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Optimal Risk-Sharing Mechanism to Enhance Resilience of Communities

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ABSTRACT: Resilience of urban communities hit by extreme events relies on the prompt access to financial resources needed for recovery. Therefore, the functioning of physical infrastructures is strongly related to that of the financial system, where agents operate in the markets of insurance contracts. When the financial capacity of an agent is lower than the requests for funds from the communities, it defaults and fails at providing these requests, slowing down the recovery process.
In this work, we investigate how the resilience of urban communities depends on the reliability of the financial agents operating in the insurance markets, and how to optimize the mechanism adopted by these agents to share the requests for funds from the policyholders. We present results for a set of loss functions that reflect the costs borne by society due to the default of the financial agents.

1. INTRODUCTION

The design of communities that are resilient to natural and/or man-made shocks is the central focus of modern urban design. When an extreme event strikes a community, it causes damages and failures to its physical assets and infrastructures, that, in their turn, determine a slowdown of the economic activities in the involved geographic area. To overcome this, in the aftermath of such a disruption, the recovery process aims at restoring or improving the preexisting functionality of the affected community. The outcome of the recovery process relies on the access to financial resources, including public aids from the government and insurance payments from financial agents operating in insurance markets. The demands for funds from the affected communities, in the form of insurance claims, pose a risk of default for the financial agents. Indeed, these agents satisfy their debts, that we call *demands* in the continuation of the paper, using their availabilities of assets, that we call *financial capacity*. If the demand for funds towards an agent overpasses its capacity, it defaults and fails to provide the whole amount of funds, or part of them, to the communities. This causes further losses and slowdowns to the whole affected communities (Cimellaro (2016)). The economic demands caused by damages of the physical infrastructures due to the extreme events can be probabilistically predicted and modeled by engineering models, e.g. for earthquakes in the reports by Hunt and Stojadinovic (2010) and Aviram et al. (2010). In this work, we focus on the design and optimization of a *risk-sharing mechanism* among financial agents, that allows them to share with each other the demands for funds from the affected communities. The risk-sharing mechanism determines a way of cooperation among financial agents in order to maximize the amount of provided funds and to avoid their default. The main questions that we aim at addressing in this work are the following: is it better to connect the agents through the risk-sharing mechanism, or to leave them isolated to face the external demands for funds? To what extent should the financial agents cooperate in order to lower the risk of systemic default? Intuition suggests that, if agents are connected, those that face low insurance claims could help those that, instead, face a high amount of claims from the policyholders. This is the phenomenon of *diversification* of the risk. The seminal work by Allen and Gale (2000) discusses this positive aspect of risk-sharing. However, connectivity can trigger the negative phenomenon of *contagion*. Under *contagion* an agent fails because of the additional demand transferred from other agents via the mechanism, whereas it would not have failed if operating in isolation. An exhaustive survey on the topic of *contagion* in financial networks is presented in the work by Glasserman and Young (2016). The work by Elliott and Jackson (2014), analyzes the dichotomy between *contagion* and *diversification*. The possibility of contagion suggests that, in some settings, it is better not to connect the agents, because the likelihood of the default increases by joining them. A detailed analysis of how *contagion* spreads in the financial networks is presented by Eisenberg and Noe (2001).

2. PROBABILISTIC MODEL OF DEMAND AND RISK-SHARING MECHANISM

2.1. *General Settings*

Consider a set of financial agents, operating in the insurance markets, whose index set is $\mathcal{N} = \{1,2,...,n\}$. Agent i has an initial amount of external capacity, c_i, and faces external demands for funds, modeled as non negative random variables, S_i, that correspond to the claims of the policyholders (i.e. the urban communities) after the occurrence of extreme events. The random vector of *external demands* is $\mathbf{S} = [S_1, S_2, \ldots, S_n]^\top \sim p_\mathbf{S}$, where $p_\mathbf{S}$ is a known joint distribution, while the vector of deterministic capacities is $\mathbf{c} = [c_1, c_2, \ldots, c_n]^\top$. A general assumption is that, in the expected sense, the agents do not default, i.e. $\mathbb{E}[S_i] < c_i, \forall\, i \in \mathcal{N}$.

