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Synchronization of complex-valued dynamic networks with intermittently adaptive coupling: A direct error method ☆

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ABSTRACT

In this paper, the models of intermittently coupled complex-valued networks (ICCVNs) are presented to reveal the mechanism of intermittent coupling, where the nodes are connected merely in discontinuous time durations. Instead of the common weighted average technique, by proposing a direct error method and constructing piecewise Lyapunov functions, several intermittently adaptive designs are developed to update the complex-valued coupling weights. Especially, an adaptive pinning protocol is designed for ICCVNs with heterogeneous coupling weights and the synchronization is ensured by piecewise adjusting the complex-valued weights of edges within a spanning tree. For ICCVNs with homogeneous coupling weights, based on a connected dominating set, an intermittently adaptive algorithm is developed which just depends on the information of the dominating set with their neighbors. At the end, the established theoretical results are verified by providing two numerical examples.

1. Introduction

Complexity, called "a science of 21st century", is one of key and leading scientific topics currently. As a sort of typical complex systems, complex networks visually demonstrate complex interactions among different units. Actually, complex networks are pervasive and considerable practical systems or problems in sociology, engineering and physics can be represented and explored by means of complex networks (Albert & Barabasi, 2002; Strogatz, 2001). Numerous facts, such as 2004 Blackout in Western Europe and 2012 major power outages in India (Rampurkar, Pentayya, Mangalvedekar, & Kazi, 2016), indicate that the investigation of complex networks is conductive not only to understand the complexity of the real world, but also to transform the objective world by providing a new methodology and powerful weapon.

An important and meaningful topic in dynamical networks is synchronization of nodes dynamics, which characterizes a representative behavior of networks and indicates mathematically that the trajectories of coupled subsystems are asymptotically identical (Arenas, Díaz-Guilera, Kurths, Moreno, & Zhou, 2008; Chen & Zhu, 2007; Wang & Chen, 2002). Besides, synchronization has many promising applications in multiple rigid bodies (Ren, 2010), social networks (Angeli & Manfredi, 2019) and mobile agents (Tanner, Jadbabaie, & Pappas, 2007). Up to now, synchronization of dynamical networks has caused extensive concern and many valuable results have been reported. For instance, the authors in Pecora and Carroll (1998) studied local synchronization of coupled systems based on master stability function. Lu and Chen (2006) proposed a new approach called weighted average method or projection approach to investigate synchronization of linear coupled networks. By defining a projection of the spatial states on the synchronization manifold and introducing the distance between the spatial states and its projection, the authors proved that the synchronization can be ensured provided that the distance is convergent to zero. Nowadays, the weighted average method has become a mainstream approach and has been widely generalized to study the synchronization of diverse networks such as dynamic networks with coupling delays (Liu & Chen, 2007; Lu, Chen, & Chen, 2006), nonidentical coupled systems (Lu, Liu, and Chen, 2010), nonlinearly coupled networks (Chen & Zhu,

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2007; Liu & Chen, 2008), spatiotemporal networks (Wang, Wu, & Guo, 2014) and fractional-order networks (Lv, Hu, Yu, Jiang, & Huang, 2018). On the other hand, to achieve and enhance synchronization of dynamic networks, various control means have been proposed, such as pinning control (Chen, Liu, & Lu, 2007; Zhou, Lu, & Lü, 2008), event-triggered control (Li, Shen, Wang, Huang, & Luo, 2019), impulsive control (Guan, Liu, Feng, & Wang, 2010; Lu, Ho, and Cao, 2010), intermittent control (Hu & Jiang, 2015; Liu & Chen, 2015) and adaptive control (Radenković & Krstić, 2018; Sorrentino & Ott, 2008).

As an effective control method, adaptive control has received considerable concern in the latest decades because of its distinctive adaptability to the evolution or disturbance of system. Self-adaptation is also ubiquitous in many real networks such as biology networks, social networks and economic networks (Fewell, 2003). In DeLellis, diBernardo, and Garofalo (2008), based on local information of the networks, edge-based distributed adaptive strategy was developed to update the weights of edges. Nowadays, combining with pinning control, the edge-based distributed adaptive control has been vastly investigated to discuss the synchronization of diverse coupled networks such as undirected networks (DeLellis & Garofalo, 2009; Su et al., 2013; Yu, DeLellis, Chen, diBernardo, & Kurths, 2012), directed networks (Yu, Lü, Yu, & Chen, 2015), nonidentical networks (Lu, Liu, and Chen, 2010) and coupled neural networks (Wang et al., 2014; Zhang, Pal, Sheng, and Zeng, 2019).

Nonetheless, the results mentioned above mainly concentrate on real-valued dynamic networks. In fact, many practical systems, such as reaction–advection–diffusion system (Bolognani, Smyshlyaev, & Krstic) and laser system (Gibbon & McGuinness, 1982), can be more effectively modeled by differential systems involving complex variables. For instance, the amplitude and phase information of the transmitted signals can be simultaneously accessed and handled by using complex-valued networks. Moreover, it is revealed that the signals transmitted by complex-variable networks possess stronger anti-attack ability (Adali, Schreier, & Scharf, 2011). Nowadays, by utilizing impulsive control (Wu, Liu, & Ye, 2015), delayed feedback control (Zhang, Wang, Wang, Zhou, and Xia, 2018; Zhang, Yang, Xu, & Feng, 2017), intermittent control (Wang, Jin, & Su, 2018; Wu & Leng, 2017) and adaptive control (Wu & Fu, 2013; Xu, Zhou, Fang, Sun, & Pan, 2015), the synchronization was extensively discussed for various kinds of complex-valued networks. Nevertheless, these results were obtained based on a common condition that the couplings among nodes are always continuous, which are inapplicable for complex networks with discontinuous coupling.

Actually, many practical systems, such as species diffusion models in biology (Dhar & Jatav, 2013) and memristor-based circuits (Strukov, Snider, Stewart, & Williams, 2008), are coupled by discontinuous connections instead of continuous coupling. Moreover, in communication networks, different components may not be always linked due to noises, packet losses and data congestion (Imer, Yüksel, & Başar, 2006). In 2008, impulsively coupled networks were introduced as a class of complex networks with discontinuous coupling to reveal the instantaneous interaction in real networks, in which all network nodes are coupled only at some isolate time instants and the dynamic behaviors of the entire networks are described by impulsive differential equations (Han, Lu, & Wu, 2008). At present, parametric uncertainties (Yang, Wang, Xiao, & Wang, 2010), time delays (Sun, Austin, Lü, & Chen, 2011), disconnected topology (Chen, Yu, Li, & Feng, 2013; Chen, Yu, Tan, & Zhu, 2016) have been respectively introduced into the modeling and synchronization control of impulsively coupled complex networks.

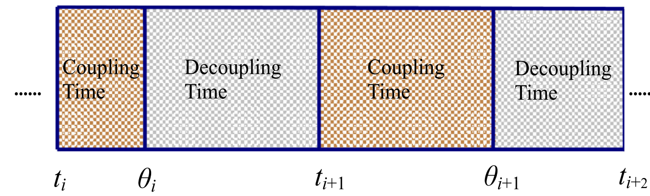

Fig. 1. The simplified intermittent coupling sketch.

As discussed above, the impulsively coupled networks can be applied to accurately describe the transient information communications among different components in practical networks. However, the communications and couplings among nodes are neither continuous nor instantaneous but intermittent in some practical applications. For example, the phenomenon of intermittent bidirectional dispersal in population models with multiple patches (Zhang, Xu, & Teng, 2016), in networks of mobile robots responsible for accomplishing task (Kantaros & Zavlanos, 2017), robots are enforced to intermittently exchange information at predetermined time durations and operate in disconnect mode in the rest of time. Moreover, it was found recently that each agent in multi-agent systems only shares its information with its neighbors on some disconnected time intervals because of the limitation of sensing ranges, equipment failures and communication obstacles (Cheng, Yu, Wan, & Cao, 2016; Li, Phillips, & Sanfelice, 2018; Wang & Yang, 2018). Evidently, the mechanism of intermittent coupling can be described by Fig. 1, in which each node only communicates with its neighbors in coupling or communication time and there is no interaction among different nodes in the rest of time called decoupling time. Compared with the traditional continuous communication, intermittent coupling provides more flexibility to each unit to accomplish its task or update its state behavior because it is not bound by communication requirements in decoupling time. In contrast to impulsive coupling, intermittent coupling facilitates communication and cooperation among network nodes as the nodes share their information with each other in entire coupling time intervals rather than at some discrete time points.

