Microwave Circuits with Exceptional Points and Applications in Oscillators and Sensors

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***Abstract*— We investigate wave properties in coupled transmission lines (CTLs) under a special condition known as the exceptional point of degeneracy (EPD) at which two or more of the supported eigenmodes of the system coalesce. At an EPD, not only the eigenvalues (resonances or wavenumbers) of the system (a resonator or a waveguide) coalesce but also the *eigenvectors* (polarization states) coalesce, and the number of coalescing eigenmodes defines the order of the degeneracy. We investigate different structures, either periodic or uniform CTLs, that are capable of exhibiting EPDs in their dispersion diagram. Secondly, we show an experimental verification of the existence of EPDs through measuring the dispersion of microstrip-based CTLs in the microwave spectrum. For antenna array configurations, we discuss the effect of CTLs radiative and dissipative losses on EPDs and how introducing gain to the CTLs compensate for such losses restoring the EPD in a fully radiating array, in what we define as the *gain and distributed-radiation balance* regime. Therefore, we show how to obtain large linear and planar arrays that efficiently generate microwave oscillations, and by spatial combination they are able to generate collimated beams with large radiation intensity. Finally, we show other promising applications based on the concept of EPDs in ultra-sensitive sensors or reconfigurable antennas.**

Keywords*— *Exceptional points, coupled transmission lines, periodic structures, PT-symmetry, oscillators and sensors.

I. Introduction

The emerging concept of exceptional point of degeneracy (EPD) and Parity-Time (PT)-symmetry have raised the attention to interesting phenomena in electromagnetics and physics [1]–[3]. Exceptional points of degeneracy (EPDs) are points at which two or more eigenmodes coalesce in both their eigenvalues and eigenvectors. For instance, at an EPD, the matrix describing the mode evolution in the structure is similar to a Jordan block or to a matrix containing at least a non-trivial Jordan block. On the other hand, PT-symmetry is a condition at which the system possesses real eigenvalues despite having losses and gain; and is implemented in CTLs by using balanced and symmetrical gain and loss or even with asymmetric distributions of gain and loss [4] where loss in our case is actually representing radiation leaking away from a microwave structure. These EPDs can also exist in coupled resonator structures whose system evolution is described in time [5], [6].

The EPD is defined by its order which is equivalent to the number of coalescing eigenmodes. For instance, the system exhibits 2nd order EPD if only two eigenmodes are coalescing and we refer to this case as the regular band edge (RBE). Similarly, in a lossless system we define the 3rd and 4th order EPDs as the stationary inflection point (SIP), and the degenerate band edge (DBE), respectively [7], [8]. The DBE based on two coupled microstrips was shown in [9], whereas an experimental demonstration of the DBE at microwave frequencies was shown in [10] in the case of cylindrical waveguides with periodic inclusions. Here we show an experimental demonstration of the DBE in CTLs made of two coupled microstrip lines as shown in Figs. 1(a) and 2.

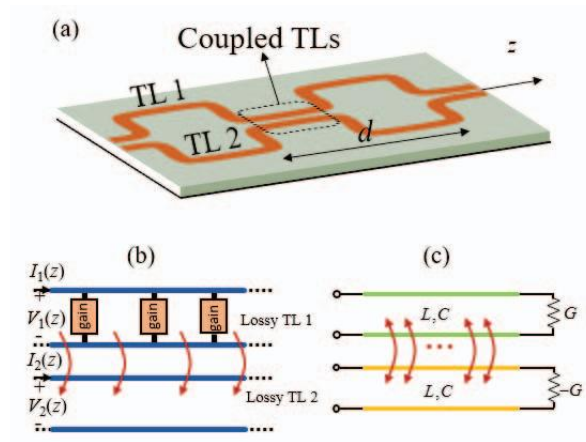

Fig. 1. (a) Periodic coupled TLs (CTLs), capable of exhibiting EPDs of orders 2 and 4. In this case the two CTLs are made of two coupled microstrip lines on a grounded dielectric slab. (b) Uniform coupled waveguides with balanced gain and loss (not necessarily topologically-symmetric) develop an EPD in their dispersion diagram. (c) Two coupled resonators made of two finite length CTLs where one is terminated with loss and the other with gain. These circuits can form an oscillator based on EPD or a sensor based on EPD.

