

Space Ants: Constructing and Reconfiguring Large-Scale Structures with Finite Automata

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Abstract

In this video, we consider recognition and reconfiguration of lattice-based cellular structures by very simple robots with only basic functionality. The underlying motivation is the construction and modification of space facilities of enormous dimensions, where the combination of new materials with extremely simple robots promises structures of previously unthinkable size and flexibility. We present algorithmic methods that are able to detect and reconfigure arbitrary polyominoes, based on finite-state robots, while also preserving connectivity of a structure during reconfiguration. Specific results include methods for determining a bounding box, scaling a given arrangement, and adapting more general algorithms for transforming polyominoes.

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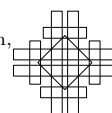
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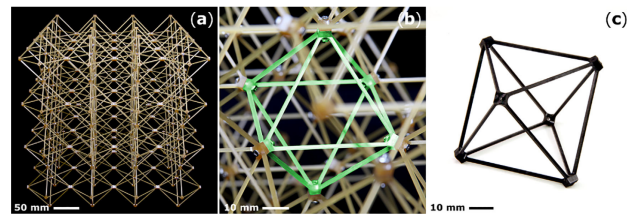
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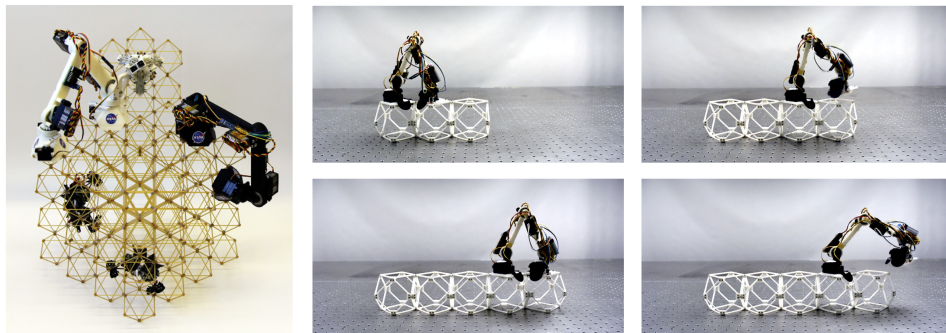
1 Introduction

Building and modifying large-scale structures is an important and natural objective in a vast array of applications. In many cases, the use of autonomous robots promises significant advantages, but also a number of additional difficulties. This is particularly true in space, where the difficulties of expensive supply chains, scarcity of building materials, dramatic costs and consequences of even small errors, and the limitations of outside intervention in case of malfunctions pose a vast array of extreme challenges.

In recent years, a number of significant advances have been made to facilitate overall breakthroughs. One important step has been the development of ultra-light and scalable composite lattice materials [29] that allow the construction of modular, reconfigurable, lattice-based structures [35]; see Figure 1. A second step has been the design of simple autonomous robots [32, 34] that are able to move on the resulting lattice structures and move their cell components, allowing the reconfiguration of the overall edifice; see Figure 2.



■ **Figure 1** (a) An assembled cuboctahedral lattice specimen, made from octahedral unit cells (highlighted), termed *voxels*. (c) A single injection molded voxel. (See [29].)



■ **Figure 2** (Left) Modular reconfigurable 3D lattice structure and mobile robots; note how robots are similar in size to lattice cells, and the parallel use of multiple robots. (See [7].) (Right) A sequence of images from the video: a BILL-E robot moving on an expanding row of voxels. (See [31].)

We address the next step in this hierarchy: Can we enable extremely simple robots to perform a more complex spectrum of construction tasks for cellular structures in space, such as patrolling and marking the perimeter, scaling up a given seed construction, and a number of other design operations? As we demonstrate, finite automata can achieve these tasks.

2 Related Work

The structures considered in this work are based on *ultra-light material*, as described by Cheung and Gershenfeld [6] and Gregg et al. [29]. Modular two-dimensional elements mechanically link in 3D to form reversibly assembled composite lattices. This process is

not limited by scale, and it enables disassembly and reconfiguration. As shown by Cramer et al. [8] and Jenett et al. [33], large but light-weight structures can be built from these components. Jenett et al. have developed autonomous robots that move on the surface [32, 31] or within the cellular structure [34]. With the help of these robots, individual cells can be attached to an existing assembly, or moved to a different location [31]. An approach for global optimization of a corresponding motion plan has been described by Costa et al. [7], while the design of hierarchical structures was addressed by Jenett et al. [36].

