Reactive Power Optimization for Flat Voltage Profiles in Distribution Networks

Vanja G. Švenda, Mai H. Vu and Alex M. Stanković
Dept. of Electrical and Computer Engineering, Tufts University, Medford, MA, USA
vanja.svenda@tufts.edu; maivu@ece.tufts.edu; astankov@ece.tufts.edu

***Abstract*—This paper considers achieving flat voltage profiles in a distribution network based on reactive power optimization (RPO) through voltage regulation devices (VRD). These devices include capacitor banks, load-tap-changing and regulating transformers, whose statuses can only assume pre-determined integer value levels, making this a non-convex problem. Two RPO-based algorithms are proposed, which can be applied to any initial states, node priority, topology and load model types. The first algorithm focuses on finding a practical solution by ensuring the VRD constraints are observed at each step. The second one focuses on finding the globally optimal solution by applying a convex relaxation technique and solving the resulting problem with the barrier interior point method. Here, the gradients are computed numerically, thus requiring no analytical functions of voltages in terms of VRDs. Numerical results and their analysis are examined on two test networks: 1) single feeder; and 2) network with laterals.**

***Index Terms*—Convex optimization, Distribution network, Flat voltage profile, Voltage regulation devices.**

I. Introduction

Modern power distribution utilities (DPUs) are emerging as independent entities, and as such must address energy reliability, quality and market requirements, while trying to limit additional investments in transmission and production capacities. Such requirements result in DPUs working close to the boundary of technical possibilities while relying on improving classic or developing novel power system methods. One such method is reactive power optimization (RPO). By controlling reactive power compensators and transformers, RPO helps reduce resistive power losses, control system voltage levels and improve power factors [1], [2].

Reactive power optimization has been the focus of numerous recent papers which investigate various problems in distribution networks. For instance, an RPO centralized system is defined in order to maintain targeted bus voltages and power factors as well as reduce power cable losses [1]. A two-stage robust RPO is defined for minimizing power losses while considering uncertain wind power generation [2]. Another RPO which relies on distributed generators to uphold the requirements set by power markets is formulated in [3]. Voltage regulation can be achieved by optimally setting reactive power of distributed energy sources [4]. Finally, RPO through STATCOM can play a role in planning of distribution networks with integrated electric vehicles [5].

This paper focuses on one typical usage of RPO—controlling voltage levels by trying to keep line voltage profiles as flat as possible. In other words, the goal is to keep voltages at all nodes as close to the nominal values as possible. Achieving such a requirement can lead to two notable advantages: 1) better economic operation under normal system conditions [6]; and 2) better voltage based load reduction when needed [7]. The first advantage comes from the fact that the vast majority of consumers are modeled for maximized efficiency when the provided voltage is close to the nominal value. Furthermore, maximum efficiency minimizes the cost of a kilowatt-hour for the consumer and the providing/delivering company [6]. The second advantage is due to the fact that most load reduction procedures focus on reducing voltage levels, either short- or long-term time periods [7], [8]. Thus, a flat voltage profile is desirable, as voltage at every node can be lowered by the same amount without breaking any constraints.

Two different RPO-based algorithms, formulated as optimizing available voltage regulation devices (VRD), are proposed in this paper:

1. Practical algorithm—focuses on finding a solution which can practically be achieved by also providing a list of steps towards achieving it.
2. Global optimal algorithm—focuses on finding the global optimal solution, but does not provide the steps for achieving it. The original problem is reformulated as a convex one.

The remainder of this paper is organized as follows. Section II formulates the problem of achieving a flat voltage profile by defining its cost function and all corresponding constraints. Two proposed algorithms are described in Section III. Numerical results and their analysis are performed in Section IV. Conclusion in Section V is followed by a list of references.

II. Problem Formulation

The problem is defined as achieving a flat voltage profile by optimizing the statuses of the following VRD [6]:

- Capacitor banks (CB)—used for reactive power series compensation of lines. Status of a CB can either be on or off.
- Load-tap-changing transformers (LTC)—designed for large voltage adjustments, like changes in voltage levels, defined by the turn ratio. This ratio can be changed by shifting tap changer positions which present the status of a LTC.
- Regulating transformers (RT)—designed for small voltage adjustments, defined by the turn ratio. This ratio can be changed by shifting tap changer positions, which present the status of a RT.

