A Novel Radar Imaging Method Based on Random Illuminations Using FMCW Radar

Prateek Nallabolu and Changzhi Li

Department of Electrical and Computer Engineering, Texas Tech University, Lubbock, Texas, USA.

***Abstract* — Compressed Sensing (CS) has provided a viable approach to undersample a sparse signal and reconstruct it perfectly. In this paper, the simulation results of a frequency-modulated continuous-wave (FMCW) radar, which employs a CS based data acquisition and reconstruction algorithm to recover a sparse 2-D target frame using fewer number of scans are presented. A 16-element antenna array based on digital beamforming approach is used on the receiver end to obtain random spatial measurements of the target frame, which is the key to compressed sensing. A linear relationship is established between the total received FMCW beat signal for each scan and the 2-D sparse target frame using a basis transform matrix. Simulations of the proposed radar are performed in MATLAB and the reconstruction results for different noise levels are presented.**

***Index Terms* — compressed sensing, digital beamforming, frequency-modulated continuous-wave radar, radar imaging, random illumination, target localization.**

I. INTRODUCTION

Nowadays, compressed sensing is being used extensively for sparse signal reconstruction [1]. In contrast to the traditional sampling theorem, compressed sensing allows for reconstruction of a sparse signal by randomly under sampling the signal and posing it as an optimization based inverse problem. Compressed sensing has found increasing applications in radars, including target localization [2],[3], and subsurface imaging [4].

In [2] and [3], the need for a matched filter and high sampling rate analog-to-digital (A/D) converter on the receiver end was eliminated by transmitting a pseudo noise (PN) or chirp sequence and an Alltop sequence (provides good incoherence), respectively, and introducing CS algorithms in the signal processing stage. In the author's previous publication [5], the concept of a radio frequency (RF) front-end compressed sensing radar for 2-D localization was proposed, which was intended to reduce the number of scans. However, the results presented in [5] did not take into consideration any radar parameters or the antenna directivity. The response of each target was mathematically represented in 1's and 0's. Moreover, the range of the targets was assumed to be known beforehand and the CS algorithm was used to determine only the angular positions of the targets. Since the entries of the measurement matrix were represented as 1's and 0's, with 1 representing the presence of a beam and 0 representing the absence of a beam, the most significant limitation in [5] was that there should be at least one beam radiated in the direction of each target during the scanning time, else the information about that corresponding target would not be embodied in any of the measurements obtained. The 2-D target frame was discretized along the angular axis in steps of 1°, which was difficult to achieve practically.

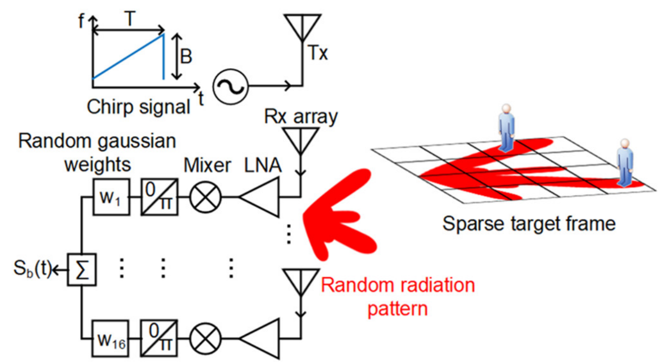

Fig. 1. Proposed radar system with random radiation pattern illuminating the target frame.

In this paper, the simulation results of a compressed sensing based FMCW radar for 2-D sparse target reconstruction are presented. The radar operates at a center frequency of 24.15 GHz, bandwidth of 300 MHz, and chirp duration of 10 ms. Random spatial measurements of the 2-D target frame are obtained by using a 16-element $\lambda/2$ spaced antenna array at the receiver end, fed with random gaussian weights using the digital beamforming approach. The directivity of the radiation pattern is considered as the measurement matrix. A practically realizable angular step size of 6° is used. The received FMCW beat signal for each scan is measured and a linear relationship is established with the 2-D target frame using a basis transform matrix. CS algorithms are used to recover both the range and the angular positions of the sparse targets.

II. THEORY

A. Compressed Sensing

Compressed sensing provides a significant advantage over existing data compression methods by simultaneously compressing the signal while sampling it. This compression while sampling approach allows for reduced sampling time/rate when dealing with sparse signals. Compressed

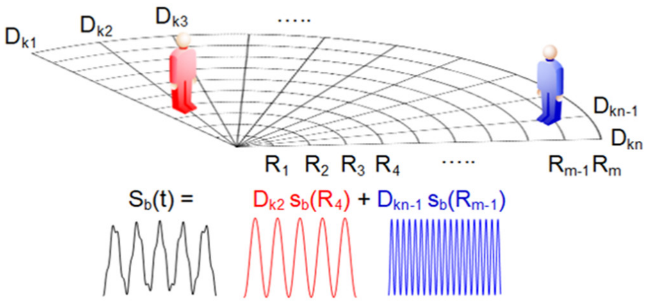

Fig. 2. Graphical representation of the 2-D discretized grid and the total received beat signal for the k[th] measurement.