The risk-sharing mechanism transforms the external demands in the *nominal demands*, through the *redistribution function*, $\mathbf{f} : \mathbb{R}^n_+ \to \mathbb{R}^n_+$, $\mathbf{f} = [f_1, f_2, \ldots, f_n]$. The *nominal demand* for agent i, d_i, is equal to:

$$d_i = f_i(\mathbf{S}) = S_i + \sum_j l_{ij} \tag{1}$$

so that, in vector notation, Eq.(1) reads:

$$\mathbf{d} = \mathbf{S} + \mathbf{L} \cdot \mathbf{1}_{n\times 1} \tag{2}$$

where $\mathbf{1}_{n\times 1} = [1, 1, \ldots, 1]^\top$ and the term l_{ij} belongs to the *matrix of internal liabilities*, i.e. debts, $\mathbf{L}$, defined as follows:

$$\mathbf{L} = \begin{bmatrix} 0 & l_{12} & \cdots & l_{1n} \\ l_{21} & 0 & \cdots & l_{2n} \\ \vdots & \cdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & 0 \end{bmatrix} \tag{3}$$

l_{ij} represents the payment due by agent i to agent j, which we call *internal demands* and depends on the chosen redistribution mechanism. Indeed if, according to the mechanism, agent j transfers 10% of its external demand to agent i, then $l_{ij} = 0.1 \cdot S_j$.

The diagonal of matrix $\mathbf{L}$ is composed of zeros, meaning that there are no reflexive debts. The elements on the i-th row of matrix $\mathbf{L}$ represent the debts of agent i towards each of the other financial agents, while the elements on the i-th column correspond to credits of agent i. Eq.(1) shows that when an agent is part of the risk-sharing mechanism it faces a demand that is higher than its external one, because it is increased by the internal demands claimed by other agents. On the other side, the mechanism also increases the capacities. The nominal amount of increased capacity of agent i due to the demand transfered to the other agents in the system is:

$$\Delta c_i = \sum_j l_{ji} \tag{4}$$

We call it *internal capacity*, to differentiate it with the external one, c_i, that is not directly affected by the mechanism. However, the internal capacity depends on the ability of the other financial agents to satisfy their debts towards agent i. We define the *available internal capacity*, $\widetilde{\Delta c_i}$, as the actual internal capacity available to agent i, depending on the possible default of some of its debtors. We always have $\widetilde{\Delta c_i} \leq \Delta c_i$, where the equality stands when no debtor of agent i defaults.

We define the vector of *nominal demands*, $\mathbf{d}$, whose i-th element, d_i, defined in Eq.(1), is the amount of demand that agent i must satisfy in favor of both the policyholders and the other financial agents. If the *available capacity* of agent i, $\tilde{c}_i = c_i + \widetilde{\Delta c_i}$, is lower than d_i, then it defaults. On the other side, let $\widetilde{\mathbf{d}} = \left[\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right]^\top$ be the vector of *satisfied demands*, where $\tilde{d}_i$ represents the demand that agent i is able to effectively satisfy. If agent i does not default $\tilde{d}_i = d_i$, while if it does $\tilde{c}_i = \tilde{d}_i < d_i$. An agent that is not in default satisfies its *nominal demand* in full, while an agent that defaults serves the demands with its total capacity corrected for some cost, the so-called *liquidation costs*.

Let $\mathbf{v} = [v_1, v_2, \ldots, v_n]^\top$ be the vector of net values for all agents. The net value of agent i, $v_i = \tilde{c}_i - d_i$, is the difference between its *available capacity* and its *nominal demand*. In reliability analysis, because of its role in defining the agent's default, the net value of an agent corresponds to its *limit state function*. Let $\mathbf{h} : \mathbb{R}^n \to \mathbb{B}^n$ be the *default state function*, where $\mathbb{B} = \{0, 1\}$, and let $\mathbf{y}$ be the *default state vector*. We have $\mathbf{y} = \mathbf{h}(\mathbf{v})$, or element-wise:

$$y_i = h_i(v_i) = \mathbb{1}\{v_i < 0\} \tag{5}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function that takes value 1 if the condition in the parenthesis is satisfied and zero otherwise. We assume a *proportionality principle* in distributing the assets of a defaulted agent. After the defaults of a financial agent in the system, its available assets are distributed to its internal and external debtors according to the proportion of their demand with respect to the total demands of the defaulted agent: for example, if agent j defaults, the actual demand paid to agent i is equal to the fraction $\frac{l_{ji}}{d_j} \cdot \tilde{d}_j < l_{ji}$. If agent j is not in default then, as $\frac{l_{ji}}{d_j} \cdot \tilde{d}_j = l_{ji}$, it satisfies all the nominal debt towards agent i.

