

Secondary Voltage Control Via Demand-Side Energy Storage with Temporal Logic Specifications

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Abstract—This paper proposes a secondary voltage control method that can handle temporal logic specifications (TLSs). The control objective is to schedule a control signal for an energy storage system (ESS) such that the voltage variation of a defined critical bus satisfies TLSs. The TLS allows definitions of set and timing constraints at the same time, such as a finite-time restoration. A simplified analytical model is derived to describe the voltage variation of critical buses. Based on this model, a numerical optimal control problem is formulated as a mixed integer linear program to generate the proper control input for the ESS. The proposed control is verified on a lumped distribution system model. With this control diagram, supportive controllers can easily be designed to make voltage behaviors comply with grid codes and avoid unnecessary relay actions.

Index Terms—Secondary voltage control, finite-time restoration, numerical optimal control, temporal logic specification.

I. INTRODUCTION

Secondary voltage control of critical buses is important to regulate equipment operation at nominal values, especially under intermittent renewable generations and constantly changing load demands [1]. Many works have been devoted to this area [1]–[7], among which finite-time restoration has become a desired specification [1]. On the other hand, most specifications of power system control design are focused on set constraints [8], [9]. However, relay settings and grid codes have simultaneous set, logic, and temporal constraints [10]. To avoid unnecessary relay actions and comply with grid codes, constraints of dwell time on specified sets need consideration. Failure to consider such critical timing constraints may lead to cascading outages [11]. Thus, finite-time restoration and dwell time constraints are pressing and require a novel control diagram and algorithm.

The temporal logic specifications (TLSs) allow richer descriptions of specifications including set, logic, and time-related properties. For example, to guarantee the proper operation of microgrids, the speed deviation of the synchronous generator should not exceed ± 1.5 Hz for 0.1 second [12]. The pioneering work in [13] introduces the TLSs for controller synthesis of energy storage systems, where a finite-time restoration is guaranteed. Ref. [14] derives a provable probabilistic guarantee in the stochastic environment of wind

power generation. In [15], a numerical optimal control (NOC)-based control synthesis approach is proposed to schedule a controller for frequency support to satisfy the TLSs.

This paper will investigate the secondary voltage control with TLSs. The NOC-based method in [15] will be employed and the voltage at a particular bus is selected as the control objective. During a disturbance, the voltage is required to be restored back above a certain value within a required time utilizing the support of the energy storage system (ESS). The controller will measure the voltage, estimate the size of disturbance, and compute the control input for the ESS. The ESS will adjust its power output such that the voltage variation satisfies the TLSs.

The reminder of the paper is organized as follows. Section II briefly introduces the preliminaries on the TLSs. Section III introduces the control diagram and NOC formulation of the secondary voltage control problem. Section IV illustrates the simulation results, followed by the conclusion in Section V.

II. PRELIMINARIES ON TEMPORAL LOGIC SPECIFICATIONS

Let φ be an atomic proposition (AP), which is a statement on the system variables that maps to the Boolean domain $\mathbb{B} = \{True, False\}$. The temporal logic formulas combine the APs according to a logical grammar and temporal specification. The standard Boolean connectives are: \neg (negation), \wedge (conjunction), \vee (disjunction), \Rightarrow (implication), and \Leftrightarrow (equivalence). The standard temporal operators used in TLSs are $\Diamond_{[t_1, t_2]}$ (eventually), $\Box_{[t_1, t_2]}$ (always), and $\mathcal{U}_{[t_1, t_2]}$ (until), where $[t_1, t_2]$ is the time interval, such that $\Box_{[t_1, t_2]}\varphi$ means φ holds at every time step between t_1 and t_2 , and $\Diamond_{[t_1, t_2]}\varphi$ means φ holds at some time step between t_1 and t_2 . Additionally, we define $\varphi\mathcal{U}_{[t_1, t_2]}\psi$, if φ holds at every time step before the AP ψ holds and ψ holds at some time step between t_1 and t_2 . We denote the fact that ξ satisfies an TLS formula φ by $\xi \models \varphi$.

