Hybrid Model of Cascading Failure in Power Systems using Trajectory Sensitivity-based Classification

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***Abstract*—A new hybrid model of cascading failure is proposed, which employs a detailed dynamic model to identify and correct the cascade propagation path of QSS model. At every event or failure, trajectory sensitivities for a few seconds following that failure are computed using the detailed dynamic model. Classification based on computed sensitivities reveal the accuracy of QSS model's prediction. If necessary, corrections are made by simulating the detailed dynamic model for a sufficient time. The proposed hybrid model along with a review of trajectory sensitivity is presented. The effectiveness of the proposed model is validated through case studies on IEEE 4-machine and 16-machine systems.**

***Index Terms*—Hybrid Model, Trajectory Sensitivity, Cascading Failure, QSS Model, Dynamic Model, Blackout**

I. Introduction

Cascading failures, wherein the initial failure of one or more components causes subsequent failure of other components in the system and finally results in the entire system collapse, are a major cause of blackouts in electric power grids. According to a recent report [1], cascading failure-based blackouts, such as Indian nationwide blackout in 2012 have caused an amount of customer-hours of lost electricity service that is comparable to major natural disasters, such as the Hurricane Maria that devastated Puerto Rico.

In order to understand, predict and prevent such cascading failure phenomenon, appropriate modeling is required. Most of the existing literature on this topic utilize quasi-steady-state (QSS) DC power flow models [2]–[4] and QSS AC power flow models [5]–[8] for modeling cascade propagation. These models are capable of capturing only certain cascade propagation paths, which are determined by thermal overloads in post-contingency steady-state conditions. Reference [9] proposed a DC load flow model augmented by a more accurate representation of under-voltage and under-frequency load-shedding. However, this model can not represent the small-signal stability and voltage stability issues. Additionally, other kinds of static models including statistical models [10]–[12] and topological models [13], [14] are also suggested. Despite their usefulness, they cannot produce insights into the cascading mechanism similar to that of physics-based models. In practical systems, cascading involves different complex mechanisms that are governed by nonlinear dynamics, which could potentially lead to mid- and long-term stability issues.

As an attempt to produce practical models for cascading failures, dynamic models in [15], [16] and numerical methods for faster simulation of these models [17], [18] were proposed. However, these models did not consider the protection elements, which play a critical role during cascading outages. Although Song et-al [19] included such protective elements in their proposed COSMIC model, this framework is too slow for many large-scale statistical analyses.

This paper proposes a new hybrid model of cascading failure, which takes advantage of both the computational ease of a QSS model and accuracy of a detailed dynamic model. The key idea is to employ a detailed dynamic model to identify and correct the cascade propagation path of its QSS counterpart model. At every event or failure, trajectory sensitivities for a few seconds following that failure are computed using the detailed dynamic model. Classification based on the computed sensitivities reveal the accuracy of QSS model's prediction. If necessary, corrections are made by simulating the detailed dynamic model for a sufficient time. The effectiveness of the proposed hybrid model is validated through case studies on IEEE 4-machine and 16-machine systems.

II. Proposed Hybrid Model of Cascading Failure in Power System

First, the typical structure of the QSS model as described in [20] is reviewed. The proposed model is then presented as an augmentation to this structure. The method of computing trajectory sensitivities and classification-based on the same is elaborated in Section III.

A. Existing QSS Model-based Methodology [20]:

Figure 1 shows both the typical methodology employed in literature for simulating QSS model-based cascaded failure, and the proposed methodology, which has the dynamic model-based validation and correction step. In the existing QSS model-based methodology, after applying the corresponding outages,

