Quantum Information Processing with Frequency-Comb Qudits

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***Abstract*—Classical optical frequency combs have revolutionized a myriad of fields, from optical spectroscopy and optical clocks to arbitrary microwave synthesis and lightwave communication. Capitalizing on the inherent robustness and high dimensionality of this mature optical platform, their nonclassical counterparts, so-called "quantum frequency combs," have recently begun to display significant promise for fiber-compatible quantum information processing (QIP) and quantum networks. In this review, the basic theory and experiments of frequency-bin QIP, as well as perspectives on opportunities for continued advances, will be covered. Particular emphasis is placed on the recent demonstration of the quantum frequency processor (QFP), a photonic device based on electro-optic modulation and Fourier-transform pulse shaping that is capable of realizing high-fidelity quantum frequency gates in a parallel, low-noise fashion.**

***Index Terms*—Frequency combs, quantum computing, electrooptic modulators, phase modulation, optical pulse shaping.**

I. Introduction

Due to their unprecedented stability, inherent parallelizability, and scalability, optical frequency combs have enabled a host of metrological applications from ultraprecise optical clocks to spectroscopy, ranging, and low-noise RF photonics [1], [2]. Yet other, more nascent applications are ripe for the utilization of frequency combs as well. For example, recent advances in generating multiphoton entangled states over discrete spectral modes [3], coupled with the advent of spectral mode manipulation tools based on linear [4], [5], [6] and nonlinear [7], [8] optical techniques, have opened an exciting new field of possibilities for all-optical information encoding, decoding, and processing with frequency combs.

Here, we describe theoretical and experimental progress on quantum state manipulation for frequency-comb-based quantum information processing (QIP). The very stability which makes frequency such a useful degree of freedom (DoF) for *encoding* information presents an array of unique challenges for quantum state *processing*, such as mixing and manipulating frequency modes at the single-photon level or even conditioned on the presence of a photon in a specific mode. But through the use of the newly developed quantum frequency processor (QFP), a device incorporating both electro-optic modulation and pulse shaping techniques, frequency comb lines (or "bins" for short), that otherwise do not interact, can be coupled in a controlled fashion with remarkable precision. Our objectives in the following will be to provide an intuitive understanding of the basic features of frequency-bin manipulations, overview recent experimental progress, and predict opportunities for continued advances. We intentionally omit discussion of quantum frequency-bin state generation, assuming that multiphoton quantum frequency combs can be prepared experimentally; for a recent overview of integrated quantum frequency combs, see Ref. [3]. Additionally, we concentrate on discrete-variable, gate-based QIP, which relies on quantum information encoding in single photons. For information on continuous-variable encoding with nonclassical combs, we refer the reader to, e.g., Refs. [9], [10].

II. General Approach

At a high level, any system for discrete-variable-based QIP requires first and foremost well-defined d-dimensional qudits (qubits for $d = 2$), which in the case of photonic QIP consists of selecting a particular DoF (set of modes or, in the parlance of quantum mechanics, the Hilbert space). Quantum information can be encoded in a scalable and interferometrically stable fashion using broadband frequency combs whereby every spectral comb line becomes an information carrier. The available mode space thus comprises equispaced bins centered at frequencies $\omega_n = \omega_0 + n\Delta\omega$ ($n \in \mathbb{Z}$). A single-qubit state can be represented as a photon in a superposition of two modes indexed n_0 and n_1: $\alpha\,|1_{n_0}\rangle + \beta\,|1_{n_1}\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$, and $|1_n\rangle$ denotes a single photon residing in bin with frequency ω_n. More generally, a single-qudit state can be defined as a d-mode superposition $\sum_{k=0}^{d-1} c_k\,|1_{n_k}\rangle$ where $\sum_{k=0}^{d-1} |c_k|^2 = 1$.

Then, to manipulate qubits one must be able to implement arbitrary frequency-bin operations with high fidelity, such that superpositions of various frequency bins can be converted to other superpositions defined by a unitary input/output matrix. One could envision performing unitaries on frequency bins through nonlinear optical interactions, and indeed, quantum frequency mixers based on $\chi^{(2)}$ [7] and $\chi^{(3)}$ [8] nonlinearities have been demonstrated on two-dimensional Hilbert spaces.

