RF Compressed Sensing Based Radar for 2-D Localization and Mapping

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***Abstract*—This paper presents the simulation results of a radio frequency (RF) compressed sensing (CS) radar for 2-D localization and mapping. In contrast to many existing literatures, this paper deals with the RF front-end compressive sensing, which is achieved by illuminating the target scenario with pseudo random multi-beam radiation patterns. The spatial sparsity of the target frame enables the use of compressed sensing to recover the scene from fewer number of scans compared with a conventional beam scanning radar. Preliminary simulations are performed, and the results of the reconstructed target frame are presented.**

***Index Terms*— 2-D localization, compressed sensing, multi-beam radiation pattern, spatial sparsity.**

I. Introduction

With the need for more precise and accurate localization in the internet of things era, modern radars [1] require a large signal bandwidth and complex front-end circuits for beam scanning mechanism. To provide accurate localization, the RF front-end must steer a very narrow beam over a small step size, increasing the scanning time and the number of samples.

Compressed sensing based radar offers the possibility to provide accurate localization using a reduced spatial scanning time. CS exploits the sparsity of a signal to reconstruct it from far fewer number of samples in contrast to the conventional Shannon-Nyquist sampling criteria. This also ensures less complexity of the RF front-end system, thereby reducing the system cost and increasing the energy efficiency. Compressed sensing has found significant applications in cameras [2] and radars [3,4]. Existing literatures on compressed sensing radars introduce randomness, which is the key to compressed sensing, using different methods such as random filtering and convolution based on digital signal processing. These were typically carried out in the baseband computational unit.

In this paper, preliminary simulation of an RF front-end compressed sensing radar in a simplified 2-D localization and mapping scenario is discussed. The proposed radar introduces randomness directly in the RF front-end, which is one of the most expensive and power-hungry blocks of the radar system. Randomness is employed by illuminating the target space with pseudo random multi-beam radiation patterns, which can be generated using a digital beamforming architecture. Compared to conventional beam scanning radar, the proposed RF front-end compressed sensing radar reconstructs the target scene using a fewer number of scans, thereby having the potential to reduce the scanning time and front-end energy consumption. Simulations are performed using MATLAB software and the results for simplified 2-D localization for sparse target frames are discussed.

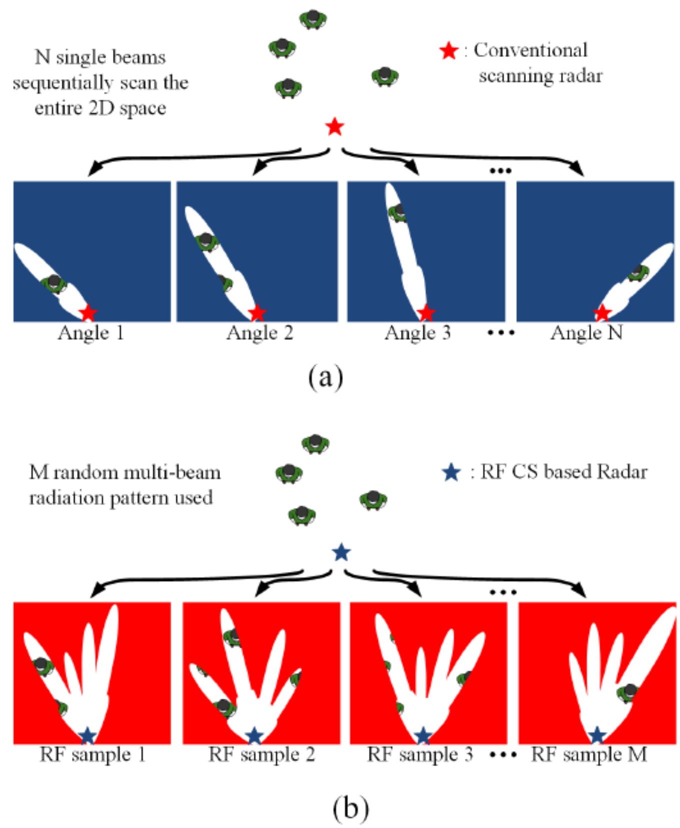

Fig. 1. Indoor localization and mapping. (a) Conventional beam scanning radar. (b) Proposed RF front-end compressed sensing based radar.