Since intermittent coupling is an objective phenomenon in real networks different from continuous or impulsive coupling, the following challenging problems would be interesting and meaningful: how to establish dynamical models to describe the intermittent coupling? What kinds of effective adaptive strategies can be designed to update the intermittently coupling weights? How to determine the controlled edges in pinning adaptive schemes to achieve synchronization? What kinds of analytical tools and mathematical methods can be employed to handle the characteristics of discontinuous coupling and complex variables? To the authors' knowledge, unfortunately, in contrast to extensive efforts on continuously or impulsively coupled real-valued networks, there seem to be no reported results at present to explore these challenging problems.

Enlightened by the above analysis, the models of ICCVNs with heterogeneous or homogeneous coupling weights are proposed in this paper and the synchronization is discussed by developing intermittently adaptive schemes for coupling weights. The innovative contents are concluded below:

(1) Two types of ICCVNs, namely, ICCVNs with heterogeneous and homogeneous complex-valued coupling weights, are presented to describe the mechanism of intermittent interaction, which are distinguishable utterly from the complex network models with continuous coupling (Chen et al., 2007; DeLellis et al., 2008; Lu & Chen, 2006; Pecora & Carroll, 1998; Wang & Chen, 2002; Yu et al., 2012; Zhang, Pal, Sheng, and Zeng, 2019)

and impulsive coupling (Chen et al., 2013, 2016; Han et al., 2008; Sun et al., 2011; Yang et al., 2010).

(2) An edge-based adaptive law and its pinning strategy are proposed for ICCVNs with heterogeneous coupling weights and the synchronization is achieved by intermittently adjusting the complex-valued strengths of edges within a spanning tree. Particularly, the result derived in Yu et al. (2012) is included and extended. For ICCVNs with a homogeneous complex-valued weight, based on a connected dominating set, an intermittently adaptive law is developed, which just depends on the information of the dominating set and their neighbors. The fully centralized control schemes given in Hu, Yu, Jiang, and Teng (2011) and Li, Jiao, and Lee (2008) are essentially improved.

(3) In theoretical analysis, by proposing a direct error method, some piecewise continuous functions are constructed to analyze synchronization of ICCVNs, which are totally different from the weighted average technique and the analysis method based on continuous Lyapunov functions employed in Chen and Zhu (2007), Liu and Chen (2008), Lu and Chen (2006), Lu et al. (2006), Lv et al. (2018), Wang et al. (2014), Yang et al. (2010) and Yu et al. (2012).

The structure for the remaining of this paper is arranged as follows. In Section 2, the models of ICCVNs and some preliminaries are introduced. The synchronization for ICCVNs is studied respectively in Section 3 and Section 4 for heterogeneous and homogeneous weights. Some numerical simulations are given in Section 5 to show the feasibility of the theoretical results. A brief summary of this paper is drawn at the end.

Notations: In this paper, C, R^m, C^m and $C^{m\times m}$ denote the sets of all complex numbers, and m-dimensional real vectors, complex vectors and complex matrices, respectively. For $\delta = Re(\delta) + iIm(\delta) \in C$, $Re(\delta)$ and $Im(\delta)$ are severally the real and imaginary parts of δ, $\delta^R = \delta + \overline{\delta}$, where $\overline{\delta}$ denotes its conjugate, i is the imaginary unit and $i^2 = -1$. For a complex-valued matrix A, A^H denotes its conjugate transposition. For $\gamma \in C^m$, the norm is defined as $\|\gamma\| = \sqrt{\gamma^H\gamma}$. Let Z^+ be the set of all non-negative integers and $\mathscr{N} = \{1, 2, \ldots, N\}$. I_m denotes an m-dimensional unit matrix, $\mathbf{1}_m$ represents an m-dimensional vector where all entries are equal to 1. $G = (\mathcal{V}, E)$ represents an undirected graph consisting of node set $\mathcal{V} = \{1, 2, \ldots, N\}$ and edge set $E = \{(r, j),\ r, j \in \mathscr{N}\}$. $\Xi = (c_{ij})_{N\times N}$ denotes the coupling matrix of G, in which $c_{rj} = c_{jr} = 1\ (j \neq r)$ if the rth node and the jth node are linked by an edge (r, j), otherwise $c_{rj} = c_{jr} = 0\ (j \neq r)$, $c_{rr} = -\sum_{j=1,j\neq r} c_{rj}$ with $r \in \mathscr{N}$. $\omega_{rj}(t) = \omega_{jr}(t) \neq 0\ (j \neq r)$ denotes the complex-valued weight of the edge (r, j). The networks addressed in this paper is assumed to be connected in all coupling time intervals.

2. The model of ICCVNs and preliminaries

Consider a type of ICCVNs involving N identical nodes, and the dynamics of each node in isolation is depicted by

$$\dot{z}_r(t) = h(z_r(t)),$$

where $z_r = (z_r^1, z_r^2, \ldots, z_r^n)^T \in C^n$ is the state variable of the rth node, $h : C^n \to C^n$ is a nonlinear vector function.

In consideration of the intermittent coupling among nodes described in Fig. 1, the model of ICCVNs can be depicted by

$$\dot{z}_r(t) = h(z_r(t)) + \sum_{k=0}^{+\infty}\Big[\sum_{p=1,p\neq r}^{N} c_{rp}\omega_{rp}(t)\big(z_p(t) - z_r(t)\big)\Big]\Delta_k(t), \tag{1}$$

where $r \in \mathscr{N}$, for each $k \in Z^+$, $\Delta_k(t)$ is an index function of intermittent coupling defined by

$$\Delta_k(t) = \begin{cases} 1, & t_k \le t \le \theta_k, \\ 0, & \text{otherwise}, \end{cases}$$

in which $t_k < \theta_k < t_{k+1}$ and $\lim_{k\to+\infty} t_k = +\infty$ for all $k \in Z^+$, the intervals $[t_k, \theta_k]$ are called coupling or communication time and the intervals $[\theta_k, t_{k+1}]$ are called decoupling time for $k \in Z^+$.

Remark 1. In the last few decades, complex-valued dynamical networks with continuous coupling have been extensively investigated and numerous excellent results on synchronization have been reported (Wang et al., 2018; Wu & Fu, 2013; Wu & Leng, 2017; Wu et al., 2015; Xu et al., 2015; Zhang, Wang, Wang, Zhou, and Xia, 2018; Zhang et al., 2017). Different from these work, intermittently coupled complex-valued network (1) is introduced in this paper as a different type of network models to describe the phenomenon of intermittent communication in real-world networks.

Remark 2. Note that the weights of different edges may be nonidentical in the model (1), namely, the weights are heterogeneous. Particularly, if they are degenerated to be homogeneous, the network (1) is rewritten as

$$\dot{z}_r(t) = h(z_r(t)) + \omega(t)\sum_{k=0}^{+\infty}\sum_{p=1}^{N} c_{rp}z_p(t)\Delta_k(t), \quad r \in \mathscr{N}, \tag{2}$$

where $\omega(t)$ is renamed as the coupling weight of the overall network.

Definition 1. The network (1) is said to be asymptotically synchronized if

$$\lim_{t\to+\infty} \|z_j(t) - z_r(t)\| = 0,\ \ r, j \in \mathscr{N}.$$

Let $\xi_{rj}(t) = z_j(t) - z_r(t)$ be the synchronization error between the rth node and the jth node and $c_{rr}\omega_{rr}(t) = -\sum_{j\neq r} c_{rj}\omega_{rj}(t)$ for $r, j \in \mathscr{N}$, then the following error system is obtained:

$$\dot{\xi}_{rj}(t) = \begin{cases} \hat{h}(\xi_{rj}(t)) + \displaystyle\sum_{p=1}^{N} c_{jp}\omega_{jp}(t)\xi_{jp}(t) \\ \quad - \displaystyle\sum_{p=1}^{N} c_{rp}\omega_{rp}(t)\xi_{rp}(t),\ \ t_k \le t \le \theta_k, \\ \hat{h}(\xi_{rj}(t)),\ \ \theta_k < t < t_{k+1},\ \ k \in Z^+, \end{cases} \tag{3}$$

where $\hat{h}(\xi_{rj}(t)) = h(z_j(t)) - h(z_r(t))$.