Notice that for a discrete-time TLS, timing intervals cannot be added with the temporal operators as $\Box(0 < x \leq 2)$, which states the x will always be positive and upper bounded by two without specifying a timing constraint for when the condition should be fulfilled. Here, the continuous-time TLSs will be employed.

There are mainly two branches of control synthesis methods for continuous-time TLSs [15]. On the one hand, in [13], [16],

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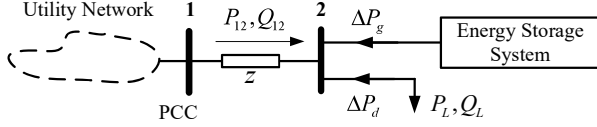


Fig. 1. A lumped distribution system model.

the temporal logic constraints are substituted into the optimization objectives, leading to an unconstrained problem that can be solved by some functional gradient descent algorithms. On the other hand, the authors in [17] introduce an approach using mixed-integer convex optimization to encode the TLSs as constraints. First, the safe or unsafe sets are represented as polyhedrons (by a finite number of hyperplanes). An AP like $x \in P$ can be formulated as a linear program. Second, some integer variables are introduced to indicate whether the condition holds or not. The "if" and "else" conditions can be formulated in the linear program using the big-M technique. Finally, the overall problem becomes a mixed-integer linear program (MILP). The encoding procedure has been implemented in the toolbox BluSTL [18], which is employed for problem conversion here. The detailed procedure of encoding TLSs into MILP will not be described in this paper.

III. SECONDARY VOLTAGE CONTROL WITH TEMPORAL LOGIC SPECIFICATION

A. Problem Statement

In this paper, a lumped distribution system model similar to the one in [19] is considered and shown in Fig. 1. Bus 1 is considered to be an infinite bus, where $\theta_1 = 0$ and $V_1 = 1$. Bus 2 is considered the critical bus, the voltage of which is required to satisfy the TLS. An aggregated net load and an ESS are connected to the critical bus. The net load may change due to the stochastic nature of solar photovoltaic (PV) generation and abrupt demand change. The net load change is denoted as ΔP_d and is considered as the disturbance. Assume the power variation of the ESS under the control input is governed by the following state-space model

$$\begin{aligned} \dot{x} &= Ax + Bu \\ \Delta P_g &= Cx \end{aligned} \quad (1)$$

The nonlinear power flows from Bus 1 to 2 can be expressed as

$$\begin{aligned} P_{12} &= G_{12}V_1^2 - V_1V_2(G_{12}\cos\theta_{12} + B_{12}\sin\theta_{12}) \\ Q_{12} &= -B_{12}V_1^2 - V_1V_2(G_{12}\sin\theta_{12} - G_{12}\cos\theta_{12}) \end{aligned} \quad (2)$$

where $\theta_{12} = \theta_1 - \theta_2$ and

$$G_{12} + jB_{12} = \frac{1}{Z_{12}} = \frac{1}{R_{12} + jX_{12}} = \frac{R_{12}}{Z_{12}^2} - j\frac{X_{12}}{Z_{12}^2}. \quad (3)$$

The linearized power flow in [20] is employed as follows

$$\begin{aligned} P_{12} &= G_{12}(V_1 - V_2) - B_{12}(\theta_1 - \theta_2) \\ Q_{12} &= -B_{12}(V_1 - V_2) - G_{12}(\theta_1 - \theta_2) \end{aligned} \quad (4)$$

With $V_1 = 1$ and $\theta_1 = 0$, we have

$$\begin{aligned} P_{12} &= G_{12}(1 - V_2) - B_{12}\theta_2 \\ Q_{12} &= -B_{12}(1 - V_2) - G_{12}\theta_2 \end{aligned} \quad (5)$$

Power balance at Bus 2 can be expressed as

$$\begin{aligned} P_{12} &= P_L - \Delta P_d - \Delta P_g \\ Q_{12} &= Q_L \end{aligned} \quad (6)$$

