

Risk-oriented PMU placement approach in electric power systems

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Abstract: The objective of traditional phasor measurement unit (PMU) placement approaches is to find the minimal number of PMUs to achieve full system observability. Very few consider the probability and the consequence severity of observability loss. For example, following cascading failures, lines are tripped, and the topology of the grid changes, which may result in observability loss for parts of the grid. With limited number of PMUs, it is desired that the PMUs should still sustain observability for the power grid after top-severe cascading failures, other than after those cascading failures of low severity. To address the above issues related with the consequence severity of observability loss, this study proposes a risk-oriented PMU placement approach. The proposed approach uses two indices, namely the probability of observability loss and the severity of its corresponding consequence to design the optimal PMU placement strategy. The authors show that the proposed approach is able to capture the critical events, while traditional approaches fail to do so. Extensive simulation results carried out on the IEEE 39-bus and 118-bus systems demonstrate the effectiveness of the proposed risk-oriented approach.

1 Introduction

In electric power systems, observability is a prerequisite for state estimation, online security assessment, and decision making. Since the early 1990s, phasor measurement units (PMUs)-based wide-area measurement systems (WAMS), with synchronised sampling and high data update rates, have been evolving. Through the global positioning system, WAMSs enable us to monitor wide area power systems in the same time reference frame with accuracy of 1 ms. Compared with the low data update rates (between 2 and 10 s) of traditional supervisory control and data acquisition systems, WAMSs are able to provide nearly real-time snapshots of power system dynamics for online monitoring, security assessment, and control [1–5]. This evokes many researchers' interests on WAMS-based power system dynamic monitoring, such as dynamic state estimation [6–9], generator model calibration [10, 11], damping oscillations [12, 13], design of adaptive protection schemes [14], and enhancement of traditional state estimators [15–17].

To achieve the aforementioned benefits, system observability provided by PMUs is typically required. For PMU optimal placement (OPP), a traditional strategy is to find optimal placement locations so that a minimal number of PMUs allows us to get the maximal observability of the system. The existing approaches in this line of research can be categorised into two main groups: stochastic-intelligent-search-based approaches [18–21] and integer programming-based approaches [22–25]. Different from continuous system optimisation algorithms, the first group does not need to calculate the derivatives of cost functions or determine the gradient direction. Their main ideas are summarised as follows: in each step, they randomly generate a group of solution candidates and select good candidates by screening out the bad ones, and then generate another group candidate again according to the selected solution candidates in the previous step. By doing it continuously, the best solution candidate can converge to the same one, which will be considered as the optimal solution. Among them, genetic algorithm [18], simulated annealing [19], Tabu search [20], particle swarm optimisation [21], and so on, are the most popular ones. However, they are computationally expensive and cannot guarantee the global optimal solution because OPP is a non-convex

optimisation problem. The second group is the integer-programming-based approaches [22–25]. Different from the first group, they formulate observability as a constraint while treating the number of PMUs as cost functions. These approaches are more computationally attractive if zero injections are treated in an appropriate way.

Recently, researchers have been studying PMU placement from a perspective of measurement reliability and power system reliability [26–37]. Some papers discussed the issue that PMUs are not fully reliable devices and it is necessary to redundantly observe critical substations of power systems through different paths to improve reliability of the monitoring system. Different methods were proposed to consider measurement redundancy in PMU placement [26, 27]. Wang *et al.* [28] proposed to use additional PMUs to enhance the reliability of the system by an incremental placement approach, while the authors of [29, 30] investigated the method of placing PMUs on more reliable buses to decrease the probability of losing observability. The other point of PMU placement is that the PMUs should be allocated at less reliable areas of power grids to minimise the probability of unobservability of such areas, and thus the whole system reliability status could be improved as the vulnerable areas are always observable. The authors of [31, 32] ranked the vulnerability of buses from reliability assessments and kept the buses with lower reliability monitored by the PMUs. Furthermore, some recent papers also considered the fact that the PMUs should not only make the power grids observable during normal operation conditions but also in contingencies. For example, the authors of [33, 34] investigated the optimal PMU placement considering measurement losses and branch outages. However, only single measurement loss and single-branch outage conditions are considered. To address these issues, the authors of [35, 36] considered the probabilistic nature of power systems and proposed an observability-probability-based PMU placement method. Specifically, an iterative algorithm considering random power system outages has been developed to obtain the probability of observability at buses for different outages. While in [36], a monetary value for contingencies and their likelihood are developed to limit the outages to be considered