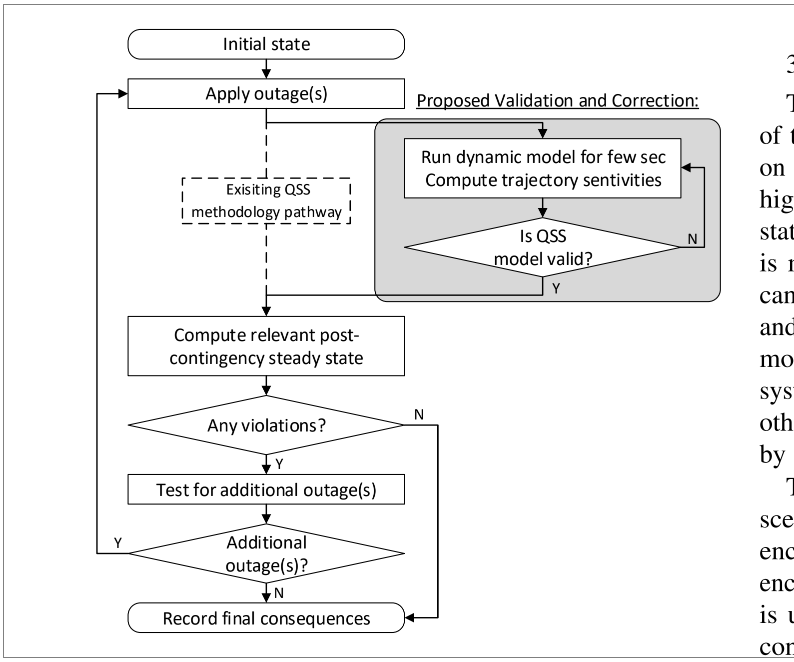

Fig. 1. Existing and proposed methodology for modeling cascading failure using QSS model [20] and hybrid model, respectively.

i.e, changes to bus and line data, QSS model is directly used to predict the post-contingency steady state condition. This is followed by a check for additional outages and if required, the outage or contingency list is updated and the process is repeated.

A major concern for accuracy of existing QSS model-based methodology lies in the computation of post-contingency steady state condition. For a practical system, after the given outage(s) is/are applied, the system could face stability issues. These stability issues, if faced, could push the system away from the steady state condition computed in existing QSS model-based methodology.

B. Proposed Hybrid Model-based Methodology:

Proposed hybrid methodology for modeling cascade failure utilizes both dynamic and QSS model for accurate and faster simulation of the cascade propagation path. From the discussion on shortcomings of the existing QSS model-based methodology, it is clear that a major source of error in this method is the prediction of the post-contingency steady state condition. Proposed hybrid methodology employs dynamic model to validate and correct the cascade propagation pathway of the QSS model.

As shown in Fig. 1, at every iteration, dynamic model is first initialized to pre-contingency steady state condition and the required outage(s) is/are applied at some time t_0. Trajectory sensitivities are computed in parallel to the simulation of dynamic model for few seconds after t_0. These trajectory sensitivities give an early indication about the condition of stress and stability in the system. Therefore, based on trajectory sensitivities, system can be classified into one the following three conditions:

1) Asymptotically stable with high damping
2) Marginally stable/very poor damping
3) Unstable

The stability condition of the system reveals the accuracy of the post-contingency steady state condition computed based on QSS model. When the system is asymptotically stable with high damping, most likely the system will reach the steady state condition predicted using QSS model. When the system is marginally stable or very poorly damped, the oscillations can trip other protection systems, for example zone 3 relays, and the propagation path could potentially deviate from QSS model. When the system is unstable and detailed protection systems are in place, then the system will definitely go through other cascade propagation paths, which can only be captured by dynamic model.

Therefore, in the case of marginally stable and unstable scenarios, dynamic model is ran until a stable scenario is encountered. When an asymptotically stable condition is encountered, the dynamic model is abandoned and QSS model is used to predict the relevant post-contingency steady state condition. Therefore, dynamic model as shown in Fig. 1 is employed to validate and correct the propagation pathway of the QSS model in this way. Note that trajectory sensitivity-based classification can be done much quicker than the time taken for the system to reach steady state, which reduces the computation time.

III. Trajectory Sensitivity-based Classification

In this Section, we first give and overview of trajectory sensitivity [21] and later present a numerical approximation that was used in this paper to compute the same. Finally, the method of classification implemented using computed trajectory sensitivity is described.