II. Theory

Compressed sensing [5] technique can recover a sparse signal from a small set of random linear measurements. A signal is said to be k-sparse if it contains utmost k non-zero elements in it. Compressed sensing in mathematical terms can be expressed as $y = \phi x$, where y represents the measured vector of length m, x represents the k-sparse input signal of length n, and ϕ is called the measurement matrix of length $m \times n$. The fact that $k << n$ makes it feasible to use $m < n$ random measurements to reconstruct the signal x from y using compressed sensing reconstruction algorithms such as basis pursuit (BP), orthogonal matching pursuit (OMP) and many more. However, to recover the signal x from y, the measurement matrix ϕ must

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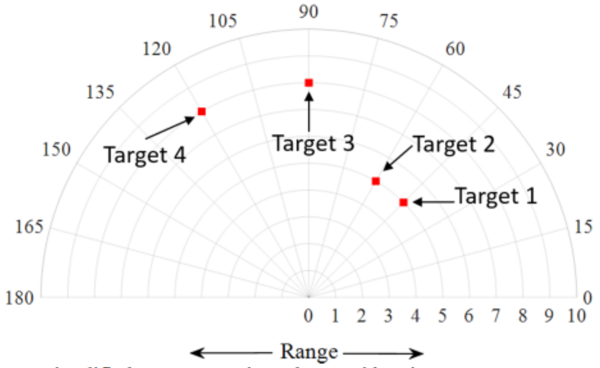

Fig. 2. A simplified target scenario under consideration.

have a low mutual coherence μ. The mutual coherence μ is given as:

$$\mu(\phi) = max_{i \neq j} |\langle \varphi_i, \varphi_j \rangle|, \tag{1}$$

where φ_i represents the normalized columns of ϕ.

For 2-D localization application, where the target frame is usually sparse in spatial domain, the proposed compressed sensing radar can recover the target frame using lesser number of scans. Fig. 1 shows the difference between a conventional scanning radar and the proposed RF front-end compressed sensing based radar. In a conventional scanning radar as shown in Fig. 1 (a), the radar scans the 2-D space using N single beams. In the proposed radar as shown in Fig. 1 (b), the 2-D space is illuminated with M ($< N$) scans of pseudo random multi-beam radiation patterns.

Any target in a 2-D space can be represented in polar co-ordinates by its range r and angle θ. For convenience, the 2-D frame under consideration is represented using polar co-ordinates (r,θ). Further, the range axis and the angular axis are divided into discrete ranges and discrete angles respectively.

For each discretized range, a corresponding row vector X_p is formulated, where p represents the range. Each element of the range vector corresponds to the presence of the target at the polar co-ordinate (p,θ), where θ represents each discrete angle. Each element is represented as 1 denoting the presence of a target or 0 otherwise. Thus, X_p is row vector of size $1 \times L$, where L represents the number of discrete angles. The target frame is then represented as the input matrix X whose rows correspond to each discretized range vector. As an example, consider the angular axis to be divided into discrete angles from 30 deg to 90 deg in steps of 10 deg. The range axis is divided into discrete ranges from 1 m to 5 m in steps of 1 m. The target frame has three targets at positions (3,40), (3,60), and (5,30). The range vector X_3 for the discretized range of 3 m is given as X_3 = [0,1,0,1,0,0,0]. The range vector X_5 for the discretized range of 5 m is given as X_5 = [1,0,0,0,0,0,0]. The remaining range vectors X_1, X_2, X_4 for the discretized ranges of 1 m, 2 m, and 4 m are all given as [0,0,0,0,0,0,0]. The range axis can be discretized into further smaller steps at the cost of computational time.