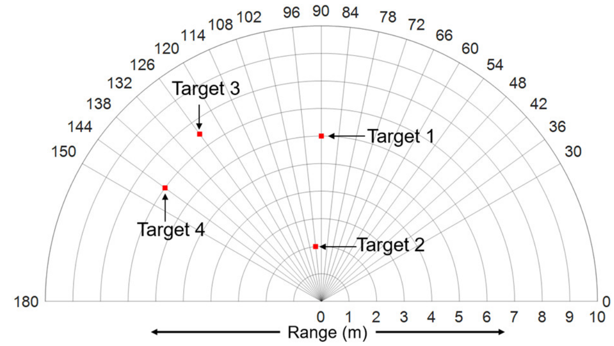

Fig. 3. The 2-D target frame to be reconstructed.

sensing states that a K-sparse signal x of length N can be under sampled and perfectly reconstructed using M random measurements, where $M << N$. The signal x can have a sparse representation in another orthobasis ψ, i.e. x can be represented as a linear combination of basis vectors from ψ

$$x = \sum_{i=1}^{N} s_i \psi_i \tag{1}$$

where s_i represents the weighting coefficients, such that only K of them are non-zero.

To reconstruct the signal x with high probability, the measurements y should be random in nature. Mathematically, the measurement matrix ϕ and the orthobasis ψ should be highly incoherent for proper reconstruction. An interesting choice for the measurement matrix is i.i.d. random variables from a gaussian distribution or uniform Bernoulli distribution. It is proven that these random distributions are incoherent with any orthobasis. The random measurements y can be represented in matrix notation as

$$y = \Phi x = \Phi \psi s \,. \tag{2}$$

The signal x can be exactly recovered from the measurements y using the basis pursuit algorithm, which is a l_1 minimization problem given as

$$s' = \text{argmin } \|s\|_1 \text{ s.t. } y = \Phi\psi s \tag{3}$$

where s' represents the reconstructed weighting coefficients. If the measurements are corrupted with noise, the algorithm above is slightly modified and given as

$$s' = \text{argmin } \|s\|_1 \text{ s.t. } \|y - \Phi\psi s\|_2 < \epsilon \tag{4}$$

where ϵ depends on the noise power level.

B. Proposed Compressed Sensing Based Radar

Fig. 1 shows the overview of the proposed FMCW radar with a digital beamforming architecture, which illuminates the target frame with a random radiation pattern for every scan. In FMCW mode of operation, the radar transmits a linear frequency-modulated continuous-wave chirp signal for a given duration T with a bandwidth B and center frequency f_c. The down-converted received signal (beat signal) s_b corresponding to a target at a given range R is given as

$$s_b(R) = exp(j(\frac{4\pi BRt}{Tc} + \frac{4\pi f_c R}{c})) \tag{5}$$

where c is the speed of light in vacuum and $t \epsilon [0,T]$.

To apply compressed sensing, the 2-D target frame must be discretized into a 2-D grid and a linear relation must be established between the total received beat signal and the targets across the 2-D grid. Consider that the target frame is discretized into m discrete ranges along the range axis and n discrete angles along the angular axis. The grid has a total of $m \times n$ points and σ_{ij} represents the radar cross section (RCS) of the target at the i^{th} range and j^{th} angle. Each target across the range axis can be uniquely identified by the corresponding beat signal it generates. To distinguish the targets across the angular axis for a given discrete range, the directivity D of the antenna array is useful. For the k^{th} measurement, the total received beat signal S_b corresponding to all the targets on the 2-D grid can be represented using a linear relationship given by

$$S_b(t) = \sum_{i=1}^{m} [s_b(R_i) \sum_{j=1}^{n} D_{kj} \sigma_{ij}] \tag{6}$$

where R_i represents the i^{th} discretized range and D_{kj} represents the directivity along the j^{th} discrete angle for the k^{th} measurement. Fig. 2 shows the pictorial representation of the discrete grid and the total received beat signal for the k^{th} measurement. The total received beat signal corresponding to each of the k measurements can be represented in a matrix notation given by

$$[S_b(t)]_{k\times 1} = [D]_{k\times n} \times [\psi]_{n\times mn} \times [\sigma]_{mn\times 1} \tag{7}$$

where ψ is the basis transform matrix represented as

$$\psi = \begin{bmatrix} s_b(R_1) & \ddots & 0 & s_b(R_2) & \ddots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \ddots & 0 & 0 & \ddots & 0 \\ 0 & \ddots & s_b(R_1) & 0 & \ddots & s_b(R_m) \end{bmatrix}_{n\times mn} . \tag{8}$$

When the 2-D grid to be reconstructed is sparse, resemblance can be noticed between (7) and (2), where the directivity matrix D is equivalent to the measurement matrix ϕ, σ represents the sparse coefficients, and ψ is the orthobasis. For proper reconstruction, the directivity D must be random in nature. The digital beamforming method is used to