A. Definition of Trajectory Sensitivity [21]:

A dynamic model of power system can be represented using a set of Differential and Algebraic equations (DAEs). The DAEs have the following form,

$$\begin{aligned} \dot{x} &= f(x, z, \lambda) \\ 0 &= g(x, z, \lambda) \end{aligned} \tag{1}$$

where, x, z and λ are vectors constructed from state variables, algebraic variables and parameters, respectively. The trajectory sensitivity matrices along the trajectories of x and z with respect to the parameters λ are defined as,

$$x_\lambda = \frac{\partial x}{\partial \lambda} \quad \text{and} \quad z_\lambda = \frac{\partial z}{\partial \lambda} \tag{2}$$

Differentiating the equations in (1) with respect to λ on both sides gives us an analytical expression to compute trajectory sensitivities,

$$\begin{aligned} \dot{x}_\lambda &= \frac{\partial f}{\partial x} x_\lambda + \frac{\partial f}{\partial z} z_\lambda + \frac{\partial f}{\partial \lambda} \\ 0 &= \frac{\partial g}{\partial x} x_\lambda + \frac{\partial g}{\partial z} z_\lambda + \frac{\partial g}{\partial \lambda} \end{aligned} \tag{3}$$

here, at the initial time t_0, the sensitivities are set to zero, i.e., $x_\lambda(t_0) = 0$ and $z_\lambda(t_0) = 0$.

B. Numerical Approximation for Computing Trajectory Sensitivity:

For a large dynamical system, obtaining a closed form expression for partial derivatives in eq (3) becomes a challenging task. One way to get around this is to redefine parameters as state variables whose derivative is always zero i.e., they remain constant – and then compute trajectory sensitivities w.r.t. initial condition of all state variables. In that scenario, the partial derivatives required in eq (3) are directly obtained as a byproduct of solving (1). However, this is true only if implicit integration method were used to solve (1).

In this paper, for computing trajectory sensitivities w.r.t. one parameter, the approximation described below, which was presented in [21] is used,

$$\begin{aligned} x_\lambda(t) &= \frac{\partial x(t)}{\partial \lambda} \approx \frac{x(t,\lambda+\Delta\lambda)-x(t,\lambda)}{\Delta\lambda} \\ z_\lambda(t) &= \frac{\partial z(t)}{\partial \lambda} \approx \frac{z(t,\lambda+\Delta\lambda)-z(t,\lambda)}{\Delta\lambda} \end{aligned} \tag{4}$$

This approximation is valid when $\Delta\lambda$ is small. In order to realize this approximation, two DAEs of the form (1) are simulated with two different parameter values $\lambda + \Delta\lambda$ and λ, respectively. The resulting trajectories of x and z are then used as shown in eq (4) to compute approximate trajectory sensitivities.

C. Classification based on Trajectory Sensitivity:

As stated in Section II, trajectory sensitivities are employed to classify post-contingency system's stability. This is essential to validate the accuracy of post-contingency steady state condition obtained from QSS model. First, a specific sensitivity norm S_λ in the context of the power system is defined. Certain features of this sensitivity norm S_λ will be used for classification.

The sensitivity norm S_λ in the context of the power system is defined as,

$$S_\lambda = \left[\sum_{i=1}^{n}\left(\left(\frac{\partial\delta_i}{\partial\lambda}-\frac{\partial\delta_{ref}}{\partial\lambda}\right)^2+\left(\frac{\partial\omega_i}{\partial\lambda}\right)^2\right)\right]^{\frac{1}{2}} \tag{5}$$

here, n represents number of generators, δ_i and ω_i are the angle and angular speed corresponding to the ith generator, respectively; and δ_{ref} is the reference angle w.r.t. which all other angles are measured.

The classification of the system into one of the two, (i) asymptotically stable or (ii) marginally stable or unstable is done based on the location of occurrence of the peak in sensitivity norm S_λ. For asymptotically stable systems, the peak in sensitivity norm S_λ occurs at an earlier stage, i.e, in few seconds. For marginally stable or unstable systems, the peak in sensitivity norm S_λ occurs at a much later stage. Especially for unstable systems, it is theoretically proven that S_λ will reach infinity.

IV. Simulation Results of Case Studies

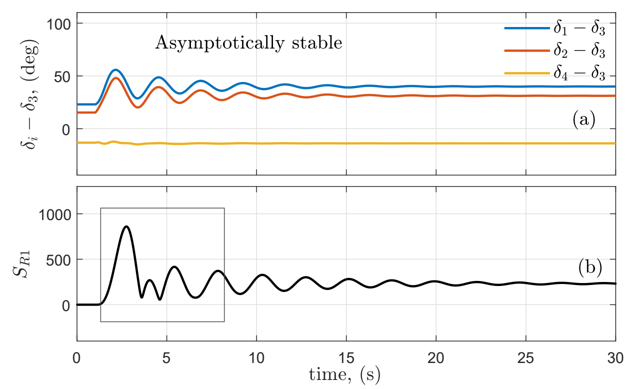

Fig. 3. System I - Case I: IEEE 4-machine system with D = 0.015 pu: (a) Damping oscillations in relative angles and (b) Diminishing trajectory sensitivity norm indicating asymptotic stability.

