International Journal of Modern Physics E
Vol. 28, No. 8 (2019) 1950072 (21 pages)

DOI: 10.1142/S0218301319500721

Lorentz breaking and $SU(2)_L \times U(1)_Y$ gauge invariance for neutrinos

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Received 4 August 2019
Accepted 13 September 2019
Published 21 October 2019

Conceivable Lorentz-violating effects in the neutrino sector remain a research area of great general interest, as they touch upon the very foundations on which the Standard Model and our general understanding of fundamental interactions are laid. Here, we investigate the relation of Lorentz violation in the neutrino sector in light of the fact that neutrinos and the corresponding left-handed charged leptons form $SU(2)_L$ doublets under the electroweak gauge group. Lorentz-violating effects thus cannot be fully separated from questions related to gauge invariance. The model dependence of the effective interaction Lagrangians used in various recent investigations is explored with a special emphasis on neutrino splitting, otherwise known as the neutrino-pair Cerenkov radiation and vacuum-pair emission (electron–positron-pair Cerenkov radiation). We highlight two scenarios in which Lorentz-violating effects do not necessarily also break electroweak gauge invariance. The first of these involves a restricted set of gauge transformations, a subgroup of $SU(2)_L \times U(1)_Y$, while in the second where differential Lorentz violation is exclusively introduced by the mixing of the neutrino flavor and mass eigenstates. Our study culminates in a model which fully preserves $SU(2)_L \times U(1)_Y$ gauge invariance, involves flavor-dependent Lorentz-breaking parameters, and still allows for Cerenkov-type decays to proceed.

Keywords: Neutrinos; Lorentz-violation; gauge invariance; spontaneous Lorentz-symmetry breaking; relation of Lorentz-symmetry breaking and gauge invariance; fundamental symmetries.

PACS number(s): 95.85.Ry, 11.10.−z, 03.70.+k.

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1. Introduction

Recently, tight bounds on Lorentz-violating parameters for neutrinos have been derived from astrophysical observations,[1–3] based on the notion that neutrino that decays into electron–positron pairs (called "lepton-pair Cerenkov radiation", LPCR, or "vacuum-pair emission") becomes kinematically allowed under a Lorentz noninvariance of the neutrino dispersion relation. One observes that even a slight violation of Lorentz invariance at high energy would lead to a large deviation of the dispersion relation from the light cone ("virtuality") $E^2 - \mathbf{p}^2 = \mathbf{p}^2(v^2 - 1)$, where $v > 1$ is the velocity parameter, because of the multiplicative prefactor $\mathbf{p}^2 \approx E^2$ (in front of $v^2 - 1$) which grows without bound at high energy. For high energy, the quantity $q^2 = E^2 - \mathbf{p}^2$ exceeds the electron–positron-pair production threshold.

Very recently, these calculations have been supplemented by an analysis of the neutrino splitting process[3] ($\nu \to \nu\bar{\nu}\nu$, neutrino-pair Cerenkov radiation, NPCR), which in contrast to charged-lepton-pair Cerenkov radiation[4,5] ($\nu \to \nu e e^+$) has negligible threshold and can serve to set even tighter bounds on the Lorentz-violating parameters.

On the theoretical side, corresponding calculations are mainly based on the notion that Lorentz noninvariance is restricted to the neutrino sector, while the Lorentz-violating parameter $\delta_e = v_e^2 - 1$ is set equal to zero for electrons and positrons. That is one assumes that the maximum attainable velocity for electrons is exactly equal to $v_e = c$, where c is the speed of light.[1,2,4–6] One might argue, though, that electrons and neutrinos enter an $SU(2)_L$ doublet, so that in addition to Lorentz violation, also the gauge symmetry is violated if one assumes a different propagation velocity for charged leptons and neutrinos in the high-energy limit.

We should point out that Lorentz violation for charged particles can often be studied more easily using other kinds of processes (involving electrons, positrons, and photons, for example). The fact that the Lorentz-violation coefficients for charged leptons and neutrinos are not independent, due to the existence of $SU(2)_L \times U(!1)_Y$ gauge symmetry, has already been used in the literature.[7,8] However, for a number of reasons, it has been of prime interest to infer bounds for Lorentz-violating parameters in the neutrino sector, even if the models involved might be considered as "kinematics-only" approaches and lead to a slight, perturbative breaking of gauge invariance, to the extent to be discussed below.

Two models have been investigated in this context by Bezrukov and Lee[5] in order to analyze the decay of superluminal neutrinos by electron–positron-pair emission; one ("model I") in which the normal Lorentz metric enters the interaction Lagrangian, and another one ("model II") in which the same Lorentz-violating "metric"

$$\widetilde{g}^{\mu\nu}(v) = \widetilde{g}_{\mu\nu}(v) = \mathrm{diag}(1, -v, -v, -v), \quad v > 1, \tag{1}$$

enters as in the dispersion relation of the decaying neutrino. (The "metric" is noncovariant, hence there is no distinction between upper and lower indices.) Potentially, this "metric" (or "pseudo-metric") also enters the effective interaction Lagrangian describing the decay process, where $v \geq 1$ is a Lorentz-violating parameter, which can be different for the maximum velocities of the initial ($v = v_i$) and final ($v = v_f$) particles in the decay process. (In this paper, we have $\hbar = c = \epsilon_0 = 1$.)

In Ref. 3, an even more general approach is taken, and the metric entering the interaction Lagrangian is taken in the form

$$\widetilde{g}^{\mu\nu}(v_{\rm int}) = {\rm diag}(1, -v_{\rm int}, -v_{\rm int}, -v_{\rm int}), \tag{2}$$

where $v_{\rm int}$ is not necessarily equal to v_i or v_f.

Here, we show in detail how the parameters of the models used by Cohen and Glashow,[4] by Bezrukov and Lee[5] and by us in Ref. 3 are related to the gauge invariance under the $SU(2)_L$ group, and how the formulation of the gauge sector relates to the individual Lorentz-violating parameters of the neutrino flavor and mass eigenstates, and those of the charged leptons.

In particular, we can anticipate that the model used by Bezrukov and Lee[5] for vacuum-pair emission turns out to be gauge-invariant (GI) only with respect to a restricted subgroup of the set of $SU(2)_L$ gauge transformations; we will investigate the respective subgroup.

This is important for a general understanding of the relation of potential Lorentz symmetry breaking in the neutrino sector, to fundamental symmetries and to other conceivable nonstandard interactions.[9] (Note that corresponding questions do not occur in Lorentz-symmetry conserving models.[10]) The underlying question is the following: Is the existence of the LPCR process compatible with electroweak gauge invariance, or do the tight bounds derived in Refs. 1–3 (and the theoretical calculations reported in Refs. 3–5) additionally depend on the possibly problematic assumption of a breaking of $SU(2)_L$ symmetry, in addition to Lorentz symmetry? These considerations are quite crucial for the clarification of the status of the derived astrophysical bounds on the Lorentz-violating parameters.[1–3]

We can likewise anticipate that our discussion will lead to a fully $SU(2)_L \times U(1)_Y$ gauge-invariant model, with differential Lorentz-symmetry breaking across generations, that can still accommodate for the possibility of neutrino decay, via both NPCR and LPCR processes, in the high-energy limit (see Sec. 3.3).

This paper is organized as follows. In Sec. 2, we introduce a modified Dirac algebra, adapted to the description of Lorentz-symmetry breaking spin-1/2 particles. Our investigations continue in Sec. 3.1 with the discussion of a manifestly Lorentz- and gauge-symmetry breaking model for the interaction of (conceivable) superluminal neutrinos with electroweak gauge bosons; this model has recently been used in Refs. 4 and 5. In Sec. 3.2, we continue with the investigation of a model which breaks Lorentz symmetry and (partially) restores electroweak gauge symmetry, within a restricted electroweak-symmetry group. We continue in Sec. 3.3 with the discussion of a Lorentz-breaking model which fully restores the electroweak-symmetry group. Neutrino decay in a fully gauge-invariant model is discussed in Sec. 4. Conclusions are reserved for Sec. 5, and some additional general remarks on the relation of (spontaneous) Lorentz-symmetry breaking and gauge invariance are relegated to Appendix A.