Combining Eqs. (5) and (6) yields

$$P_L - \Delta P_d - \Delta P_g = G_{12}(1 - V_2) + B_{12}\theta_2 \quad (7)$$

$$Q_L = -B_{12}(1 - V_2) + G_{12}\theta_2 \quad (8)$$

In this paper, only active support is considered for simplicity. From (8) we will have

$$\theta_2 = \frac{Q_L + B_{12}(1 - V_2)}{G_{12}} \quad (9)$$

Substituting (9) into (7) yields

$$\begin{aligned} V_2 &= \frac{G_{12}}{G_{12}^2 + B_{12}^2} \Delta P_g + \frac{G_{12}}{G_{12}^2 + B_{12}^2} \Delta P_d \\ &+ \frac{G_{12}^2 + B_{12}Q_L + B_{12}^2 - G_{12}P_L}{G_{12}^2 + B_{12}^2} \end{aligned} \quad (10)$$

Let

$$\begin{aligned} c_1 &= \frac{G_{12}}{G_{12}^2 + B_{12}^2} \\ c_2 &= \frac{G_{12}^2 + B_{12}Q_L + B_{12}^2 - G_{12}P_L}{G_{12}^2 + B_{12}^2} \end{aligned} \quad (11)$$

The voltage deviation of Bus 2 is expressed by

$$\Delta V_2 = c_1 \Delta P_g + c_1 \Delta P_d \quad (12)$$

Based on (1) and (12), the simplified predictive model in state-space form can be expressed as follows

$$\begin{aligned} \dot{x} &= Ax + \begin{bmatrix} B & 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \\ \Delta V_2 &= c_1 Cx + \begin{bmatrix} 0 & c_1 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \end{aligned} \quad (13)$$

B. Control Diagram

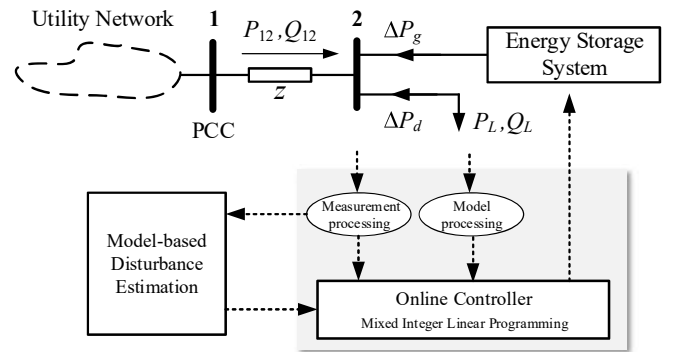


Fig. 2. Secondary voltage control diagram with temporal logic specification satisfaction.

The secondary voltage control diagram with temporal logic specification satisfaction is illustrated in Fig. 2. The voltage at Bus 2 will be measured and used to estimate the size

of the disturbance according to Eq. (12) with ΔP_g . The estimated disturbance is sent to the online controller, where the derived model in (13) is embedded. With both information of the disturbance and analytical model received, the online controller formulates the control synthesis problem as a mixed integer linear program, and solves it with an off-the-shelf solver to obtain a sequence of control input signals for the ESS.

Since the occurrence of disturbance cannot be predicted, the controller will only be activated once the voltage has dropped. Thus, the runtime of the overall control process, including voltage measurement, disturbance estimation, formulating and solving the MILP, as well as communication delay, needs to be considered when designing the temporal specification. For example, if the runtime of the controller is t_c , and the finite-time restoration requirement is t_f , the temporal specification should be given as $t_a = t_f - t_c$. Here, we assume that t_c is negligible under the studied scenarios.