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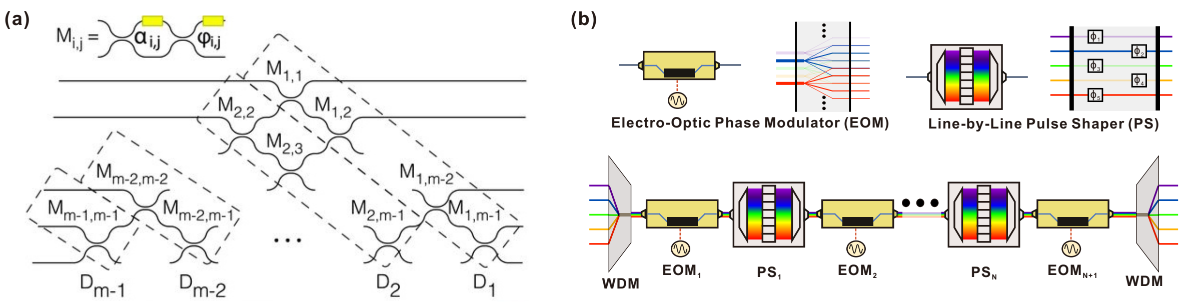

Fig. 1. Comparing path- and frequency-based transformations. (a) Path-based unitaries can be constructed from two-mode Mach–Zehnder interferometers. (Carolan *et al.* [11]) (b) Frequency-bin unitaries build instead on two fundamental components, EOMs and PSs, producing a complete network via an alternating sequence (the QFP). The various horizontal paths shown correspond to distinct frequencies, all residing in a single spatial mode.

However, scaling up this approach to many bins is unclear, as it would require additional pump fields, carefully engineered phase-matching conditions, and aggressive pump filtering.

Alternatively, frequency bins can be manipulated via electro-optic phase modulators (EOMs) driven by RF signals periodic at the spacing $\Delta\omega$. The optical sidebands produced by such an EOM cause states across distinct input bins to overlap and interfere in a complex fashion set by the specific RF waveform [12], [13], [14]. Importantly, EOMs are both optically linear and unitary: an ideal phase modulator spectrally redistributes optical energy but does not absorb it, aside from technical issues such as insertion loss. Accordingly, EOMs represent plausible candidates for arbitrary spectral transformations. However, in contrast to path/polarization DoFs, where an arbitrary transformation on N modes can be decomposed into beamsplitters and single-mode phase shifters with a total of $\mathcal{O}(N^2)$ elements [15], [16], EOMs operate on a formally infinite-dimensional space of frequency bins, so that any single mixing process will unavoidably scatter photons outside of the computational space, i.e., the predefined subset of frequency bins used to encode quantum information.

Yet as recently discovered [4], this roadblock can be overcome with the addition of another frequency-bin operation: Fourier-transform pulse shaping [17], [18]. Applying arbitrary phase shifts to each frequency bin, a line-by-line pulse shaper (PS) acts as a diagonal unitary matrix; by cascading EOMs and pulse shapers in an alternating sequence—what we call a QFP—the spectral spreading caused by a single EOM can be compensated through successive frequency-bin interference, thereby enabling arbitrary finite-dimensional $N \times N$ unitaries, with the number of EOM/PS elements scaling as $\mathcal{O}(N)$ [4]. In this way, any processing tasks possible in path-based encoding can be directly realized in frequency bins. Termed "spectral linear-optical quantum computation (LOQC)" after the original *spatial-mode* LOQC paradigm [19], this formalism confirmed the feasibility of frequency-bin operations as a comprehensive approach to quantum computation. Figure 1 furnishes a visual comparison of the basic elements of frequency-bin QIP compared to more traditional path-encoded QIP. Whereas a collection of two-mode interferometers can produce an arbitrary operation on a discrete set of spatial paths, spectral LOQC relies on fundamental elements which operate on a single spatial mode but many frequency bins simultaneously.

III. Experimental Demonstrations

Following the introduction of spectral LOQC, a series of proof-of-principle experiments using a 2EOM/1PS QFP have realized fundamental single- and two-photon quantum gates [20], [21], [22]. For maximum generality in the original proposal [4], it was assumed each EOM could be driven by arbitrary modulation patterns repeating at $\Delta\omega$. However, reproducing the required phase modulation bandwidth can pose a significant practical challenge to both electronic waveform generators and EOMs. A major finding in these experimental demonstrations is that much simpler single-tone drive signals are sufficient to construct many basic quantum gates with an extremely small reduction in performance.

