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Mimetic Hořava gravity

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ABSTRACT

We show that the scalar field of mimetic gravity could be used to construct diffeomorphism invariant models that reduce to Hořava gravity in the synchronous gauge. The gradient of the mimetic field provides a timelike unit vector field that allows to define a projection operator of four-dimensional tensors to three-dimensional spatial tensors. Conversely, it also enables us to write quantities invariant under space diffeomorphisms in fully covariant form without the need to introduce new propagating degrees of freedom.

It has been recognized for some time that in order to improve the UV behaviour of the graviton propagator and, thus, the renormalizability of gravity, it is necessary to add higher spatial derivatives to its Lagrangian but no higher time derivatives. Because this seems to contradict the relativistic local Lorentz invariance, it was thought necessary to break the symmetry between space and time. The most notable attempt is the one by Hořava [9], who constructed a model of quantum gravity with explicitly broken Lorentz symmetry, which allowed him to add to the action terms dependent on the spatial Ricci tensor and curvature scalar and their space derivatives (see e.g. [10] and references therein). This is a high price to pay because, although the Hořava model is renormalizable when projected into the product space $\mathbb{R}\times\Sigma_3$, this property is lost when the model is made covariant by adding one new field [6]. Various attempts were made to keep renormalizability of the models while restoring Lorentz invariance by adding a dynamical scalar or vector [8]. Such models exhibit additional propagating degrees of freedom, which limited their acceptance as a solution to the problem of renormalizability of gravity.

Mimetic gravity was proposed as a way of separating the scale factor from the metric and resulted in reproducing Einstein gravity in addition to half a degree of freedom which could be used to mimic dark matter [1]. The main observation is that one can define the metric tensor $g_{\mu\nu}$ in terms of an auxiliary metric $\widetilde{g}_{\mu\nu}$ by the relation

$$g_{\mu\nu}=\widetilde{g}_{\mu\nu}\left(\widetilde{g}^{\kappa\lambda}\partial_\kappa\phi\partial_\lambda\phi\right), \tag{1}$$

where ϕ is a scalar field. The metric $g_{\mu\nu}$ is invariant under the scale transformation $\widetilde{g}_{\mu\nu}\to\Omega^2\widetilde{g}_{\mu\nu}$ and, as can be easily shown, satisfies the constraint

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi=1, \tag{2}$$

governing the evolution of ϕ. Thus, instead of introducing the mimetic field ϕ through the reparametrization (1), it is easier to consider directly the physical metric $g_{\mu\nu}$ together with a constrained scalar field, enforcing (2) through a Lagrange multiplier [2]. This implies that out of the 11 variables $g_{\mu\nu}$ and ϕ there are only 10 independent fields. In the ADM decomposition of $g_{\mu\nu}$,

$$ds^2=N^2dt^2-\gamma_{ij}\left(dx^i+N^idt\right)\left(dx^j+N^jdt\right),\quad i=1,2,3 \tag{3}$$

where N is the lapse function, N^i is the shift vector, and $\gamma_{ij}=-g_{ij}$ is the metric on the spatial 3d hypersurface, the constraint (2) can be solved for N in terms of the 10 variables N_i, γ_{ij} and ϕ, yielding

$$N^2=\frac{\left(\partial_0\phi-N^i\partial_i\phi\right)^2}{\left(1+\gamma^{ij}\partial_i\phi\partial_j\phi\right)}. \tag{4}$$

In the synchronous gauge $N=1$, $N_i=0$, a solution of (2) is given by

$$\phi=t+A, \tag{5}$$

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where A is a constant. Since there exists a whole family of synchronous coordinate systems, corresponding to the freedom of choice of an initial hypersurface of constant time, this solution is not unique. On the other hand, ϕ can always be used as one particular synchronous time coordinate, fixing a unique $3+1$ slicing that we will use from now on. The timelike unit vector $n_\mu = \partial_\mu \phi$ points in this time direction. In particular, we can define the projection operator

$$P^\nu_\mu = \delta^\nu_\mu - \partial_\mu \phi \partial_\kappa \phi g^{\nu\kappa}, \tag{6}$$

satisfying the relations

$$P^\rho_\mu P^\nu_\rho = P^\nu_\mu, \qquad P^\nu_\mu \partial_\nu \phi = 0. \tag{7}$$