Let $\beta_i \in [0, 1]$ be the *loss-given-default* rate for agent i. When agent i defaults, agent j with internal demand towards agent i, receives a smaller fraction of its *nominal internal demand* from the defaulted agent, i.e. $(1 - \beta_i) \cdot \frac{l_{ij}}{d_i} \cdot \tilde{d}_i$. We, therefore, assume that the *liquidation costs* are linear in the total amount of available capacity of the agents. It results that $\tilde{c}_i = c_i + \sum_j (1 - \beta_j \cdot y_j) \frac{l_{ji}}{d_j} \cdot \tilde{d}_j$ and that $\tilde{d}_i = \min\{d_i, \tilde{c}_i\}$. The overall framework is, hence, recursive, so that to compute the vectors $\mathbf{y}$, $\mathbf{v}$ and $\widetilde{\mathbf{d}}$, we rely on the *fictitious defaults* algorithm presented in the work by Eisenberg and Noe (2001). It works as follows. Given the realization of $\mathbf{S}$, we construct the matrix $\mathbf{L}$ according to the risk-sharing mechanism and we derive $\mathbf{d}$ by Eq.(1). Then, we proceed as follows:

Step 0) Assign $\widetilde{\mathbf{d}}^0 = \mathbf{d}$ and $\widetilde{\Delta c}_i^0 = \Delta c_i$, $\forall i \in \mathcal{N}$. Compute the net value of each agent given by:

$$v_i^0 = c_i + \Delta c_i - d_i \tag{6}$$

Apply the *default state function* to the vector of initial net values to obtain the vector of initial defaults at step 0, $\mathbf{y}^0 = \mathbf{h}(\mathbf{v}^0)$.

Step t) Compute:

$$\widetilde{\Delta c}_i^t = \sum_j (1 - \beta_j \cdot y_j^{t-1}) \frac{l_{ji}}{d_j} \cdot \tilde{d}_j^{t-1} \tag{7}$$

so that $\tilde{c}_i^t = c_i + \widetilde{\Delta c}_i^t$. Compute the elements of the vector $\widetilde{\mathbf{d}}^t$ as follows:

$$\tilde{d}_i^t = \min\{d_i, \tilde{c}_i^t\} \tag{8}$$

Compute the net value of the agent as follows:

$$v_i^t = \tilde{c}_i^t - d_i \tag{9}$$

Apply the default state function to the vector $\mathbf{v}^t$ to obtain the vector of defaults at step t,

$$\mathbf{y}^t = \mathbf{h}(\mathbf{v}^t)$$

The algorithm stops when $\mathbf{y}^t = \mathbf{y}^{t-1}$, for some $t \in \{1, 2, \ldots, n\}$. When the algorithm ends we obtain the vector of satisfied demands, $\widetilde{\mathbf{d}} = \widetilde{\mathbf{d}}^t$ and the vector of final defaults, $\mathbf{y} = \mathbf{y}^t$. Both vector $\tilde{\mathbf{d}}$ and vector $\mathbf{y}$ exist and are unique given a certain realization of the external demands, as shown by Eisenberg and Noe (2001).

Let $\ell(\mathbf{y}, \mathbf{d}) : \mathbb{B}^n \times \mathbb{R}^n_+ \to \mathbb{R}$ be the *loss function*, that returns the total cost for society. For example, ℓ can be the number of defaulted agents or the total amount of unsatisfied policies due to the default of some agents in the system. Note that the cost function ℓ is a function of the random vector $\mathbf{S}$, and, thus, it is a random variable itself.

We assume that a central decision-maker aims at finding the best risk-sharing mechanism among financial agents, minimizing the expectation of the chosen loss function:

$$\mathbf{f}^* = \arg\min_{\mathbf{f} \in \mathscr{F}} \mathbb{E}_{\mathbf{S}} \left[\ell(\mathbf{h}(\mathbf{v}(\mathbf{S}), \mathbf{d}(\mathbf{S}))) \right] \qquad (10)$$

In this work, we restrict the optimization problem in Eq.(10) to the set of linear redistribution functions: accordingly, $l_{ij} = \alpha_{ji} \cdot S_j$, where α_{ji} is the linear redistribution coefficient that corresponds to the fraction of external demand transferred from agent j to agent i.