Generally, the system dispersion relation at an EPD of order m has an asymptotic behavior of $(\omega-\omega_e)\propto(k-k_e)^m$ near the EPD, where ω and k are the angular frequency and the guided wavenumber, respectively, and the EPD is designated with the subscript e. Such dispersion behavior is accompanied by severe reduction in the group velocity of waves propagating in those structures. When designing a cavity based on CTL with an EPD, we get giant scaling of the loaded quality factor (Q_{Loaded}) compared to a cavity without an EPD according to the scaling law [7], [11], [12]

$$Q_{Loaded}\propto N^{m+1} \tag{1}$$

where N is the number of unit cells in a periodic CTL [Fig 1(a)]. This giant scaling of Q has several interesting applications such as making oscillators with low threshold or with high power efficiency.

II. Exceptional points of Degenracy in CTLs

To explain the existence of EPDs in CTLs, we start by representing field amplitudes in the two CTLs by voltage and current vectors $\mathbf{V}(z)=\begin{bmatrix}V_1(z) & V_2(z)\end{bmatrix}^T$ and

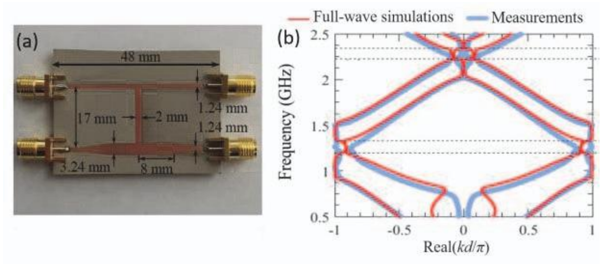

Fig. 2. (a) The fabricated microstrip unit cell of a periodic CTL that exhibits fourth order EPDs, namely the DBEs. (b) Dispersion relation measurement versus full-wave simulation of the unit cell of a periodic CTL. This structure exhibits DBEs occurring at various frequencies as shown in the dispersion diagram. Only the real part of the wavenumber versus frequency is shown here.

$\mathbf{I}(z)=\left[I_1(z) \quad I_2(z)\right]^T$ where T denotes the transpose operation, as shown in Fig.1 (b). It is convenient to describe the system evolution by defining the four-dimensional state vector $\boldsymbol{\psi}(z)=\left[\mathbf{V}^T(z) \quad \mathbf{I}^T(z)\right]^T$. The system evolution equation along the z-direction is written as $\partial\boldsymbol{\psi}/\partial z=-j\underline{\mathbf{M}}(z)\boldsymbol{\psi}$, with $\underline{\mathbf{M}}(z)$ being the 4×4 CTL system matrix, where we have adopted the $e^{j\omega t}$ time convention.

Uniform CTLs: For uniform (i.e., z-invariant) CTLs as shown if Fig. 1(b), the EPD is a point in the system parameter space at which there is an eigenvalue of the system matrix $\underline{\mathbf{M}}$ whose geometric multiplicity is less than its algebraic multiplicity [13]. At the EPD the system evolution cannot be described using regular eigenvectors therefore we use the generalized eigenvectors instead. Hence, the eigenmodes algebraically diverge along the z-direction as $\boldsymbol{\Psi}(z)\propto z^{q-1}e^{-jkz}\,\boldsymbol{\Psi}_q(0)$, where $q=2,3,...,m$ with m being the order of the EPD. In this case we can get the EPD based on the PT-symmetry concept by introducing gain in one transmission line while the other has losses.

Periodic CTLs: The EPD condition may also arise in periodic coupled waveguides as those shown in Fig. 1(a). The state vector across the unit cell is translated between two points z and $z+d$, where d is the period, as $\boldsymbol{\psi}(z+d)=\underline{\mathbf{T}}(z+d,z)\boldsymbol{\psi}(z)$. Similar to the uniform case, the EPD emerges in this periodic system when the transfer matrix $\underline{\mathbf{T}}$ is similar to a matrix possessing a non-trivial Jordan block, i.e., it is non-diagonalizable. Moreover, the degenerate eigenmode solution in such periodic systems algebraically diverges along the z-direction. In this case the system may have an EPD also when it is lossless and gainless, and still $\underline{\mathbf{T}}$ is similar to a Jordan block.