2. Relation to Modified Dirac Algebra

We would like to briefly discuss a connection of the common Lagrangian used in the description of superluminal, Lorentz-violating neutrinos to a generalized Dirac algebra. According to Eq. (16) of Ref. 3, one may use the following Lagrangian, for a Lorentz-violating, massless, left-handed Dirac particle:

$$\mathcal{L} = \mathrm{i}\bar{\psi}\gamma_\mu \widetilde{g}^{\mu\nu}(v)\partial_\nu \psi, \tag{3}$$

where we assume that $\psi = \nu_\ell^{(\mathrm{m})}$ is a left-handed neutrino mass eigenstate, i.e., $[(1-\gamma^5)/2]\psi = \psi$. (In the course of the current investigations, we attempt to keep the notation as concise as possible and avoid any superfluous superscripts, or subscripts.) We can write the Lagrangian for the massless case as

$$\mathcal{L} = \bar{\psi}\mathrm{i}\widetilde{\gamma}^\mu \partial_\mu \psi, \tag{4}$$

where $\widetilde{\gamma}^\mu$ are given as

$$\widetilde{\gamma}^0 = 1, \quad \widetilde{\gamma}^i = v\gamma^i. \tag{5}$$

These fulfill the anti-commutator relation

$$\{\widetilde{\gamma}^\mu, \widetilde{\gamma}^\nu\} = 2\widetilde{g}^{\mu\nu}(v^2) = 2\mathrm{diag}(1, -v^2, -v^2, -v^2), \tag{6}$$

where we note the square of the velocity. Note that the given pseudo-metric implies a spatially isotropic breaking of Lorentz invariance. For bounds on coefficients with directional dependence, we refer to the data compilation presented in Ref. 11.

One can in fact relate this formalism to the so-called *vierbein* coefficients (cf. Refs. 12–17). Namely, in a more general context, one can define the generalized Dirac matrices

$$\widetilde{\gamma}^\mu = e^\mu_A \gamma^A, \tag{7}$$

where the Einstein summation convention is used, and γ^A with $A = 0, 1, 2, 3$ are the ordinary Dirac γ matrices, while e^μ_A take the role of the so-called "vierbein" in general relativity, with the property

$$\widetilde{g}^{\mu\nu}(v^2) = e^A_\mu g_{AB} e^B_\nu = e^A_\mu e_{\nu A}. \tag{8}$$

This implies that the "vierbein" takes the role of the square root of the metric.[13] Capital Latin indices can be raised with the flat-space metric g^{AB}. One can then easily show that

$$\{\widetilde{\gamma}^\mu, \widetilde{\gamma}^\nu\} = e^\mu_A e^\nu_B \{\gamma^A, \gamma^B\} = e^\mu_A e^\nu_B (2g^{AB}) = 2\widetilde{g}^{\mu\nu}(v^2). \tag{9}$$

The analogy to the formalism of general relativity implies that $\widetilde{g}^{\mu\nu}(v^2)$ takes the role of a modified Lorentz "metric", but without curvature (because we assume that the coefficients are constant). The word "metric" should be understood with a grain of salt (hence the apostrophes), because it does not constitute a space-time metric in the sense of general relativity, that is used to measure space-time intervals, but rather, a mathematical object used to parameterize the dispersion relation of a Lorentz-violating particle. One might thus call it a "pseudo-metric". Because of the lack of curvature, the pseudo-metric $\widetilde{g}_{\mu\nu}(v^2)$ is still characterizing a flat "space-time". For a truly curved space, the notation $\overline{g}_{\mu\nu}$ has been proposed in Refs. 12–17 in order to distinguish the curved-space quantities from the flat-space ones. [As a remark, we here note that the superluminal, Lorentz-violating neutrino model is different from Lorentz-conserving, γ^5-Hermitian ("pseudo-Hermitian") models discussed in the literature.[18]]

For a modified "metric" of the form (6), one can choose the vierbein coefficients as

$$e^0_0 = 1, \quad e^0_i = e^i_0 = 0, \quad e^i_j = v\delta^i_j, \quad i,j = 1,2,3. \tag{10}$$

The modified Dirac equation describing the Lorentz violation, now with a mass term, can be written as

$$(\mathrm{i}\widetilde{\gamma}^\mu \partial_\mu - m)\,\psi = 0. \tag{11}$$

We now assume that ψ stands for a Majorana neutrino field. One can multiply from the left by the operator $(\mathrm{i}\widetilde{\gamma}^\nu \partial_\nu + m)$, and use the operator identity

$$(\mathrm{i}\widetilde{\gamma}^\nu \partial_\nu + m)\,(\mathrm{i}\widetilde{\gamma}^\mu \partial_\mu - m) = -\widetilde{g}^{\mu\nu}(v^2)\partial_\mu\partial_\nu - m^2. \tag{12}$$

For the metric (6), one can use the identity

$$-\widetilde{g}^{\mu\nu}(v^2)\partial_\mu\partial_\nu - m^2 = E^2 - v^2\mathbf{p}^2 - m^2, \tag{13}$$

where E is the energy and $\mathbf{p}$ is the momentum operator. This leads to the dispersion relation

$$E = \pm\sqrt{\mathbf{p}^2 v^2 + m^2}. \tag{14}$$

The Lagrangian (11) is then seen to describe a Lorentz-violating particle with the dispersion relation (14).

In order to draw a connection to the basis of the fermions of the Standard Model Extension (SME), we refer to the classification of operators given in Eq. (9) of Ref. 19. Our isotropic Lorentz-violating model corresponds, in the notation of Eq. (9) of Ref. 19, to the case

$$\hat{c}^{\mu\nu}_{F'F''} = \delta_{F'F''} c^{\mu\nu}, \quad c^{\mu\nu} = (v-1)(g^{\mu\nu} - t^\mu t^\nu), \tag{15}$$

where the diagonality in the fermion flavor indices F' and F'' simply means that our Lorentz-violating model does not involve additional flavor mixing. The time-like unit vector $t^\mu = (1,0,0,0)$ is used throughout this paper. Note, incidentally, that the $\hat{\Gamma}^\mu_{AB}$ matrices defined in Eq. (9) of Ref. 19 correspond to our $\widetilde{\gamma}$ matrices, in the sense that

$$\hat{\Gamma}^\mu_{F'F''} = \Gamma^\mu \delta_{F'F''}, \quad \Gamma^\mu = c^{\mu\nu}\gamma_\nu = \widetilde{\gamma}^\mu. \tag{16}$$

Note also that, as pointed out in the text following Eq. (9) of Ref. 19, the Lorentz breaking implied by the parameters $c^{\mu\nu}$ is CPT even and thus leaves the CPT symmetry intact. (It is still interesting to discuss possible connections of Lorentz-symmetry breaking and CPT violation; a few remarks on this point will be given in the following.) Also, Sec. II of Ref. 19 addresses the problem of defining Majorana fermions in a Lorentz-violating extension of the Standard Model. Further considerations on this point can be found in Ref. 20.

3. Lorentz Violation and Gauge Coupling

3.1. *Lorentz violation and gauge* (*Non*)*invariance*

In this subsection, we discuss how the coupling to the electroweak gauge sector has to be modified in order to obtain the effective interaction Lagrangian used by Cohen and Glashow,[4] which is equivalent to "model I" used by Bezrukov and Lee.[5]

Let us keep the notation as simple as possible, and start from the standard generalized Dirac Lagrangian (11), which we recall for convenience,

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\widetilde{\gamma}^{\mu}\partial_{\mu} - m)\psi, \tag{17}$$

and assume that ψ stands for a (Majorana) neutrino field. [Questions related to the $SU(2)_L$ doublet will be answered below.] We can write this Lagrangian as

$$\mathcal{L} = \bar{\psi}[\mathrm{i}\gamma^{\mu}\partial_{\mu} - m + \underbrace{\mathrm{i}(\widetilde{\gamma}^{\mu} - \gamma^{\mu})\partial_{\mu}}_{\equiv\mathcal{Q}}]\psi, \tag{18}$$

where $\mathcal{Q}$ is the Lorentz-violating perturbation and constitutes a special case of Eqs. (3), (8), and (9) of Ref. 19.