Let $\mathbf{A}$ be the matrix of linear redistribution coefficients, whose ij-th entry is α_{ij}. Under the assumption of linear redistribution mechanism, the optimization problem is equivalent to finding the optimal matrix $\mathbf{A}^*$, as matrix $\mathbf{A}$ completely defines the redistribution network. Indeed, for a given matrix $\mathbf{A}$ and a realization of the external demands, $\mathbf{S}$, we can compute the matrix of internal debts $\mathbf{L}$. We impose the conditions that $\alpha_{ij} \in [0, 1]$, $\forall\, i, j \in \mathscr{N}$ and $\sum_{j=1}^n \alpha_{ij} = 1$, because the agents cannot transfer more than their total external demand to a single agent or to the set of financial agents; we also impose the *"fairness"* condition, $\sum_{j=1}^n \alpha_{ji} = 1$, to avoid that a large portion of the total external demands is transferred towards a single agent or a small group of agents. In summary, we assume that matrix $\mathbf{A}$ belongs to the set of doubly stochastic matrices.

2.2. *Loss Function*

The loss function represents a measure of the overall costs borne by the communities and/or financial agents, due to the extreme events. In this section, we list three loss functions that we consider in this work, namely: (i) $X := \|\mathbf{y}\|_1$, is the number of agents in default state; (ii) $m := n^{-1} \cdot \left[\sum_i \left(1 - \frac{\tilde{d}_i}{d_i}\right) \cdot S_i \right]$, are the unpaid insurance claims; (iii) $\delta := n^{-1} \cdot \left[\sum_i \left(1 - \frac{\tilde{d}_i}{d_i}\right) \cdot S_i + \gamma \cdot X \right]$, is a mix of the previous loss functions.

Loss function X measures the implicit costs related to the default of the agents. Function m is a measure of the losses on insurance claims of the policyholders due to the default of financial agents. Function δ combines the losses suffered by the policyholders and the penalty for the default of the agents, and parameter γ weights the importance of the two contributions.

2.3. *Some Configurations of Risk-Sharing Mechanisms*

In this subsection, we describe some special configurations of risk-sharing mechanisms. The first two are *extreme configurations*. (i) In the *isolated agents* configuration, each agent faces only its external demand for funds: this configuration corresponds to matrix $\mathbf{A}^{\mathrm{IS}} = \mathbf{I}_{n\times n}$, where $\mathbf{I}_{n\times n}$ is the identity matrix, and to the set of isolated nodes in a network. (ii) In the *perfect team* configuration, each agent transfers equal parts of its external demand to each other financial agent and to itself. In network representation, the *perfect team* corresponds to the complete graph with homogeneous weights on each arc. In this case, the redistribution matrix is $\mathbf{A}^{\mathrm{PT}} = n^{-1} \cdot \mathbf{1}_{n\times n}$, where $\mathbf{1}_{n\times n}$ is the matrix of ones. (iii) The *intermediate mechanism* is an intermediate configuration between the two extreme ones. In this case, the matrix is $\mathbf{A}^{\mathrm{IM}} = (1-\varphi)\mathbf{I}_{n\times n} - (n-1)^{-1} \cdot \varphi \cdot (\mathbf{1}_{n\times n} - \mathbf{I}_{n\times n})$, where $\varphi \in \left(0, \frac{1}{2}\right]$ is the *transferred part*.

3. PROBABILISTIC MODELING OF THE DEMANDS

3.1. General Settings: Bernoulli External Demands

In this section, we present results for the Bernoulli demand distribution, $p_{\mathbf{S}}$. Let the external demands be independent and identically distributed Bernoulli random variables, i.e. $S_1, S_2, \ldots, S_n \sim \text{Ber}(p)$. This models n insurance companies underwriting policies in separated geographical regions/insurance markets with same probability, p, of occurrence of the independent extreme events, i.e. the occurrence of a shock in one area does not influence the likelihood of occurrence in another area. We assume that the agents have the same external capacity, $c_i = \bar{c} < 1, \forall\, i \in \mathcal{N}$, where 1 is the magnitude of the costs caused by each shock, expressed in normalized units. Under these settings, we are referring to the *homogeneous case*.