In analogy to systems where the mode evolution is described in space, along the z direction, EPDs also exist in systems with temporal mode evolution [5], [6]. An example of such system is the finite length coupled resonators made of two CTLs in Fig. 1(c) where one of them is terminated by a load (it could be a resistor or an antenna), represented by positive conductance G, while the other TL is terminated by gain, represented by a negative conductance $-G$.

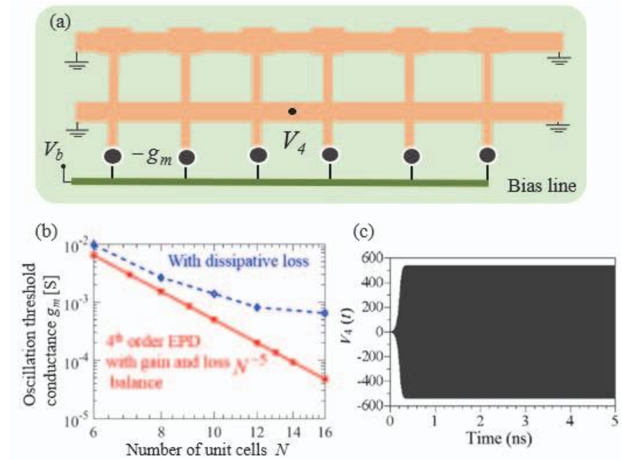

Fig. 3. (a) CTL with gain and loss balance made of a finite number of unit cells. The gain in each unit cell is represented by a negative conductance g_m. The structure is terminated in short circuits and oscillates for sufficiently large g_m. (b) Minimum gain (i.e., the oscillating threshold) lumped conductance $-g_m$ needed to start oscillations is calculated for different number of unit cells N. The oscillation threshold shows the N^{-5} trend with N while due to dissipative losses this trend ceases for large values of N. (c) Steady state oscillation voltage $V(t)$ at the center of the array where it oscillates at a single frequency coinciding with the DBE one.

III. Experimental Verfication of the Existence of EPDs

In this section, we show experimentally the existence of the EPD in the unit cell shown in Fig. 2(a). The unit cell is fabricated on Rogers RT/Duroid 6010.8 laminate, with dielectric constant of 10.8, loss tangent 0.001, and the substrate height of 1.27 mm over a ground plane. The TL appearing at the top of the figure is designed to have a characteristic impedance of 50 Ohms. To verify the existence of the EPD in the unit cell, we perform the scattering (S)-parameter measurement of the four-port unit cell using a four-port Rohde & Schwarz Vector Network Analyzer (VNA) ZVA 67. The measured scattering parameters are then elaborated with Matlab and the solution for the 4×4 transfer matrix eigenvalues is found numerically. The dispersion diagram in Fig. 2(b) shows only the real part of the wavenumber versus frequency and it is in good agreement with the results based on full-wave simulations of the S-parameters performed using Keysight Technologies ADS based on the Method of Moments (MoM). Fig. 2(b) shows different DBEs at different microwave frequencies. The perfect DBE exists in lossless structures [1], [7], [10], [12], [13]. Here losses, fabrication tolerances and connectors have an impact on the ideal EPD condition, though the general structure of the DBE is still visible.

IV. Applications of CTLs With EPDs

The existence of EPDs in CTLs is associated with unique characteristics due to the severe reduction of the group velocity which makes such structures very promising towards many applications. From the various applications of CTLs with EPDs, we mainly focus here on two applications which are the low-threshold oscillators and the highly sensitive sensors.

The DBE exists in lossless structures where the presence of losses would deteriorate such condition. Here we aim at

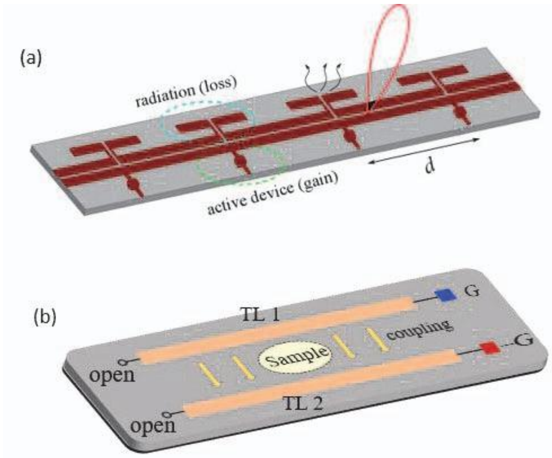

Fig. 4. (a) Schematic of a radiating oscillator based on EPDs using two coupled microstrips where one of them has gain. (b) Schematic of a sensor with EPD based on two finite CTLs; one is terminated with loss and the other with gain, where the under test sample changes the coupling parameter between the CTLs. The EPD enhances the sensitivity of the sensor.