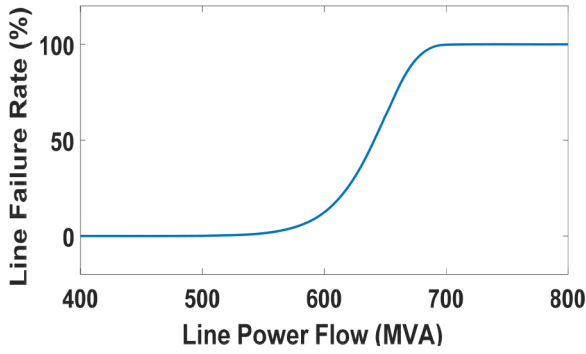


Fig. 1 Line failure rate with line power flow

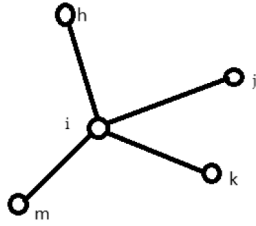


Fig. 2 Illustration for probability of observability loss

in the PMU placement problem. However, to the authors' best knowledge, the consequence severity of observability loss caused by power grid contingencies was not well discussed in existing research, which is critical in the presence of cascading events. Different contingencies may have different severity for the power grid, for example, some contingencies may lead to load/generator trips or line overloading, while other contingencies may not lead to any system abnormality. With limited number of PMUs, it is desired that the PMUs should be useful for monitoring top-severity contingencies, other than monitoring those contingencies with no/low severity. Another critical point is that, only considering single-branch outage is not enough, as some of the initial single branch outage may lead to cascading failures. It is also desired that with limited number of PMUs, the PMUs should be useful for maintaining the observability and monitoring the power grid after severe cascading failures happen, other than just monitoring the power grid before cascading failures.

To ensure the system situation awareness in the presence of cascading events, this paper proposes a risk-oriented PMU placement approach. It contains the following salient features:

- (i) Two indices are presented in a unified way to characterise the probabilistic nature of a power system; namely the probability of observability loss and the severity of its corresponding consequence. To the best of our knowledge, this is the first time these two important factors are unified when designing PMU placement in the presence of multiple system contingencies.
- (ii) We formulate the risk-oriented PMU placement strategy as a constrained non-linear programming problem by treating the risk for observability loss as the objective function while guaranteeing the risk of observability loss for each bus is less than a given threshold as the inequality constraints. As a result, it fits the framework of constrained non-linear programming solution methodologies and can be solved extensively by many software packages.
- (iii) Through extensive case studies, we show that our proposed approach is able to capture the critical events while the traditional probability of observability based approaches fails to do so.

The rest of this paper is organised as follows. In Section 2, a risk-oriented PMU placement strategy is formulated, in which both the probability and severity of observability loss are considered. In Section 3, case studies are presented to demonstrate the effectiveness of the proposed approach. Finally, conclusions are presented in Section 4.

2 Formulation of observability risk

2.1 Relationship between overload rate and line trip probability

In the power grid, the line failure probability L_{ij} is associated closely with the overloading rate of the transmission line. In this paper, we use the following line failure probability that is defined in [37]:

$$L_{ij} = 1 - e^{-(L_{Aij}/\alpha L_{Rij})^\beta T_s}, \quad (1)$$

where L_{Aij} and L_{Rij} are the actual power flow and power rating of the line between bus i and bus j , respectively; α and β are parameters characterizing the sharpness of the line failure rate curve; T_s represents the time interval in which the line trip happens. For example, a typical curve relating line failure rate with the line power flow is shown in Fig. 1, where the power rating of the line is assumed to be 600 MVA.

2.2 Probability of observability

The observability and probability can be linked through the probability of a line trip and the location of PMU placement. Let us take a simple topology shown in Fig. 2 to illustrate the observability probability of the power grid. Here we assume that if a PMU is installed at bus i , the PMU collects three-phase voltage measurements at bus i , as well as all three-phase current measurements in the branches connected to bus i .

In the topology shown in Fig. 2, the probability PU_i of losing observability of bus i can be formulated as follows:

$$PU_i = (1 - u_i) \times \prod_{j \in I_i} (1 - u_j(1 - L_{ij})), \quad \forall i, \quad (2)$$

where $u_j = 1$ if there is PMU measurement at bus j , otherwise $u_j = 0$; L_{ij} is the failure probability of the line between bus i and bus j , which is defined in (1); I_i is the set of lines connected to bus i . $u_i = 1$ if there is PMU measurement at bus i , otherwise $u_i = 0$; then the probability of observability loss of the whole system is

$$PU = \frac{1}{N} \sum_{i=1}^N PU_i, \quad (3)$$

where N is the total number of buses in the system and the probability of whole system observability is

$$PO = 1 - PU = 1 - \frac{1}{N} \sum_{i=1}^N PU_i. \quad (4)$$

Note that the value of PU and PO are in the interval of $[0, 1]$.