In Refs. 21 and 22, the $\mathcal{Q}$-term is advocated to be the sub-Planck limit of a nonlocal theory with spontaneous Lorentz and CPT violations,[21] or in more general terms, as the low-energy limit of new physics originating from the Planck scale. Indeed, as we investigate possible violations of Lorentz invariance, we explore the limits of validity of our current understanding of fundamental quantum field theory. For example, it is well known that Lorentz invariance is one of the assumptions underlying the proof of the CPT theorem.[23] A violation of Lorentz invariance therefore allows for violations of CPT, and indeed, some of the operators in the full *ansatz* for $\mathcal{Q}$, as discussed in Ref. 19, are CPT odd. For a long time, one has held the belief that CPT violation automatically implies a violation of Lorentz invariance,[24] while conversely, broken Lorentz invariance allows for, but does not require, broken CPT invariance.[24] Recently,[25,26] invoking additional concepts like nonlocal interactions, the conclusions of Ref. 24 have been questioned, and it has been claimed that scenarios exist where CPT invariance is broken, but Lorentz invariance still holds. In general, the questions regarding the ultimate limits of the validity of our current understanding of fundamental physical laws must include bounds on Lorentz-violating terms, and terms that allow for other broken fundamental symmetries, like the CPT.

Furthermore, the Lorentz-violating operators are assumed to be the sub-Planck limit of new physics originating at the Planck scale, where the fundamental interactions will be completely different from "low" energy physics [where "low" energy could even extend to the PeV scale, which is still 12 orders of magnitude below the (reduced) Planck scale of $\sqrt{1/(8\pi G)} = 2.4 \times 10^{18}\,\text{GeV}$]. At "low" energy, the coupling to the electroweak gauge bosons proceeds by the substitution

$$\partial_\mu \to D_\mu, \tag{19}$$

where the operator D_μ constitutes the $SU(2)_L$-covariant derivative, applied to an $SU(2)_L$ doublet, as discussed below. It is therefore permissible, or suggested, to experiment with the idea that the substitution (19) applies *only* to the unperturbed Lagrangian in Eq. (18), but leaves the perturbative $\mathcal{Q}$-term unchanged. In this case, the perturbative term does not participate in the electroweak interaction [$SU(2)_L$-doublet], while modifying the free propagation of the neutrino [once the $\mathcal{Q}$-operator is written so that it applies only to the upper component of the $SU(2)_L$ doublet, i.e., only to the neutrino].

To be specific, let us start from the doublet

$$L_e = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \tag{20}$$

[see Eq. (12.227) of Ref. 27], where ν_e is the electron neutrino field and e_L is the left-handed electron–positron field, and consider the coupling to the electroweak sector, as in Eq. (12.232) of Ref. 27, concentrating on the terms that couple to the electroweak gauge fields (in the Lorentz-covariant theory)

$$\begin{aligned} \mathcal{L}_G &= \bar{L}_e (\mathrm{i}\gamma^\mu D_\mu) L_e \\ &= \bar{L}_e \left[\mathrm{i}\gamma^\mu \left(\partial_\mu - \frac{g'}{2} B_\mu - g \frac{\tau_i}{2} A_{i,\mu} \right) \right] L_e, \end{aligned} \tag{21}$$

where the B and the A_i $(i = 1, 2, 3)$ fields transform into the photon, Z_0, and $W^\pm$ gauge bosons under electroweak unification (for details, see the discussion below). The Pauli matrices are τ_i, and they act within the $SU(2)_L$ doublet. The charge e and the electroweak couplings including the Weinberg angle are related to g and g' [see Eq. (33) below]. If we add to $\mathcal{L}_G$ the mass term

$$\begin{aligned} \mathcal{L}_M &= -\bar{L}_e \cdot \mathbf{M} \cdot L_e \\ &= \begin{pmatrix} \bar{\nu}_e \\ \bar{\psi}_e \end{pmatrix} \begin{pmatrix} -m_\nu + \mathcal{Q} & 0 \\ 0 & -m_e \end{pmatrix} \begin{pmatrix} \nu_e \\ \psi_e \end{pmatrix}, \end{aligned} \tag{22}$$

then the metric to be used for the effective interaction (Fermi interaction) at the electroweak vertex remains the unperturbed Lorentz metric $g_{\mu\nu}$, while the propagation of free neutrinos acquires a Lorentz-breaking term $\mathcal{Q}$, as specified in Eq. (18). In writing Eq. (22), we use an oscillation-free neutrino model, and assume, furthermore, that the neutrino mass term is of the Majorana type, i.e., $\nu_e = \nu_e^{(\mathcal{C})}$ where

$\mathcal{C}$ denotes the charge conjugate. We also supplement the right-handed component of the electron field, $\psi_e = e_L + e_R$, for the Dirac mass term of the electron. An inspection shows that the Lagrangian

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M \tag{23}$$

directly leads to the interaction Lagrangian used by Cohen and Glashow[4] and in "model I" of Bezrukov and Lee.[5] Strictly speaking, the Lagrangian $\mathcal{L}_G + \mathcal{L}_M$ breaks electroweak gauge invariance due to the presence of partial (not covariant) derivative operators in $\mathcal{Q}$, but the gauge and Lorentz-breaking terms enter at the same perturbative level, namely, at first order in $\mathcal{Q}$ (see also Appendix A).

3.2. *Lorentz-violation and gauge coupling: One-flavor model*

In this subsection, we investigate which Lagrangian should be used in the calculation of vacuum-pair emission and neutrino splitting if we intend to preserve electroweak gauge invariance to the extent possible. We intend to show that it is possible to preserve $SU(2)_L$ gauge invariance under a restricted set of gauge transformations in the electroweak sector, specifically, the sector related to the Z_0 exchange, and still break Lorentz invariance differentially, i.e., with different values for the Lorentz-breaking parameters, for neutrinos compared to charged fermions. We first calculate this in an "oscillation-free" environment (using only one-particle generation), where we neglect the mixing of neutrino mass eigenstates and weak interaction eigenstates, due to the off-diagonal entries of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix.

We again emphasize that τ_i matrices in Eq. (21) act in the $SU(2)_L$ doublet, while the $\widetilde{\gamma}^\mu$ matrices act on the electrons and neutrinos separately. The first observation is that one can choose the free Lagrangian as follows (we ignore the mass terms which are irrelevant for the considerations that follow):

$$\mathcal{L}_F \sim \begin{pmatrix} \bar{\nu}_e \\ \bar{e}_L \end{pmatrix} \begin{pmatrix} \mathrm{i}\widetilde{\gamma}^\mu_{\nu_e}\partial_\mu & 0 \\ 0 & \mathrm{i}\widetilde{\gamma}^\mu_e\partial_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \tag{24}$$

for the $SU(2)_L$ doublet. In this case, it is immediately clear that free neutrinos and free electrons obtain different maximal velocities, according to the anti-commutation relations

$$\{\widetilde{\gamma}^\mu_{\nu_e}, \widetilde{\gamma}^\rho_{\nu_e}\} = 2\widetilde{g}^{\mu\nu}(v^2_{\nu_e}) = 2\ \mathrm{diag}(1, -v^2_{\nu_e}, -v^2_{\nu_e}, -v^2_{\nu_e}\}, \tag{25a}$$

$$\{\widetilde{\gamma}^\mu_{e}, \widetilde{\gamma}^\rho_{e}\} = 2\widetilde{g}^{\mu\nu}(v^2_{e}) = 2\ \mathrm{diag}(1, -v^2_{e}, -v^2_{e}, -v^2_{e}\}. \tag{25b}$$

As discussed in Sec. 2, these lead to dispersion relations $E_e = |\mathbf{p}_e| v_e$ and $E_{\nu_e} = |\mathbf{p}_{\nu_e}| v_{\nu_e}$ for the electron and electron neutrino, respectively.