The first experimental demonstration in this paradigm was the Hadamard (H) gate [20]—or equivalently, a frequency-bin beamsplitter. Experimentally implemented on a 2EOM/1PS QFP driven by a single microwave tone, this gate performs transformations such as that shown in Fig. 2(a) (left), which depicts how an in-phase, equiamplitude superposition input transforms into a single frequency bin output at mode 0. The measured operation fidelity is $\mathcal{F} = 0.99998 \pm 0.00003$, and with only $\sim$2.61% probability does the photon land outside of the computational space (small bumps in adjacent modes -1 and 2); this "leakage" represents the only negative consequence of employing pure-sinewave phase modulation. Since an ideal EOM is invariant with optical frequency translation, given the 25 GHz mode spacing and suitable guardband, a total number of 33 parallel frequency beamsplitters can be realized in this single QFP across the full 5 THz shaper bandwidth, with negligible performance degradation. Indeed, the fact that all frequency modes share the same EO modulation pattern restricts the diversity of parallel operations, i.e., only a specific set of functions can be implemented in different frequency bands in parallel [21]. However, our results still highlight an important synergy between frequency-bin QIP and classical optical networking, in which a single fiber-optic spatial mode can support multiple qubit operations in a parallel fashion.

Another advantage of frequency-bin QIP is its inherent high dimensionality, occurring naturally in broadband quantum frequency combs [3]. Incidentally, by incorporating an additional harmonic to the microwave drive, the same three-element QFP was found capable of realizing a fundamental qutrit (three frequency-bin) operation as well: the three-point discrete

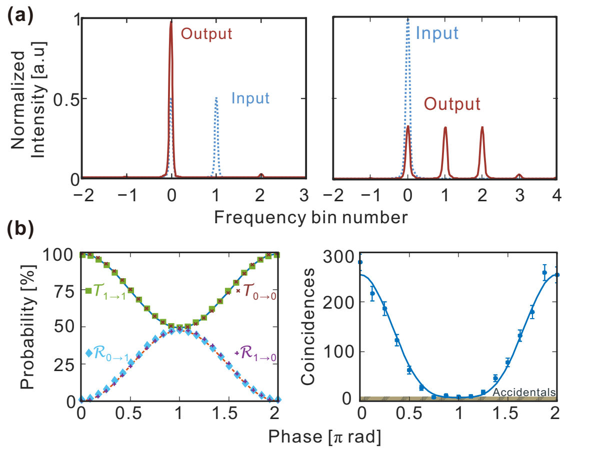

Fig. 2. Experimental demonstrations of frequency-bin gates. (a) Examples of output spectra for a frequency beamsplitter (left) and tritter (right). (Lu *et al.* [20]) (b) Tunable frequency beamsplitter (left) and its application to frequency-bin HOM interference (right). (Lu *et al.* [21])

Fourier transform or "tritter." The measured performance likewise attains near-unity fidelity with minimal scattering outside of the computational space. Figure 2(a) (right) presents an example measured spectrum, in which a superposition of three equiamplitude bins results from an input in bin 0.

To establish the QFP as a reliable device for scalable QIP with frequency-encoded qubits, one must move beyond single-photon gates (irrespective of dimensionality) and demonstrate verifiably quantum features relying on multiphoton interference. Hong–Ou–Mandel (HOM) interference [23], for example, can be viewed as the foundational quantum effect underlying two-qubit gates in the LOQC paradigm. In conventional HOM, one mixes two photons on a 50/50 spatial beamsplitter while scanning some parameter to control their overlap, observing photon bunching at the exit ports as a result of quantum interference between indistinguishable two-photon probability amplitudes. In the case of photons sharing a single spatial mode but different colors, the quantum interference can instead be realized with a frequency mixer [7], [24], [25] where in the case of the QFP above, the frequency mixing probability is scanned directly [21]. Specifically, controlling the depth of the phase jump imparted by the QFP pulse shaper, one can tune the beamsplitter "reflectivity" between bins 0 and 1 smoothly from 0–50% as illustrated in Fig. 2(b) (left). This tunability enables frequency-bin HOM when a photon pair $|\Psi\rangle = |1_0, 1_1\rangle$ is sent into the QFP. The visibility of the coincidences between the output frequency bins, 0.971 ± 0.007 [Fig. 2(b)], far surpasses any previous demonstrations in frequency encoding, mainly due to the fine controllability and fidelity of the operation as well as the absence of extra optical noise sources in the electro-optic approach.

Such high-visibility HOM interference bodes well for realizing a full two-photon entangling gate, and for the first experimental demonstration, a postselected controlled-NOT (CNOT) was considered: this gate flips the state of one photon conditioned on that of the other, and the success of the operation is marked by the presence of one photon each in the control and target output frequency bins (the so-called coincidence basis [26]). Figure 3 shows experimentally obtained coincidences for inputs in the computational basis for this frequency-bin CNOT, also realized on a 2EOM/1PS QFP [22]. The characteristic bit flip pattern is observed when the first photon is in the logical 1 state, highlighting the pivotal quantum functionality of a single-photon frequency flip conditioned on the frequency of another single photon.