2.3 Probability of cascading failure path

A cascade of failures is usually the result of randomly or intentionally tripping one or multiple components in a power transmission network. The early outage may cause the re-distribution of power flows that may further force some lines to exceed their transfer limits or encounter overloading. These overloaded lines can be tripped by their protection relays, inducing another round of power flow re-distribution. The cascading process will continue until no more line outages occur. Note that after cascading failures, the system may be split into several islands. In this paper, we assume that the line tripping events are mainly caused by overloading.

To characterise the randomness of the power grid state change (line flows) in a probability manner during cascading process, the Markov chain approach [38] can be used. Assuming the relays operate independently, the state transition probability between two events is thus a function of the line transition rates. For example, in Fig. 3, when one line is lost because of overloading and relay

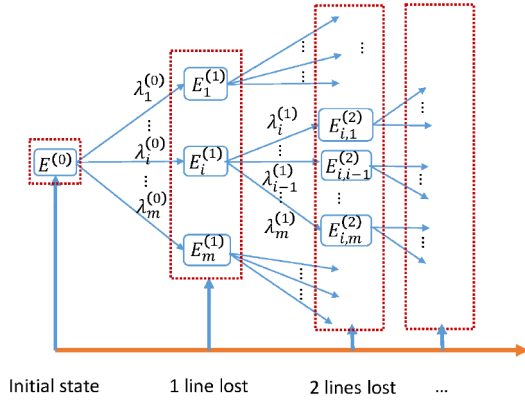


Fig. 3 Markov transition model for power grid cascading failures

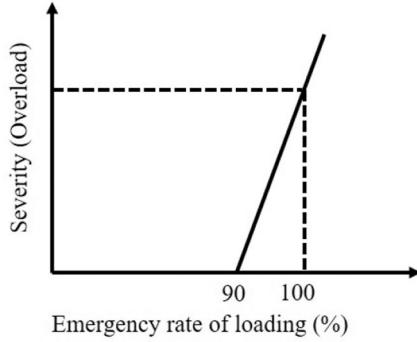


Fig. 4 Severity function of transmission line

protection, the probability of the system state to transit from $E^{(0)}$ to $E_i^{(1)}$ is $\lambda_i^{(0)}$; here $\lambda_i^{(0)}$ is the probability of the i th line trip.

If one more line is tripped, the probability of the system state to transit from $E^{(0)}$ to $E_{i,j}^{(2)}$ is $\lambda_i^{(0)}\lambda_j^{(1)}$, where $\lambda_j^{(1)}$ is the probability of the j th line trip following the i th line trip. Via this Markov transition model, we can identify several cascading paths with high probability.

2.4 Severity and risk of cascading failure paths

Different cascading failure path results in different severity of consequence. Certain indices should be presented to quantify them. In this paper, we present two types of severity indices. The first one is for overloaded lines while the second one is for load or generation loss.

For the first type, the severity function proposed by Xiao and McCalley [39] is used to quantify the post-contingency severity caused by overloading. The relationship between emergence rate of line loading and its associated severity is shown in Fig. 4. It can be observed that overload severity increases linearly as the line flow increases near its emergency rating. The linearly increasing severity functions capture non-zero risk for performance close to, but within a performance limit, reflecting the realistic sense that such a situation is in fact risky. In this case, the post-contingency severity is given by the following equation:

$$S = \sum_{i=1}^N s_i, \quad (5)$$

where s_i is the severity function for overloading line defined according to Fig. 4, and N is the total number of lines in the system.

For the second type of severity, which evaluates the possible load or generation loss during and after the cascading failure, the following method is developed to quantify the post-contingency performance caused by load or generation loss. Assuming after the cascading failure, the tripping of the overloaded lines leads the system being split into Q different separated areas or islands $Q = \{Q_1, Q_2, \dots, Q_q\}$, each area is not connected with others. The total loss of generation or load S of the system is given by (see (6)), where Q_i represents the i th separated area; S_{Q_i} represents the load/generation loss of area Q_i ; $P_{load,j}$ represents the j th load in area Q_i ; $G_{max,k}$ and $G_{min,k}$ represent the maximum and minimum real power limits of the k th generator in area Q_i , respectively.

The risk of cascading failure k is defined as

$$R_k = \lambda_k S_k, \quad (7)$$

where λ_k is defined in Section 2.3, while S_k is defined by (5) or (6).