In the synchronous slicing from above we have

$$P^0_0 = 0, \qquad P^i_0 = 0, \qquad P^0_i = 0, \qquad P^j_i = \delta^j_i, \tag{8}$$

showing that P^ν_μ projects space-time vectors to space vectors. It is then clear that in mimetic gravity, using the projection operator and the vector $n_\mu = \partial_\mu \phi$, it is possible to construct four-dimensional tensors whose only non-zero components in the synchronous gauge are along space directions. For example, as we will show in the following, the expression

$$\widetilde{R} := 2R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - R - (\Box\phi)^2 + \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi \tag{9}$$

coincides with the spatial curvature scalar ${}^3R$ of synchronous slices.

In previous works we have shown that in mimetic gravity, without the need to introduce any additional fields, we can build cosmological models [3] and solve the singularity problem for Friedmann, Kasner [4] and Black hole [5] solutions by using the idea of limiting curvature. More recently we have shown that the idea of asymptotic freedom can be implemented in mimetic gravity by introducing a $\Box\phi$ dependent effective gravitational constant which vanishes at the limiting curvature [11]. Moreover, it was shown that such a dependence does not introduce higher time derivatives.

The purpose of this letter is to show that within mimetic gravity we can construct all the terms needed in Hořava gravity using four-dimensional tensors that reduce to the desired form in the synchronous gauge. We will thus show that in mimetic gravity it is possible to formulate Hořava gravity in a diffeomorphism invariant way without introducing ghost-like degrees of freedom.

The basic fields that appear in Hořava gravity are the three-dimensional tensors and scalars κ_{ij}, κ, ${}^3R_{ij}$, ${}^3R$, $D_k{}^3R_{ij}$, and their contractions needed to form space diffeomorphism invariant expressions. The extrinsic curvature of the synchronous slices $\phi = const.$ is given by

$$\kappa_{ij} = \frac{1}{2}\dot{\gamma}_{ij}, \qquad \kappa^j_i = \gamma^{jl}\kappa_{il}, \qquad \kappa = \kappa^i_i = (\ln\sqrt{\gamma})^{\cdot}, \tag{10}$$

where dot denotes t derivative and γ is the metric determinant. Using ϕ, it can be expressed covariantly as

$$\nabla_i\nabla_j\phi = -\kappa_{ij}, \quad \nabla_i\nabla^j\phi = \kappa^j_i, \quad \Box\phi = \kappa. \tag{11}$$

The non-vanishing components of the four-dimensional Riemann tensor are determined by

$$R^0{}_{kij} = D_i\kappa_{kj} - D_j\kappa_{ki}, \tag{12}$$
$$R^0{}_{k0j} = \dot{\kappa}_{jk} - \kappa_{jn}\kappa^n_k, \tag{13}$$
$$R^l{}_{kij} = {}^3R^l{}_{kij} + \kappa^l_i\kappa_{jk} - \kappa^l_j\kappa_{ik}, \tag{14}$$

where D_i and ${}^3R^l{}_{kij}$ are the covariant derivative and the Riemann tensor belonging to the metric γ_{ij}. With the help of the above identities, we can construct the four-dimensional tensor

$$\widetilde{R}^\sigma{}_{\rho\mu\nu} := P^\sigma_\delta P^\gamma_\rho P^\alpha_\mu P^\beta_\nu R^\delta{}_{\gamma\alpha\beta} + \nabla_\mu\nabla^\sigma\phi\nabla_\rho\nabla_\nu\phi - \nabla_\nu\nabla^\sigma\phi\nabla_\rho\nabla_\mu\phi \tag{15}$$

whose only non-zero components are ${}^3R^l{}_{kij}$ in the synchronous gauge. Next, we compute the Ricci tensor components

$$R_{00} = -\dot{\kappa} - \kappa_{ij}\kappa^{ij} \tag{16}$$
$$R_{0i} = D_l\kappa^l_i - D_i\kappa \tag{17}$$
$$R_{ij} = {}^3R_{ij} + \kappa\kappa_{ij} - \kappa^n_i\kappa_{nj} + R^0_{i0j}. \tag{18}$$

These relations allow us to define the tensor

$$\widetilde{R}_{\mu\nu} := P^\alpha_\mu P^\beta_\nu R_{\alpha\beta} + \Box\phi\nabla_\mu\nabla_\nu\phi - \nabla_\mu\nabla^\rho\phi\nabla_\nu\nabla_\rho\phi - R^\gamma{}_{\mu\delta\nu}\nabla^\delta\phi\nabla_\gamma\phi, \tag{19}$$

whose non-zero components coincide with ${}^3R_{ij}$ in the synchronous gauge. Contracting with $g^{\mu\nu}$, we arrive at (9).

