$ were bombarded with deuterons with energies between 0.1 to 7.2 MeV produced by the Stanford 3-MeV Van de Graaff and FN Tandem. The thin targets were prepared by evaporation of isotopically enriched

metal onto heavy metal backings *in situ* and had energy losses for deuterons of from 10 to 50 keV.

The 429- and 478-keV γ rays from the first excited states in ${}^7\text{Be}$ and ${}^7\text{Li}$, respectively, were observed with a small, 6-cm³ ($1 \times 2 \times 3$ cm) Ge(Li) detector placed at 90 deg from the target so as to yield nearly symmetric γ -ray peaks and thereby simplify data analysis. Detector resolution was 5.0-keV FWHM at 511 keV; the Doppler broadened peaks from the de-excitation γ rays were significantly broader. In addition to the γ rays from ${}^7\text{Be}$ and ${}^7\text{Li}$, there is a positron annihilation peak at 511 keV and, at the higher energies, a peak at 495 keV from the ${}^{16}\text{O}(d, n){}^{17}\text{F}$ reaction.

The statistical error of each ratio measurement was less than $\pm 1.5\%$ except for $E_d < 350$ keV. The total error in $(d, n')/(d, p')$ was estimated from various trial analyses and by varying parameters; it is usually around 2% but varies slowly with energy as the peak shapes change with bombarding energy. Other systematic errors, such as those related to beam integration, uneven target thickness, and dead-time effects, do not affect $(d, n')/(d, p')$; The separate neutron and proton cross sections are subject to them and uncertainties are larger, typically $\pm 5\%$.

Several phenomena do affect the cross-section ratio. The ${}^7\text{Li}(d, d')$ reaction on the small amount of ${}^7\text{Li}$ in the enriched ${}^6\text{Li}$ target contributes to the ${}^7\text{Li}$ (478-keV) peak. The ${}^7\text{Li}(d, d')$ excitation function was measured from threshold to 8 MeV and the results are used to correct the measured ratios for the 0.5% ${}^7\text{Li}$ impurity.

This leads to a 3% increase in the ratio at 4 MeV and a smaller correction elsewhere. At energies below 1 MeV, the effect is negligible.

${}^7\text{Be}$ ($t_{1/2} = 53.22$ days) builds up during the measurement and decays about 10% of the time to the first excited state in ${}^7\text{Li}$. The effect was minimized by measuring the smaller cross sections first, and was always less than 0.5% of the prompt value.

Finally, the observed ratio has to be corrected for the difference in efficiency of the 6-cc planar Ge(Li) detector for the 429- and 478-keV γ rays. The efficiency ratio obtained was $\epsilon(429)/\epsilon(478) = 1.19$, which divides the raw results.

The corrected ratio is shown in Fig. 2. The cross section ratio averages about 1.07 over the range from 2 to 7.2 MeV, with fluctuations of at least twice the systematic uncertainties near the positions of known levels in ${}^8\text{Be}$, and falls off steeply at very low bombarding energies. This fall-off is consistent with other low-energy experiments, as shown in Fig. 4.

III. PROTON-INDUCED REACTIONS

While the procedures were mostly the same as those just discussed, the measurements did not extend to such low bombarding energies because the ${}^7\text{Li}(p, n')$ reaction is endoergic with a Q value of -1.644 MeV. Self-supporting rolled ${}^7\text{Li}$ targets were used at the higher energies.

The values of $(p, n')/(p, p')$ are generally much less than one, owing to contributions from direct processes. Measurements in the 20- to 50-MeV range [9] find values of about one third because the effective interaction mediating inelastic scattering is significantly larger than that mediating charge

¹A very brief preliminary description of some of the results was given in Ref. [8].

