Generation of a non-separable two-qudit state using a time-frequency SUM operation

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Abstract: We demonstrate a deterministic two-qudit SUM gate with up to 16-dimensional qudits encoded in the time and frequency degrees of freedom of a single photon. Using this SUM gate, we generate non-separable time-frequency qudit states.

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The high-speed nature of photonic quantum states, combined with their minimal interactions with their surroundings make them a favorable candidate for quantum communication [1]. To manipulate such states, two categories of optical quantum operations are required: rotations of a single quantum information unit and interactions between two of these units. So far, most of the advances in this field have focused on two-level quantum states (qubits), for which single-qubit and two-qubit gates are required to realize arbitrary quantum operations. Multi-dimensional quantum states (qudits), however, have shown a promising approach for encoding more information in each photon [2]. To process these high-dimensional states, single- and two-qudit gates are needed, which are just now beginning to be realized [3-5]. By encoding two three-dimensional qudits in the time and frequency degrees of freedom (DoFs) of a single photon, we have recently demonstrated elementary single- and two-qudit gates, showing the ability of our platform to perform arbitrary qudit-based quantum operations [4]. These two-qudit operations can be used to demonstrate a qudit teleportation scheme [6], where more quantum information can be teleported compared to conventional qubit teleportation [7].

In this abstract, we significantly expand our previous approach and encode 16-dimensional qudits in both time and frequency bins, for an equivalent of 4 bits of quantum information in each DoF. We then build on the general approach from our recent work [4] to demonstrate a 16×16 two-qudit SUM gate, where the qudit encoded in the frequency DoF is added modulo 16 to the qudit encoded in the time DoF. In this realization, an equivalent of 8 qubits are encoded in a single photon, showing the value of high-dimensionality in each DoF. Using a three-dimensional SUM gate, we then generate a three-dimensional non-separable state between time and frequency qudits, which shows the coherence of our SUM gate scheme.

The experimental setup is depicted in Fig. 1. We use a continuous-wave laser with around 773 nm wavelength to pump a periodically poled lithium niobate (PPLN) crystal, generating a broadband spectrum of time-frequency entangled photons [8], with the bandwidth of ~5 THz centered around 1546 nm. After using filters to attenuate the pump frequency, an intensity modulator driven by an arbitrary waveform generator is used to carve out sixteen time bins with a full width at half maximum of ~200 ps and 1.2 ns spacing between them, to generate the time-bin qudits. Then, a pulse shaper is used to carve out the frequency of these entangled photons to generate sixteen 22 GHz wide frequency bins on both the signal and idler side of the spectrum, each 75 GHz spaced from each other. This pulse shaper is also used as a wavelength router to separate the signal and idler photons. The idler photons are then sent directly to a superconducting nanowire single-photon detector (SNSPD), to herald single photons in the signal arm. These heralded signal photons carry two 16-dimensional qudits in their time and frequency DoFs.

To operate the SUM gate on these heralded single photons, a dispersion module is used to delay the adjacent frequency bins by one time bin. We use a chirped fiber Bragg grating with a dispersion of −2 ns/nm, which puts a 1.2 ns delay on frequency bins 75 GHz spaced from each other, which adds the value of the frequency qudit to the time qudit. This leaves us with 31 time bins, half of which are now outside of our computational space. To

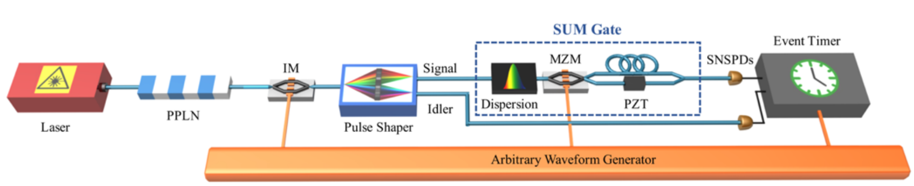

Fig. 1. The experimental setup. PPLN: periodically poled lithium niobate, IM: intensity modulator, PZT: piezo-electric phase shifter, MZM: Mach-Zehnder modulator, SNSPD: superconducting nanowire single-photon detector.

demonstrate the modulo 16 additions, a 1×2 switch (a Mach- Zehnder modulator) is used to separate time bins that are now placed later than the 16th time bin (which is the last bin in our computational space) and delay the computational space by 16 time bins with respect to these latter bins to bring them back into the computational space. In principle, recombination of these time bins can be done in a lossless way using switches and wavelength division multiplexers, but in this proof-of-principle experiment, we use a 50/50 beam combiner which probabilistically returns the spatially separated bins back into one spatial mode.