2.5 Formulation for observability risk optimisation

Assume we have identified m cascading paths with high probability and each of them has probability $p_{path_1}, \dots, p_{path_m}$ associated with severity index $S_1, \dots, S_m$. Given the number of PMUs, according to (1)–(4), the observability probability will change after the lines trip in cascading paths. Suppose PO_0 is the observability probability without line trip, we then obtain

$$PO_0 = 1 - \frac{1}{N} \sum_{i=1}^N \left[(1 - u_i) \prod_{j \in I_i} (1 - u_j (1 - L_{ij}^b)) \right], \quad (8)$$

where L_{ij}^b is the line trip probability before cascading failure happens.

After line trips in cascading path p_{path_r} , the observability probability of the system can be calculated through

$$PO_r = 1 - \frac{1}{N} \sum_{i=1}^N \left[(1 - u_i) \prod_{j \in I_i} (1 - u_j (1 - L_{ij}^{ar})) \right], \quad (9)$$

where L_{ij}^{ar} is the line trip probability after line trips in cascading path p_{path_r} . Then, we have the observability probability change as

$$\Delta PO_r = PO_0 - PO_r. \quad (10)$$

Define observability probability change for the first m cascading paths as $\Delta PO_1, \dots, \Delta PO_m$, the risk for observability loss is therefore expressed as

$$R_k^O = \Delta PO_k S_k, \quad k = 1, \dots, m, \quad (11)$$

where S_k is defined in (5) or (6).

In summary, the objective of the proposed approach is to minimise the risk of observability loss while guaranteeing that the risk of observability loss of each bus is less than a given threshold and number of PMUs. Formally, we have

$$S = \sum_{Q_i \in Q} S_{Q_i} S_{Q_i} = \begin{cases} \left(\sum_{j \in Q_i} P_{load,j} - \sum_{k \in Q_i} G_{max,k} \right), & \text{if } \sum_{k \in Q_i} G_{max,k} < \sum_{j \in Q_i} P_{load,j}, \\ 0, & \text{if } \sum_{k \in Q_i} G_{min,k} < \sum_{j \in Q_i} P_{load,j} < \sum_{k \in Q_i} G_{max,k}, \\ \left(\sum_{k \in Q_i} G_{min,k} - \sum_{j \in Q_i} P_{load,j} \right), & \text{if } \sum_{k \in Q_i} G_{min,k} > \sum_{j \in Q_i} P_{load,j}, \end{cases} \quad (6)$$

$$\min J = \sum_{k=1}^m w_k R_k^O \quad (12)$$

Subject to

$$\begin{cases} UR_k \leq UR_k^{\min}, \forall k \\ \sum_{j=1}^N u_j \leq u_{\max} \end{cases}, \quad (13)$$

where

$$UR_k = S_i(1 - u_i) \prod_{j \in I_i} (1 - u_j(1 - L_{ij}^b)), \quad (14)$$

and w_k is defined as follows:

$$w_k = p_{\text{path}_k} / \sum_{k=1}^m p_{\text{path}_k} \quad (15)$$

2.6 Flowchart of the proposed method

Fig. 5 shows the flowchart of the implementation of the proposed risk-observability-based PMU placement method. Given a certain power grid model, the method will first scan all the possible cascading failures by iteratively tripping one line in the power grid model a time as the initial event, followed by a power flow analysis. If the power flow analysis shows there are additional lines suffering overloading (cascading failure) in the system after the previous round of line tripping, the line failure rate will be computed based on (1), and the severity will be computed based on (5) and (6), and the lines with overloading conditions will be tripped. The new round of power flow analysis and searching for new overloading lines will start again, unless the power flow analysis shows that the system suffers an islanding (separation) or an un-converged power flow. The risk of cascading failure path will be evaluated based on (7). At the end of cascading failure path scanning, the method can identify the cascading failure paths with the highest severities and risks. The next step is to formulate the risk-oriented PMU placement optimisation based on (8)–(15) by selecting and including the cascading failure paths with the highest risks. Note that the inputs for the PMU placement optimisation problem are m selected cascading failure paths associated with computed severity for each path, line failure rate for each line for the base case and for each cascading failure, as well as the pre-defined unobservable risk threshold for each cascading failure.

3 Case study

In this section, we demonstrate the proposed PMU placement method through the IEEE 39-bus and IEEE 118-bus systems. The optimisation problem in (12)–(15) is formulated and solved by GAMS using the Knitro optimisation engine [40]. It is noted that the optimisation problem in (12)–(15) is formulated as a mixed-integer programming problem, and the global optimal solution can be found by using Branch and Bound method [41] in the Knitro optimisation engine.