In order to validate the proposed model, two systems, IEEE 4-machine and IEEE 16-machine systems are considered. For both the systems, parameter λ is chosen as governor droop constant $R1$ corresponding to generator $G1$.

For System I, i.e., 4-machine system, two cases are considered. In one case, Case I, a damping coefficient of D = 0.015pu is assumed for all generators. In other case, Case II, the damping coefficient is assumed to be D = 0.001pu for

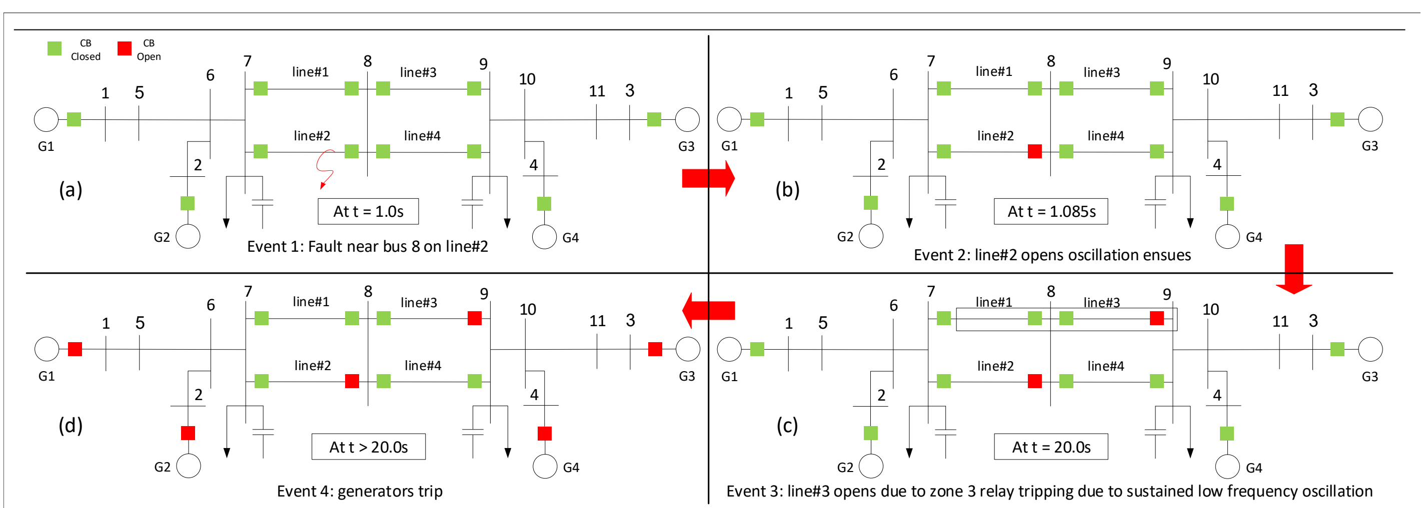

Fig. 2. System I - Case II: IEEE 4-machine, 2-area system with sequence of events in the dynamic model. Generator damping coefficient D = 0.001pu is assumed. Power-flow from bus 7 to 9: 400MW.

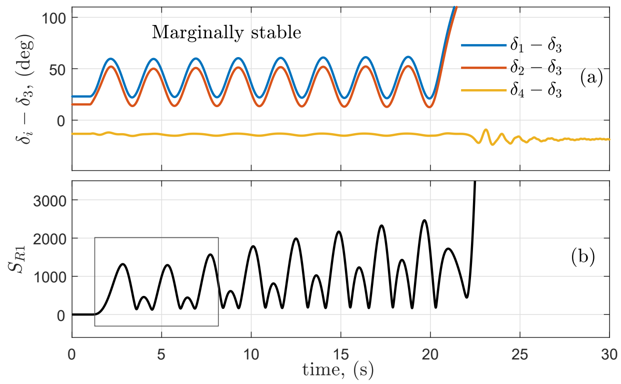

Fig. 4. System I - Case II: IEEE 4-machine system with D = 0.001 pu: (a) Undamped oscillations in relative angles and (b) increasing trajectory sensitivity norm indicating marginal oscillations.