The second observation is that one can replace the partial derivatives in Eq. (24) by covariant derivatives, according to Eq. (21). The covariant derivation, under the $SU(2)_L$ gauge group, is matrix-valued and the substitution $\partial_\mu \to D_\mu$ will lead to off-diagonal entries in Eq. (24), coupling the electron to the neutrino by what is later identified as the W boson. Furthermore, the diagonal matrix [diagonal with regard to the $SU(2)_L$ doublet] with entries

$$\begin{pmatrix} \widetilde{\gamma}^{\mu}_{\nu_e} & 0 \\ 0 & \widetilde{\gamma}^{\mu}_{e} \end{pmatrix} \tag{26}$$

does not necessarily commute with the W interaction Lagrangian, which is proportional to the terms involving the τ_1 and τ_2 matrices in Eq. (21). However, one can formulate a restricted set of gauge transformations, which pertain only to the

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{27}$$

matrix in Eq. (21), and restrict the covariant derivative to

$$\mathrm{i}\partial_\mu \to \mathrm{i}D_\mu = \mathrm{i}\partial_\mu - \frac{g'}{2} B_\mu + \frac{g}{2} \tau_3 A_{3,\mu}. \tag{28}$$

The gauge coupling Lagrangian $\mathcal{L}_{Z,A}$ which is to be added to $\mathcal{L}_F$ under the restricted set of gauge transformations, reads as follows:

$$\mathcal{L}_{Z,A} = \bar{L}_e \cdot \mathbf{G} \cdot L_e, \tag{29a}$$

$$\mathbf{G} = \begin{pmatrix} \frac{1}{2}\widetilde{\gamma}^{\mu}_{\nu_e}(gA_{3,\mu} - g'B_\mu) & 0 \\ 0 & -\frac{1}{2}\widetilde{\gamma}^{\mu}_{e}(gA_{3,\mu} + g'B_\mu) \end{pmatrix}. \tag{29b}$$

Defining, as in Eq. (12.238) of Ref. 27, the Z_0 and A_3 fields as

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(-gA_{3,\mu} + g'B_\mu), \tag{30}$$

$$A_{3,\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(gB_\mu + g'A_{3,\mu}), \tag{31}$$

one obtains the following couplings:

$$\begin{aligned} \mathcal{L}_{Z,A} = &-\frac{e}{2}\left[\tan\theta_W(\bar{\nu}_e \widetilde{\gamma}^{\mu}_{\nu_e} \nu_e + \bar{e}_L \widetilde{\gamma}^{\mu}_{e} e_L) \right. \\ &\left. - \cot\theta_W(\bar{e}_L \widetilde{\gamma}^{\mu}_{e} e_L - \bar{\nu}_e \widetilde{\gamma}^{\mu}_{\nu_e} \nu_e)\right] Z_\mu \\ &- e\widetilde{\gamma}^{\mu}_{e} \bar{e}_L A_\mu e_L, \end{aligned} \tag{32}$$

where

$$\begin{aligned} e &= \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad \tan\theta_W = \frac{g'}{g}, \\ e &= g'\cos\theta_W = g\sin\theta_W, \end{aligned} \tag{33}$$

and θ_W is the Weinberg angle and e is the electron charge. (Adding the right-handed component of the charged fermion field restores the full QED Lagrangian for the coupling of the electron–positron field.) The result (32) is exactly equivalent to the corresponding terms in Eq. (12.240) of Ref. 27, with the replacement $\gamma^\mu \to \widetilde{\gamma}^\mu_{\nu_e}$ for the neutrino couplings to the Z_0 boson, and $\gamma^\mu \to \widetilde{\gamma}^\mu_e$ for the electron couplings to the Z_0 boson. The resulting modified effective Fermi Lagrangian describing the coupling of electrons and neutrino is thus exactly the one of "model II" used by Bezrukov and Lee, and by us in Ref. 3. (We recall that the dominant contributions to both neutrino- as well as lepton-pair Cerenkov radiations proceed by Z_0 exchange.)

For the calculation of neutrino splitting,[3] it means that, for example, if $\widetilde{\gamma}^\mu_{\nu_\mu}$ for muon neutrinos are different from those of electrons neutrinos, $\widetilde{\gamma}^\mu_{\nu_e}$, because of a different maximum velocity for the two species, then the neutrino splitting process becomes kinematically allowed (for $v_{\nu_\mu} = v_f > v_{\nu_e} = v_i$). Furthermore, the effective interaction Lagrangian describing the four-fermion vertex receives a correction from the two Z_0 vertices, leading to the appropriate replacement

$$v_{\text{int}} = v_i v_f \tag{34}$$

for the pseudo-metric to be used in the effective Lagrangian in Eq. (18) of Ref. 3, within a gauge-invariant formulation. The same is done in "model II" of Ref. 5. Here, v_i is the Lorentz-violating velocity parameter for the initial (oncoming) particle, while v_f is that of the emitted (final) particle.

In the context of Lorentz breaking, one often finds that symmetry groups are broken down to smaller subgroups (see also the discussion in Appendix A). Here, we observe that the Lorentz-breaking terms change the gauge group from $SU(2)_L \times U(1)_Y$ to $U(1)_L \times U(1)_Y$.

3.3. *Lorentz violation and gauge coupling: Three-flavor model*

In the above considerations, we have shown that it is possible to formulate differential Lorentz violation in the same $SU(2)_L$ doublet, to obtain different Lorentz-breaking parameters for electron neutrinos as compared to (left-handed) electrons, while preserving gauge invariance with respect to a restricted subset of $SU(2)_L$ gauge transformations. This consideration required the use of different $\widetilde{\gamma}^\mu$ matrices for the upper and lower components of the same $SU(2)_L$ doublet. One might ask the question if different Lorentz-breaking parameters could be obtained for different neutrino species, as compared to electrons, and among the neutrino mass eigenstates, even if one uses the same $\widetilde{\gamma}^\mu$ matrices for the upper and lower components of the same $SU(2)_L$ doublet, and only assumes a dependence of the $\widetilde{\gamma}^\mu$ matrices on the fermion generations. In contrast to the model discussed in Sec. 2, we here preserve full $SU(2)_L$ gauge invariance.

We thus start from the Lagrangian (ignoring the free mass terms)

$$\mathcal{L}_{3G} = \begin{pmatrix} \bar{\nu}_e \\ \bar{e}_L \end{pmatrix} \begin{pmatrix} \mathrm{i}\widetilde{\gamma}^\mu_{\nu_e} D_\mu & 0 \\ 0 & \mathrm{i}\widetilde{\gamma}^\mu_{\nu_e} D_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} + (e \leftrightarrow \mu) + (e \leftrightarrow \tau), \tag{35}$$

where "$3G$" refers to the three generations, and we assume a uniform Lorentz violation within the first generation, and uniform within the second generation, but with different overall parameters,

$$\widetilde{\gamma}^{\rho}_{\nu_e} = \widetilde{\gamma}^{\rho}_{e} \neq \widetilde{\gamma}^{\rho}_{\nu_\mu} = \widetilde{\gamma}^{\rho}_{\mu} \neq \widetilde{\gamma}^{\rho}_{\nu_\tau} = \widetilde{\gamma}^{\rho}_{\tau}. \tag{36}$$

(Note that, in the sense of Sec. 3.2, we now have $\widetilde{\gamma}^{\mu}_{\nu_e} = \widetilde{\gamma}^{\mu}_{e}$.) As the matrix

$$\begin{pmatrix} \widetilde{\gamma}^{\mu}_{\nu_e} & 0 \\ 0 & \widetilde{\gamma}^{\mu}_{\nu_e} \end{pmatrix} \tag{37}$$

is proportional to the unit matrix [from within the $SU(2)_L$ doublet], full gauge invariance is preserved.