Assumption 1. There exists a real constant F such that for any $z_1,\ z_2 \in C^n$,

$$(z_1 - z_2)^H(h(z_1) - h(z_2)) + (h(z_1) - h(z_2))^H(z_1 - z_2) \le F(z_1 - z_2)^H(z_1 - z_2).$$

Assumption 2. There exist constants $T > \beta > 0$ such that

$$\inf_{k\in Z^+}\{\theta_k - t_k\} = \beta, \quad \sup_{k\in Z^+}\{t_{k+1} - t_k\} = T.$$

Remark 3. If $h : R^n \to R^n$, Assumption 1 is reduced to the following form

$$(z_1 - z_2)^T(h(z_1) - h(z_2)) \le \hat{F}(z_1 - z_2)^T(z_1 - z_2),$$

where $z_1, z_2 \in R^n$, $\hat{F} = \frac{F}{2}$. It is obviously equivalent to the QUAD condition which has been widely utilized to analyze synchronization of real-valued networks (Chen & Zhu, 2007; DeLellis, diBernardo, & Russo, 2011; Liu & Chen, 2008; Lu & Chen, 2006; Yu et al., 2012, 2015). From this point, Assumption 1 can be regarded as an extension of the QUAD condition to complex-valued vector fields.

Definition 2. (Bondy & Murty, 1976). For a connected graph G, an acyclic connected subgraph containing all vertices of G is called to be a spanning tree of G.

Definition 3. (Sampathkumar & Walikar, 1979). In a graph $G = (\mathcal{V}, E)$, a subset $S \subseteq \mathcal{V}$ is called a dominating set of it if every vertex not in S is adjacent to at least one vertex in S. Furthermore, the dominating set S is said to be a connected dominating set if it induces a connected subgraph in G.

Lemma 1. *(Sampathkumar & Walikar, 1979). For a graph G, a connected dominating set exists if and only if G is connected.*

Lemma 2. *(Horn & Johnson, 1985). Denote $L = (l_{rj})_{N\times N}$ as the Laplacian matrix of an undirected graph, then*

(1) L has a unique zero eigenvalue and the rest of the eigenvalues are positive if and only if the network is connected.

(2) The smallest nonzero eigenvalue $\lambda_2(L)$ satisfies

$$\lambda_2(L) = \min_{u\neq 0, u\perp \mathbf{1}_N} \frac{u^H L u}{u^H u}.$$

(3) If a vector denoted by $u = (u_1^H, u_2^H, \ldots, u_N^H)^H \in C^{nN}$ satisfies $\sum_{r=1}^N u_r = 0$ with $u_r \in C^n$, then for any symmetric and positive semidefinite matrix $\Sigma \in R^{n\times n}$,

$$u^H(L\otimes \Sigma)u \geq \lambda_2(L) u^H(I_N \otimes \Sigma)u.$$

3. Synchronization for ICCVNs with heterogeneous coupling weights

In this section, some intermittently adaptive schemes for coupling weights are developed to achieve synchronization.

Proposition 1. *For the synchronization errors, the following equality is true:*

$$\begin{aligned}&\sum_{r=1}^N \sum_{j=1,j\neq r}^N \sum_{p=1}^N \xi_{rj}^H(t)\Big(c_{jp}\omega_{jp}(t)\xi_{jp}(t) - c_{rp}\omega_{rp}(t)\xi_{rp}(t)\Big)\\ &+\sum_{r=1}^N \sum_{j=1,j\neq r}^N \sum_{p=1}^N \Big(c_{jp}\omega_{jp}(t)\xi_{jp}(t) - c_{rp}\omega_{rp}(t)\xi_{rp}(t)\Big)^H \xi_{rj}(t)\\ &= -N\sum_{r=1}^N \sum_{j=1,j\neq r}^N c_{rj}\omega_{rj}^R(t)\xi_{rj}^H(t)\xi_{rj}(t). \end{aligned} \tag{4}$$

Particularly, if $\omega_{jp}(t)$ and $\xi_{jp}(t)$ are real-valued for $j, p \in \mathcal{N}$,

$$\begin{aligned}&\sum_{r=1}^N \sum_{j=1,j\neq r}^N \sum_{p=1}^N \xi_{rj}^T(t)\Big(c_{jp}\omega_{jp}(t)\xi_{jp}(t) - c_{rp}\omega_{rp}(t)\xi_{rp}(t)\Big)\\ &= -N\sum_{r=1}^N \sum_{j=1,j\neq r}^N c_{rj}\omega_{rj}(t)\xi_{rj}^T(t)\xi_{rj}(t). \end{aligned} \tag{5}$$

Proof. See Appendix A. ■

Proposition 2. *Suppose that function $U(t)$ is nonnegative and continuous on $[t_0, +\infty)$, nonnegative function $W(t)$ is continuous on $[t_0, +\infty)$ except for a time set $\{t_k\}$ and is right continuous at these time points. Denote $V(t) = U(t)+W(t)$, under Assumption 2, if there exist constants $\mu > 0$ and $\lambda > 0$ satisfying $\varpi = \mu\beta - \lambda(T-\beta) > 0$ such that*

$$D^+V(t) \leq \begin{cases} -\mu V(t), & t_k \leq t \leq \theta_k,\\ \lambda V(t), & \theta_k < t < t_{k+1},\end{cases} \tag{6}$$

and

$$W(t_{k+1}) \leq e^{\mu(\theta_k - t_k) - \lambda(t_{k+1}-\theta_k)} W_-(t_{k+1}), \tag{7}$$

then $\lim_{k\to+\infty} U(t_k) = 0$, where $W_-(t_{k+1})$ denotes the left limit of $W(t)$ at time t_{k+1}. Furthermore, $\lim_{t\to+\infty} U(t) = 0$ if there exists a constant σ such that

$$D^+U(t) \leq \sigma U(t),\ \ t_k \leq t < t_{k+1}. \tag{8}$$

Proof. See Appendix B. ■

Theorem 1. *Based on Assumptions 1–2, the network (1) reaches synchronization under the following intermittently adaptive scheme:*

$$\begin{cases} \dot{\omega}_{rj}(t) = \alpha_{rj}c_{rj}\big(z_r(t) - z_j(t)\big)^H\big(z_r(t)-z_j(t)\big),\ t_k \leq t \leq \theta_k,\\ \omega_{rj}(t_{k+1}) = \omega_{rj}(\theta_k),\\ \omega_{rj}(t) = 0,\ \ \theta_k < t < t_{k+1},\ \ k \in Z^+,\ \ (r, j) \in E,\end{cases} \tag{9}$$

where $\alpha_{rj} = \alpha_{jr} \in C$ with $Re(\alpha_{rj}) > 0$, and $Re(\omega_{rj}(t_0)) \geq 0$, $(r, j) \in E$, $k \in Z^+$.

Proof. Choose a positive constant μ such that $\mu\beta - F(T-\beta) > 0$. Obviously, such constant μ always exists. Construct a piecewise function described by

$$W(t) = \begin{cases} \dfrac{N}{2}\displaystyle\sum_{r=1}^N \sum_{j=1,j\neq r}^N \frac{e^{-\mu(t-t_k)}}{\alpha_{rj}^R}\Big(\omega^* - \omega_{rj}^R(t)\Big)^2,\ \ t_k \leq t \leq \theta_k,\\ \dfrac{N}{2}\displaystyle\sum_{r=1}^N \sum_{j=1,j\neq r}^N \frac{e^{F(t-\theta_k)-\mu(\theta_k - t_k)}}{\alpha_{rj}^R}\Big(\omega^* - \omega_{rj}^R(\theta_k)\Big)^2,\\ \qquad\qquad \theta_k < t < t_{k+1},\end{cases} \tag{10}$$

where $k \in Z^+$, ω^* is a positive constant. Obviously, $W(t)$ is continuous except on $t = t_{k+1}$ and

$$W(t_{k+1}) = W_+(t_{k+1}) = e^{\mu(\theta_k - t_k) - F(t_{k+1}-\theta_k)} W_-(t_{k+1}), \tag{11}$$

in which $W_-(t_{k+1})$ and $W_+(t_{k+1})$ denote the left and right limits of $W(t)$ at time t_{k+1}, respectively.

