# Super Sensitive Phase Measurement Deflectometry with effective fringe periods beyond the MTF limit

**Luke A. DeMars, Ana Hiza Ramirez-Andrade, Rosario Porras-Aguilar, Konstantinos Falaggis**
*University of North Carolina at Charlotte, 9201 University City Blvd., Charlotte, NC, USA 28223*
*Author e-mail address: ldemars@uncc.edu*

**Abstract:** A super-sensitive phase measuring Deflectometry technique is presented to obtain effective fringe periods beyond the MTF limit. This allows decreasing significantly the random component of the uncertainty in the measured slope. © 2019 The Author(s)
**OCIS codes:** 120.3940, 120.5050.

## 1. Introduction

In Phase Measuring Deflectometry (PMD) a series of phase shifted sinusoidal patterns are generated on a display, where a portion of the out-going rays are deliberately reflected off a specular surface and the distorted reflected pattern is captured by a camera. The principle is shown in Fig. 1. The deformed sinusoidal patterns hold the slope information of the part under test [1]. A limiting factor is the Modulation Transfer Function (MTF) of the Imaging system. For incoherent light, the MTF is linearly decreasing across spatial frequency and has a cut off frequency that is dependent on various system parameters [2].

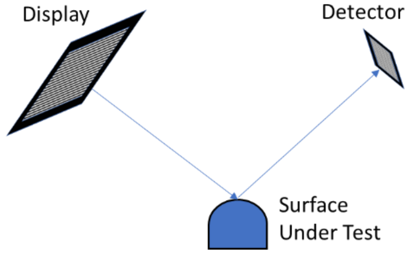


Fig. 1. Principle of a PMD system

This linear dependence of the MTF to spatial frequency affects the uncertainty in the slope measurement [3]. To improve on the MTF limit imposed by Deflectometry we propose using a Multi-Frequency PMD technique, where (a) the maximum fringe frequency is chosen to have an acceptable fringe modulation but be closest to MTF limit, and (b) the effective synthetic periods generated [4] have an effective fringe frequency that is higher than the highest fringe frequency. This reduces the slope uncertainty, but comes at an expense, because high-frequency patterns have a highly wrapped phase map [5]. A hierarchical phase unwrapping technique is then implemented to unwrap the phase map with a minimum number of frames [6].

## 2. Breaking the MTF Limit in Deflectometry

Multi-Frequency Phase Unwrapping (MFPU) is a field with many different applications in science and technology. Commonly, as for interferometry, MFPU is used to increase the unambiguous measurement range (UMR) of the measurement system [3]: the individual fringe frequencies $f_i$ (or fringe period $p_i$) are combined to generate a longer fringe period $\Lambda$ that is larger than the largest period of the measurement system with $\Lambda = \frac{p_1 p_0}{p_1 - p_0}$. In contrast, Tilford who proposed the use of multiple frequency sums and differences to make beat frequencies of arbitrary values as shown in Eq. (1), where $C_n$ is an integer, N is the total number of periods, and $\Lambda_s$ is the unique beat period generated, and $\varphi_s$ is the synthetic phase at $\Lambda_s$ with

$$\Lambda_s = \frac{1}{\sum_{n=0}^{N-1} \frac{C_n}{p_n}} \quad \text{and} \quad \varphi_s = mod\left(\sum_{n=0}^{N-1} C_n \varphi_n + \pi, 2\pi\right) - \pi, \tag{1}$$

where "*mod(x+π, 2π)- π*" wraps the phase into the interval [- $\pi$, $\pi$]. Servin [5] highlighted that if $C_n = 1$, the resulting phase is smaller than the smallest period $p_n$ and $\varphi_s$ reduces to the sum of all phases. We therefore refer to this technique as the sum-of-phase (SOP) approach. The SOP technique has one important advantage: the distance uncertainty reduces by the same amount as $p_n$. However, $\varphi_s$ is highly wrapped when using SOP. This work proposes a hybrid multi-frequency approach to obtain a full unambiguous measurement range while preserving the uncertainty level of

SOP. An optimum hierarchical MFPU is the so-called GOMF technique developed by Towers [6] that uses a minimum number of periods, where the upper limit of the ratio between two consecutive periods determined by the phase noise. In this way, an MFPU is achieved near the theoretical performance limit. In this work we incorporate the SOP approach by adding two additional fringe periods near the shortest period.