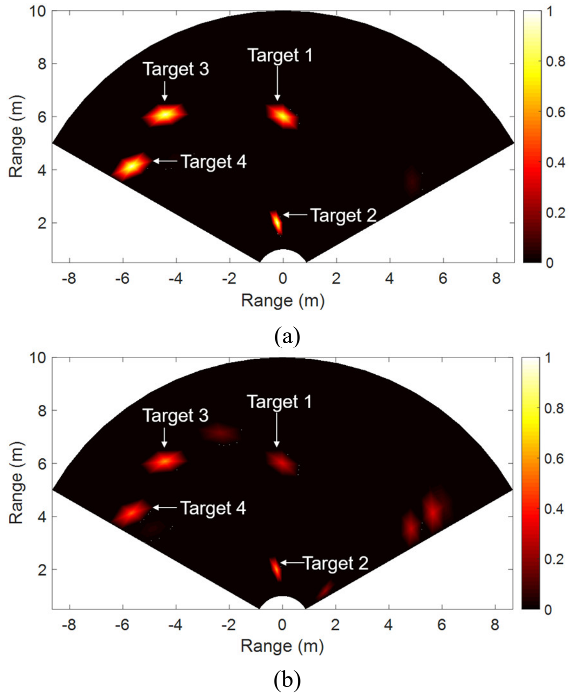

Fig. 4. Reconstructed 2-D target frame when measurements are corrupted with noise of (a) standard deviation = 0.1 and (b) standard deviation = 0.5.

generate random multi-beam radiation patterns, with three main beams pointing at random discretized angles for each measurement. To further improve the randomness, the amplitude and phase of each element of the antenna array are derived from a standard gaussian distribution. To recover the target positions, k scans of the target frame are performed, and the sparse targets are recovered using (3) for a noiseless scenario or (4) when the measurements are corrupted with noise.

III. Simulation Results

Simulations for the proposed radar are performed in MATLAB software. The FMCW radar parameters considered are: center frequency (f_c) of 24.15 GHz, chirp duration (T) of 10 ms, bandwidth (B) of 300 MHz, and a sampling frequency of 5 KHz. A 16-element $\lambda/2$ spaced antenna array is used to generate the random radiation patterns, where λ is the free space wavelength. A random number generator is used to decide the direction of the three main beams for each measurement. The 2-D frame of interest is discretized along the range axis from 1 m to 10 m in steps of 0.5 m, and from 30° to 150° in steps of 6° along the angular axis. $\sigma_{ij} = 1$ indicates the presence of a target at the i^{th} discretized range and j^{th} discretized angle, and $\sigma_{ij} = 0$ indicates the absence of a target. The effect of the radar range equation on the power of the received signal is not considered in the simulation. The reconstruction algorithms are implemented in CVX [6], a convex optimization program supported in MATLAB.

Fig. 3 shows the 2-D target frame that must be reconstructed. Four targets are considered at locations (6m,90°), (2m,96°), (7.5m,126°), and (7m,144°). Fig. 4 (a) represents the perfectly reconstructed targets when noise with standard deviation of 0.1 is added to the measurements, and 10 scans are performed. When the standard deviation of the noise is increased to 0.5, the targets are not fully reconstructed, and some false targets appear as shown in Fig. 4 (b). In the similar conditions presented above, a conventional beam scanning radar would require 21 scans to reconstruct the target frame, whereas the proposed radar can reconstruct the target frame in 10 scans, offering 50% reduction in the scanning time. The simulation was repeated 250 times in a noise free scenario, with varying target locations and antenna directivity for each iteration. Random outliers were observed twice, where the targets were not properly reconstructed. The cause of these random outliers is still being researched.

IV. Conclusion

A basis transform matrix was used to obtain a linear relationship between the total received FMCW beat signal for each scan and the sparse targets across the discretized 2-D grid. A 16-element antenna array employing digital beamforming technique was used on the receiver end to obtain random spatial measurements. Each antenna element was fed with random gaussian weights. Basis pursuit algorithm was applied on the obtained data to recover the sparse targets from fewer number of scans. The effect of noise on the target reconstruction was illustrated.

Acknowledgement

The authors wish to acknowledge the National Science Foundation (NSF) for funding support under grant 1808613 and 1718483.

References

[1] D. L. Donoho, "Compressed sensing," *IEEE Transactions on information theory*, vol. 52, no. 4, pp. 1289-1306, 2006.

[2] R. Baraniuk and P. Steeghs, "Compressive radar imaging," in *2007 IEEE Radar Conference*, 2007, pp. 128-133: IEEE.

[3] M. Herman and T. Strohmer, "Compressed sensing radar," *2008 IEEE Radar Conference*, Rome, 2008, pp. 1-6.

[4] A. C. Gurbuz, J. H. McClellan and W. R. Scott, "A compressive sensing data acquisition and imaging method for stepped frequency GPRs," in *IEEE Transactions on Signal Processing*, vol. 57, no. 7, pp. 2640-2650, July 2009.

[5] P. Nallabolu and C. Li, "RF compressed sensing based radar for 2-D localization and mapping," *2019 IEEE MTT-S International Microwave Biomedical Conference (IMBioC)*, Nanjing, China, 2019, pp. 1-3.

[6] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1." http://cvxr.com/cvx, March 2014.