Define the following Lyapunov function

$$V(t) = U(t) + W(t), \tag{12}$$

where

$$U(t) = \sum_{r=1}^N \sum_{j=1,j\neq r}^N \xi_{rj}^H(t)\xi_{rj}(t).$$

For $t_k \leq t \leq \theta_k$, from Assumptions 1–2 and Proposition 1,

$$\begin{aligned} D^+V(t) \leq& \Big(F+\mu\Big)\sum_{r=1}^N \sum_{j=1,j\neq r}^N \xi_{rj}^H(t)\xi_{rj}(t) - \mu V(t)\\ &- Ne^{-\mu T}\omega^* \sum_{r=1}^N \sum_{j=1,j\neq r}^N c_{rj}\xi_{rj}^H(t)\xi_{rj}(t). \end{aligned} \tag{13}$$

Let $\hat{\xi}(t) = (\hat{\xi}_1^H(t), \hat{\xi}_2^H(t) \ldots, \hat{\xi}_N^H(t))^H$ and

$$\hat{\xi}_r(t) = z_r(t) - \frac{1}{N}\sum_{k=1}^N z_k(t),\quad r \in \mathcal{N},$$

then by Lemma 2,

$$\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t) = 2\sum_{r=1}^{N}\sum_{j=1}^{N} l_{rj}z_r^{H}(t)z_j(t) = 2\hat{\xi}^{H}(t)(L\otimes I_n)\hat{\xi}(t) \geq 2\lambda_2(L)\hat{\xi}^{H}(t)\hat{\xi}(t), \tag{14}$$

$$\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N} \xi_{rj}^{H}(t)\xi_{rj}(t) = 2\hat{\xi}^{H}(t)(M\otimes I_n)\hat{\xi}(t), \tag{15}$$

where $M=(m_{rj})_{N\times N}$, $m_{rj}=-1$ for $r\neq j$ and $m_{rr}=N-1$.

Choose a suitable constant ω^* such that

$$(F+\mu)(M\otimes I_n)-\omega^* Ne^{-\mu T}\lambda_2(L)I_{nN}\leq 0,$$

then from (13)–(15), for $t_k\leq t\leq\theta_k$,

$$D^+V(t)\leq -\mu V(t). \tag{16}$$

On the other hand, evidently, for $\theta_k<t<t_{k+1}$,

$$D^+V(t)\leq FV(t). \tag{17}$$

Note that $\omega_{rj}^{R}(t)\geq 0$ for $r,j\in Z^+$ and $t\geq t_0$, by Proposition 1,

$$D^+U(t)\leq FU(t),\quad t_k\leq t<t_{k+1}. \tag{18}$$

By (16)–(18) and Proposition 2, $\lim_{t\to+\infty}U(t)=0$. ■

Remark 4. In continuously coupled networks (Lu, Liu, and Chen, 2010; Lv et al., 2018; Wang et al., 2014; Yang et al., 2010; Yu et al., 2012), it suffices to assign the initial conditions to the adaptive gains at time t_0. However, the proposed coupling is intermittent in this paper and the evolutions of weights are interrupted in decoupling intervals (θ_k, t_{k+1}). So, in the following coupling intervals $[t_{k+1}, \theta_{k+1}]$, the values $\omega_{rj}(t_{k+1})$ must be provided according to the theory of Cauchy problem of differential equations to guarantee the evolution of the adaptive weights during the coupling intervals. Hence, the values $\omega_{rj}(t_{k+1})=\omega_{rj}(\theta_k)$ are essential and indispensable in the intermittent adaptive scheme (9).

Note that all coupling weights are updated in Theorem 1 under the intermittent adaptive law (9). In the following, only a fraction of weights is adjusted to realize synchronization.

Let $\hat{G}=(\mathcal{V},\hat{E})$ be a spinning tree of the network (1), which always exists since the network (1) is connected. Evidently, the elements of the Laplacian matrix $\hat{L}$ of $\hat{G}$ can be described by

$$\hat{l}_{rj}=\begin{cases}-c_{rj}, & \text{if } (r,j)\in\hat{E},\\ -\sum_{j=1,j\neq r}^{N}\hat{l}_{rj}, & \text{if } r=j,\\ 0, & \text{otherwise.}\end{cases} \tag{19}$$

Denote $\hat{\Omega}=(\hat{\omega}_{rj})_{N\times N}$ and

$$\hat{\omega}_{rj}=\begin{cases}-c_{rj}\omega_{rj}^{R}(t_0), & \text{if } (r,j)\in E\setminus\hat{E},\\ -\sum_{j=1,j\neq r}^{N}\hat{\omega}_{rj}, & \text{if } r=j,\\ 0, & \text{otherwise.}\end{cases}$$

Theorem 2. *Based on Assumptions 1–2, the network (1) is asymptotically synchronized under the following intermittently adaptive pinning scheme*

$$\begin{cases}\dot{\omega}_{rj}(t)=\alpha_{rj}c_{rj}\big(z_r(t)-z_j(t)\big)^{H}\big(z_r(t)-z_j(t)\big),\ t_k\leq t\leq\theta_k,\\ \omega_{rj}(t_{k+1})=\omega_{rj}(\theta_k),\\ \omega_{rj}(t)=0,\ \theta_k<t<t_{k+1},\ k\in Z^+,(r,j)\in\hat{E},\end{cases} \tag{20}$$

where $\alpha_{rj}=\alpha_{jr}\in C$ with $Re(\alpha_{rj})>0$ and $(r,j)\in\hat{E}$, $Re(\omega_{rj}(t_0))\geq 0$ with $(r,j)\in E$.

Proof. Construct the following Lyapunov function

$$\hat{V}(t)=U(t)+\hat{W}(t), \tag{21}$$

where

$$\hat{W}(t)=\begin{cases}\frac{N}{2}e^{-\mu(t-t_k)}\sum_{r=1}^{N}\sum_{(r,j)\in\hat{E}}\frac{1}{\alpha_{rj}^{R}}\Big(\hat{\omega}^*-\omega_{rj}^{R}(t)\Big)^2,\ t_k\leq t\leq\theta_k,\\ \frac{N}{2}e^{F(t-\theta_k)-\mu(\theta_k-t_k)}\sum_{r=1}^{N}\sum_{(r,j)\in\hat{E}}\frac{1}{\alpha_{rj}^{R}}\Big(\hat{\omega}^*-\omega_{rj}^{R}(\theta_k)\Big)^2,\\ \qquad\theta_k<t<t_{k+1},\end{cases}$$

in which $k\in Z^+$, $\hat{\omega}^*$ is a positive constant.

When $t_k\leq t\leq\theta_k$, from Proposition 1, one can easily obtain

$$D^+\hat{V}(t)\leq F\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\xi_{rj}^{H}(t)\xi_{rj}(t)-\mu W(t) - N\sum_{r=1}^{N}\sum_{(r,j)\in E\setminus\hat{E}}c_{rj}\omega_{rj}^{R}(t_0)\xi_{rj}^{H}(t)\xi_{rj}(t) - N\hat{\omega}^*e^{-\mu T}\sum_{r=1}^{N}\sum_{(r,j)\in\hat{E}}c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t). \tag{22}$$

Similar to (14), one has

$$\sum_{r=1}^{N}\sum_{(r,j)\in E\setminus\hat{E}}c_{rj}\omega_{rj}^{R}(t_0)\xi_{rj}^{H}(t)\xi_{rj}(t)=2\hat{\xi}^{H}(t)(\hat{\Omega}\otimes I_n)\hat{\xi}(t), \tag{23}$$

$$\sum_{r=1}^{N}\sum_{(r,j)\in\hat{E}}c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t)\geq 2\lambda_2(\hat{L})\hat{\xi}^{H}(t)\hat{\xi}(t). \tag{24}$$

Note that the spanning tree $\hat{G}=(\mathcal{V},\hat{E})$ is connected, $\lambda_2(\hat{L})>0$ by Lemma 2 and a suitable $\hat{\omega}^*$ can be chosen such that

$$(F+\mu)(M\otimes I_n)-\hat{\Omega}\otimes I_n-\hat{\omega}^*Ne^{-\mu T}\lambda_2(\hat{L})I_{nN}\leq 0, \tag{25}$$

then from (22)–(24),

$$D^+\hat{V}(t)\leq-\mu\hat{V}(t)\quad t_k\leq t\leq\theta_k.$$

The rest of the steps are the same as that of Theorem 1. ■

Remark 5. Unlike the full control of edges in Theorem 1, an intermittently adaptive pinning scheme is proposed to update the weights of partial edges in Theorem 2, which indicates that the synchronization of the network (1) is still achieved by only adjusting the weights of edges within a spanning tree. In addition, from the intermittently adaptive strategies (9) and (20), when the synchronization of the network (1) is achieved, the derivative $\dot{\omega}_{rj}(t)$ approaches to zero and each weight $\omega_{rj}(t)$ converges to a positive constant in coupling time intervals.