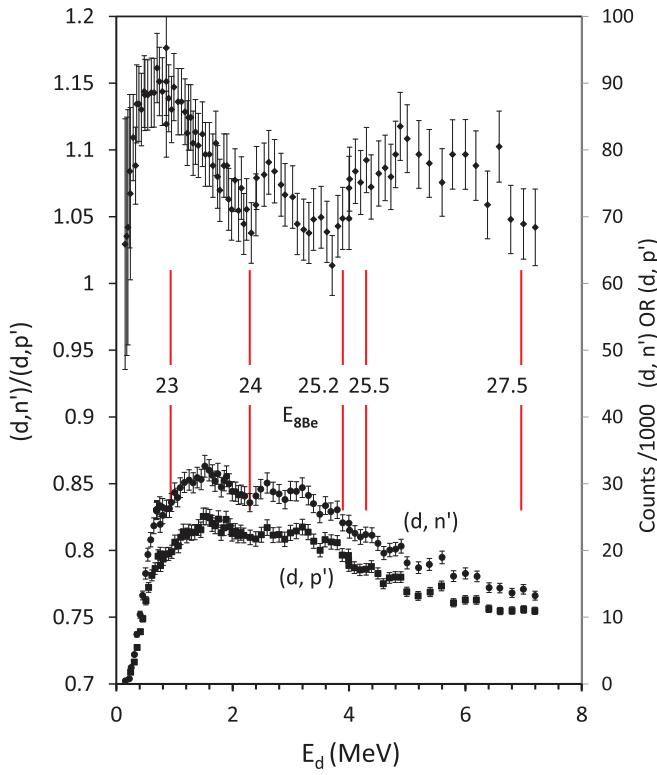


FIG. 2. The ${}^6\text{Li}(d, n')$ and ${}^6\text{Li}(d, p')$ angle integrated excitation functions and their ratio for deuteron energies between 0.15 and 7.2 MeV. The ratio is corrected for detector efficiency (a factor of 1.19 as described in the text) but the excitation functions are not. The error bars shown for the ratio, $\pm 2\%$, include slowly varying systematic uncertainties of around $\pm 1.5\%$. The uncertainties for the individual cross sections are $\pm 5\%$, for reasons noted in the text. Also shown are the energies of known states in ${}^8\text{Be}$.

exchange. The dominant feature of Fig. 3 is a strong decrease in the ratio from a peak at 3.31 MeV [$E_x({}^8\text{Be}) = 20.2$ MeV] and dominant neutron emission, to a valley at 5.5 MeV [$E_x({}^8\text{Be}) = 22.1$ MeV] and dominant proton emission.

IV. DISCUSSION

A. General comments

States of pure isospin in ${}^8\text{Be}$ have specific decay properties: For example, states with $T = 2$ cannot emit protons or neutrons to the first excited states of ${}^7\text{Li}$ or ${}^7\text{Be}$, states with $T = 0$ can emit α particles, and those with $T = 0$ or 1 can emit protons and neutrons. A state with a mixture of $T = 0$ and $T = 1$ can emit α particles and protons or neutrons, with their relative intensity depending on phase and phase-space relationships.

The present experiments were motivated by Wilkinson [10], who noted that if a compound state is initially formed with well-defined isospin, then isospin mixed states occur only if the relatively weak Coulomb matrix elements (CME) have time to mix states in ${}^8\text{Be}$ of the same J^π and different isospin. Mixing would occur neither at low excitation energy because the states are too far apart for the CME to be effective nor at very high excitation energies because the

states are too short lived for there to be time for mixing. While these arguments are crude and do not take in account qualitative structure changes in different nuclei, they may give rough guidance. For a more general point of view, see Ref. [11].

Wilkinson found strong mixing between 10 and 30 MeV in ${}^{16}\text{O}$. The best studied isospin mixed pair in ${}^8\text{Be}$ [1,12] is the 2^+ doublet at 16.6 and 16.9 MeV (see shell model discussion in Sec. IV C). The present experiments indicate that mixed states may occur in the $E_x = 20$ –26 MeV range.

B. ${}^6\text{Li}(d, n')/{}^6\text{Li}(d, p')$

Assuming charge symmetry, no Coulomb effects, and equal Q values, the ratio $(d, n')/(d, p')$ should be unity. To evaluate the effects of the Q value differences and the Coulomb interaction requires a reaction model. For low deuteron energies, $E_d \lesssim 0.4$ MeV, the observed differential cross sections are nearly isotropic, presumably indicating that compound nuclear precesses dominate, while at higher energies the data are forward peaked, indicating that direct reactions (stripping) are important [13]. A detailed analysis in terms of the Wigner-Eisenbud formalism [14] showed that both compound and direct (stripping) processes are important in the $E_d = 0.1$ to 1.0 MeV range and that direct processes are increasingly important at higher energies.