3.2. Risk metric: Expected number of defaults

Considering the loss function $\ell = X$, we restrict the set of possible risk-sharing mechanisms to the *extreme configurations*. Let $\Lambda = \left\{\mathbf{A}^{\text{IS}}, \mathbf{A}^{\text{PT}}\right\}$ be that set. When the agents are *homogeneous*, minimizing the expected number of defaulted agents is the same as minimizing the probability that a single agent defaults, i.e. the probability of failure P_f. The probability of failure of agent i in the isolated agents configuration is equal to $P_f^{\text{IS}} = \mathbb{P}(S_i > \bar{c}_i) = \mathbb{P}(S_i = 1) = p$, while for the *perfect team*, it is equal to the probability that the sum of external demands is greater than the team total capacity, that we called *pooled capacity*. We thus conclude that:

$$P_f^{\text{PT}} = \mathbb{P}\left(\sum_i S_i > n \cdot \bar{c}\right) = 1 - F_Y\left(\lfloor n \cdot \bar{c} \rfloor\right) \quad (11)$$

where we define $Y = \sum_i S_i \sim \text{Bin}(n, p)$ and F_Y being its cumulative distribution function.

Fig.(1) plots P_f vs the number, n, of agents in the system under the *perfect team* configuration, where all the agents participate in a single team. More generally, it is possible to show the following.

Proposition 1. *If* $\lfloor n \cdot \bar{c} \rfloor < n_S$*, where* n_S *is chosen*

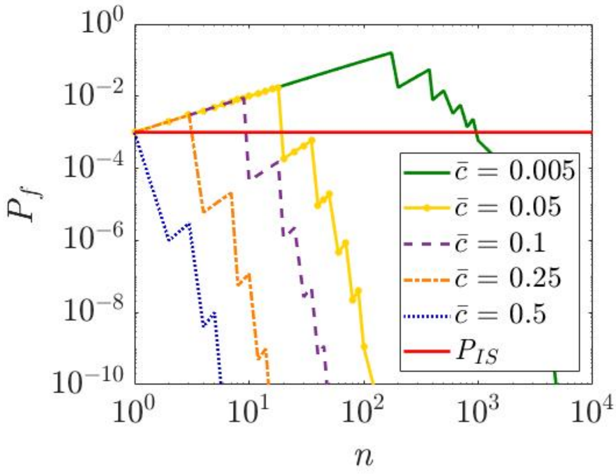

Figure 1: Comparison of probability of failure for the isolated agents (horizontal red line) and for the perfect team for several values of $\bar{c} < 1$ (other lines), as n increases.

such that:

$$n_S = \min\left\{k : \mathbb{P}\left(\sum_{j=1}^{n} S_j > k\right) < p,\, k \in \mathcal{N}\right\} \quad (12)$$

then the probability of failure P_f is lower for the isolated agents than for the perfect team.

In the above proposition, n_S is the number of shocks hitting the team so that the probability of having more than this number is less than the probability that a shock hits an agent.

Proposition 1 states that if there are enough agents in the team so that their *pooled capacity*, $\lfloor n \cdot \bar{c} \rfloor$, is able to face more than a certain number, n_S, of shocks, then the team configuration is to be preferred over the isolated one.

It is important to note that the above proposition is valid for any value of β, because the probability of default both of the isolated agents and of a member of the *perfect team*, does not depend on the value of the *loss-given default* rate.

3.3. Risk metric: Expected loss on insurance payments

Function $\mathbb{E}[m]$ measures the costs for the communities via the expected lack of insurance payments. Therefore, by designing an optimal risk-sharing mechanism that minimizes this risk-based