Remark 6. In Chen and Zhu (2007), Liu and Chen (2008), Lu and Chen (2006), Lu et al. (2006), Lv et al. (2018), Wang et al. (2014), Yang et al. (2010) and Yu et al. (2012), to obtain the synchronization of real-valued networks with continuous coupling, the weighted average $\bar{z}=\frac{1}{N}\sum_{j=1}^{N}z_j$ was defined and the synchronization is ensured by proving the errors $\xi_r=z_r-\bar{z}$ are convergent to zero. Different from the weighted average method, a new equality about the direct errors $\xi_{rj}=z_j-z_r$ is established in Proposition 1 and the synchronization of networks is obtained by directly showing the convergence of the error $\xi_{rj} = z_j - z_r$. Hence, this paper develops a more direct approach to discuss the synchronization of complex networks.

If $\theta_k = t_{k+1}$ for $k \in Z^+$, the ICCVNs (1) are reduced to the following complex networks with continuous coupling

$$\dot{z}_r(t) = h(z_r(t)) + \sum_{p=1}^{N} c_{rp}\omega_{rp}(t)z_p(t),\ r \in \mathcal{N}. \tag{26}$$

The following result is directly established by Theorem 2.

Corollary 1. *Under Assumption 1, the network (26) is synchronized under the following pinning adaptive strategy*

$$\dot{\omega}_{rj}(t) = \alpha_{rj}c_{rj}\big(z_r(t) - z_j(t)\big)^H\big(z_r(t) - z_j(t)\big),\ (r,j) \in \hat{E}, \tag{27}$$

where $\alpha_{rj} = \alpha_{jr} \in C$ with $Re(\alpha_{rj}) > 0$ and $(r,j) \in \hat{E}$, $Re(\omega_{rj}(t_0)) \geq 0$ with $(r,j) \in E$.

Remark 7. Evidently, Corollary 1 is still true if the states of all nodes and other parameters in system (26) are simplified to real-valued variables. In fact, the synchronization of the simplified real-valued networks has been investigated in Yu et al. (2012) by means of distributed adaptive control. It is easy to see that the main result of Theorem 4 in Yu et al. (2012) is the same as Corollary 1 when system (26) is degraded to the real-valued system.

4. Synchronization for ICCVNs with homogeneous coupling weights

In this section, we investigate the synchronization of ICCVNs with homogeneous weights (2).

Obviously, from Proposition 1,

$$\begin{aligned}
&\omega(t)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}\xi_{rj}^H(t)\Big(c_{jp}\xi_{jp}(t) - c_{rp}\xi_{rp}(t)\Big)\\
&+\overline{\omega}(t)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}\Big(c_{jp}\xi_{jp}(t) - c_{rp}\xi_{rp}(t)\Big)^H\xi_{rj}(t)\\
&= -N\omega^R(t)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}c_{rj}\xi_{rj}^H(t)\xi_{rj}(t).
\end{aligned}\tag{28}$$

Theorem 3. *The network (2) is asymptotically synchronized under Assumptions 1–2 and the following intermittently adaptive law:*

$$\begin{cases}\dot{\omega}(t) = \alpha\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}c_{rj}\big(z_r(t) - z_j(t)\big)^H\big(z_r(t) - z_j(t)\big),\\
\qquad\qquad\qquad\qquad t_k \leq t \leq \theta_k,\\
\omega(t_{k+1}) = \omega(\theta_k),\\
\omega(t) = 0,\ \theta_k < t < t_{k+1},\ k \in Z^+,\end{cases}\tag{29}$$

where $Re(\omega(t_0)) \geq 0$, $\alpha \in C$ with $Re(\alpha) > 0$.

Proof. Define the following Lyapunov function

$$\check{V}(t) = U(t) + \check{W}(t), \tag{30}$$

where $U(t)$ is the same as (12) and

$$\check{W}(t) = \begin{cases}\dfrac{N}{2\alpha^R}e^{-\mu(t-t_k)}\big(\check{\omega}^* - \omega^R(t)\big)^2,\ t_k \leq t \leq \theta_k,\\
\dfrac{N}{2\alpha^R}e^{F(t-\theta_k)-\mu(\theta_k-t_k)}\big(\check{\omega}^* - \omega^R(\theta_k)\big)^2,\ \theta_k < t < t_{k+1},\end{cases}$$

in which $k \in Z^+$, $\check{\omega}^*$ is a positive constant determined later.

When $t_k \leq t \leq \theta_k$, applying Assumptions 1, 2 and (28),

$$\begin{aligned}
D^+\check{V}(t) \leq&\Big(F + \mu\Big)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\xi_{rj}^H(t)\xi_{rj}(t) - \mu\check{V}(t)\\
&- Ne^{-\mu T}\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\check{\omega}^*c_{rj}\xi_{rj}^H(t)\xi_{rj}(t).
\end{aligned}\tag{31}$$

The rest of the proof is similar to that of Theorem 1. ■

Actually, the designed intermittent adaptive scheme (29) in Theorem 3 is a centralized control, which depends on the information of all nodes and edges. In the following, a modified adaptive strategy is developed to achieve the synchronization, in which the partial information of the network is required.

Let $\hat{\mathcal{V}}$ be a connected dominating set of the network (2), it follows from Lemma 1 that it always exists since the network (2) is assumed to be connected. For any node $r \in \hat{\mathcal{V}}$, $\check{G}_r$ denotes the graph consisting of it and its neighbors. Evidently, the graph $\check{G} = \cup_{r\in\hat{\mathcal{V}}}\check{G}_r$ is connected and the elements of the Laplacian matrix $\check{L}$ of the graph $\check{G}$ are given by

$$\check{l}_{rj} = \begin{cases}-c_{rj}, & \text{if } r \in \hat{\mathcal{V}} \text{ or } j \in \hat{\mathcal{V}},\ r \neq j,\\
-\sum_{j=1,j\neq r}^{N}\check{l}_{rj}, & \text{if } r = j,\\
0, & \text{otherwise}.\end{cases}\tag{32}$$

Denote $\check{\Omega} = (\delta_{rj})_{N\times N}$ and

$$\delta_{rj} = \begin{cases}-c_{rj}, & \text{if } r \notin \hat{\mathcal{V}} \text{ and } j \in \hat{\mathcal{V}},\ \text{ or } r \in \hat{\mathcal{V}} \text{ and } j \notin \hat{\mathcal{V}},\\
-\sum_{j=1,j\neq r}^{N}\delta_{rj}, & \text{if } r = j,\\
0, & \text{otherwise}.\end{cases}$$

Let $\bar{\Omega} = (\bar{\delta}_{rj})_{N\times N}$ and

$$\bar{\delta}_{rj} = \begin{cases}-c_{rj}, & \text{if } r \text{ or } j\ \in \mathcal{V}\setminus\hat{\mathcal{V}},\ r \neq j,\\
-\sum_{j=1,j\neq r}^{N}\bar{\delta}_{rj}, & \text{if } r = j,\\
0, & \text{otherwise}.\end{cases}$$

Theorem 4. *Under Assumptions 1–2, the network (2) is synchronized via the following intermittently adaptive scheme*

$$\begin{cases}\dot{\omega}(t) = \alpha\sum_{r\in\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N}c_{rj}\big(z_r(t) - z_j(t)\big)^H\big(z_r(t) - z_j(t)\big),\\
\qquad\qquad\qquad\qquad t_k \leq t \leq \theta_k,\\
\omega(t_{k+1}) = \omega(\theta_k),\\
\omega(t) = 0,\quad \theta_k < t < t_{k+1},\quad k \in Z^+,\end{cases}\tag{33}$$

where $Re(\omega(t_0)) \geq 0$, $\alpha \in C$ with $Re(\alpha) > 0$.

