INTEGRATING TEMPORAL INFORMATION TO SPATIAL INFORMATION IN A NEURAL CIRCUIT

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Abstract. In this paper, we consider a network of spiking neurons with a deterministic synchronous firing rule at discrete time. We propose three problems – "first consecutive spikes counting", "total spikes counting" and "k-spikes temporal to spatial encoding" – to model how brains extract temporal information into spatial information from different neural codings. For a max input length T, we design three networks that solve these three problems with matching lower bounds in both time $O(T)$ and number of neurons $O(\log T)$ in all three questions.

1. Introduction

Algorithms in the brain are inherently distributed. Although each neuron has relatively simple dynamics, as a distributed system, a network of neurons shows strong computational power. There have been many attempts to model the brain computationally. At a single-neuron level, theoretical neuroscientists were able to model the dynamics of a single neuron to high accuracy with the Hodgkin-Huxley model [HH52]. At a circuit level, to make the analysis tractable, neuroscientists approximated detailed dynamics of neurons with simplified models such as the nonlinear integrate-and-fire model [FTHvVB03] and the spiking response model [WWvJ97]. Recently, Lynch et al. used stochastic neurons firing at discrete time to solve problems such as winner-take-all and similarity testing [LMP17a, LMP17b]. These models vary in their assumptions about spike/rate code, deterministic/stochastic response, and continuous/discrete time. In this paper, we consider a network of spiking neurons with a deterministic synchronous firing rule in discrete time to simplify the analysis and focus on the computational principles.

One of the most important questions in neuroscience is how humans integrate information over time. Sensory inputs such as visual and auditory stimulus are inherently temporal; however, brains are able to integrate the temporal information to a single concept, such as a moving object in a visual scene, or an entity in a sentence. There are two kinds of neuronal codings: rate coding and temporal coding. Rate coding is a neural coding scheme assuming most of the information is coded in the firing rate of the neurons. It is most commonly seen in muscle in which the higher firing rates of motor neurons correspond to higher intensity in muscle contraction [AZ26]. On the other hand, rate coding cannot be the only neural coding brains employ. A fly is known to react to new stimuli and change its direction of flight within 30-40 ms. There is simply not enough time for neurons to compute averages [RWdRvSB96]. Therefore, neuroscientists propose the idea of temporal coding, assuming the information is coded in the specific temporal firing patterns. One of the popular temporal codings is the first-to-spike coding. It has been shown that the timing of the first spike encodes most information of an image in retinal cells [GM08]. We propose three toy problems to model how brains extract information from different coding. "First consecutive spikes counting" (FCSC) counts the first consecutive interval of spikes, which is equivalent to counting the distance between the first two spikes, a prevalent neural coding scheme in sensory cortex. "Total spikes counting" (TSC) counts the number of the spikes over an arbitrary interval, which is an example of rate coding. Lastly, "k-spikes temporal to spatial encoding" (kSTS) is a generalization of "first

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consecutive spikes counting" and an example of temporal coding. In particular, TSC contains an interesting difficulty: there are conflicting objectives between maintaining the count when no spike arrives and updating the count when a spike arrives. To overcome this difficulty, we allow the network to enter an unstable intermediate state which carries the information of the count. The intermediate state then converges to a stable state that represents the count after a computation step without inputs. Hitron and Parter, in a newly-submitted paper [HP19], propose a different solution to our TSC problem.

In this paper, we design three networks that solve the above three problems by translating temporal information into spatial information with matching lower bounds in both time $O(T)$ and number of neurons $O(\log T)$ for all three questions.

2. Problem Statements/Goals

In this section, we cover the model definition and the following three problems: first consecutive spikes counting (FCSC), total spikes counting (TSC) and k-spikes temporal to spatial encoding (kSTS). In particular, we will use FCSC networks as subroutines on a kSTS network.

2.1. **Model.** In this paper, we consider a network of spiking neurons with deterministic synchronous firing at discrete times. Formally, a neuron x consists of the following data with $t \geq 1$

$$x^{(t)} = \Theta(\sum_{y \in P_x} w_{yx} y^{(t-1)} - b_x)$$

where $x^{(t)}$ is the indicator function of neuron x firing at time t, b_x is the threshold (bias) of neuron i, P_x is the set of presynaptic neurons of i, w_{yx} is the strength of connection from neuron y to neuron x and Θ is a nonlinear function. Here we take Θ as the Heaviside function given by $\Theta(z) = 1$ if $z > 0$ and 0 otherwise. At $t = 0$, we let $x^{(0)} = 0$ if x is not one of the input neurons.