To determine whether a stripping model might explain the decrease in $(d, n')/(d, p')$ at low energies shown in Fig. 4, Czerski *et al.* [4] calculated the ratio in the Distorted Wave Born Approximation. They found, instead, that it increases as the bombarding energy decreases, reaching about 1.4 at $E_d = 0.12$ MeV, while the observed ratio is about 1.0 and is constant or decreasing as energy decreases.

Some decrease at low energies would be expected in a stripping model because the deuteron is polarized in the field of the ${}^6\text{Li}$ nucleus, so the proton is further from the nuclear center: the Oppenheimer-Phillips effect (OPE) [15]. An attempt to describe this effect phenomenologically [3] led to ambiguous results. Recent experiments [16,17] and more detailed but still simplified theoretical approaches indicate that these effects are too small to explain the $(d, n')/(d, p')$ data. For example, Koonin [18] found 5–10% decreases, with increasing energy, for both the ${}^6\text{Li}(d, n')$ and ${}^6\text{Li}(d, p')$ cross sections owing to these effects. The relative decreases were roughly independent of energy. As a result, the ratio changed by much less than observed at low energies and in the wrong direction. Other processes [19] may increase the ratio at somewhat higher energies, but there have been no detailed calculations.

This led to the suggestion [4] that the bulk of the decrease in the ratio at lower energies is owed to a known $2^+ T = 0$, subthreshold level ($E_x = 22.2$ MeV, width 800 keV) in ${}^8\text{Be}$ that emits primarily protons and yields an enhanced (d, p') cross section and decreased ratio. This appears to provide a reasonable explanation of the observed decrease at low energies, although in detail it depends on the questionable validity of calculations of stripping processes at these low energies. If this explanation is correct, the 22.2-MeV state, although previously assigned isospin $T = 0$ [1] because it