C. Numerical Optimal Control Formulation

Let the analytical model in (13) be discretized at a sample time of t_s and expressed compactly as follows

$$x(k+1) = A_d x(k) + B_{d1} u(k) + B_{d2} \hat{d}(k) \quad (14)$$

$$\Delta V_2(k+1) = C_d x(k) + D_{d1} u(k) + D_{d2} \hat{d}(k) \quad (15)$$

Let the scheduling horizon be denoted as $k \in \mathcal{T} = [1, \dots, T]$. The control input should not exceed a certain limit in any time, that is,

$$|u(k)| \leq U_{\lim}, \quad \forall k \in \mathcal{T} \quad (16)$$

Then, the voltage is required to satisfy the following TLS φ to enhance the performance

$$\Delta V_2(k) \models \varphi, \quad \forall k \in \mathcal{T} \quad (17)$$

where

$$\varphi = \Box[(|\Delta V_2(k)| \geq \Delta V_c) \Rightarrow \Diamond_{[0, t_a]} \Box(|\Delta V_2(k)| \leq \Delta V_c)] \quad (18)$$

The above TLS states that whenever the voltage deviation is larger than ΔV_c , then it should become less than ΔV_c within t_a seconds.

The objective is to minimize the control efforts. The total control effort can be represented as the summation of all decision variables as

$$C_U = \sum_{k=1}^T |u(k)| \quad (19)$$

Then, the scheduling problem can be summarized as follows

$$\begin{aligned} & \min \quad C_U \\ & \text{s.t.} \quad \forall k \in \mathcal{T} \\ & x(k+1) = A_d x(k) + B_{d1} u(k) + B_{d2} \hat{d}(k) \\ & \Delta V_2(k+1) = C_d x(k) + D_{d1} u(k) + D_{d2} \hat{d}(k) \\ & |u(k)| \leq U_{\lim}, \quad \forall k \in \mathcal{T} \\ & \Delta V_2(k) \models \varphi, \quad \forall k \in \mathcal{T} \\ & \varphi = \Box[(|\Delta V_2(k)| \geq \Delta V_c) \Rightarrow \Diamond_{[0, t_a]} \Box(|\Delta V_2(k)| \leq \Delta V_c)] \end{aligned} \quad (20)$$

The TLS can be encoded into a MILP using the toolbox BluSTL [18]. Then, the overall problem is converted into a MILP, written in the format of Yalmip [21] and solved by efficient solvers like Mosek [22] and Gurobi [23].

IV. SIMULATION AND RESULTS

A. System Parameters

The base of the system is given as follows

$$\begin{aligned} P_b &= 100 \times 10^3 \text{ [W]} \\ V_b &= \frac{480}{\sqrt{3}} = 391.9184 \text{ [V]} \\ V_{blu} &= 480 \text{ [V]} \\ I_b &= \frac{P_b}{\frac{3}{2} V_b} = 170.1035 \text{ [A]} \\ Z_b &= \frac{V_b}{I_b} = 2.304 \text{ [ohms]} \end{aligned} \quad (21)$$

The network parameters are given as follows

$$\begin{aligned} R_{12} &= 0.15 \text{ [ohms]} \\ X_{12} &= 0.7 \times 10^{-3} \text{ [H]} \\ Z_{12} &= 0.15 + j0.2639 \text{ [ohms]} \\ \bar{Z}_{12} &= \frac{Z_{12}}{Z_b} = 0.0651 + j0.1145 \text{ [p.u]} \\ Y_{12} &= G_{12} + jB_{12} = \frac{1}{\bar{Z}_{12}} = 3.7525 - j6.6001 \end{aligned} \quad (22)$$

The load demand is given as

$$P_L = 0.6, \quad Q_L = 0.5 \quad (23)$$

The parameters of the responsive model of the ESS are assumed to be

$$A = -0.3914, \quad B = -0.3121, \quad C = 0.4127. \quad (24)$$

B. Case Study

Consider the state space model (13), with the parameters in (24), and the step disturbance $d = -0.5$, such that $c_1 = 0.0651$. Let the scheduling problem be as in (20), and we propose to constraint the voltage deviation to satisfy the TLS formula $\varphi = \Box(\varphi_t)$ for finite-time voltage restoration.