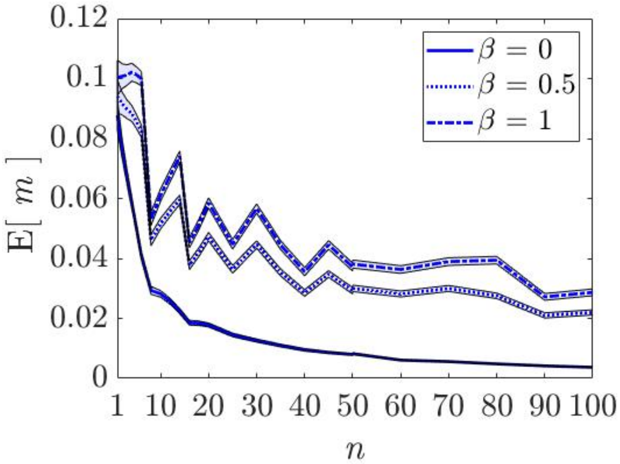

Figure 2: Analysis of $\mathbb{E}[m]$ *for* β *equal to 0, 0.5 and 1 in the perfect team, when* $\bar{c} = 0.125$, $p = 0.1$.

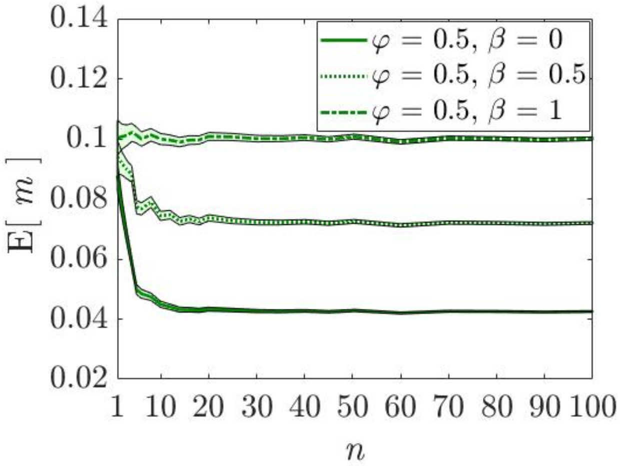

Figure 3: Analysis of $\mathbb{E}[m]$ *in the intermediate mechanism, when* $\bar{c} = 0.125$, $p = 0.1$.

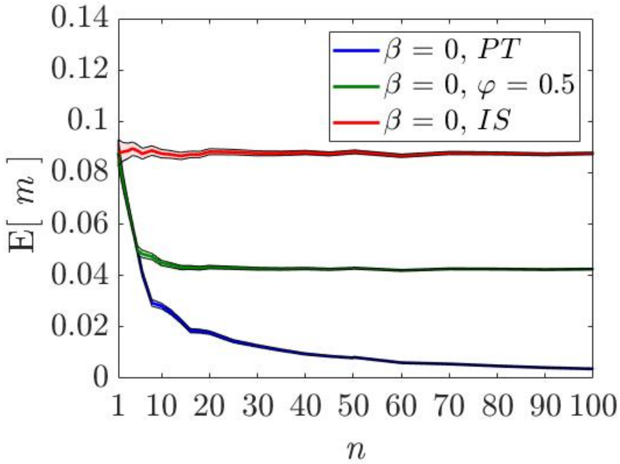

Figure 4: Metric $E[m]$*. Comparison of perfect team, isolated agents and intermediate mechanism as n increases and* $\beta = 0$. $\bar{c} = 0.125$, $p = 0.1$.

metric, the central decision-maker achieves the goal of increasing the short-term resilience of the urban communities.

In Fig.(2) and Fig.(3) we assume that $\beta_i = \beta$, $\forall i$, and we show how the value of this parameter affects the risk-based metric for the *perfect team* and the intermediate configuration, as presented in section (2.3). The metric for the *isolated agents* does not vary with n, so that we do not plot the analysis for this configuration. The results of this section are obtained by averaging on 10,000 simulated scenarios. In the figures, we also include the 95% confidence bounds for the estimated metric based on Monte Carlo simulations.

As we would expect, the larger the *loss-given-default* rate, β, the larger the losses for the policyholders in the communities, for any fixed number of agents in the financial system, n. This is true for any mechanism. In Fig.(4) and (5) we compare the three mechanisms, for $\beta = 0$ and $\beta = 1$.

We note that the *isolated agents* perform the worst among the three mechanisms compared in the figure, while the *perfect team* is the best mechanism for any β. It is also worth noting that, for any group dimension, we never prefer the *isolated agents* to the *perfect team*. This is due to the fact that the value of the metric $\mathbb{E}[m]$ for the isolated agents is equal to the following value:

$$\mathbb{E}\left[m^{\mathrm{IS}}\right] = [1 - (1 - \beta) \cdot \bar{c}] \cdot p \qquad (13)$$

No *diversification* effect takes place in Eq.(13). On the other hand, for the *perfect team*, as the number of agents increases, the losses for the policyholders diminish on expectation. This is because the probability of default of the agents decreases with n, due to the *diversification* effect, so that we observe the metric going to zero as $n \to \infty$.