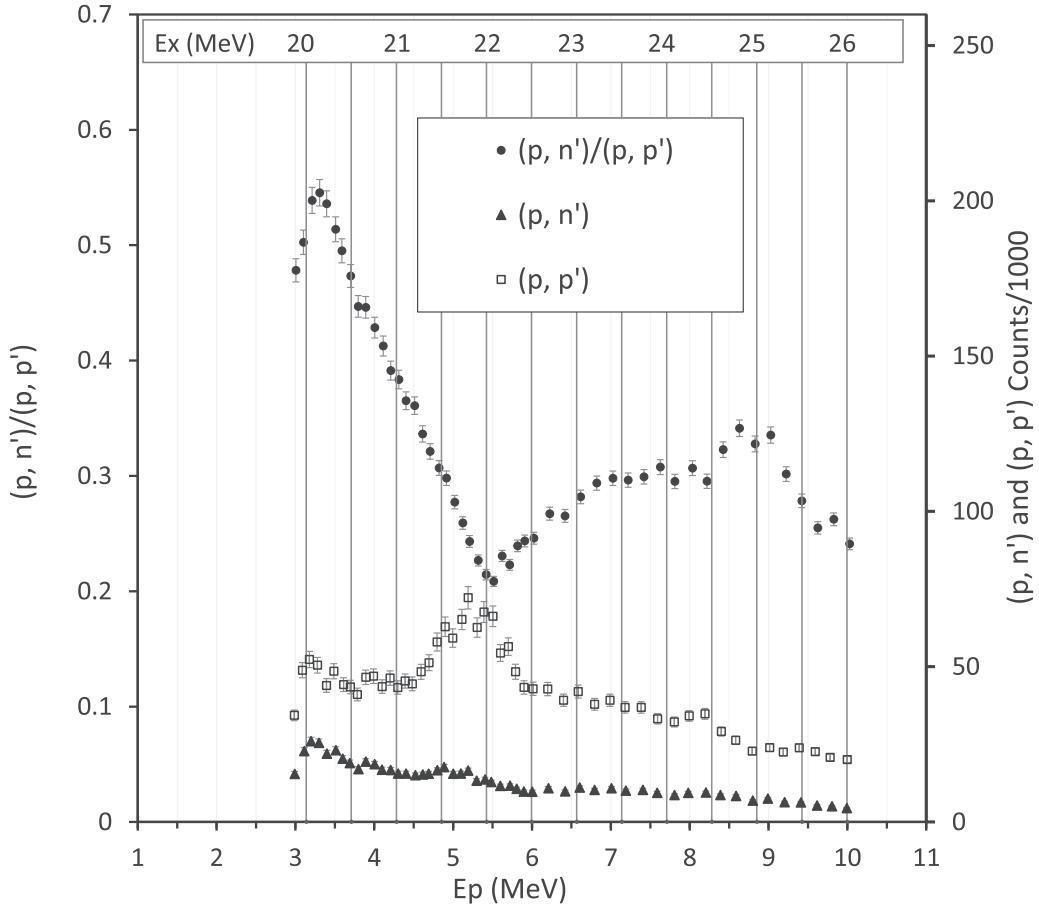


FIG. 3. The ${}^7\text{Li}(p, n')$ and ${}^7\text{Li}(p, p')$ angle integrated excitation functions for proton energies between 3 and 10 MeV and their ratio. The ratio is corrected for detector efficiency (a factor of 1.19 as described in the text) but the excitation functions are not. The statistical uncertainties in the ratio are 1–1.5% with the larger values above 8 MeV. The plotted errors include systematic effects and are about twice the statistical error as shown in the figure. The uncertainties in the individual cross sections are larger, around $\pm 5\%$, for reasons noted in the text. Also shown as vertical lines are the relationships between bombarding energy and specific values of excitation energy.

emits α particles, must be an isospin mixed state, and there must be a nearby 2^+ isospin mixed level that emits mostly neutrons. There are no experimentally known 2^+ states nearby but there are uncharacterized states at 22.6 and 22.98 MeV that might be candidates for this state. The shell model calculations shown below do have a close-lying pair of 2^+ levels at 20 ($T = 0$) and 20.5 ($T = 1$) MeV, a result consistent with these conclusions. We return to this discussion in Sec. IV C.

We next examine whether, as an alternative explanation, the observed ratios could be explained, at least partially, by penetrability effects for unmixed states. If a level has well-defined T , its intrinsic decay amplitudes for neutrons and protons will be equal, and the observed ratio will be given by the ratio of the penetration factors for the neutron and proton channels: $(d, n')/(d, p') = P_n/P_p$. We have calculated these cross-section ratios using the AZURE [20] code. From 0.5 to 0.1 MeV, the measured ratio decreases by around 15–20%. The $L = 1$ and $L = 3$ ratios decrease by 2% and 9%, which is not sufficient to explain the observed low-energy decrease, especially since $L = 1$ transitions are expected to dominate when they are allowed.

The higher lying variations in the ratio seen in Fig. 2 also lie near observed states in ${}^8\text{Be}$: a very broad giant resonance state with width of 7000 keV ($1, 2)^- T = 1$ at 24 MeV, $2^+ T = 0$ at 25.2 MeV, and $4^+ T = 1$ at 25.5 MeV. It is not known whether there are nearby states that could mix with any of these and cause the observed ratio increases owing to isospin impurities.

C. ${}^7\text{Li}(p, n')/{}^7\text{Li}(p, p')$

The ratio data of Fig. 3 have a strong neutron enhanced peak at $E_x = 20.2$ MeV. This is near the $2^- T = 1$ level at 20.8 MeV predicted by Barker [21] in an analysis of data for the $2^- T = 0$ state at 18.91 MeV [22,23]. That data were interpreted as resulting from isospin mixing with a higher lying $2^- T = 1$ state. The R -matrix analysis was difficult [12,21–23], since the $T = 0$ state is near the neutron separation energy of 18.899 MeV. The deduced isospin impurity ranges from $\approx 7\%$ [23] to $\approx 24\%$ [12,23]. The two-level R -matrix analysis of Barker [21] required a $T = 1$ state at $E_x = 20.8$ MeV with a mixing matrix element of 0.58 MeV and strongly enhanced neutron emission.