Proof. When $t_k \leq t \leq \theta_k$, it is evident that

$$\begin{aligned}
D^+\check{V}(t) \leq&\Big(F + \mu\Big)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\xi_{rj}^H(t)\xi_{rj}(t) - \mu\check{V}(t)\\
&- N\check{\omega}^*e^{-\mu T}\sum_{r\in\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N}c_{rj}\xi_{rj}^H(t)\xi_{rj}(t)\\
&- N\omega^R(t_0)\sum_{r\in\mathcal{V}\setminus\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N}c_{rj}\xi_{rj}^H(t)\xi_{rj}(t).
\end{aligned}\tag{34}$$

By the definition of $\check{L}$, similar to (14),

$$\sum_{r\in\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t)+\sum_{r\notin\hat{\mathcal{V}}}\sum_{j\in\hat{\mathcal{V}}} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t) \geq 2\lambda_2(\check{L})\hat{\xi}^{H}(t)\hat{\xi}(t). \tag{35}$$

Since $c_{rj}=c_{jr}$, then

$$\sum_{r\notin\hat{\mathcal{V}}}\sum_{j\in\hat{\mathcal{V}}} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t)=\sum_{r\in\hat{\mathcal{V}}}\sum_{j\notin\hat{\mathcal{V}}} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t),$$

which combines with the definition of $\check{\Omega}$, one has

$$\sum_{r\notin\hat{\mathcal{V}}}\sum_{j\in\hat{\mathcal{V}}} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t)=\hat{\xi}^{H}(t)(\check{\Omega}\otimes I_n)\hat{\xi}(t). \tag{36}$$

Similarly,

$$\sum_{r\in\mathcal{V}\setminus\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N} c_{rj}\xi_{rj}^{H}(t)\xi_{rj}(t)=\hat{\xi}^{H}(t)(\bar{\Omega}\otimes I_n)\hat{\xi}(t).$$

The rest of the proof is similar to that of Theorem 1. ∎

If $\theta_k = t_{k+1}$ for $k\in Z^{+}$, the ICCVNs (2) are reduced to the following complex networks with continuous coupling

$$\dot{z}_r(t)=h(z_r(t))+\omega(t)\sum_{p=1}^{N} c_{rp}z_p(t),\ r\in\mathcal{N}. \tag{37}$$

The following result is directly obtained from Theorem 4.

Corollary 2. *The network (37) is asymptotically synchronized under Assumption 1 and the following adaptive strategy*

$$\dot{\omega}(t)=\alpha\sum_{r\in\hat{\mathcal{V}}}\sum_{j=1,j\neq r}^{N} c_{rj}\Big(z_r(t)-z_j(t)\Big)^{H}\Big(z_r(t)-z_j(t)\Big),$$

where $Re(\omega(t_0))\geq 0$, $\alpha\in C$ with $Re(\alpha)>0$.

Remark 8. In fact, the synchronization of real-valued networks with homogeneous continuous coupling has been studied by designing adaptive schemes for coupling weights in Chen et al. (2007), Hu et al. (2011) and Li et al. (2008), where the adaptive strategies depend on all information of the network. Different from these, the synchronization of intermittently coupled complex-valued networks (2) is considered in Theorem 4 by developing an intermittently adaptive scheme for the coupling weight which just depends on the information of a connected dominating set with their neighbors.

Remark 9. In Wang et al. (2018), Wu and Fu (2013), Wu and Leng (2017), Wu et al. (2015), Xu et al. (2015), Zhang, Wang, Wang, Zhou, and Xia (2018) and Zhang et al. (2017), the synchronization of diverse complex-valued networks was discussed, in which continuous coupling is considered and some additional control inputs are designed. Especially, the complex-valued networks are separated into two subsystems with real values in Zhang, Wang, Wang, Zhou, and Xia (2018) to achieve synchronization. Unlike these results, a type of complex-valued dynamical networks with intermittent coupling is proposed in this paper and some intermittently adaptive schemes for coupling weights are developed based on the theory of complex-variable functions instead of the separation method to realize the synchronization of the overall networks.

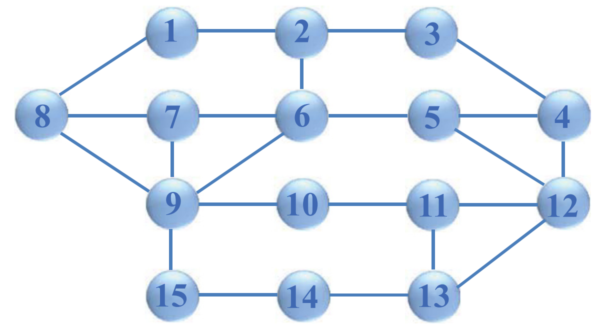

Fig. 2. The topological structure of the network (38) in coupling time.

5. Numerical simulations

Two numerical examples are presented in this section to support the derived theoretical results.

Example 1. Consider the following ICCVNs with heterogeneous weights consisting of 15 nodes:

$$\dot{z}_r(t)=h(z_r(t))+\sum_{k=1}^{+\infty}\Big[\sum_{j=1,j\neq r}^{15} c_{rj}\omega_{rj}(t)\big(z_j(t)-z_r(t)\big)\Big]\Delta_k(t), \tag{38}$$

here $z_r(t)=(z_r^1(t),z_r^2(t))^T$, $h(z_k)=-(1+i)z_k+Qg(z_k)$, $g(z_k)=(g_1(z_k^1),g_2(z_k^2))^T$, $g_j(z_k^j)=\tanh(z_k^j)+i\sin(z_k^j)$ with $j=1,2$ and

$$Q=\begin{pmatrix} 1.8-3.6i & -0.6+1.5i \\ 0.85-0.1i & 1.5-2i \end{pmatrix},$$

the time sequences $\{t_k\}$ and $\{\theta_k\}$ are given by

$$\{t_k\}=\{0,4,8.5,13,17,22,26.5,31.5,34,38,42,\\ 46,51,54,58.5,62,65,69,72,\ldots\},$$

$$\{\theta_k\}=\{2,5,10,15,18,24,28,33,35.5,40.5,43.5,\\ 47.5,53,55.5,60,63,67.5,71,\ldots\},$$

and the topology of the network (38) in coupling time durations is given in Fig. 2.

Choose $\hat{E}=\{(r,j),\ 1\leq r<j\leq 15\}$, then the network (38) is synchronized according to Theorem 2 based on the intermittently adaptive law (20), which is shown in Figs. 3–5, where $\alpha_{rj}=1+0.3i$, $\omega_{rj}(0)=\phi_{rj}-\psi_{rj}i$ with $(r,j)\in\hat{E}$, ϕ_{rj} and ψ_{rj} are randomly chosen on [0,0.5], $z_{rj}(0)=\lambda_{rj}+\zeta_{rj}i$, λ_{rj} and ζ_{rj} are selected randomly on $[-3,3]$. Fig. 3 reveals the evolution of synchronization error $U(t)$ defined in (12) and the evolutions of intermittent coupling weights are shown in Figs. 4 and 5.

Example 2. Consider the following ICCVNs with homogeneous weights described by

$$\dot{z}_r(t)=h(z_r(t))+\omega(t)\sum_{k=1}^{+\infty}\sum_{j=1}^{15} c_{rj}z_j(t)\Delta_k(t), \tag{39}$$

where h, Δ_k and the topology of the network in coupling time are the same as that of Example 1.

Select $\hat{\mathcal{V}}=\{2,5,6,9,12,13\}$, it is evident that it is a connected dominating set. By virtue of Theorem 4, based on the intermittently adaptive scheme (33), the network (39) is synchronized, which is demonstrated in Figs. 6–8, where $\alpha=0.007+0.001i$, $\omega(0)=0.1+0.1i$, $z_{rj}(0)=\lambda_{rj}+\zeta_{rj}i$, λ_{rj} and ζ_{rj} are selected randomly on $[-5,5]$.