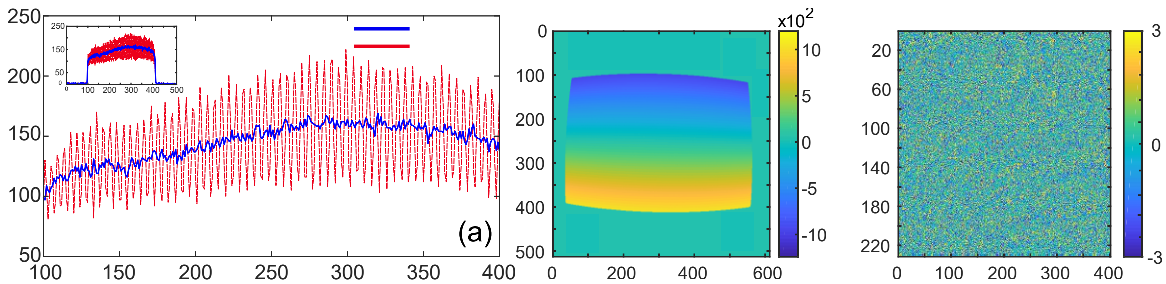


Fig. 2. (a) measured intensity for p = 7 and p = 24. (b) absolute phase for the periods 7, 49. 343, and 2401pixels.

With this concept in mind, an experiment was conducted in which the fringes projected on the display was directly imaged by a camera. The resulting intensity for a fringe patterns with the periods of p = 24 and p = 7 is shown in Fig. 2a. The results show, that p = 7 is well below the MTF limit and the modulation is too low to obtain an accurate phase map, see Fig. 2b. However, an accurate phase map for the GOMF based fringe periods p = 24, 240, and 2400 could be achieved with no difficulty. In a second experiment, the fringe periods p = 27, 30, 33 pixels have been added to incorporate the SOP approach. Fig. 3 shows the unwrapped phase map for the effective period near with p = 6.81. A comparison in gives experimental proof that with the proposed methodologies the MTF uncertainty limit can be surpassed.

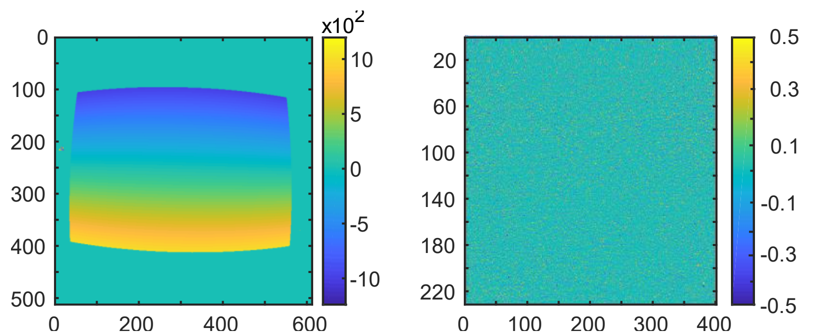


Fig. 3. Unwrapped phase for the fringe periods 24, 27, 30, 33, 240, and 2400 pixels (effective period of 6.81 pixels).

**3. Conclusion**

What has been shown is that by using the SOP approach paired with the GOMF based MFPU it is possible to surpass the uncertainty limit of the MTF. Direct experimental evidence has been shown that with a shortest period of p = 24 we were able to achieve an effective period of p = 6.81 pixels. This approach is interesting for future work when using the ART technique [7] that does not need long fringe periods: for the periods 18, 20, and 22 ART unwraps correctly 7920 pixels while SOP provides the sensitivity of an effective period of 6.6 pixels.

**4. References**

[1] Häusler, G., Faber, C., Olesch, E., Ettl, S. and Staudtstr, B., "Deflectometry vs . Interferometry," 13–16 (2013).
[2] Joseph W. Goodman, "Frequency Analysis of Optical Imaging Systems" in *Introduction to Fourier Optics,* Third ed. (New York: McGraw-Hill, 1988.).
[3] Konstantinos Falaggis, Ana Hiza Ramirez-Andrade, David Towers, Catherine Towers, Rosario Porras-Aguilar, "Multi-wavelength phase unwrapping: a versatile tool for extending the measurement range, breaking the Nyquist limit, and encrypting optical communications," Proc. SPIE 10749, Interferometry XIX, 1074913 (18 August 2018); doi: 10.1117/12.2321987
[4] Tilford, C. R., "Analytical procedure for determine lengths from fractional fringes," Appl. Opt. 16(7), 1857– 1860 (1977).
[5] Servin, M., Padilla, M. and Garnica, G., "General theory for super-sensitive dual-wavelength phase metrology: error-free unwrapping and signal-to-noise ratio" (2017).
[6] Towers, C. E., Towers, D. P. and Jones, J. D. C., "Generalized frequency selection in multifrequency interferometry.," Opt. Lett. 29(12), 1348–1350 (2004).
[7] Konstantinos Falaggis, David P. Towers, and Catherine E. Towers, "Algebraic solution for phase unwrapping problems in multiwavelength interferometry," Appl. Opt. 53, 3737-3747 (2014).