To show the effectiveness of the temporal logic specification, we will propose two different scenarios:

- Scenario 1:

$$\varphi_t = ((|\Delta V_2| \geq 0.002)) \Rightarrow (\Diamond_{[0, 8]} \Box(|\Delta V_2| \leq 0.002)) \quad (25)$$

which reads “when $|\Delta V_2|$ is larger than 0.002, it should become less than 0.002 in less than 8s for always (i.e., it will never be larger than 0.002 again).” The simulation of this temporal logic specification is illustrated in Figs. 3 and 4.

- Scenario 2:

$$\varphi_t = ((|\Delta V_2| \geq 0.002)) \Rightarrow (\Diamond_{[0, 8]} (|\Delta V_2| \leq 0.002)) \quad (26)$$

The difference here from Scenario 1, is that there is no specification of always (i.e., \Box), which say that “whenever $|\Delta V_2|$ is larger than 0.002, it should become less than 0.002 in less than 8s.” The simulation of this formulation is represented in Figs. 5 and 6. The negative control input signal in Figs. 4 and 6, indicates the power is transferred from the ESS to Bus 2.

This problem is feasible with these scenarios for a control input limit

$$U_{\lim} = 9.5$$

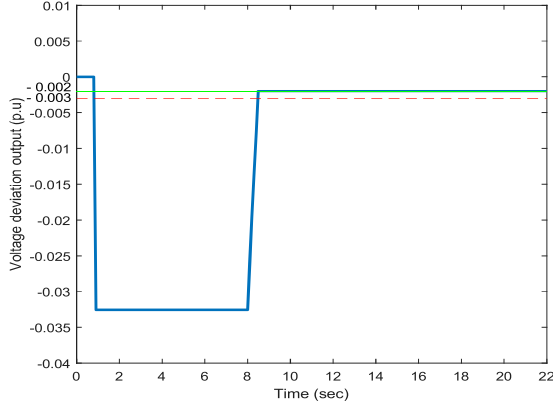


Fig. 3. Voltage deviation response under Scenario 1.

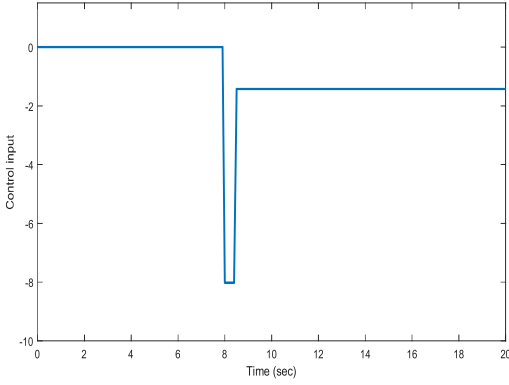


Fig. 4. Input signal under Scenario 1.

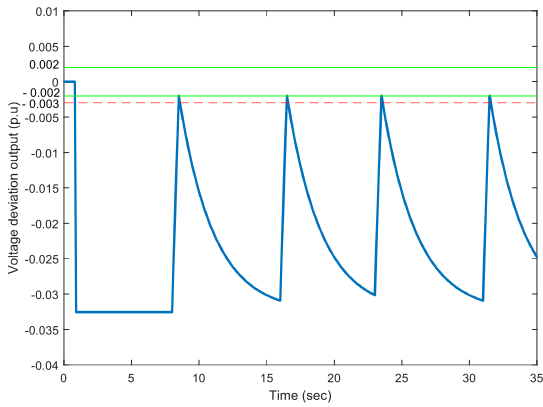


Fig. 5. Voltage deviation response under Scenario 2.