3.1 Case 1: IEEE 39-bus system

In the case of IEEE 39-bus system, as shown in Fig. 6, we consider three cascading paths with the highest probabilities. Table 1 gives details of the initial events and the line-tripping sequences for the three cascading paths, as well as the failure probability of each tripped line and the total amount of tripped generation and load for these three cascading failures, based on the method described in Section 2.4. According to Table 1, cascading path 1 has the highest failure probability 0.5654 and the lines tripped in cascading path 1 result in three islands, but with no generation loss or load tripping. The boundaries of the three islands of cascading path 1 are shown in Fig. 6 with blue colour. Cascading path 2 has the failure probability 0.0693 and the lines tripped in cascading path 2 result

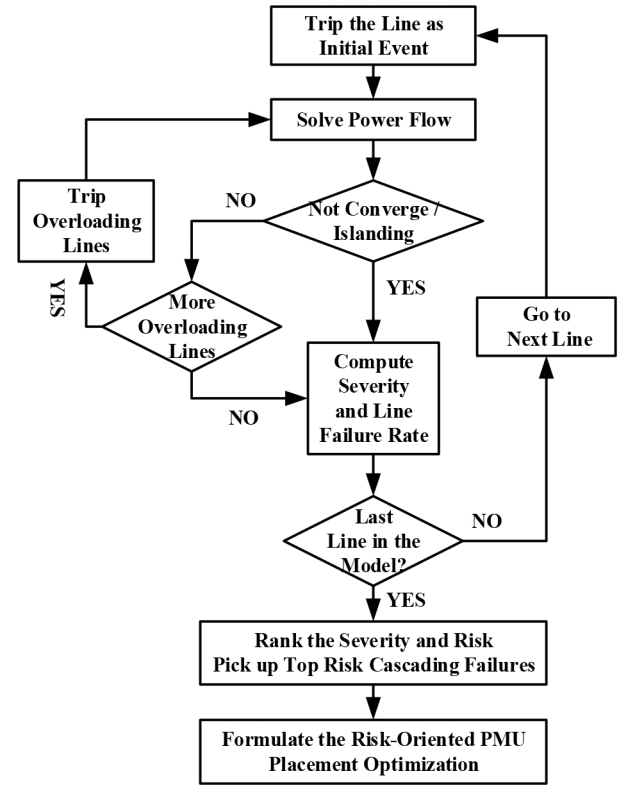


Fig. 5 Flowchart of the implementation of the proposed risk-observability-based PMU placement method

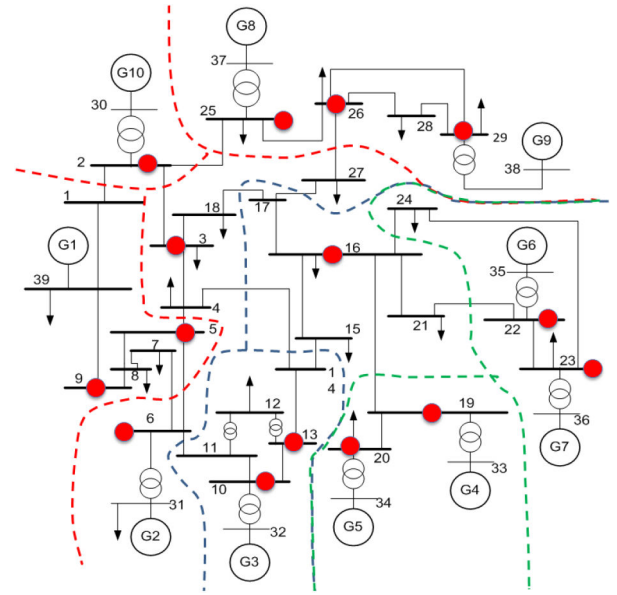


Fig. 6 IEEE 39-bus test system with cascading failure paths, probability-oriented PMU placement

in three islands, with 836.9 MW load tripping (most severe cascading path). The boundaries of the three islands of cascading path 2 are shown in Fig. 6 with red colour. Cascading path 3 has the failure probability 0.5074 and the lines tripped in cascading path 3 result in three islands, with 308.6 MW generation tripping (second most severe cascading path). The boundaries of the three islands of cascading path 3 are shown in Fig. 6 with green colour. According to Section 2.4, the severity and risk of each path are obtained and shown in Table 1.