Invoking neutrino oscillations, we can write the mass term as

$$\mathbf{M} = \bar{\nu}^{(\mathrm{m})}_k m_k \nu^{(\mathrm{m})}_k, \tag{38}$$

where $\nu^{(\mathrm{m})}_k$ are the neutrino mass eigenstates ($k = 1, 2, 3$ is summed over). In the free theory, we end up with a Lagrangian

$$\mathcal{L}_F = \mathrm{i}\bar{\nu}^{(\mathrm{m})}_k \widetilde{\gamma}^{(\mathrm{m}),\mu}_{kj} \partial_\mu \nu^{(\mathrm{m})}_j - \bar{\nu}^{(\mathrm{m})}_k m_k \nu^{(\mathrm{m})}_k, \tag{39}$$

where repeated indices are summed over the generations ($k, j = 1, 2, 3$), and the mass eigenstates $\nu^{(\mathrm{m})}_k$ and the flavor eigenstates $\nu^{(\mathrm{f})}_j$ are related by the PMNS matrix with entries U_{kj},

$$\nu^{(\mathrm{m})}_k = U_{kj} \nu^{(\mathrm{f})}_j. \tag{40}$$

The emergence of the PMNS matrix for both Dirac as well as Majorana neutrinos is discussed in detail in Ref. 28. Again, neutrino mass (m) and flavor (f) eigenstates are distinguished based on their superscripts. Of course, the mass-basis matrices

$$\widetilde{\gamma}^{(\mathrm{m}),\mu}_{kj} = U_{k\ell} \widetilde{\gamma}^{(\mathrm{f}),\mu}_{\ell} U^{-1}_{\ell j} \tag{41}$$

are effective, Lorentz-violating, modified Dirac matrices describing the (possibly off-diagonal, $k \neq j$) Lorentz violation in the neutrino mass eigenstate basis (the subscript ℓ is being summed over in the above equation). [For absolute clarity, we should reemphasize that the $\widetilde{\gamma}$ matrices used up to this point in our analysis, such as in Eq. (35), constitute flavor-basis matrices which would otherwise carry a superscript (f) once we distinguish between the mass and the flavor bases.]

Two limiting cases are of interest: (i) In the low-energy limit, the Lorentz-violating parameters play a subordinate role as compared to the mass terms, and the energy splitting for equal momenta, among the neutrinos, is given in the mass eigenstate basis. In that limit, an inspection shows that the dominant terms in the free Lagrangian (39) are just the diagonal ones in the mass basis,

$$\mathcal{L}_F \approx \mathrm{i}\bar{\nu}^{(\mathrm{m})}_k \widetilde{\gamma}^{(\mathrm{m}),\mu}_{k} \partial_\mu \nu^{(\mathrm{m})}_k - \bar{\nu}^{(\mathrm{m})}_k m_k \nu^{(\mathrm{m})}_k, \tag{42}$$

where, of course, the subscript k is being summed over $k = 1, 2, 3$. However, the matrices $\widetilde{\gamma}^{(\mathrm{m}),\mu}_{\ell}$ are being defined as in $\widetilde{\gamma}^{(\mathrm{m}),\mu}_{\ell} = \widetilde{\gamma}^{(\mathrm{m}),\mu}_{\ell\ell}$, without a summation over ℓ. Under these assumptions, the maximal attainable velocities $v_k^{(\mathrm{m})}$ of the mass eigenstates are thus related of the flavor eigenstates, by the relation

$$\sum_{\ell=1}^{3} U_{k\ell} v_\ell^{(\mathrm{f})} U_{\ell k}^{-1} = v_k^{(\mathrm{m})} \quad \text{(no summation over } k). \tag{43}$$

For the Lorentz-breaking but gauge-invariant formulation of the neutrino splitting process, the appropriate choice [for the low-energy region, as measured by Eq. (52)] for the Lorentz-violating parameter in the effective interaction Lagrangian is [see Eq. (18) of Ref. 3]

$$v_{\mathrm{int}} = v_i^{(\mathrm{m})} v_f^{(\mathrm{m})}, \tag{44}$$

where the velocities of the initial and final mass eigenstates are denoted as $v_i^{(\mathrm{m})}$ and $v_f^{(\mathrm{m})}$, respectively. Furthermore, it is clear that the effective velocities $v_k^{(\mathrm{m})}$ for the neutrino mass eigenstates, under the given assumptions, will be different from those of the electrons, which are given (due to the absence of mass mixing among the *charged* leptons) by $v_j^{(\mathrm{f})}$, thus kinematically allowing the vacuum-pair emission process [again, for the low-energy region, as measured by Eq. (52)].

(ii) In the high-energy limit, one can neglect the mass term in Eq. (39), and observe that in this limit, the flavor eigenstates approximate the mass eigenstates. Furthermore, the PMNS matrix approaches the unit matrix,

$$U_{k\ell} \to \delta_{k\ell} \quad \text{(high-energy limit with } v_e \neq v_\mu \neq v_\tau). \tag{45}$$

Here, we refer to Eq. (36) for the definition of the corresponding $\widetilde{\gamma}$ matrices, with

$$\{\widetilde{\gamma}^\rho_{\nu_e}, \widetilde{\gamma}^\sigma_{\nu_e}\} = \{\widetilde{\gamma}^\rho_e, \widetilde{\gamma}^\sigma_e\} = 2\widetilde{g}^{\rho\sigma}(v_e^2), \tag{46}$$

$$\{\widetilde{\gamma}^\rho_{\nu_\mu}, \widetilde{\gamma}^\sigma_{\nu_\mu}\} = \{\widetilde{\gamma}^\rho_\mu, \widetilde{\gamma}^\sigma_\mu\} = 2\widetilde{g}^{\rho\sigma}(v_\mu^2), \tag{47}$$

$$\{\widetilde{\gamma}^\rho_{\nu_\tau}, \widetilde{\gamma}^\sigma_{\nu_\tau}\} = \{\widetilde{\gamma}^\rho_\tau, \widetilde{\gamma}^\sigma_\tau\} = 2\widetilde{g}^{\rho\sigma}(v_\tau^2), \tag{48}$$

where the subscripts e, μ, and τ refer to the different fermion flavors (generations). For absolute clarity, we reiterate that, according to the discussion above, the velocities v_e, v_μ, and v_τ are defined, first and foremost, in the flavor eigenstate basis, with the charged fermions and neutrinos within the same generation attaining the same velocity.

The Lagrangian, in the high-energy limit, can be written as

$$\begin{aligned} &\mathcal{L}_F \approx \mathrm{i}\bar{\nu}_k^{(\mathrm{f})} \widetilde{\gamma}_k^{(\mathrm{f}),\mu} \partial_\mu \nu_k^{(\mathrm{f})} \approx \mathrm{i}\bar{\nu}_k^{(\mathrm{m})} \widetilde{\gamma}_k^{(\mathrm{m}),\mu} \partial_\mu \nu_k^{(\mathrm{m})}, \\ &\gamma_k^{(\mathrm{m}),\mu} \to \gamma_k^{(\mathrm{f}),\mu}, \quad \nu_k^{(\mathrm{m})} \to \nu_k^{(\mathrm{f})}, \quad \text{(high-energy limit with } v_e \neq v_\mu \neq v_\tau), \end{aligned} \tag{49}$$

where we remember that we started from the $\widetilde{\gamma}_k^{(\mathrm{f}),\mu}$ matrices which were diagonal in the flavor basis [see Eq. (35), with $k = e, \mu\tau$]. Under this assumption, both vacuum-pair emission (Refs. 4 and 5) as well as neutrino splitting are kinematically allowed across (but not within!) generations (flavors), provided we have

$$v_f < v_i \qquad (f, i = e, \mu, \tau). \tag{50}$$

That is to say, the "faster generation" decays into the "slower generation". We recall once more that, within the gauge-invariant model, the charged fermions offer the same Lorentz-violating parameters as the corresponding neutrino flavors, and hence, $v_f \equiv v_k^{(\mathrm{f})}$. Neutrino splitting as well as vacuum-pair emission are both kinematically allowed, because of the differences among the Lorentz-violating parameters for the different neutrino flavors, which happen to approximate the mass (energy) eigenstates under the given assumptions.