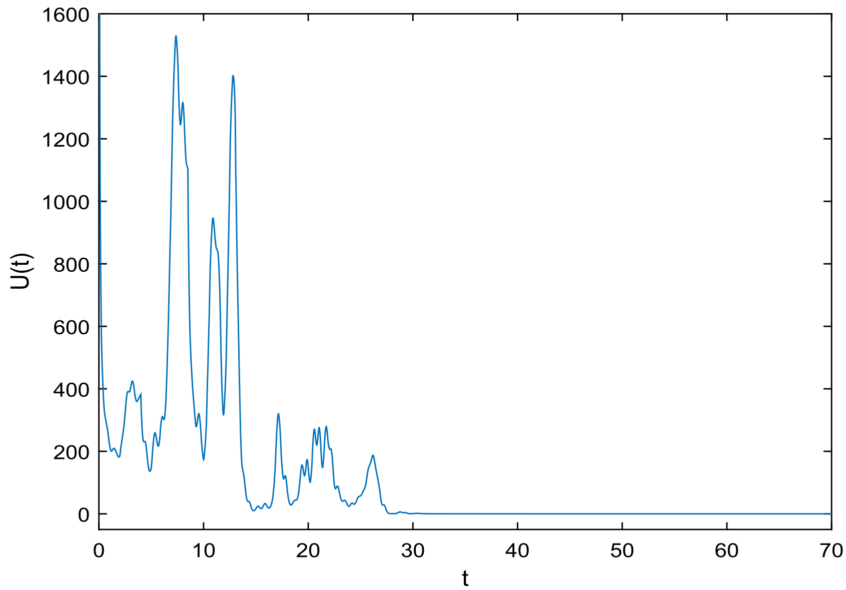

Fig. 3. The evolution of synchronization error $U(t)$ in Example 1.

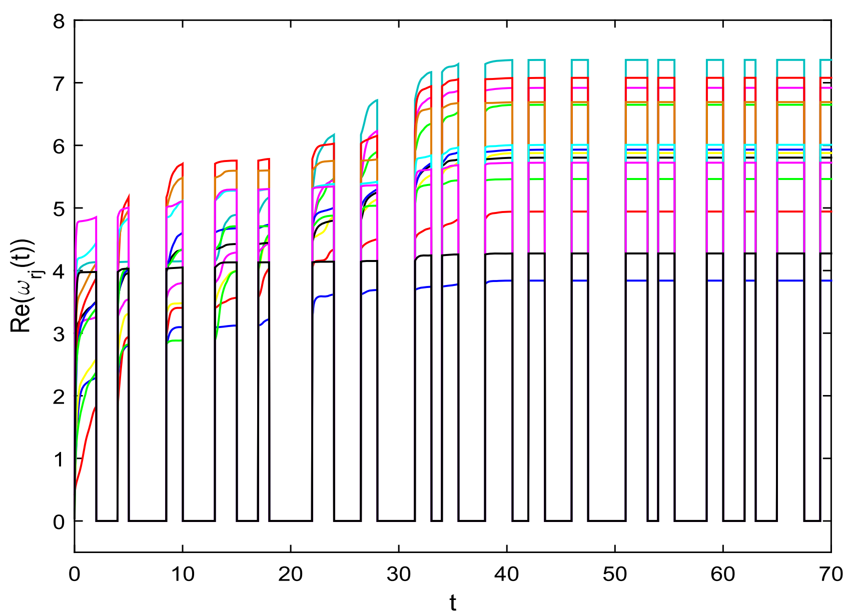

Fig. 4. The evolutions of $Re(\omega_{rj}(t))$ in Example 1.

6. Conclusion

This paper dealt with a new coupling mechanism of complex networks and it was described by the models of intermittently coupled networks with adaptive coupling weights. For ICCVNs with heterogeneous weights, the synchronization is ensured by designing intermittently adaptive strategies to update the weights of those edges within a spanning tree. For ICCVNs with homogeneous weights, it was found that the synchronization can be reached by designing an intermittently adaptive control which just depends on the information of a connected dominating set with their neighbors. In theoretical analysis, different from the weighted average method used in Chen and Zhu (2007), Lu and Chen (2006), Lu et al. (2006), Lv et al. (2018), Wang et al. (2014), Yang et al. (2010) and Yu et al. (2012), a direct error method is developed to discuss synchronization of complex networks.

Nowadays, directed and switching topologies have been considered in complex networks (Chen et al., 2013, 2016; Yang & Lu, 2016; Yang et al., 2010; Yu et al., 2015), it would be meaningful to explore the synchronization of intermittently coupled networks involving directed or switching topology. In addition, as pointed out in Liu and Chen (2007) and Lu et al. (2006), the coupling delay plays a key role and has a great influence on the synchronization of networks, which provide a striking and valuable insight into the study of the synchronization of intermittent coupled networks with coupling delays. It will be considered in our coming research.

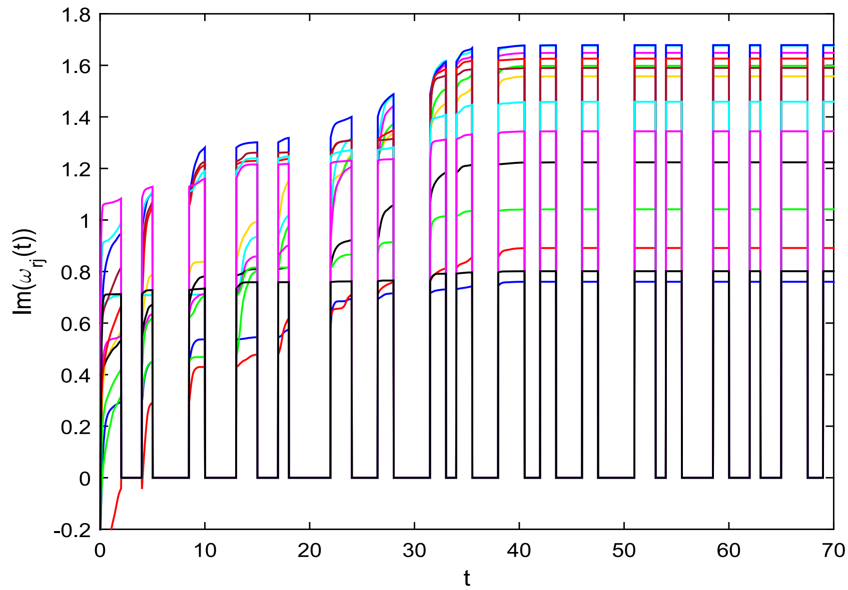

Fig. 5. The evolutions of $Im(\omega_{rj}(t))$ in Example 1.

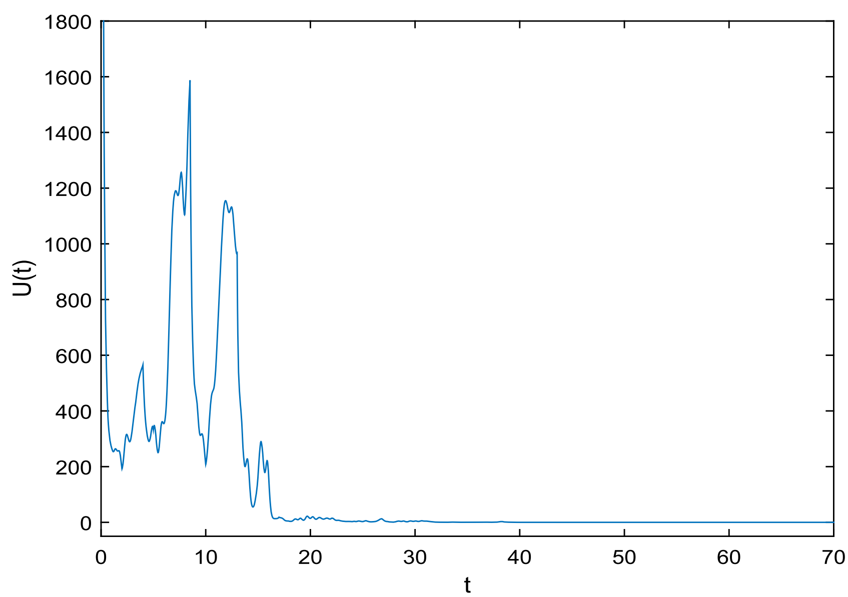

Fig. 6. The evolution of synchronization error $U(t)$ in Example 2.