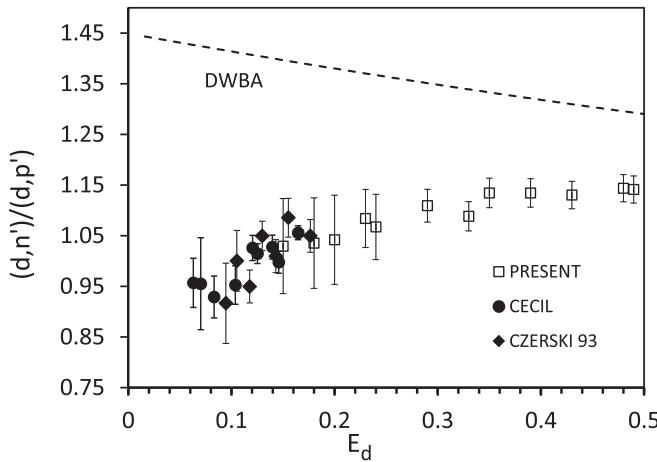


FIG. 4. Cross-section ratios for ${}^6\text{Li}+d$ reactions from previous experiments. The lower energy results from the present experiment are shown for comparison. The excitation energy in ${}^8\text{Be}$ is 22.28 MeV (22.66 MeV) at $E_d = 0$ MeV (0.5 MeV). A direct reaction (stripping) calculation [4] is shown for comparison.

The (p, n') and (p, p') excitation functions shown in Fig. 5 have a peak at 5.24 MeV ($E_x = 21.8$ MeV) with locally strong proton emission and a peak at 5.06 ($E_x = 21.7$ MeV) with locally strong neutron emission. There are also (probably) weak peaks in the ratio near 7 and 8 MeV ($E_x = 23$ and 24 MeV). The rapid change in the cross-section ratio observed in this experiment is an indication of isospin mixing.²

The properties of proton enhanced peak are consistent with those of the observed 2^+ state at $E_x = 22.2$ MeV. As noted above, Czerski [4] assumed this was a mixed isospin state that dominantly emitted protons so as to explain the low energy decrease of $(d, n')/(d, p')$. The neutron enhanced peak at 21.7 MeV is a likely candidate for the mixing state.

V. SHELL MODEL CALCULATIONS

We have carried out shell model calculations for ${}^8\text{Be}$ with the $0p$ model space for positive-parity states and then $1\hbar\omega$ excitations beyond this for the negative-parity states. For the $0p$ Hamiltonian, we use the PJT interaction from Ref. [24]. For the $1\hbar\omega$ Hamiltonian, we combine PJT with the cross-shell part of the WBT Hamiltonian from Ref. [25]. The $0p$ shell results for positive-parity states are shown in Fig. 6. The agreement with experiment is good for the 2^+ and 4^+ states with $E_x < 14$ MeV and is satisfactory for higher energies. In particular, the calculation reproduces the well-known isospin doublets for 1^+ , 2^+ , and 3^+ . The 2^+ doublet is known to have

²We made a single-level R -matrix fit to the lowest lying peak in each of these these excitation functions using the AZURE code [20] with a radius of 4.2 fm and channel spin (orbital angular momentum) of 1(2). The resulting fits are, within statistics, essentially identical to the curves of Fig. 5. These fits are not unique and the data are insufficient, but they yield widths consistent with those expected for strong excitations: less than a quarter of the Wigner limit.

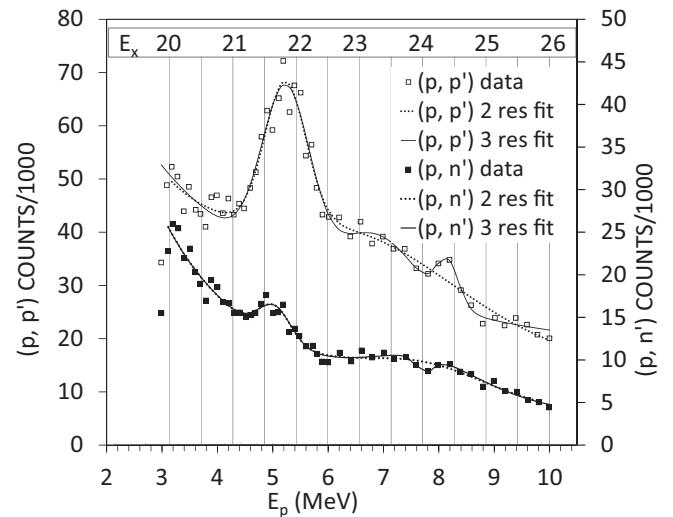


FIG. 5. Non-linear least squares fits to the (p, n') and (p, p') excitation functions shown in Fig. 3. The plotted uncertainties correspond to estimated systematic uncertainties as discussed in the text. A power law background of the form $p_1 E_p^{p_2}$ plus two or three Gaussian peaks were fitted to the data. The vertical lines show the relationships between bombarding energy and specific values of excitation energy