2.2. **First consecutive spikes counting (FCSC).** Given an input neuron x and the max input length T, we consider any input firing sequence such that $x^{(t)} = 0$ for all $t \geq T$. Define L_x in terms of this firing sequence as follows: if $x^{(t)} = 1$ for some t, then there must exist integers $\hat{t}, L$ such that $x^{(t)} = 0$ for all $t, t < \hat{t}$, $x^{(\hat{t}+i)} = 1$ for all $i, 0 \leq i < L$ and $x^{(\hat{t}+L)} = 0$. Define $L_x = L$. (i.e. L is the length of the first consecutive spikes interval in the sequence.) Otherwise, that is if $x^{(t)} = 0$ for all $t \geq 0$, then define $L_x = 0$.

Let $\{y_i\}_{0 \leq i < m}$ be m output neurons. Then we say a network of neurons solves FCSC in time t' with m' neurons if there exists an injective function $F : \{0, \cdots, n\} \to \{0,1\}^m$ such that $y^{(t)} = F(L_x)$ for all $t, x, t \geq t'$ and the network has m' neurons.

2.3. **Total spikes counting (TSC).** Given an input neuron x and the max input length T, we consider any input firing sequence such that $x^{(t)} = 0$ for all $t \geq T$. Define $L_x = |\{t : x^{(t)} = 1, 0 \leq t < T\}|$ as the total number of spikes in the sequence. Let $\{y_i\}_{0 \leq i < m}$ be m output neurons. Then we say a network of neurons solves TSC in time t' with m' neurons if there exists an injective function $F : \{0, \cdots, n\} \to \{0,1\}^m$ such that $y^{(t)} = F(L_x)$ for all $t, x, t \geq t'$ and the network has m' neurons.

2.4. **k-spikes Temporal to Spatial Encoding.** Given an input neuron x and the max input length T, we consider any input firing sequence such that $x^{(t)} = 0$ for all $t \geq T$ and $|\{t : x^{(t)} = 1, 0 \leq t < T\}| = k$ (i.e, there are spikes at k distinct time points). We also assume that there is a designated x_{end} neuron that fires at time T to notify the network that the input ends. Let $\{y_i\}_{0 \leq i < m}$ be m output neurons. Denote the set of input temporal signals of max input length T with k distinct 1 as $S_{T,k}$. Then we say a network of neurons solves kSTS in time t' with m' neurons if there exists an injective function $F : S_{T,k} \to \{0,1\}^m$ such that $y^{(t)} = F(x)$ for all $t, x, t \geq t'$ and the network has m' neurons.

Our contributions in this paper are to design networks that solve these three problems respectively with matching lower bounds.

Theorem 2.1. *There exists a network with $O(\log T)$ neurons that solves FCSC problem in $T+1$ time.*

Theorem 2.2. *There exists a network with $O(\log T)$ neurons that solves TSC problem in $T+1$ time.*

Theorem 2.3. *There exists a network with $O(k \log T)$ neurons that solves kSTS problem in $T+1$ time.*

It is easy to see that we also have the corresponding information-theoretical lower bound all being $\Omega(\log T)$ if we treat k as a constant.

3. Technical Contribution and Comparison

The main technical difficulty in this paper arises from TSC problem. To count the total number of spikes in an arbitrary interval requires persistence of neurons without external spikes. Since each spike is transient, ideally we want to toggle neural representation of a count to another count without delays. However, persistence of neurons and toggles without delays are conflicting objectives; persistence of neurons stabilizes the network while toggling without delays changes the firing patterns of the network. For example, we can use self-inhibition loop to count mod2 but if we use self inhibition to count mod 2, the neuron cannot maintain the count during intervals with no inputs. In fact, we can show that it is impossible to solve the problem in T time with only $O(\log T)$ neurons. Our main technical novelty is to circumvent this difficulty by allowing the network to enter an unstable intermediate state that still stores the information of the count when the spikes arrive; however, the network will converge to a *clean state* that according to binary representation after one step of computation without external signals, and this *clean state* is stable in an arbitrary interval with no input.