Appendix A. Proof of Proposition 1

Proof. Since $\xi_{rj}(t)=\xi_{rp}(t)-\xi_{jp}(t)$ and $\xi_{rr}(t)=0$,

$$\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}\xi_{rj}^{H}(t)\Big(c_{jp}\omega_{jp}(t)\xi_{jp}(t)-c_{rp}\omega_{rp}(t)\xi_{rp}(t)\Big)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}\Big(c_{jp}\omega_{jp}(t)\xi_{jp}(t)-c_{rp}\omega_{rp}(t)\xi_{rp}(t)\Big)^{H}\xi_{rj}(t)$$
$$=-2(N-1)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}c_{rj}\omega_{rj}^{R}(t)\xi_{rj}^{H}(t)\xi_{rj}(t)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{jp}\omega_{jp}(t)\xi_{rp}^{H}(t)\xi_{jp}(t)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{jp}\overline{\omega_{jp}(t)}\xi_{jp}^{H}(t)\xi_{rp}(t)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{rp}\omega_{rp}(t)\xi_{jp}^{H}(t)\xi_{rp}(t)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{rp}\overline{\omega_{rp}(t)}\xi_{rp}^{H}(t)\xi_{jp}(t). \tag{40}$$

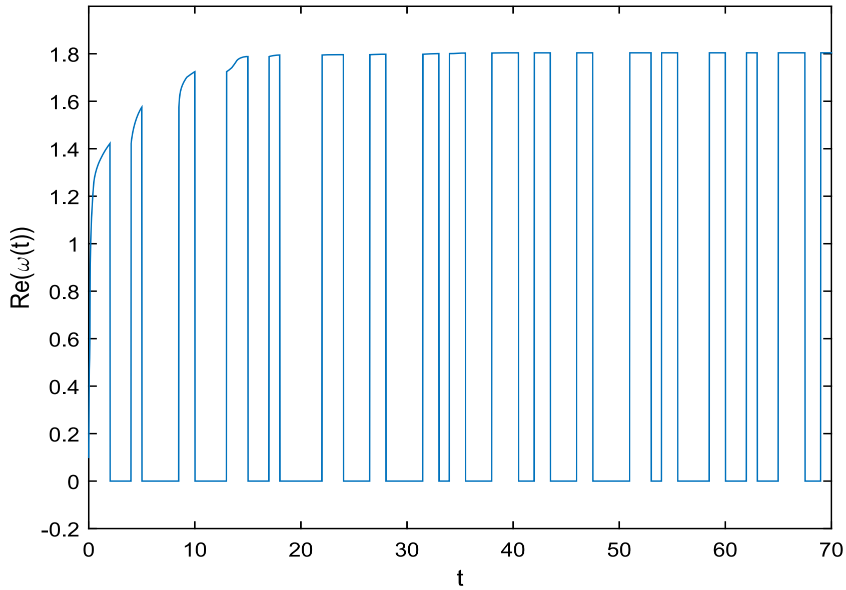

Fig. 7. The evolution of $Re(\omega(t))$ in Example 2.

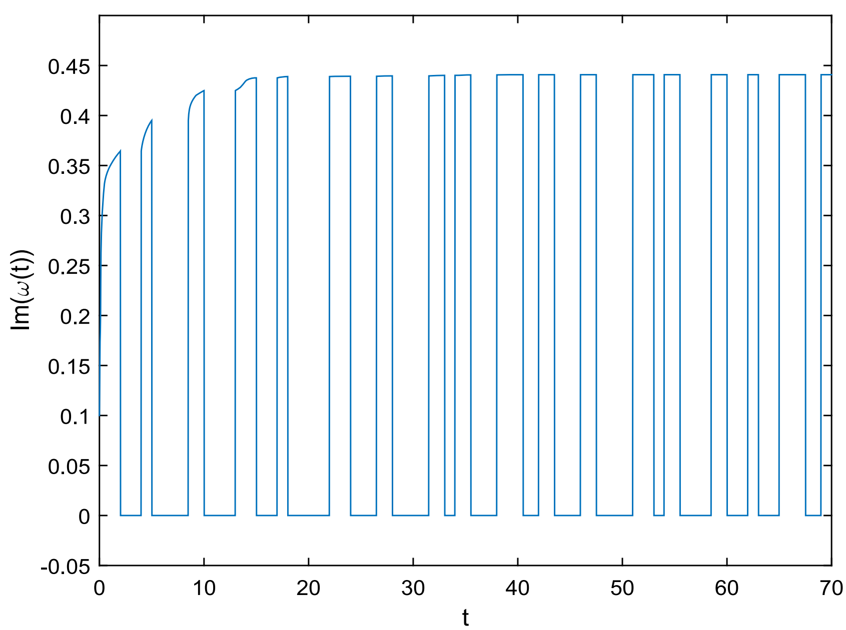

Fig. 8. The evolution of $Im(\omega(t))$ in Example 2.

Applying $c_{rp}\omega_{rp}(t) = c_{pr}\omega_{pr}(t)$, $\xi_{jr}(t) = -\xi_{rj}(t)$ and $\xi_{rr}(t) = 0$,

$$\begin{aligned}
&\sum_{r=1}^{N}\sum_{j=1}^{N}\sum_{p=1}^{N} c_{jp}\omega_{jp}(t)\xi_{rp}^{H}(t)\xi_{jp}(t) \\
&= \sum_{r=1}^{N}\sum_{j=1}^{N}\sum_{p=1}^{N} c_{pr}\omega_{pr}(t)\xi_{jr}^{H}(t)\xi_{pr}(t) \\
&= \sum_{r=1}^{N}\sum_{j=1}^{N}\sum_{p=1}^{N} c_{rp}\omega_{rp}(t)\xi_{rj}^{H}(t)\xi_{rp}(t) \\
&= \sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N} c_{rp}\omega_{rp}(t)\xi_{rj}^{H}(t)\xi_{rp}(t),
\end{aligned}$$

which implies that

$$\begin{aligned}
&\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N} c_{jp}\omega_{jp}(t)\xi_{rp}^{H}(t)\xi_{jp}(t) \\
&+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N} c_{rp}\omega_{rp}(t)\xi_{jp}^{H}(t)\xi_{rp}(t)
\end{aligned}$$

$$=\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{rp}\omega_{rp}(t)\xi_{rp}^{H}(t)\xi_{rp}(t)$$
$$-\sum_{r=1}^{N}\sum_{p=1}^{N}c_{rp}\omega_{rp}(t)\xi_{rp}^{H}(t)\xi_{rp}(t)$$
$$=(N-2)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}c_{rj}\omega_{rj}(t)\xi_{rj}^{H}(t)\xi_{rj}(t). \tag{41}$$

Similarly,

$$\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{jp}\overline{\omega_{jp}(t)}\xi_{jp}^{H}(t)\xi_{rp}(t)$$
$$+\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}\sum_{p=1}^{N}c_{rp}\overline{\omega_{rp}(t)}\xi_{rp}^{H}(t)\xi_{jp}(t)$$
$$=(N-2)\sum_{r=1}^{N}\sum_{j=1,j\neq r}^{N}c_{rj}\overline{\omega_{rj}(t)}\xi_{rj}^{H}(t)\xi_{rj}(t),$$

which combines with (40) and (41), equality (4) is derived. ■

Appendix B. Proof of Proposition 2

Proof. Evidently, $V(t)$ is continuous except on the set $\{t_k\}$ and is right continuous at these time points. Denote $V_{-}(t_{k+1})$ is the left limit of $V(t)$ at time t_{k+1}. From (7), one has

$$V(t)\le V(t_k)e^{-\mu(t-t_k)},\quad t_k\le t\le\theta_k,\ k\in Z^{+},$$

$$V(t)\le V(\theta_k)e^{\lambda(t-\theta_k)},\quad \theta_k<t<t_{k+1},\ k\in Z^{+},$$

hence,

$$V_{-}(t_{k+1})\le e^{\lambda(t_{k+1}-\theta_k)}e^{-\mu(\theta_k-t_k)}V(t_k),$$

which combines with (7),

$$\begin{aligned}V(t_{k+1})&\le U(t_{k+1})+e^{\mu(\theta_k-t_k)-\lambda(t_{k+1}-\theta_k)}W_{-}(t_{k+1})\\&\le V(t_k)+(1-e^{\varpi})U(t_{k+1}).\end{aligned}\tag{42}$$

By means of $\varpi>0$ and the nonnegativity of $V(t)$,

$$\sum_{k=1}^{+\infty}U(t_k)\le\frac{V(t_0)}{e^{\varpi}-1}.$$

From series theory, $\lim_{k\to+\infty}U(t_k)=0$. Moreover, if (7) holds,

$$U(t)\le\zeta U(t_k),\quad t_k\le t<t_{k+1}, \tag{43}$$

where $\zeta=\max\{1,e^{\sigma T}\}$. Evidently, it follows from (43) that $\lim_{t\to+\infty}U(t)=0$ since $k\to+\infty$ when $t\to+\infty$. ■

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