Consider now the optimal control without temporal logic specification by considering that the objective is to minimize the control effort and the voltage deviation. Similar to (19)

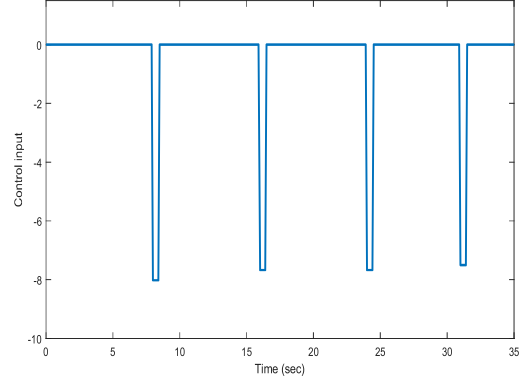


Fig. 6. Input signal under Scenario 2.

and (20), the total control effort and the voltage deviation can be represented as

$$\overline{C_U} = \sum_{k=1}^T [|u(k)| + |\Delta V_2(k)|]. \quad (27)$$

So, the scheduling is summarized as follows

$$\begin{aligned} \min \quad & \overline{C_U} \\ \text{s.t.} \quad & \forall k \in \mathcal{T} \\ & x(k+1) = A_d x(k) + B_{d1} u(k) + B_{d2} \hat{d}(k) \\ & \Delta V_2(k+1) = C_d x(k) + D_{d1} u(k) + D_{d2} \hat{d}(k) \\ & |u(k)| \leq U_{\lim}, \quad \forall k \in \mathcal{T} \\ & |u(k) - u(k-1)| \leq \Delta U_{\lim}, \quad \forall k \in \mathcal{T} \end{aligned} \quad (28)$$

For satisfaction of temporal constraints, a rate of change constraint of the control input is added. Fig. 7 represents the voltage deviation with control and without control, while Fig. 8 represents the control input. We remark that the voltage restoration takes more than 15s and the problem is feasible for a control limit bound $U_{\lim} = 2$. As shown, temporal constraints can be met by using rate of change constraints through a trial and error procedure.

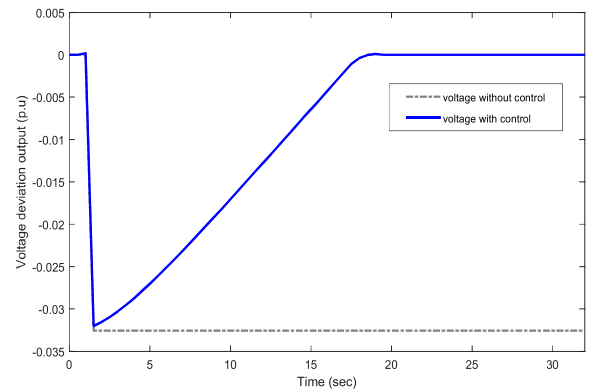


Fig. 7. Voltage deviation response using rate of change constraint instead of TLS.

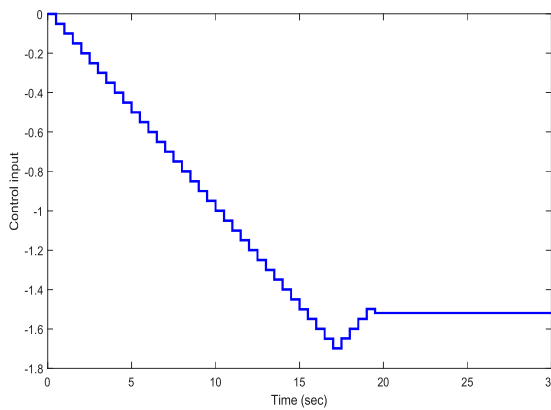


Fig. 8. Input signal using rate of change constraint instead of TLS.

V. CONCLUSION

In this paper, a NOC-based control scheduling approach is applied that enables the realization of the TLS. The controller schedules a series of control signals when large voltage deviation is measured. The scheduled signal is obtained from a MILP, where the TLS is encoded. The scheduled signal is sent to the ESS such that the voltage admits a finite-time restoration. The future work will be devoted to multiple actuator scheduling using both active and reactive power. Islanded microgrids with dynamic motor loads will be considered as well. In addition, the runtime issue will be considered and resolved and an online model predictive control will be designed.

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