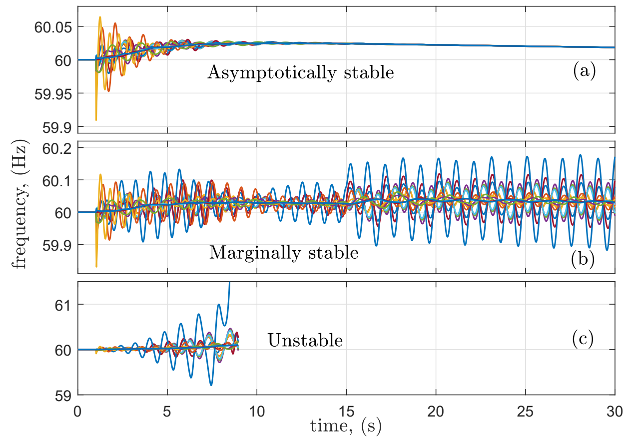

Fig. 6. System II: Frequency dynamics following fault and line outage at bus#30: (a) Stable oscillations with K_{pss} = 0.2, (b) Poorly damped oscillations

all generators. In both the cases same initial disturbance, i.e., three-phase fault near bus#8 at t = 1.0s (Fig. 2(a)) and outage of line#2 (Fig. 2(b)) is considered.

In Case I, due to the higher damping available in the system, and following the outage of line#2, the oscillations are damped out as shown in Fig. 3(a). Corresponding to the asymptotic behavior of the system, as shown in Fig. 3(b), the trajectory sensitivity norm, S_{R1} peaked early on and started to diminish. Accordingly, in hybrid model decision on validity of QSS model is made in a short time as highlighted in Fig. 3(b). The outage of line#2 increased power flow of line#1 to 400MW. However, no cascading failure is observed as the thermal limit of line#1 is assumed to be 450MW and the same is captured by the QSS model.

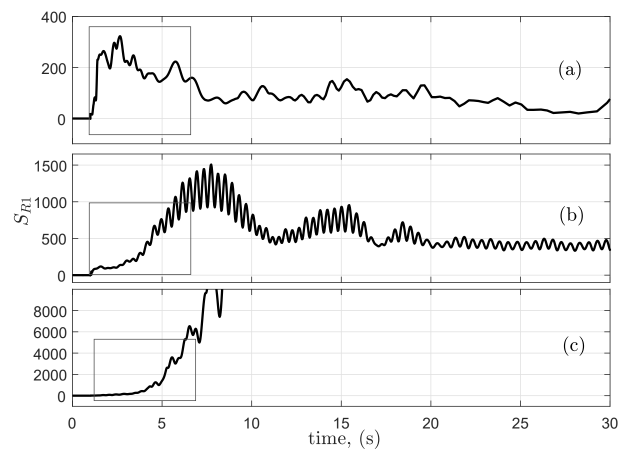

Fig. 7. System II: Trajectory sensitivity norm: (a) Diminishing when system is stable, (b) Growing when system is marginally stable and (c) Rapidly escalating when system is unstable.