The coincidence of the mass and flavor eigenstates in the high-energy limit makes the theoretical analysis easier; the appropriate choice for the parameter v_{int} (see Ref. 3) entering the interaction Lagrangian (see also Sec. 4 below) is

$$v_{\mathrm{int}} = v_i v_f = v_i^{(\mathrm{f})} v_f^{(\mathrm{f})} \quad \text{(high-energy limit with } v_e \neq v_\mu \neq v_\tau\text{)}, \tag{51}$$

and it fully preserves preserves gauge invariance.

The transition among the two regimes characterized by the Lagrangians (49) and (42) occurs at a momentum scale of the order of

$$|\mathbf{p}| = \sqrt{\delta m^2 / \delta_{f_1 f_2}}, \tag{52}$$

where δm^2 is a typical neutrino mass square difference, and of course, $\delta_{f_1 f_2}$ is a typical delta-parameter difference among the Lorentz-violating parameters for the different neutrino flavors. (We set $v^2 = 1 + \delta$, in accordance with Refs. 3–5.) For the parameter estimates of two different neutrino flavors of $\delta_{f_1 f_2} \sim 10^{-20}$ and $\delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$, the transition should occur at momenta on the order of $10^8 \cdots 10^9\,\mathrm{eV}$. (we here refer to bounds on Lorentz-violating parameters from laboratory-based experiments,[29,30] which are less strict than those derived from astrophysical observations[31,32]; the latter, though, are under less stringent external control.)

4. Gauge Invariance and Neutrino Decay

The observations made above, especially those reported in Sec. 3.3 for the high-energy limit of differential Lorentz violation across generations (flavors), but with the same Lorentz-violating parameters ascribed to charged and neutral fermions, allow us to discuss a fully gauge-symmetry conserving model, which still allows for NPCR and LPCR decays to proceed. Compared with other models studied in the literature (see Refs. 3–5), the model discussed here is most restricted in parameter space (it requires flavor-dependent Lorentz-breaking parameters), but perhaps, most stringent in its theoretical formulation, in the sense that it can be fully embedded into the Lorentz-violating SME. Because of full gauge invariance, we are also able to address, in passing, the gauge dependence not within the $SU(2)_L \times U(1)_Y$ gauge group, but within additional terms induced by the R_ξ gauge for the Z^0

boson propagator, mapped onto the effective Fermi interaction. Another question to address is whether the $SU(2)_L$ gauge symmetry protects potentially superluminal neutrinos from the NPCR and LPCR decay processes discussed in Refs. 3–5. Or one might ask if, conversely, bounds on Lorentz-violating parameters for charged fermions could universally be applied to neutrinos, if we postulate the full retention of $SU(2)_L \times U(1)_Y$ gauge invariance.

Under the assumption of flavor-dependent Lorentz breaking, if, say, electrons and electron neutrinos propagate faster than muons and muon neutrinos, the decay processes $\nu_e \to \nu_e \nu_\mu \bar{\nu}_\mu$ and $\nu_e \to \nu_e \mu^- \mu^+$ will be kinematically allowed [see Eq. (50)]. Let us give a brief account of the calculations. We define the pseudo-metric corresponding to the velocity $v_j > 1$, in full accordance with Eq. (6), as follows:

$$\widetilde{g}^{\mu\nu}(v_j) = v_j g^{\mu\nu} + (1 - v_j) t^\mu t^\nu = \mathrm{diag}(1, -v_j, -v_j, -v_j). \tag{53}$$

The massive Feynman propagator for the gauge vector boson in R_ξ gauge is given as follows:

$$D_F^{\mu\nu} = -\frac{g^{\mu\nu} + (\xi - 1) k^\mu k^\nu / (k^2 - \xi M_Z^2)}{k^2 - M_Z^2 + \mathrm{i}\epsilon}, \tag{54}$$

where $\varepsilon > 0$ denotes the infinitesimal imaginary part, and M_Z is the Z_0 boson mass. The modified Dirac matrices $\widetilde{\gamma}_j^\mu$, which are alternatively denoted as Γ_j^μ [see Eq. (16)], read as follows:

$$\Gamma_j^\mu = \widetilde{\gamma}_j^\mu = [v_j g^{\mu\nu} + (1 - v_j) t^\mu t^\nu] = \widetilde{g}^{\mu\nu}(v_j) \gamma_\nu, \tag{55}$$

where we note that there is no distinction any more between the flavor and the mass eigenstate bases. The effective Lagrangian for the decay $\nu \to \nu \Psi \overline{\Psi}$ is given as follows:

$$\begin{aligned} \mathcal{L}_{\mathrm{int}}^{2\nu 2\Psi} &= \frac{G_F}{2\sqrt{2}} [\overline{\nu}_j \widetilde{\gamma}_j^\mu (1 - \gamma_5) \nu_j] g_{\mu\nu} [\overline{\Psi}_k \widetilde{\gamma}_j^\nu (c_V - c_A \gamma_5) \Psi_j] \\ &= \frac{G_F}{2\sqrt{2}} [\overline{\nu}_j \widetilde{\gamma}_j^\mu (1 - \gamma_5) \nu_j] \widetilde{g}_{\mu\nu}(v_j v_k) [\overline{\Psi}_k \widetilde{\gamma}_j^\nu (c_V - c_A \gamma_5) \Psi_j]. \end{aligned} \tag{56}$$

For NPCR, we have

$$\Psi_k = \nu_k \Rightarrow c_V = c_A = 1, \tag{57}$$

whereas if ℓ_k is a charged lepton (electron–positron pair), then

$$\Psi_k = \ell_k \Rightarrow c_V = 0, \quad c_A = -1/2, \tag{58}$$

approximately. The invariant matrix element in the full gauge theory is

$$\begin{aligned} \mathcal{M} = \frac{g^2}{16 \cos^2 \theta_W} & [\overline{u}_i(p_3) \widetilde{\gamma}_i^\mu (1 - \gamma^5) u_i(p_1)] \frac{\mathrm{i}}{(p_2 + p_4)^2 - M_Z^2 + \mathrm{i}\epsilon} \\ & \times \left[g_{\mu\nu} + (\xi - 1) \frac{(p_2 + p_4)_\mu (p_2 + p_4)_\nu}{(p_2 + p_4)^2 - \xi M_Z^2} \right] [\overline{u}_f(p_4) \widetilde{\gamma}_f^\nu (c_V - c_A \gamma^5) u_f(p_2)], \end{aligned} \tag{59}$$

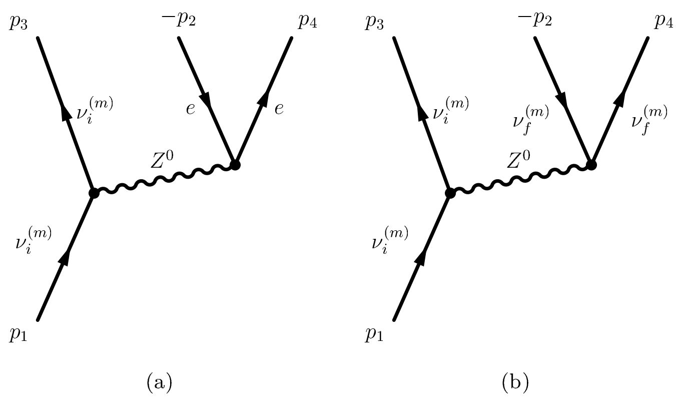

Fig. 1. Feynman diagrams for the LPCR [Panel (a)] and NPCR [Panel (b)] processes; these proceed via exchange of a virtual Z_0 boson. The diagrams are especially relevant in the high-energy region of our gauge-invariant, but Lorentz-breaking model, where the threshold conditions for LPCR are met. The "faster" generation of particles, of which we assume that ν_i is a member, decays into charged fermions [Panel (a)] or neutral fermions [Panel (b)] of a "slower" flavor (see the text of Sec. 4 for further explanations).

where, just like in Ref. 3, p_1 is the four-momentum of the oncoming particle, p_3 is the neutrino momentum after the decay, and p_2 and p_4 are the four-momenta of the created fermion–anti-fermion pair (see Fig. 1). The squared matrix element computed in this way is gauge-invariant (with respect to the electroweak gauge group), and an explicit calculation shows that terms involving the gauge parameter ξ of the R_ξ gauge do not appear in the final result. In order to arrive at this result, it is crucial though that (i) the Z_0 boson–fermion–anti-fermion vertices are proportional to $\widetilde{\gamma}_j^\mu$, (ii) the correct prescription for the spin sums [see Eq. (32) of Ref. 3] is used,

$$\sum_s \nu_{j,s} \otimes \overline{\nu}_{j,s} = \widetilde{g}_{\mu\nu}(v_j)\gamma^\mu p^\nu = v_i \not{p} + (1 - v_i)(p \cdot t)\not{t}, \tag{60}$$

and (iii) the dispersion relation in Eq. (22) is taken into account for external superluminal particles, most conveniently in the form $v_j p^2 + (1 - v_j)^2 (t \cdot p)^2 = 0$.