In this paper, we have shown that networks of neurons are capable of integrating temporal information to solve three different tasks with temporal inputs efficiently. Our paper follows similar approaches to Lynch [LMP17a, LMP17b, LM18] by treating neurons as static circuits to explore the computational power of neural circuits. There are three noteworthy points about our model. First, instead of a stochastic model, we use a deterministic one. However, it should be noted that all the results in this paper would still hold under the randomized model of Lynch [LMP17a, LMP17b, LM18] with high probability. Second, we use a model which resets the potential at every round. Therefore, to retain temporal information, many self-excitation connections are employed in our networks. At the other extreme, we can have a model in which the potential does not decay from past rounds. In that model, temporal information can be stored in potentials, but it might require different mechanisms to translate the information from potentials to spikes. The two models thus can lead to different possible computational principles in brains. Third, we used a discrete time model instead of a continuous time model, which would be more biologically plausible. However, this might not be a concern since we can use Maass's synchronization module [Maa96] to simulate our discrete time model from a continuous time model.

This paper mainly deals with the exact versions of the problems. One possible extension is to consider the approximate versions of the problems. By introducing noise into our models, we might be able to solve the approximate versions of the problems more efficiently. For example, for approximate counting, we aim to output some firing patterns corresponding to a number $\tilde{X}$ such that

$$P(|\tilde{X} - X| > \epsilon X) < \delta$$

is small. The lower bound for this question is $\Omega(\log \log T)$ and finding a matching upper bound can be an interesting future direction. However, approximate versions of the questions are tricky with temporal inputs because the network inevitably reuses random bits if they are stored inside the weights. A possible approach is to use a small number of random bits to generate a large family of k-wise independent random functions within neurons.

References

[AZ26] E. D. Adrian and Yngve Zotterman. The impulses produced by sensory nerve-endings: Part ii. the response of a single end-organ. *J Physiol*, 61(2):151–171, 1926.

[BP98] G.Q. Bi and M.M. Poo. Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type. *J. Neurosci*, 18:10464–10472, 1998.

[FTHvVB03] N. Fourcaud-Trocme, D. Hansel, C. van Vreeswijk, and N. Brunel. How spike generation mechanisms determine the neuronal response to fluctuating input. *J. Neuroscience*, 23:11628–11640, 2003.

[GM08] T. Gollisch and M. Meister. Rapid neural coding in the retina with relative spike latencies. *Science*, 319:1108–1111, 2008.

[GS06] Rober Gutig and Haim Sompolinsky. The tempotron: a neuron that learns spike timing-based decisions. *Nature Neuroscience*, 9(3):420–428, 2006.

[Heb49] D. O. Hebb. *The Organization of Behavior*. Wiley, New York, 1949.

[HH52] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J Physiol.*, 117(4):500–544, 1952.

[HKP91] J. Hertz, A. Krogh, and R. G. Palmer. *Introduction to the Theory of Neural Computation*. Addison-Wesley, Redwood City CA, 1991.

[Hop82] J. J. Hopfield. Neural networks and physical systems with emergent collective computational abilities. *Proc. Natl. Acad. Sci. USA*, 79:2554–2558, 1982.

[HP19] Yael Hitron and Merav Parter. Counting to ten with two fingers: Compressed counting with spiking neurons. *Personal Communication*, 2019.

[JJ15] Hopfield JJ. Understanding emergent dynamics: Using a collective activity coordinate of a neural network to recognize time-varying patterns. *Neural Computation*, 27:2011–2038, 2015.

[Kos88] B. Kosko. Bidirectional associative memories. *IEEE Transactions on Systems, Man, and Cybernetics*, 18:49–60, 1988.

[LM18] Nancy Lynch and Cameron Musco. A basic compositional model for spiking neural networks. *arXiv:1808.03884 [cs.DC]*, 2018.

[LMP17a] Nancy A. Lynch, Cameron Musco, and Merav Parter. Computational tradeoffs in biological neural networks: Self-stabilizing winner-take-all networks. *ITCS*, 2017.

[LMP17b] Nancy A. Lynch, Cameron Musco, and Merav Parter. Neuro-ram unit with applications to similarity testing and compression in spiking neural networks. *arXiv:1706.01382v2 [cs.NE] 21 Aug 2017*, 2017.

[Maa96] Wolfgang Maass. Lower bounds for the computational power of networks of spiking neurons. *Neural Computation*, 8:1–40, 1996.

[RWdRvSB96] F. Rieke, D. Warland, R. de Ruyter van Steveninck, and W. Bialek. *Spikes - exploring the neural code*. MIT Press, Cambridge, MA., 1996.

[WWvJ97] Kistler W.M., Gerstner W., and vanHemmen J.L. Reduction of hodgkin-huxley equations to a threshold model. *Neural Comput*, 9:1069–1100, 1997.