The measurement matrix ϕ will assume a size of $K \times L$, where K represents the number of measurements. The measurement matrix is generated using a random number

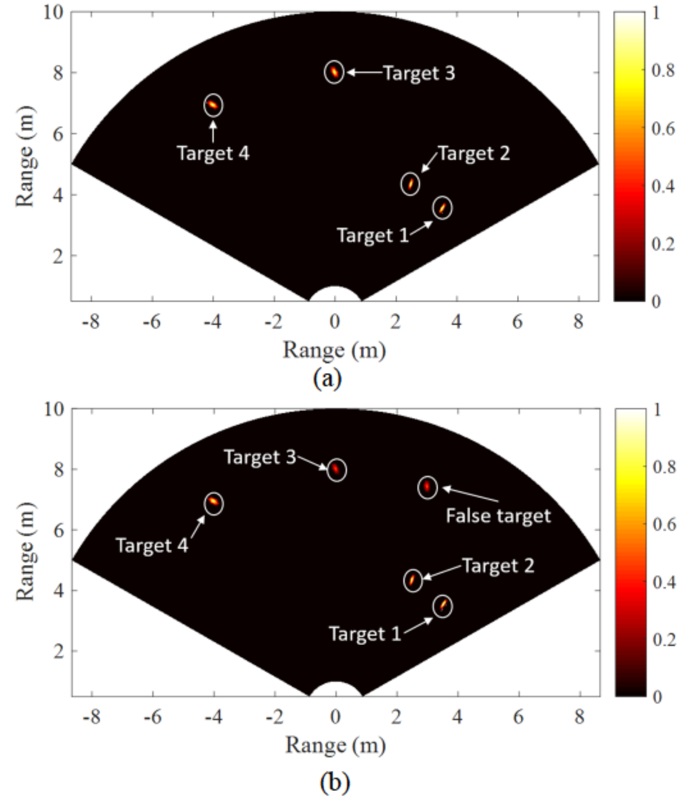

Fig. 3. (a) Reconstructed 2-D map from 100 multi-beam radiation patterns. (b) Reconstructed 2-D map from 30 multi-beam radiation patterns.

generator with values 0 and 1. Each row vector of the measurement matrix corresponds to the pseudo random multi-beam radiation pattern for each measurement. For each measurement, a fixed number of beams are radiated randomly in the direction of the discretized angles. Each element of the row vector in the measurement matrix corresponds to the radiation of the beam in the direction of the discretized angles. If the element in a row vector is set to 1, a beam is radiated in the direction of the corresponding discrete angle for that measurement. Assume the same discretized angles from the previous example and number of multiple beams per measurement to be 2. If the first row vector of the measurement matrix is [0,0,0,1,0,0,1], then two beams are simultaneously radiated at angles 60 deg and 90 deg respectively for the first measurement. If the second row vector is [1,0,0,1,0,0,0], then two beams are simultaneously radiated at angles 30 deg and 60 deg, respectively, for the second measurement.

The measured matrix Y is obtained by the matrix multiplication of the measurement matrix ϕ and the input matrix X. The measured matrix Y is equivalent to the target frame samples measured by the radar. Each column vector of Y, denoted as Y_p represent the samples for each discretized range p. To reconstruct the signal X from Y, the samples Y_p for each range vector X_p are taken separately and the corresponding range vector is reconstructed using the l1-minimization technique:

$$min \|\bar{X}_p\|_1 \quad s.t. \; Y_p = \phi \bar{X}_p, \tag{2}$$

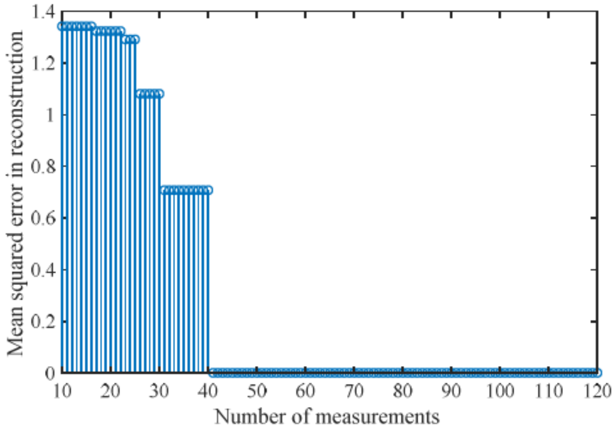

Fig. 4. Variation in mean square error in reconstruction corresponding to the number of radiation patterns used.