We note that in a spontaneously broken gauge theory, the gauge invariance of the squared matrix element computed in R_ξ gauge is usually only recovered once diagrams involving both vector and scalar boson exchanges are summed up. (This may involve exchanges of gauge bosons of the weak interaction as well as Higgs particle exchanges.) Indeed, in the Standard Model, the left-handed lepton current $\overline{\psi}\gamma_\mu(c_V - c_A\gamma^5)\psi$ is not conserved, with the non-conservation being proportional to the fermion mass. (Note that this non-conservation already occurs at tree level and can be derived on the basis of the axial component of the current.) However, the mass itself is proportional to the Yukawa coupling of the fermion (to the Higgs particle). Since we are working in the massless approximation for leptons, the Yukawa couplings in our model are set to zero, and the single diagram with only Z_0 boson exchange is gauge-invariant by itself. In particular, we have explicitly checked that terms coming from contractions involving the part of the Z_0 boson propagator proportional to $(\xi - 1)$ sum to zero upon using the superluminal dispersion relation.

For a general decay process in our gauge-invariant model, we obtain the following formula for the decay rate:

$$\Gamma_{\nu_i \to \nu_i \psi_f \overline{\psi}_f} = \left(\frac{g^2}{16 M_Z^2 \cos^2 \theta_W} \right)^2 \frac{E_1^5}{24\pi^3} \frac{c_V^2 + c_A^2}{420 n_s} \times (\delta_i - \delta_f) \left[17(\delta_i - \delta_f)^2 + 7(\delta_i + \delta_f)^2 \right]. \tag{61}$$

Here, n_s counts the allowed spin states of the neutrino ($n_s = 2$ in Ref. 4 but $n_s = 1$ in Ref. 5). Note that the factor f_e used in the notation of Ref. 3 is absorbed in the prefactor $c_V^2 + c_A^2$. The energy loss rate is obtained, in the fully gauge-invariant model, as

$$\frac{\mathrm{d}E_{\nu_i \to \nu_i \psi_f \overline{\psi}_f}}{\mathrm{d}x} = \left(\frac{g^2}{16 M_Z^2 \cos^2 \theta_W} \right)^2 \frac{E_1^6}{24\pi^3} \frac{c_V^2 + c_A^2}{672 n_s} \times (\delta_i - \delta_f) \left[22(\delta_i - \delta_f)^2 + 8(\delta_i + \delta_f)^2 \right]. \tag{62}$$

For LPCR decay, in our gauge-invariant model, we find

$$\Gamma_{\nu_i \to \nu_i e^- e^+} = a_{\mathrm{GI}} \frac{G_F^2}{192\pi^3} k_1^5, \tag{63a}$$

$$\frac{\mathrm{d}E_{\nu_i \to \nu_i e^- e^+}}{\mathrm{d}x} = -a'_{\mathrm{GI}} \frac{G_F^2}{192\pi^3} k_1^6, \tag{63b}$$

with the following results:

$$a_{\mathrm{GI}} = \frac{17(c_V^2 + c_A^2)}{420 n_s} (\delta_i - \delta_f) \left[(\delta_i - \delta_f)^2 + \frac{7}{17} (\delta_i + \delta_f)^2 \right], \tag{64a}$$

$$a'_{\mathrm{GI}} = \frac{11(c_V^2 + c_A^2)}{336 n_s} (\delta_i - \delta_f) \left[(\delta_i - \delta_f)^2 + \frac{4}{11} (\delta_i + \delta_f)^2 \right]. \tag{64b}$$

For NPCR decay, in our gauge-invariant model, we find

$$\Gamma_{\nu_i \to \nu_i \nu_f \overline{\nu}_f} = b_{\mathrm{GI}} \frac{G_F^2}{192\pi^3} k_1^5, \tag{65a}$$

$$\frac{\mathrm{d}E_{\nu_i \to \nu_i \nu_f \overline{\nu}_f}}{\mathrm{d}x} = -b'_{\mathrm{GI}} \frac{G_F^2}{192\pi^3} k_1^6, \tag{65b}$$

where the coefficients b_{GI} and b'_{GI} can be obtained from Eq. (61) by setting $c_V = c_A = 1$,

$$b_{\mathrm{GI}} = \frac{17}{210 n_s} (\delta_i - \delta_f) \left[(\delta_i - \delta_f)^2 + \frac{7}{17} (\delta_i + \delta_f)^2 \right], \tag{66a}$$

$$b'_{\mathrm{GI}} = \frac{11}{168 n_s} (\delta_i - \delta_f) \left[(\delta_i - \delta_f)^2 + \frac{4}{11} (\delta_i + \delta_f)^2 \right]. \tag{66b}$$

The change in the prefactors as compared to the kinematics-only approach pursued in Refs. 3–5 does not significantly change the conclusions drawn in Ref. 3 on astrophysically derived bounds for the Lorentz-violating parameters. We can establish that $SU(2)L \times U(1)_Y$ gauge invariance does *not* protect superluminal neutrinos from decay and energy loss processes (NPCR and LPCR).

5. Conclusions

In this paper, we have investigated the assumptions underlying the model-dependent interaction Lagrangians used in Refs. 3–5 for the formulation of the LPCR and NPCR (see Sec. 4) processes, which have led to very tight bounds on the Lorentz-violating parameters in the neutrino sector.[1–3] The main results can be summarized as follows.

Conclusion (i). The model used by Cohen and Glashow[4] and "model I" of Bezrukov and Lee[5] can be traced to an interaction Lagrangian which breaks electroweak gauge invariance, in addition to Lorentz invariance (see Sec. 3.1). However, this breaking proceeds on the same perturbative level on which the Lorentz-breaking terms themselves are formulated [see Eq. (22)]. A discussion on the implications with respect to fundamental symmetries is given in Sec. 3.1 of this paper.

Conclusion (ii). "Model II" of Bezrukov and Lee, used in the formulation of the LPCR process in Ref. 5, and also used by us in Ref. 3, is gauge-invariant under a restricted set of gauge transformations, within the $SU(2)_L$ gauge group. The use of nonuniform modified Dirac matrices, within the same $SU(2)_L$ doublet, is crucial to this observation [see Eq. (24)]. The derivation goes through even in an "oscillation-free" environment where one neglects the off-diagonal entries of the PMNS matrix, in the neutrino sector. The result given in Eq. (34) clarifies the "gauge-invariant" Lagrangian used in "model II" [see Eq. (4) of Ref. 5].

Conclusion (iii). If one invokes neutrino oscillations, then the situation is even more favorable for the gauge-invariant models (see Sec. 3.3). One can use uniform modified Dirac matrices within the same $SU(2)_L$ doublet, but assumes different Lorentz-violating parameter between generations [see Eq. (35)]. By assuming only a generation dependence, one obtains differential Lorentz violation among the neutrino mass eigenstates, and between neutrinos and charged leptons, without breaking $SU(2)_L \times U(1)_Y$ gauge-invariance. Under these assumptions [see Eq. (44)], it is useful to keep the Lorentz-violating parameter v_{int} that enters the interaction Lagrangian, separate from the ones of the initial and final states, as is done in Ref. 3. Inspired by the considerations reported in Sec. 3.3, one may devise a fully $SU(2)_L \times U(1)_Y$ gauge-symmetry conserving model, which still allows for the NPCR and LPCR decays to proceed (see Sec. 4). In the course of the calculations reported in Sec. 4, we also address the question regarding the dependence of the results on the gauge used for the massive vector boson propagator (R_ξ gauge).