about equal mixing between $T = 0$ and $T = 1$, resulting in the lower energy state with the structure of ${}^7\text{B} + \text{proton}$, the higher energy state with the structure of ${}^7\text{Be} + \text{neutron}$, and an isospin mixing matrix element about equal to the level spacing of 0.296 MeV. There is another 2^+ pair near $E_x = 20$ MeV; this pair may be isospin mixed and account for the levels near $E_p = 5$ MeV shown in Fig. 5.

The $1\hbar\omega$ results for negative-parity states are shown in Fig. 7. Again, the agreement with the known states is good, except for the first $1^- T = 0$ state. This state may be missed due to a large α -decay width.

The lowest $2^- T = 1$ state is calculated to come at $E_x = 21.5$ MeV, not far from the energy of $E_x = 20.8$ MeV required by the two-level R -matrix analysis of Barker [21] or from the 20.2-MeV peak in the ratios shown in Fig. 3. It is reasonable to identify this state with the observed peak. There are $2^- T = 0$ states at $E_x = 20.94$, 22.08, and 22.29 MeV which may also mix with this $T = 1$ peak.

Finally, there is a pair 2^- states near $E_x = 24$ MeV with the $T = 1$ state lying about 300 keV below the $T = 0$ state. Their mixing may be responsible for the structure observed near 24 MeV in $(d, n')/(d, p')$ of Fig. 2.

VI. CONCLUSIONS

A consistent, although speculative, picture follows from the previous discussion. The explanation [4] of the decrease of $(d, n')/(d, p')$ toward low E_d , was that it owed to a 2^+ state at $E_x = 22.2$ MeV, below the threshold for ${}^6\text{Li} + d$ (see Fig. 3), that is isospin mixed and emits mainly protons. It is tempting to identify this state with the proton emitting level seen in ${}^7\text{Li} + p$ at $E_p = 5.24$ MeV, $E_x = 21.8$ MeV (see Fig. 5); this state is presumably mixed with the dominantly neutron emitting level lying $E_p = 5.06$ MeV, $E_x = 21.7$ MeV. The

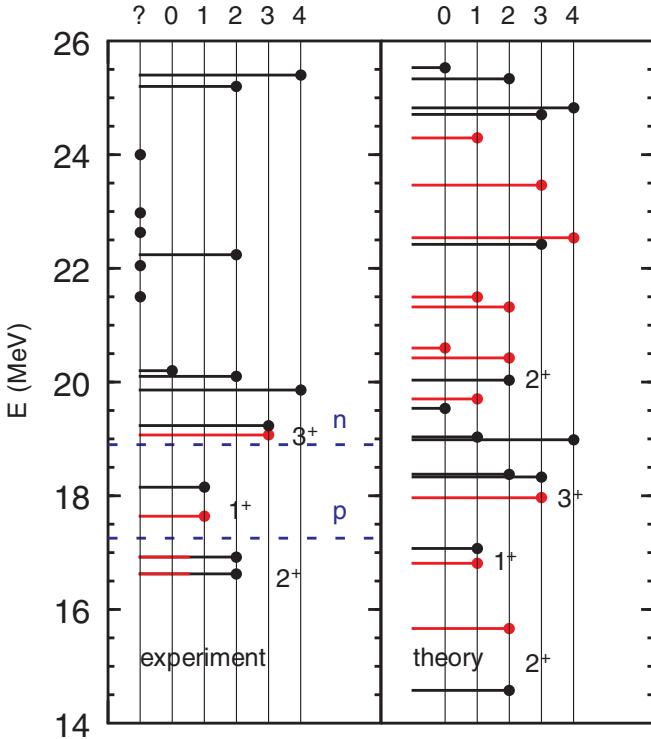


FIG. 6. Positive-parity states in ${}^8\text{Be}$. Experiment is shown on the left and the results of the calculation with the PJT Hamiltonian on the right. The dots together with the vertical lines give the J of the level; some of the J values are labeled for guidance. The vertical lines are labeled by the J of the levels; the ? indicates experimental states with uncertain J^π assignments. States with $T = 0$ are shown in black and states with $T = 1$ are shown in red. The pair of experimental 2^+ states near 17 MeV is shown half black and half red to indicate the complete isospin mixing. The neutron and proton separation energies are shown by the dashed blue lines.

shell model calculations have a 2^+ doublet with a $T = 0$ state at 20.0 MeV and a $T = 1$ state at 20.4 MeV.