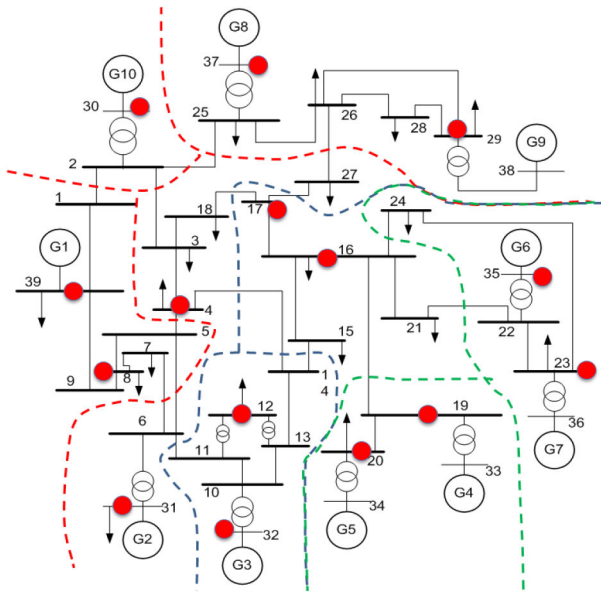
Given 15 PMUs, the proposed risk-oriented approach produces the PMU placement results displayed in Table 2 and Fig. 7. For comparison, Table 2 also gives the PMU placement with the traditional probability-oriented approach [34, 35] that maximises the observability probability, as shown in Fig. 6.

Table 1 Cascading paths for 39-bus system

	Cascading path 1	Cascading path 2	Cascading path 3
initial event	line 4 to 14 trip	line 2 to 3 trip	line 21 to 22 trip
2nd round line trip	line 6 to 11 trip with Pro. 0.983	line 6 to 7 trip with Pro. 0.821 line 26 to 27 trip with Pro. 0.671	line 16 to 24 trip with Pro. 0.727
3rd round line trip	line 4 to 5 trip with Pro. 0.823 line 5 to 6 trip with Pro. 0.916 line 14 to 15 trip with Pro. 0.957 line 17 to 18 trip with Pro. 0.964 line 17 to 27 trip with Pro. 0.827	line 1 to 2 trip with Pro. 0.494 line 2 to 25 trip with Pro. 0.623 line 4 to 5 trip with Pro. 0.761 line 5 to 6 trip with Pro. 0.537	line 16 to 19 trip with Pro. 0.698
cascading failure probability	0.5654	0.0693	0.5074
severity	no load tripping	863.9 MW load tripping	308.6 MW generator tripping
risk	0.0	59.8554	156.5978

Table 2 Comparison of PMU placement for 39-bus system

Traditional probability-oriented approach	Proposed risk-oriented approach
2, 3, 5, 6, 9, 10, 13, 16, 19, 20, 22, 23, 25, 26, 29	4, 8, 12, 16, 17, 19, 20, 23, 29, 30, 31, 32, 35, 37, 39

**Fig. 7** IEEE 39-bus test system with cascading failure paths, risk-oriented PMU placement

It can be seen from Fig. 7 that for the cascading failure paths 2 and 3, the generation and load tripping can be captured by the proposed risk-oriented PMU placement because this PMU placement achieves full observability of cascading failure paths 2 and 3. However, the system becomes unobservable (buses 14 and 18) following the cascading failure path 1. This is acceptable because cascading failure path 1 does not cause any load or generation tripping. On the contradictory side, as shown in Fig. 6, the traditional probability-oriented PMUs placement cannot capture the generation and load tripping for cascading paths 2 and 3 although the system is still observable following the cascading failure path 1. The reasons for above observations are explained as

follows: (i) the line trips in cascading failure path 1 do not result in generation and load tripping; and (ii) it is not necessary to guarantee the observability for low-risk cascading failure path, such as path 1 without any load or generation tripping; and (iii) the severity of the observability loss is not considered in the traditional probability-oriented approach.

3.2 Case 2: IEEE 118-bus system

In the case of IEEE 118-bus system, as shown in Fig. 8, we consider two cascading paths with the highest probabilities. Table 3 presents details of the initial events and the line-tripping sequences for the two cascading paths, and the failure probability of each line based on the method described in Section 2.4. Note that for this case the severity index listed in Table 3 is related with line overloading, which is defined by (5), for the two cascading paths. The risk-oriented PMU placement optimisation problem described by (8)–(15) can be formulated and solved.

Given 33 PMUs, the proposed risk-oriented approach produces the PMU placement results shown in Table 4. For comparison, Table 4 also provides the PMU placement using the conventional probability-oriented approach that maximises the observability probability. Table 5 shows the unobservable overloaded lines with the cascading failure paths 1 and 2, for both the conventional probability-oriented approach and the proposed risk-oriented approach. It can be found from Table 5 that the conventional probability-oriented approach loses the observability for several overloaded lines after the cascading failure, while the proposed risk-oriented approach could reduce the number of unobservable overloaded lines for the cascading paths, e.g. cascading path 1 with only one unobservable overloaded line and cascading path 2 with no unobservable overloaded lines. Thus, the benefits of the proposed approach can be clearly demonstrated.