We note in passing that the total derivative $\frac{1}{\sqrt{\gamma}}\partial_0\left(\sqrt{\gamma}\kappa\right)$ can be easily eliminated from the Lagrangian of Einstein-Hilbert gravity, leaving us with

$$-R - 2\nabla_\mu\left(\Box\phi\nabla^\mu\phi\right) = \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi - (\Box\phi)^2 + \widetilde{R}. \tag{20}$$

For manifolds with boundary $\partial M = \{\phi = \phi_i\} \cup \{\phi = \phi_f\}$ consisting of closed spatial hypersurfaces of constant ϕ, this has precisely the same effect as adding the Gibbons-Hawking boundary term.

Space derivatives of the above tensors can be obtained by applying the operator $P^\rho_\mu\nabla_\rho$. Note that the spatial components of $P^\gamma_\rho\nabla_\gamma\widetilde{R}_{\alpha\beta}$ coincide with $D_k{}^3R_{ij}$ in the synchronous gauge. To obtain a purely spatial tensor, one still must project all four-dimensional indices, i.e. one has to use $P^\gamma_\rho P^\alpha_\mu P^\beta_\nu\nabla_\gamma\widetilde{R}_{\alpha\beta}$. Thus, we can now define the analogue of the three-dimensional Cotton tensor

$${}^3C^i_j = \frac{1}{\sqrt{\gamma}}\epsilon^{ikl}D_k\left({}^3R_{jl} - \frac{1}{4}\gamma_{jl}{}^3R\right) \tag{21}$$

by writing

$$\widetilde{C}^\mu_\nu := -\frac{1}{\sqrt{-g}}\epsilon^{\mu\rho\kappa\lambda}\nabla_\lambda\phi\,\nabla_\rho\left(\widetilde{R}_{\nu\kappa} - \frac{1}{4}g_{\nu\kappa}\widetilde{R}\right), \tag{22}$$

whose only non-vanishing components in the synchronous gauge are ${}^3C^i_j$.

Another object that could be constructed is the Chern-Simons three form ω_P related to the Pontryagin topological invariant

$$R^\sigma_\rho \wedge R^\rho_\sigma = \mathrm{d}\omega_P\,, \tag{23}$$

$$\omega_P = \Gamma^\nu_\mu \wedge \mathrm{d}\Gamma^\mu_\nu + \frac{2}{3}\Gamma^\mu_\nu \wedge \Gamma^\nu_\rho \wedge \Gamma^\rho_\mu\,, \tag{24}$$

where $\Gamma^\nu_\mu = \mathrm{d}x^\rho\Gamma^\nu_{\rho\mu}$ and $R^\sigma_\rho = \frac{1}{2}R^\sigma_{\rho\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$ are the Christoffel connection one-form and the curvature two form, respectively. The four-form $\mathrm{d}\phi \wedge \omega_P$ is not parity invariant. Up to a boundary term, its integral is given by

$$\int \mathrm{d}\phi \wedge \omega_P = -\int \phi\, R^\sigma_\rho \wedge R^\rho_\sigma. \tag{25}$$

This shows that such a contribution to the action is covariant and invariant under global shifts of ϕ. In the synchronous gauge the integrand reduces to

$$\epsilon^{ijk}\left(\Gamma^{\nu}_{i\mu}\partial_j\Gamma^{\mu}_{k\nu}+\frac{2}{3}\Gamma^{\mu}_{i\nu}\Gamma^{\nu}_{j\rho}\Gamma^{\rho}_{k\mu}\right)={}^3\omega_P+2\epsilon^{ijk}\kappa^n_i D_j\kappa_{kn}, \qquad (26)$$

where

$${}^3\omega_P=\epsilon^{ijk}\left(\lambda^n_{im}\partial_j\lambda^m_{kn}+\frac{2}{3}\lambda^m_{in}\lambda^n_{jl}\lambda^l_{km}\right), \qquad (27)$$