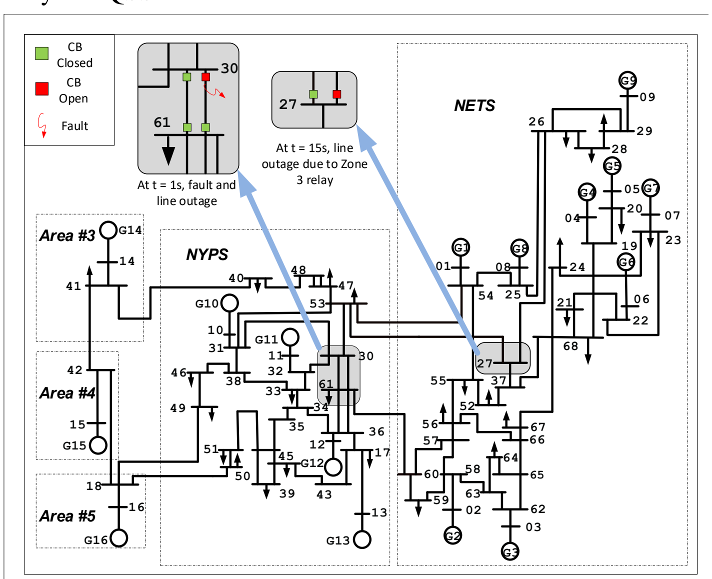

Fig. 5. System II: IEEE 16-machine system showing fault at bus #30 and opening of circuit breaker on the line between buses #30 and #61.

In Case II, due to poor damping, the system is set into marginally stable oscillations following the fault at t = 1.0s and outage of line#2. Figure 2 illustrate the sequence of events following this undamped oscillations. Due to these undamped oscillations, zone 3 relay near bus#9 trips the breaker at t = 20.0s and line#3 is taken out, see Fig. 2(c). Following the outage of the line#3, system becomes unstable and all the generators are disconnected at some t > 20.0s, see Fig. 2(d). Corresponding to the marginally stable behavior of the system, as shown in Fig. 3(b), the trajectory sensitivity norm, S_{R1} is gradually increased. Importantly the peak of trajectory sensitivity norm did not occur in the first few seconds following the initial outage as highlighted in Fig. 3(b). Also, for t > 20.0s, when the system is unstable, the trajectory sensitivity norm is tending to approach infinity.

In order to further validate the proposed hybrid methodology, System II, a larger IEEE 16-machine system as shown in Fig. 5 is considered. Three cases, each corresponding to different values of power system stabilizer (PSS) gains, K_{pss} located at generator $G9$ is considered. For Case I, K_{pss} = 0.2, for Case II, K_{pss} = 0.006 and for case III, K_{pss} = -0.02. These PSS gains set such that in Case I, Case II, and Case III following a fault near bus#30 and line (30-61) outage at t = 1s, stable, marginally stable and unstable oscillations are produced, respectively. These oscillations in frequencies of the generators are shown in Fig. 6, wherein each subplot corresponds to each case. Particularly in Case II, due to high amplitude of oscillation in $G9$ (blue trace), a zone 3 relay near bus#27 trips line (26-27) at t = 15s, as shown in Fig. 5. This resulted in higher amplitude undamped oscillations see Fig.

TABLE I
SUMMARY OF CASE STUDIES

Test System	QSS Model	Dynamic Model		
		Case I	Case II	Case III
IEEE 4-machine	line#2 outage, no cascade	line#2 outage, no cascade	line#2 outage, followed by cascade	-
IEEE 16-machine	line (30-61) outage, no cascade	line (30-61) outage, no cascade	line (30-61) outage, followed by line (26-27) outage	line (30-61) outage, followed by instability

6(b), which can potentially cause further failures (not simulated here).

Figure 7 shows the oscillatory behavior in all three cases as reflected in trajectory sensitivity norm, S_{R1}. Clearly, in Fig. 7(a), which corresponds to stable oscillatory behavior the sensitivity norm peaked early on and started diminishing. In the case of marginal stability, as shown in Fig. 7(b) the sensitivity norm is gradually growing and did not peak early on. In the unstable case, as shown in Fig. 7(c) the sensitivity norm is tending to approach infinity. Therefore, classification based on trajectory sensitivity norm is valid for System II.

As summarized in Table I, the QSS model based methodologies are valid only for Case I, in other cases a dynamic model is required. Proposed hybrid methodology employs trajectory sensitivity norm to distinguish between Case I and II, and validate the use of QSS model in a short time.

V. Conclusion

A new hybrid methodology for modeling cascading failure in power systems was proposed. The proposed methodology employed a detailed dynamic model to identify and correct the cascade propagation path of QSS model. At every event or failure, trajectory sensitivity norm as defined is computed for a few seconds. Classification based on sensitivity norm revealed the oscillatory behavior of the system, which validates the accuracy of QSS model's prediction. If necessary, corrections are made by simulating the detailed dynamic model for a sufficient time. The effectiveness of the proposed methodology was validated through case studies on IEEE 4-machine and 16-machine systems.

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