3.3 Discussion of applying the proposed method on large-scale systems

Paralleling the cascading failure path identification approach described in Section 2.6 for different initial events of line tripping on large-scale systems is very straight-forward and easy, since different initial events of line tripping will lead to different cascading failure path analysis and the analysis is totally independent of each other. We parallel the cascading failure path identification tasks for the 39-bus and 118-bus systems with the MATLAB Parallel Computing Toolbox function *parfor* on a Windows Server with 32 cores of 3.20 GHz Intel Xeon CPUs, 128 GB Memory. For 39-bus system, it only takes 6.34 s to complete the cascading failure path identification, while for the 118-bus system, it takes 72.86 s to complete the tasks.

For the scalability of the PMU placement optimisation problem, we would like to discuss it from three perspectives: (i) we treat the PMU placement optimisation problem as a planning stage problem, and we do not request that the optimisation problem to be solved within certain time limitation; (ii) in reality, for large-scale power grid with more than 1000 bus, typically the PMUs will be first installed for the high voltage level networks, such as the 500 and 230 kV substations, and thus the number of substations (buses) for PMU placement consideration is much smaller than the total number of buses in the system; (iii) in reality, for large-scale power grid with more than 1000 bus, the power grid is typically divided into several different areas and each area is managed by different utility companies, and each utility will only consider the PMU placement problem for its own area. Based on the above perspectives, we may argue that 39-bus system and 118-bus system are proper test systems to show the performance of the PMU placement optimisation method. On a laptop with 2.6 GHz CPU and Windows 7 operation system, the average solution time for the optimisation problem of the IEEE 39-bus system and IEEE 118-bus system are 10.4 and 307.6 s, respectively.

4 Conclusions

This paper proposes a risk-oriented PMU placement approach. The probability of observability loss and the corresponding