and λ^k_{ij} are the Christoffel symbols calculated for γ_{ij}. The term $2\epsilon^{ijk}\kappa^n_i D_j\kappa_{kn}$ can be written as

$$\epsilon^{ijk}\nabla_i\nabla^n\phi R^0_{njk}\,, \qquad (28)$$

which coincides in the synchronous gauge with

$$\epsilon^{\mu\nu\rho\sigma}\nabla_\mu\phi\nabla_\nu\nabla^\lambda\phi R^\tau_{\lambda\rho\sigma}\nabla_\tau\phi. \qquad (29)$$

Thus, the purely three-dimensional Chern-Simons form can be incorporated in the action by adding the term

$$\int \mathrm{d}\phi\wedge\widetilde{\omega}_P:=\int \mathrm{d}\phi\wedge\left(\omega_P-\nabla^\lambda \mathrm{d}\phi\wedge R^\tau_\lambda\nabla_\tau\phi\right). \qquad (30)$$

All of these manipulations illustrate that any expression invariant under spatial diffeomorphisms can be written as a combination of four-dimensional tensors that reduces to it in the synchronous gauge.

We conclude by writing an exemplary Hořava action in mimetic gravity, in terms of four-dimensional tensors and thus completely preserving diffeomorphism invariance, without the need for new degrees of freedom:

$$S=\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi-c_1(\Box\phi)^2+c_2\widetilde{R}\right. \qquad (31)$$
$$+c_3\widetilde{R}^2+c_4\widetilde{R}_{\mu\nu}\widetilde{R}^{\mu\nu}+c_5\widetilde{C}^\mu_\nu\widetilde{C}^\nu_\mu+c_6\,\eta^{\mu\nu\rho\sigma}\nabla_\mu\phi\,(\widetilde{\omega}_P)_{\nu\rho\sigma}$$
$$\left.+c_7\,\eta^{\mu\nu\rho\sigma}\nabla_\sigma\phi\widetilde{R}_{\mu\alpha}\nabla_\nu\widetilde{R}^\alpha_\rho+\ldots+\lambda\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-1\right)\right),$$

where $\eta^{\mu\nu\rho\sigma}=\frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}$. The case where $c_1=c_2=1$ and all other couplings vanish is just a rewritten form of General Relativity with mimetic matter. The constants $c_1,\ldots,c_7$ could also be taken as functions of $\Box\phi$ in such a way as to reproduce General Relativity in the low curvature limit.

There is no need to repeat calculations done for the Hořava models, as those could be thought of as a gauge fixed version of a diffeomorphism invariant theory in the synchronous gauge.

In the projectable Hořava models, the lapse function N is assumed to depend on time only, $N=N(t)$. These models coincide with the above family of actions in the synchronous gauge. Their renormalization analysis was carried out in references [6], [7], where they were shown to be renormalizable. When the same analysis was applied to the non-projectable case where the lapse function is $N=N\left(x^i,t\right)$, so that terms dependent on the vector $a_i=\frac{\partial_i N}{N}$ can occur, it was found that these models become non-renormalizable. Attempts were made to construct diffeomorphism invariant Hořava models by adding a unit vector field u_μ, subject to the hypersurface orthogonality condition $u_{[\mu}\nabla_\nu u_{\rho]}=0$. These models, however, have a spin-1 and spin-0 degree of freedom in addition to the graviton.

The synchronous gauge belongs to the family of temporal gauge, which for fluctuations of the metric takes the form $n^\mu h_{\mu\nu}=0$, where $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ and $n_\mu=(1,0,0,0)$. The main advantage of working in this gauge is that the model proposed above will be power counting renormalizable and that the ghosts associated with gauge fixing will decouple from the physical S-matrix. The disadvantage is the need to have an unambiguous prescription for the unphysical singularities of the form $(q.n)^{-\alpha}$, $\alpha=1,2$ (cf. [12]) and the difficulty in performing higher loop calculations. It is a challenging problem to perform a detailed analysis of the renormalization program, either in the synchronous gauge or in a covariant gauge, using the background field method and integrating out the mimetic constraint, along the lines of [6]. Even though an actual proof could be quite demanding, we expect the mimetic Hořava model presented here to be renormalizable.

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