To verify the operation, we send in different input two-qudit states, which can be in one of 256 different states, and measure the output after the gate. While this yields a total of 256×256 (2^{16}) computational input/output combinations to test, we have no active frequency-shifting elements in our setup, so we make the reasonable assumption that the frequency qudit remains unchanged through the operation. This is also enforced by the high extinction ratio of the pulse shaper (~ 40 dB), which blocks the unwanted frequency bins with a high success probability, allowing us to focus on results in the sixteen 16×16 transfer matrices measured in Fig. 2a-p (a subset with a total of 2^{12} input/output combinations). In each matrix, 16 different inputs with the same frequency and different time bins are sent into the SUM gate and the output time bins are measured without accidental subtraction. The average computational space fidelity of the whole process, with the assumption that frequencies do not leak into each other, can be calculated—using Bayesian estimation [4]—as $\bar{\mathcal{F}}_{\mathrm{C}} = 0.9589 \pm 0.0005$, which shows the high performance of our operation. To show the coherence of our SUM gate, we use this setup to perform a SUM operation on a three-dimensional input state, $|\psi\rangle_{\mathrm{in}} = 1/\sqrt{3}\,(|0\rangle_{\mathrm{f}} + |1\rangle_{\mathrm{f}} + |2\rangle_{\mathrm{f}})|0\rangle_{\mathrm{t}}$, which results in a maximally non-separable state between time and frequency DoFs: $|\psi\rangle_{\mathrm{out}} = 1/\sqrt{3}\,(|00\rangle_{\mathrm{ft}} + |11\rangle_{\mathrm{ft}} + |22\rangle_{\mathrm{ft}})$. To quantify the dimensionality of this state, we use an entanglement certification measure called *entanglement of formation* (E_{of}) using the joint spectral intensity of the state (Fig. 2q) and time-frequency projection measurements [9]. We experimentally obtain $E_{\mathrm{of}} \geq 1.19 \pm 0.12$ ebits, where 1 ebit corresponds to a maximally non-separable pair of qubits, while 1.585 ebits represents the maximum for two three-dimensional parties in exceeding the qubit limit, our state thus possesses true high-dimensional non-separability which shows the phase coherence of our SUM gate. Using the same structure as the SUM gate but comprising a dispersion module with equal and opposite dispersion, a two-qudit XOR gate can be realized, which subtracts the control qudit from the target and is a requirement for qudit teleportation [6].

In Conclusion, we have demonstrated a two-qudit SUM gate with two 16-dimensional qudits encoded in the time and frequency degrees of freedom of a single photon. We furthermore used a three-dimensional SUM gate to demonstrate a non-separable state between time and frequency qudits. These demonstrations pave the way to use these optical qudits for quantum networking proposes, such as in qudit teleportation [6,7].

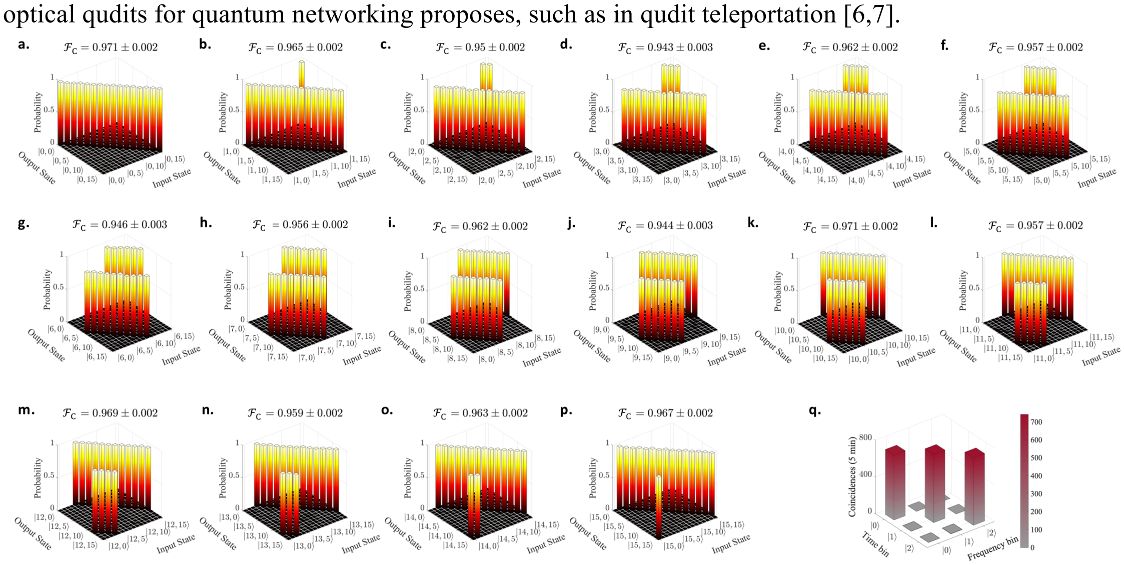

Fig. 2. a-p, The transfer matrices corresponding to each possible time-bin output for each individual input time bin without accidental subtraction. Each matrix is specified for one frequency input, where the matched frequency output for different time bins is measured. In $|m,n\rangle$ on the x and y axis, m indicates the frequency qudit and n is the time bin qudit. **q.** Joint spectral intensity of the three-dimensional non-separable state. The accidentals were subtracted in this measurement, with a coincidence to accidentals ratio of about 30.

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