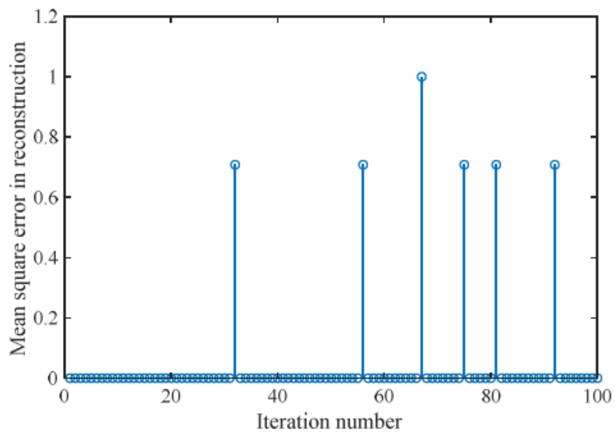

Fig. 5. Variation in mean square error in reconstruction with different measurement matrices for the case of using 50 radiation patterns.

where $\bar{X}_p$ is the reconstructed range vector for the discretized range p. The reconstructed target frame is then represented as the matrix $\bar{X}$ whose rows correspond to each reconstructed discretized range vector.

III. Simulation Results

Fig. 2 represents the target frame in polar co-ordinates. The range axis is divided into discrete ranges from 1 m to 10 m in steps of 1 m. Similarly, the angular axis is divided into discrete angles from 30 deg to 150 deg in steps of 1 deg. There are four targets at positions (5,45), (5,60), (8,90), and (8,120) respectively. The measurement matrix is generated using a random number generator with entries 0 and 1. For each range vector, the corresponding measured vector is calculated. The range vector is then reconstructed using $l1$-magic toolbox [6]. All the reconstructed range vectors combined create the reconstructed 2-D target frame.

Fig. 3 (a) shows the perfectly reconstructed target frame with 100 measurements. Fig. 3 (b) represents the scenario where false targets are reconstructed when 30 measurements are used. A simulation is carried out with two targets at polar co-ordinates (3,90) and (3,120) to observe the variation in quality of reconstruction with the number of measurements. The quality of reconstruction is represented as the mean square error (MSE), which is calculated as the l2-norm of the difference between the reconstructed matrix and the input matrix. Fig. 4 shows the variation of mean square error in reconstruction with the number of measurements. It is to be noted that MSE is a mere mathematical representation and it does not provide the physical range error or angle error in the target reconstruction. Fig. 5 represents the variation in mean square error in reconstruction with different randomly generated measurement matrices keeping the number of measurements constant, which in this case is 50. In the current scenario, a conventional beam scanning radar requires 121 scans to scan through each of the discretized angles in the 2-D space.

It should be noted that the FMCW radar parameters and the antenna array parameters were not considered in the preliminary simulation. The short coming of the above results is that there must be at least one beam in the direction of every target during the measurements, which can be resolved by considering the directivity of the antenna array. Furthermore, the reconstruction problem can be set up by directly considering the FMCW radar beat signals for different radial distances and the reflectivity of the targets. The beat signals represent the basis expansion matrix and the reflectivity of the targets represent the sparse coefficients of the basis expansion matrix that can be recovered using compressed sensing.

IV. Conclusion

In this paper, preliminary simulation for an RF front-end compressed sensing radar for 2-D localization and mapping was discussed. A pseudo random multi-beam radiation pattern was used to illuminate the target space. A simulation environment was set up to reconstruct a sparse target frame. From the simulation results, it was shown that the proposed radar was able to reconstruct the target scene from a smaller number of scans as compared to a conventional beam scanning radar. The effect of number of measurements on the quality of reconstruction was also studied. The similarity between the input matrix and the reconstructed matrix is used to evaluate the quality of reconstruction.

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