The $(d, n')/(d, p')$ data also have a pronounced structure just above $E_x = 24$ MeV that may be related to mixing of the pair of 2^- states with $T = 0$ and $T = 1$ predicted by the shell model calculations to lie near 24 MeV.

The present $(p, n')/(p, p')$ results (Fig. 3) also give evidence for an isospin-mixed, neutron-emitting state in ${}^8\text{Be}$ at an excitation energy of 20.2 MeV. This is consistent with Barker's prediction [21] of a $2^- T = 1$, mainly neutron-emitting, state at $E_x = 20.8$ MeV, and with our shell model prediction that the lowest negative-parity $T = 1$ level in ${}^8\text{Be}$ is at 21.5 MeV. A speculative explanation for the general increase in relative neutron emission for $E_x > 24$ MeV in Figs. 2 and 3, might be isospin mixing, as described in Sokolov and Zelevinsky [11], of a broad simple state (for example, the wide $(1^-, 2^- T = 1$ giant resonance state at $E_x = 24$ MeV [1,5]), and higher lying relatively narrow states.

We have not made more definitive statements about the states involved in mixing because of the limitations of the experimental data for ${}^8\text{Be}$ and of the available theoretical calculations. At present *ab initio* calculations have only

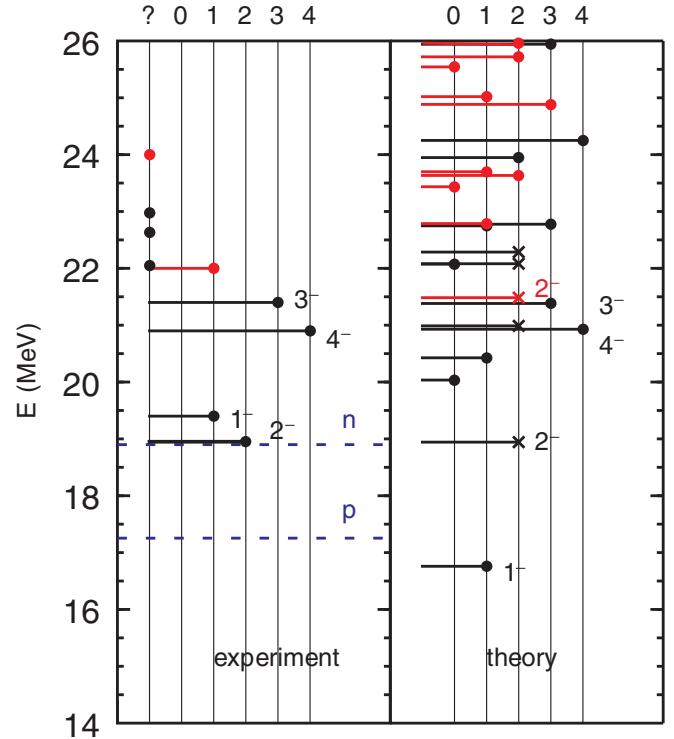


FIG. 7. Negative-parity states in ${}^8\text{Be}$. Experimental states are shown on the left and the results of the calculation with the PJT + WBT Hamiltonians on the right. For other details, see the caption of Fig. 6. The 2^- states discussed in the text are those on the right-hand side labeled by crosses; one of these has $T = 1$ and three of them have $T = 0$.

been carried out for positive-parity states that have a dominant p -shell configuration (Refs. [26,27] and Table VII in Ref. [28]). In addition, the widths of the states involved means that continuum effects must be introduced into the shell model and the description of the isospin mixing. To eliminate these limitations is a major task. Our measurements and suggestions for the energy of the lowest $T = 1$ negative-parity state near 20.3 MeV, of a pair of mixed 2^+ states near 20 MeV, and of a pair of mixed 2^- states near 24 MeV will be important constraints for these more advanced calculations.

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