According to this metric, also when $n < n_S$, we still prefer the *perfect team*. The reason is that by joining the agents we exploit more efficiently the capacities available in the financial system. Indeed, under this configuration, the financial system is able

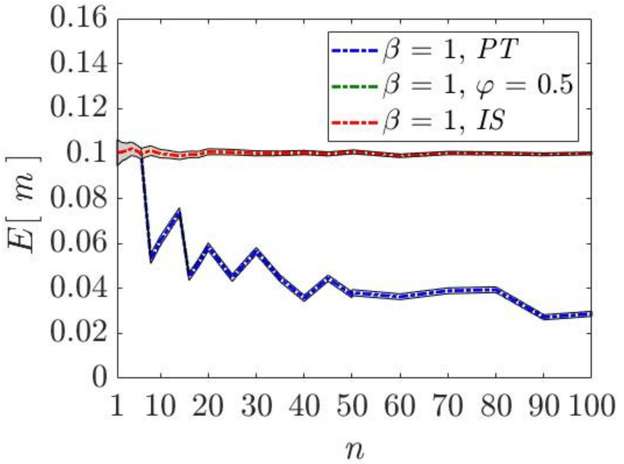

Figure 5: Metric $E[m]$. Comparison of perfect team, isolated agents and intermediate mechanism as n increases and $\beta = 1$. $\bar{c} = 0.125$, $p = 0.1$.

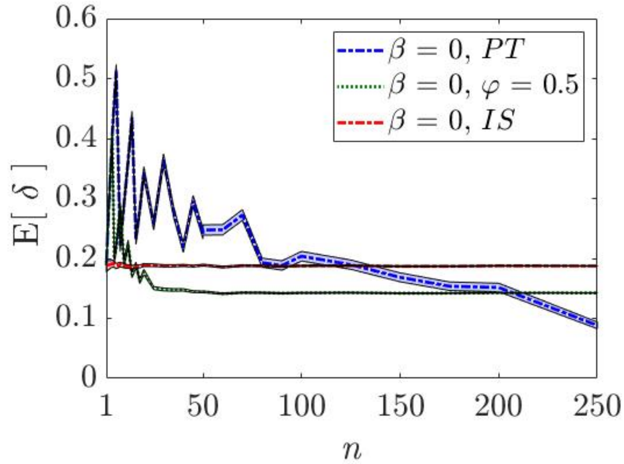

Figure 6: Metric $E[\delta]$: comparison of isolated agents, intermediate mechanism, $\varphi = 0.5$, and perfect team, for $\beta = 0$, $\bar{c} = 0.125$, $p = 0.1$ and $\gamma = 1$.

to satisfy more external demands than it would be under the *isolated agents*, even if all the agents in the *perfect team* default. It follows that we prefer to join the agents in the *perfect team*, for every group dimension n. However, this result does not quantify the losses related to the collapse of large parts of the financial system. For this reason, we investigate the behavior of the metric $\mathbb{E}[\,\delta\,]$ that takes into account some penalty for the default of groups of financial agents.

3.4. Risk metric: Mixed Losses

In this section, we present some results for the metric $\mathbb{E}[\,\delta\,]$, that takes into account both the losses on claims of the communities and the number of defaulted agents that serve the external demands. We consider this metric because it gives a more comprehensive estimate of the losses for society. Indeed, the *resilience* of the communities does not only depend on the unsatisfied demands, but also on the number of agents that survive the extreme event, so that they could provide funds to the communities if other shocks occur.