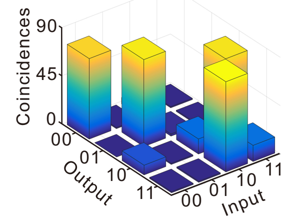

Fig. 3. Implementation of frequency-bin CNOT. Measured coincidences for all input/output two-photon states in the computational basis. (Lu *et al.* [22])

IV. Opportunities

The QFP has by this point been experimentally leveraged to realize a broad set of QIP primitives. The natural next objective, then, is to scale up such fundamental systems into larger processors designed for useful functionalities in quantum information. For example, higher-dimensional quantum operations can be implemented on a fixed-size QFP by introducing more complex temporal modulation patterns, which requires either high-speed arbitrary waveform generators or finer spectral resolution. Cascading additional EOMs and PSs enables even more complex operations, but the insertion loss from these off-the-shelf components, which already accounts for ∼12.5 dB in the three-element QFP [20], hampers the possibility of building larger circuits with the current designs. Importantly, though, frequency-bin encoding and the QFP operations themselves prove well suited for on-chip integration. EOM and pulse shaper photonic circuits have the potential to attain not only greater scalability, but also better raw performance at the component level, than the discrete fiber-optic devices employed in experimental demonstrations thus far. For example, recent developments in thin-film lithium niobate have enabled integrated EOMs with a unique combination of ultralow loss (<0.5 dB) and high bandwidth [27].

Figure 4(a) shows optical sidebands produced from such an EOM driven at 30 GHz [28]; the low half-wave voltage and high power handling result in ∼40 lines from a single EOM—extremely valuable for frequency-bin operations that can couple widely separated modes. On the pulse shaping front, line-by-line control of discrete frequency bins is possible with designs such as that in Fig. 4(b), where tuned microring resonators isolate and address individual frequency components, then combine them back into a single waveguide [29], [30]. Existing silicon-photonic foundries are capable of producing microring pulse shaper designs with losses $\lesssim$0.5 dB which, combined with low-loss EOMs, should make the QFP throughput competitive with purely passive photonic circuits. Admittedly, there remain significant challenges associated with

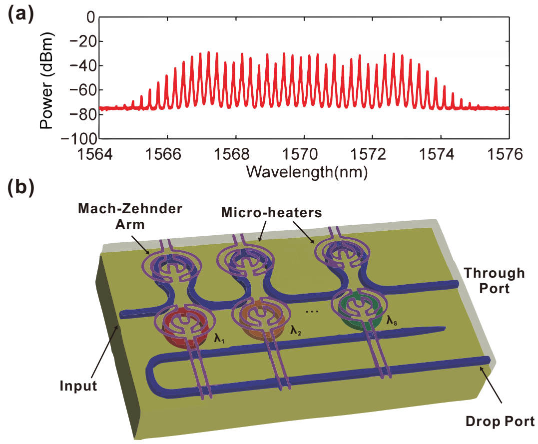

Fig. 4. Demonstrations of integrated EOM and pulse shaper. (a) Broadband electro-optic comb from a single thin-film lithium niobate modulator. (Ren *et al.* [28]) (b) Silicon-based integrated pulse shaper. (Khan *et al.* [29])

either (i) heterogeneously integrating the distinct material platforms optimal for photon sources, EOMs, and pulse shapers; or (ii) attaining superior performance using foundry-compatible processes only. Still, QFPs with many more elements, lower loss, and wider bin-to-bin connectivity than possible with fiber-optic versions certainly appear feasible in the near future.

Such large-scale integrated QFPs would then be well positioned for the application spaces originally envisioned for frequency-bin processors, such as interconnecting matter qubits with mismatched frequencies, frequency-bin quantum key distribution, and single-spatial-mode photonic quantum computation. Indeed, analog quantum simulations have already been conducted in the current generation of QFPs, where the wide frequency parallelization facilitates higher dimensionality and faster runtimes [31]. And by combining the pure QFP paradigm described here with time-frequency hyperentanglement [32], [33], a variety of interesting directions for comb-based photonic processing should emerge as well.

V. Conclusion

Quantum frequency combs represent a broadband, phase-stable, and scalable resource for encoding quantum information, compatible with single-mode optical fiber as well as on-chip photonics. A key challenge with such encoding, however, is how to coherently and efficiently manipulate frequencies that otherwise would not interact with each other. We have reviewed one solution to this problem—the QFP—which utilizes EOMs and pulse shapers to realize unitary operations in frequency space. Bolstered by the prospect of fully on-chip devices, the QFP looks to continue to expand impact for a variety of QIP tasks, adding yet another slate of intriguing applications to the frequency comb's repertoire.

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