Assembly by simple robots has also been considered at the micro scale, where global control is used for supplying the necessary force for moving agents, e.g., see Becker et al. [2] for the corresponding problem of motion planning, Schmidt et al. [39] for using this model for assembling structures, and Balanza-Martinez et al. [1] for theoretical characterizations. On the algorithmic side, work dealing with *robots or agents on graphs* includes Blum and Kozen [4], who showed that two finite automata can jointly search any unknown maze. Other work has focused on exploring general graphs (e.g., [38, 23, 20]), as a distributed or collaborative problem using multiple agents (e.g. [3, 21, 9, 5]) or with space limitations (e.g. [22, 23, 17, 24, 25]).

From an algorithmic view, we are interested in *different models representing programmable matter* and further recent results. Inspired by the single-celled amoeba, Derakhshandeh et al. introduced the Amoebot model [11] and later a generalized variant, the general Amoebot model [15]; see [13, 10, 16, 14, 12] for various results in this model. Other models with active particles were introduced in [40] as the Nubot model and in [30] with modular robots. In [26], Gmyr et al. introduced a model with two types of particles: active robots acting like a deterministic finite automaton and passive tile particles. Furthermore, they presented algorithms for shape formation [28] and shape recognition [27] using robots on tiles.

3 Results for Finite Automata

We consider a set of N two-dimensional orthogonal *tiles* that form a *polyomino* P of total width w and height h . We use *robots* as active particles, which work like *finite deterministic automata* that can move between adjacent grid positions, where they can place or remove a tile. We assume that different robots cannot occupy the same position at the same time, and communication between robots is limited to adjacent positions. A basic step for recognizing and possibly reconfiguring P is based on constructing its bounding box $bb(P)$, which is the boundary of the smallest axis-aligned rectangle enclosing but not touching P ; this implies that there is a gap of one tile between the two, so we use a robot to keep the two parts connected.

The first result demonstrated in the video deals with constructing the bounding box, and thus recognizing the extent of a shape. See [19, 18, 37] for technical details.

► **Theorem 1.** *Given a polyomino P of width w and height h , we can build a bounding box surrounding P with the boundary and P always being connected, with two finite-state robots in $O(\max(w, h) \cdot (wh + k \cdot |\partial P|))$ steps, where k is the number of convex corners in P .*

The second result demonstrated in the video achieves *scaling* of a given shape.

► **Theorem 2.** *After building $bb(P)$, scaling a polyomino P of width w and height h by a constant scaling factor c without loss of connectivity can be done with one finite-state robot in $O(wh \cdot (c^2 + cw + ch))$ steps.*

Further reconfiguration results mentioned in the video are as follows.

► **Theorem 3.** Copying a polyomino P columnwise can be done within $\mathcal{O}(wh^2)$ steps using $\mathcal{O}(N)$ of auxiliary particles and $\mathcal{O}(wh)$ additional space in $\mathcal{O}(h)$ extra rows and columns.

► **Theorem 4.** Reflecting a polyomino P horizontally can be done in $\mathcal{O}(w^2h)$ steps, using $\mathcal{O}(w)$ of additional space and $\mathcal{O}(w)$ auxiliary particles.

► **Theorem 5.** There is a strategy to rotate a polyomino P by $\pm\frac{\pi}{2}$ within $\mathcal{O}((w+h)wh)$ steps, using $\mathcal{O}(w+h+|w-h|/h)$ of additional space in $\mathcal{O}(|w-h|+1)$ extra rows and columns and $\mathcal{O}(w+h)$ auxiliary particles.

Finally, the video demonstrates how we can carry out any geometric transformation by finite-state robots, if and only if there is a corresponding Turing machine for transforming the corresponding one-dimensional string $S(P_1)$ (arising from a row-wise scan of P_1) into $S(P_2)$.

► **Theorem 6.** Let P_1 and P_2 be two polyominoes with $|P_1| = |P_2| = N$. There is a strategy transforming P_1 into P_2 if there is a Turing machine transforming the corresponding one-dimensional string $S(P_1)$ into $S(P_2)$. The finite-state robot needs $\mathcal{O}(\partial P_1 + \partial P_2 + S_{TM})$ auxiliary particles, $\mathcal{O}(N^4 + T_{TM})$ steps, and $\Theta(N^2 + S_{TM})$ of additional space, where T_{TM} and S_{TM} are the number of steps and additional space needed by the Turing machine.

4 The Video

The video starts with a discussion of the problems faced when building large-scale structures in space, and an introduction of digital, ultra light-weight materials and simple robots currently developed at MIT and NASA. This is followed by a description of finite automata corresponding to finite-state robots. As a first algorithmic demonstration, the connected construction of the bounding box of a given polyomino shape is shown, followed by producing a scaled copy of a shape. Then we show how general constructions can be built based on methods of Turing machines. The video concludes with a 3D simulation.

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