We have thus clarified the cryptic remark of the "gauge invariance" of "model II" of Bezrukov and Lee [see Eq. (4) of Ref. 5], and provided additional motivation for the functional form of the various model-dependent interaction Lagrangians used in Refs. 3–5.

The very stringent bounds on the Lorentz-violating parameters in the neutrino sector, based on astrophysical observations,[1–3] thus do not require models in which electroweak gauge invariance is broken. This observation is quite crucial because it implies that one cannot "argue away" the tight bounds derived in Refs. 1–3 for the Lorentz-breaking parameters in the neutrino sector, based on the notion that the preservation of electroweak gauge invariance would otherwise preclude the existence of the decay processes on which the bounds are based.

Acknowledgments

The authors would like to acknowledge support from the National Science Foundation (Grant PHY-1710856). This work was also supported by the ÚNKP-17-3 New National Excellence Program of the Ministry of Human Capacities of Hungary and by the Grant K 125105 of the National Research, Development and Innovation Fund in Hungary.

Appendix A. Spontaneous Lorentz-Symmetry Breaking: Models and Implications

Although the *ansatz* of the current paper is completely phenomenological, and we do not discuss the possible mechanism behind Lorentz violation in any greater detail, it is still instructive to mention a specific model of spontaneous Lorentz invariance violation, which has been discussed in rather great detail in the literature.

Namely, according to Refs. 33–35, the photon could potentially be formulated as the Nambu–Goldstone boson linked to spontaneous Lorentz invariance violation. (This *ansatz* was originally formulated before electroweak unification.) Interest in this approach has recently been revived, and the theory has been worked out in greater detail.[33–44] Both Abelian as well as a nonAbelian gauge theories have been discussed.[37] In the case of an Abelian gauge theory, one assumes that the gauge field A_μ obtains a nonvanishing vacuum expectation value according to [see text after Eq. (1) of Ref. 43]

$$\langle A_\mu \rangle = n_\mu M, \tag{A.1}$$

where M is a (possibly large) energy scale at which the breaking of Lorentz symmetry occurs. The Lorentz group restricts itself to $SO(1,2)$ if n_μ is space-like ($n_\mu n^\mu = -1$), and into $SO(3)$ if n_μ is time-like ($n_\mu n^\mu = 1$).

The dynamical constraint [see Eq. (1) of Ref. 43]

$$A_\mu A^\mu = n^2 M^2 \tag{A.2}$$

is imposed on the A_μ field. One then parameterizes the A_μ field as [see Eq. (3) of Ref. 43]

$$A_\mu = a_\mu + \frac{n_\mu}{n^2}(n \cdot A), \tag{A.3}$$

where a_μ takes the role of the photon field. The following Lagrangian is eventually obtained after an expansion in leading order in $1/M$ [see Eq. (3) of Ref. 43]:

$$\begin{aligned} L(a,\psi) = &-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2}\delta(n\cdot a)^2 \\ &+ \bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi - e a_\mu \bar{\psi}\gamma^\mu\psi \\ &- \frac{1}{4} f_{\mu\nu} h^{\mu\nu} \frac{n^2 a_\rho a^\rho}{M} + \frac{e n^2 a_\rho a^\rho}{2M}\bar{\psi}(\gamma\cdot n)\psi. \end{aligned} \tag{A.4}$$

Here, a_μ takes the role of the (quantized) electromagnetic field, while $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the field strength tensor. Also, $h^{\mu\nu} = n^\mu \partial^\nu - n^\nu \partial^\mu$ is an oriented Lorentz-violating tensor. The orthogonality condition $n \cdot a = 0$ is explicitly introduced in the Lagrangian through a gauge-fixing term with parameter δ. Note that the Lagrangian (A.4) is obtained after a suitable redefinition of the fermion field, as given explicitly in Eq. (6) of Ref. 43.

We note that the sum of the terms

$$\bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi - e a_\mu \bar{\psi}\gamma^\mu\psi \tag{A.5}$$

in Eq. (A.4) adds up to the gauge-invariant quantum electrodynamic interaction (e is the electron charge).

In the limit $M \to \infty$, the entire Lagrangian (A.4) approximates the ordinary QED Lagrangian. However, for finite M, the fifth and sixth terms on the right-hand side of Eq. (A.4), which are initially generated by spontaneous Lorentz breaking in the electromagnetic sector, explicitly break electromagnetic gauge invariance, in addition to breaking Lorentz invariance. The fifth term generates a three-photon vertex, while the sixth term generates a two-fermion, two-photon interaction (see, for example Ref. 40).

In Eq. (22) of Ref. 40, it is shown that the contributions of both of the Lorentz-breaking terms to the electron–photon scattering amplitude vanish (due to mutual cancellations) if we take the matrix element between on-shell spinors. Around Eq. (32) of Ref. 40, it is argued that the same cancellation occurs for the one-loop amplitude, if the specific photon propagator integral given in Eq. (32) of Ref. 40 is evaluated in dimensional regularization. These considerations show that the Lorentz-violating terms in Eq. (A.4) do not necessarily lead to observable effects at low energy.

The generalization to spontaneous Lorentz-symmetry breaking in non-Abelian gauge fields involves the assumption [see Eq. (9) of Ref. 37]

$$\langle A^i_\mu \rangle = n^i_\mu M, \tag{A.6}$$

where the upper index i describes the component within the non-Abelian gauge group, for example, $SU(N)$, in which case $i = 1, \ldots, N$. For the Lorentz-breaking terms to vanish in the low-energy limit, one then has to make additional assumptions regarding the masses of the particles in a given $SU(N)$ multiplet; for example, according to Eq. (19) of Ref. 37, one needs to assume these masses to be equal.

For the context of the current paper, two observations are relevant:

(i) The approach taken in Refs. 37–44 starts from a spontaneous Lorentz-symmetry breaking at some high-energy scale M, involving a gauge boson field. This assumption is quite natural, because symmetry breaking for a vector field automatically singles out a specific direction in space-time (it would not necessarily do so for a spinor). However, as a comparison to Eq. (A.4) shows, for the case of spontaneous symmetry breaking in the gauge boson sector, the fermion sector is largely unaffected by the Lorentz-symmetry breaking, which initially occurs only in the gauge boson sector. [We observe that the third and fourth terms in Eq. (A.4) add up to the fully Lorentz-covariant Lagrangian for the electromagnetically coupled electron.] Hence, the *ansatz* discussed in Refs. 33–44 is not directly applicable to the models constrained by our calculations, which pertain to Lorentz violation in the fermion (neutrino) sector.

(ii) A very important observation can be made. Namely, Lorentz violation and gauge invariance violation are intimately intertwined. The term

$$\frac{en^2 a_\rho a^\rho}{2M}\bar{\psi}(\gamma\cdot n)\psi \tag{A.7}$$

in Eq. (A.4) manifestly breaks the electromagnetic $U(1)_{\rm EM}$ gauge symmetry. We remember that a gauge transformation in quantum electrodynamics works as $a_\mu \to a_\mu - \partial_\mu\Lambda$ and $\psi \to \psi\exp(\mathrm{i}e\Lambda)$, where $\Lambda = \Lambda(x)$ is the gauge function and e is the electron charge. Under this gauge transformation, the term (A.7) is manifestly noninvariant. Lorentz violation has thus created a term that violates gauge invariance, on the perturbative level (in first order in the $1/M$ expansion). Analogously, the model used by Cohen and Glashow in Ref. 4 assumes a breaking of gauge invariance on the perturbative level, in the latter case, of the electroweak gauge symmetry. Based on our comparison with the approach taken in Refs. 33–44, this is a perfectly permissible assumption.

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