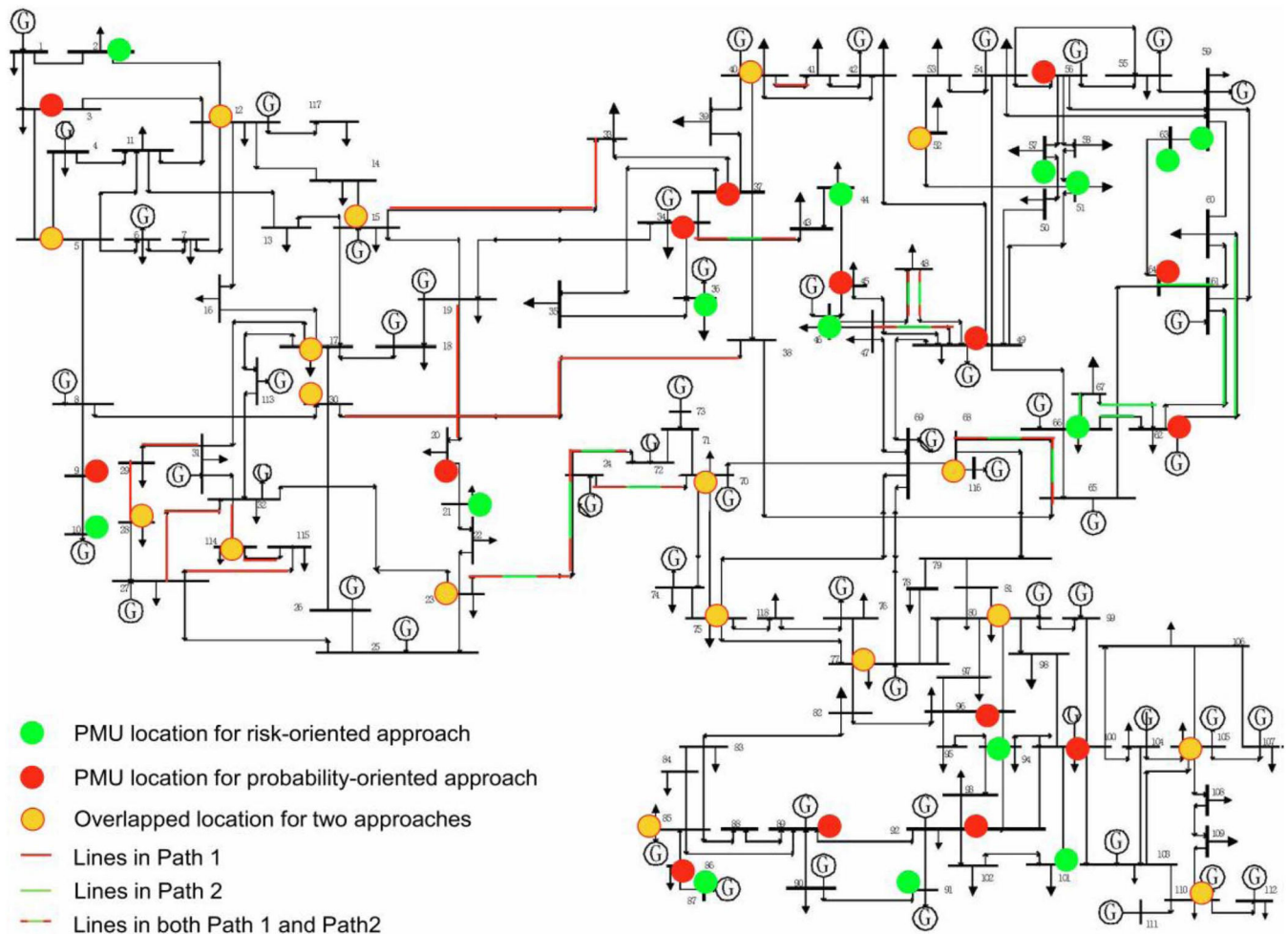


Fig. 8 IEEE 118-bus test system

Table 3 Cascading paths for 118-bus system

	Cascading path 1	Cascading path 2
initial event	line 27 to 115 trip	line 62 to 66 trip
2nd round line trip	line 27 to 32 trip with Pro. 0.999 line 32 to 114 trip with Pro. 0.999 line 114 to 115 trip with Pro. 0.999	line 61 to 62 trip with Pro. 0.173 line 61 to 64 trip with Pro. 0.232 line 60 to 62 trip with Pro. 0.999 line 62 to 67 trip with Pro. 0.999 line 66 to 67 trip with Pro. 0.157 line 65 to 68 trip with Pro. 0.999
3rd round line trip	line 24 to 72 trip with Pro. 0.999 line 28 to 29 trip with Pro. 0.999 line 34 to 43 trip with Pro. 0.353 line 29 to 31 trip with Pro. 0.999 line 65 to 68 trip with Pro. 0.159	
4th round line trip	line 23 to 24 trip with Pro. 0.999 line 24 to 70 trip with Pro. 0.999 line 47 to 49 trip with Pro. 0.999	
5th round line trip	line 15 to 33 trip with Pro. 0.999 line 19 to 20 trip with Pro. 0.651 line 30 to 38 trip with Pro. 0.826 line 40 to 41 trip with Pro. 0.683 line 46 to 48 trip with Pro. 0.999 line 48 to 49 trip with Pro. 0.517	line 23 to 24 trip with Pro. 0.999 line 24 to 72 trip with Pro. 0.999 line 47 to 49 trip with Pro. 0.999 line 24 to 70 trip with Pro. 0.999 line 34 to 43 trip with Pro. 0.999 line 46 to 48 trip with Pro. 0.999 line 48 to 49 trip with Pro. 0.999
6th round line trip		
cascading failure probability	0.0104	0.0062
severity of line overloading	164.8480	109.9518
risk	1.7219	0.6859

Table 4 Comparison of PMU placement for 118-bus system

Traditional probability-oriented approach	Proposed risk-oriented approach
3, 5, 9, 12, 15, 17, 20, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 89, 92, 96, 100, 105, 110, 114	2, 5, 10, 12, 15, 17, 21, 23, 28, 30, 36, 40, 44, 46, 51, 52, 57, 59, 63, 66, 71, 75, 77, 80, 85, 87, 91, 94, 101, 105, 110, 114, 116

Table 5 Comparison of PMU placement for unobservable over-loaded lines after cascading failures with 118-bus system

Traditional probability-oriented approach		Proposed risk-oriented approach	
Cascading failure path 1	Cascading failure path 2	Cascading failure path 1	Cascading failure path 2
line 41 to 42	line 43 to 44	line 41 to 42	no overloaded lines unobservable
line 43 to 44	line 46 to 47		
line 46 to 47	line 47 to 49		
line 47 to 49	line 59 to 60		
	line 59 to 61		
	line 60 to 61		

consequence severity are considered for PMU placement. The key point is that we can tolerate the observability loss as long as there are no severe consequences (e.g. overloaded lines or generation trip or load trip can still be observed) following the observability loss. Simulation results show that the proposed approach captures the risk nature associated with observability loss in power grid while the traditional approach may fail to do so.

5 References

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