Figs.(6 - 7) show that, according to this metric, the *perfect team* is not always as good as other mechanisms for any group dimension, as happened for metric m in Figs.(4 - 5). We observe that both for $\beta = 0$ and $\beta = 1$ there is some threshold of group dimension below which we prefer the *isolated agents* to the *perfect team*. When we compare the two *extreme* configurations, for low n *contagion* prevails on *diversification*, so that the likelihood of the failure of all the members of the *perfect team* is large enough to make the metric larger than in the *isolated agents*, even if the *perfect team* fulfills more external demands, i.e. $\mathbb{E}\left[\delta^{\mathrm{PT}}\right] \geq \mathbb{E}\left[\delta^{\mathrm{IS}}\right]$. As soon as the threshold of group dimension is crossed, *diversification* in the *perfect team* prevails, so that less agents default on average in this configuration, and the direction of the inequality between the metrics for the two extreme mechanisms is reversed, i.e. $\mathbb{E}\left[\delta^{\mathrm{PT}}\right] < \mathbb{E}\left[\delta^{\mathrm{IS}}\right]$. When β increases the threshold increases, since the losses due to the default of the agents are exacerbated.

From Figs.(6 - 7), we also observe that the intermediate mechanism is preferred to the *perfect team*, for some group dimensions n. In the intermediate mechanism the agents retain a larger amount of their external demands, so that the level of connectivity in the associated network of internal demands is lower than that in the *perfect team*. This feature of the intermediate configuration limits *contagion* and, as a consequence, for the group dimensions n for which *contagion* prevails in the *perfect team*, by choosing an intermediate configuration we limit this phenomenon and we are better off. This happens both for $\beta = 0$, as shown in Fig.(6), and for $\beta = 1$, as we see in Fig.(7). The lower level of

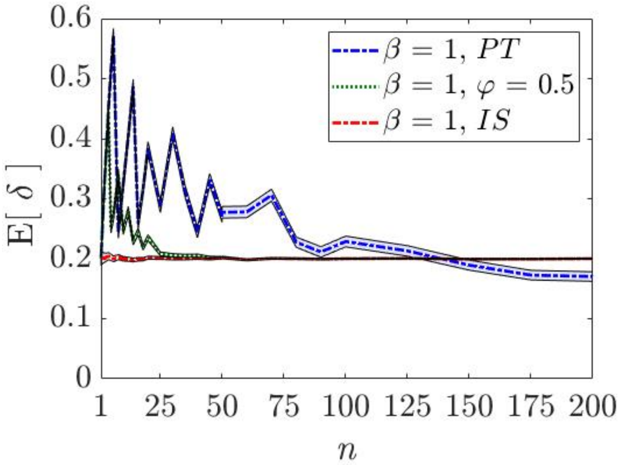

Figure 7: Metric $E[\delta]$: comparison of isolated agents, intermediate mechanism, $\varphi = 0.5$, and perfect team, for $\beta = 1$, $\bar{c} = 0.125$, $p = 0.1$ and $\gamma = 1$.

connectivity of the intermediate configuration also limits the *diversification* of the risk if compared to the *perfect team*. This translates in the preference of the latter when $n \to \infty$, or else when the group dimension of the financial system is large enough.

When β is close to one, the characteristics of the intermediate mechanism, low *diversification* and a level of connectivity that allows for a large shock to be redistributed to all the agent in the system, make this intermediate configuration *fragile* to the shocks, so that it performs poorly with respect to both the extreme ones.

4. CONCLUSIONS

Our results show that, in some settings, *contagion* prevails on the *diversification* of the risk, making us prefer a risk-sharing mechanism with a lower level of redistribution with respect to the *perfect team*. This is clear for the metric $\mathbb{E}[X]$ that, however, does not quantify how much the affected communities lose after the default of some financial agents. The metric $\mathbb{E}[m]$, instead, is a measure of the unpaid insurance claims by the defaulted agents and it tends to favor the mechanisms that, as the *perfect team*, fully exploits the capacity of the agents even at the price of the default of all its members. This metric, however, fails at taking into account disruptions due to the defaults of agents, so that we have proposed the metric $\mathbb{E}[\delta]$ that combines the costs suffered by the communities due to the defaulted agents with the number of defaults in the financial system. The *resilience* of a community, indeed, depends also on the number of agent that do not default after the extreme event and are able to fulfill insurance claims if other shocks occur. The result is that, according to metric $\mathbb{E}[\delta]$, the *perfect team* is not always at least as good as the other mechanisms, especially for low n when the *diversification effect* is small and *contagion* prevails. Moreover, in some cases, we prefer intermediate mechanisms to the extreme configurations.

5. ACKNOWLEDGMENTS

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