recovering EPDs by introducing gain in each unit cell until we reach the condition of loss and gain balance. We demonstrate an application of a 4th EPD with gain and loss balance to conceive a new class of distributed single-frequency oscillators especially grid oscillators [7]. We design the CTL with a unit cell as in Fig. 2(a) to have a DBE at kd = 0. Such finite structure when terminated by short circuits, as shown in Fig. 3(a), forms an oscillator and it oscillates at a single frequency, in the close proximity of the EPD frequency. The time domain simulations of this finite structure is shown in fig. 3(c). Interestingly, the oscillation threshold (defined as the minimum negative conductance needed to start the oscillation) scales with the number of unit cells N in the finite structure as $1/N^5$, as shown in Fig. 3(b). Although this scaling behavior is shown for lossless structure [7], it is still valid for lossy CTLs for small number of unit cells where the N^{-5} trend ceases to exist for large number of unit cells [12]. In this oscillating structure, the loss needed in one of the transmission lines can be designed to be the radiation from an antenna, therefore we can use the same concept to design a radiating oscillator array as an attractive application. As shown in Fig. 4(a), the antennas can be attached to each unit cell such that the array radiates at the oscillation frequency.

Another unique feature of CTLs with EPDs is its ultra-sensitivity to perturbations which make such CTLs very promising to conceive extremely sensitive devices, like tunable antennas and sensors. The enormous and desirable sensitivity enhancement to perturbation at EPDs is due to the coalescence of eigenvectors of the system [14]. In general, the induced perturbation, proportional to a small number δ, leads to a perturbed transfer matrix, and as a consequence it leads to perturbed eigenvalues λ_p ($p = 1, 2, ...m$ and m is the order of EPD). The small perturbation δ could represent, for example, the variation in the dielectric constant of a capacitor of an LC circuit or the variation in the mutual capacitance of the two coupled resonators in Fig. 1(c) or Fig. 4(b). When the resonator or two coupled resonators work at a second order EPD, the perturbed eigenvalues are proportional to the perturbation as $\lambda_p \propto \sqrt{\delta}$. Note that, when $\delta << 1$ then one has $\sqrt{\delta} >> \delta$ that implies much higher sensitivity to a small variation δ. If the resonator works at an EPD of m^{th} order, then the perturbed eigenvalue is proportional to the m^{th} root of the perturbation δ, i.e., $\lambda_p \propto \sqrt[m]{\delta}$ based on the Puiseux series expansion near an EPD. As an example of a sensing device based on EPDs, we can use the two CTLs terminated with gain and loss as shown in Fig. 4(b), where the eigenvalues are directly related to the resonance frequency of the oscillating structure as $\lambda_p = j\omega_p$. In such system, the under test sample will perturb the coupling parameter between the two CTLs which results in a huge variation of the eigenvalues, hence the resonance frequencies.

V. Conclusion

We have illustrated the general conditions that lead to the existence of exceptional points of degeneracy (EPDs) in coupled transmission lines (CTLs). We have also shown an experimental verification of the existence of EPD (4th order EPD) in CTLs at microwave frequencies through the measurement of the unit cell dispersion relation. More importantly, we have shown two promising applications of CTLs with EPDs. Importantly, we have demonstrated a novel paradigm for radiating array oscillators formed by coupled-waveguides utilizing EPDs. The first application is in radiating array oscillators that are based on gain and radiation-loss balance (losses represent radiation, from a circuit point of view). This has led to a radiating oscillator with a single frequency oscillation and broadside radiation that shows unprecedented scaling of its oscillation threshold with the array length as $1/N^5$. This could be used in various devices requiring coherent emission at microwaves and millimeter waves and it may lead to very power-efficient radiation at millimeter waves. The other application is in highly senstive sensors based on EPDs where we have shown that the system eigenvalues greatly change with the introduced perturbation as $\lambda \propto \sqrt[m]{\delta}$ where m is the order of the EPD.

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