Mathematically, this can be formulated as an optimization problem, with the goal of minimizing node voltage deviations from the nominal value with respect to node priorities:

$$\mathrm{C}(\boldsymbol{x}_{\mathrm{VRD}}) = \min_{\boldsymbol{x}_{\mathrm{VRD}}} \left\{ \sum_{i=1}^{N} w_i \cdot (V_n - V_i)^2 \right\}, \tag{1}$$

where:

$\mathrm{C}(\cdot)$ – cost function;

This work has been supported by NSF under grant ECCS-1710944, by CURENT Engineering Research Center of the National Science Foundation and the Department of Energy under NSF Award Number EEC-1041877, and by ONR under grant N00014-16-1-3028.

978-1-7281-0407-2/19/$31.00 ©2019 IEEE

$\boldsymbol{x}_{\text{VRD}}$ –vector of controlled variables (VRD statuses);
N – number of nodes in a network;
V_n – nominal voltage value, taken as 1 relative unit (r.u.);
V_i – voltage value at node i;
w_i – priority of node i.

Three node types and corresponding priorities are examined: 1) nodes with low priority (no consumers); 2) nodes with normal priority consumers (such as households) and 3) nodes with high priority consumers (such as hospitals). Priorities are presented by different weights (i.e. high weight for high priority level). Note that the result of this optimization should provide the optimal value of $\boldsymbol{x}_{\text{VRD}}$, denoted as $\boldsymbol{x}^*_{\text{VRD}}$.

Additionally, as we want to make the solution applicable in real-life distribution system implementation, certain practical constraints must be included. These are split into two groups:

- Basic constraints—due to standards set for an observed network. In this paper, the focus is on the maximum allowed voltage deviation from the nominal value:

$$V^{\min} \le V_i \le V^{\max}, \forall i, \tag{2}$$

  where $V^{\min}$ and $V^{\max}$ present the minimum and maximum allowed voltages, respectively.
- Technical constraints—due to limitations of the VRD:

$$T_{\text{LTC}i} \in \left(p_1^{\text{LTC}}, \ldots, p_{ki}^{\text{LTC}}\right), \forall i; \tag{3}$$

$$T_{\text{RT}i} \in \left(p_1^{\text{RT}}, \ldots, p_{ki}^{\text{RT}}\right), \forall i; \tag{4}$$

$$S_{\text{CB}i} \in \left(0, 1\right), \forall i. \tag{5}$$

where:
$T_{\text{LTC}i}$ – tap changer position (status, p^{LTC}) of LTC i.
$T_{\text{RT}i}$ – tap changer position (status, p^{RT}) of RT i.
$k\ i$ – number of tap changer positions for a LTC/RT;
$S_{\text{CB}i}$ – status of CB i (0 – off; 1 – on).

Note that statuses can only be integer values and are ordered from the lowest value with an increment of 1 (i.e., $p_j^{\text{LTC}} = p_{j-1}^{\text{LTC}} + 1$, and $p_j^{\text{RT}} = p_{j-1}^{\text{RT}} + 1$);

It is important to note the following factors which further complicate solving the problem formulated by (1)-(5):

- Acquiring voltages at each node has to be done numerically by solving Power Flow equations—a set of nonlinear, simultaneous equations which can only be solved iteratively [6]. Thus, a closed form solution for node voltages, and consequentially the cost function (1), cannot be derived.
- Due to constraints (3)-(5), this optimization is an integer problem and hence is non-convex [9].

III. Practical and Global Optimal Algorithms

For the formulated problem, two algorithms, which focus on different solutions, are proposed:

- Practical algorithm (denoted PA, Section III.A)—a practically achievable solution is derived, but might not be the best possible.
- Global optimal algorithm (denoted GOA, Section III.B)—the best possible solution is derived, but might not be practically achievable.

So, even though GOA will provide the global optimal solution, certain practical factors favor PA, like the actual transition from the current to the optimal state. Practically, determining only the final optimal solution is not sufficient—to fully solve the problem we need to know how to reach it from the current state, without violating constraints (2)-(5) in any step. Thus, we need to know in what order to change the statuses of VRD—form the Switching Order List (SOL), which is possible in PA, but not in GOA. This is where the practical problem of GOA lies, as without the SOL we cannot guarantee that the solution is achievable without breaking any constraints in the process.

For both solutions, it is assumed that the initial state is known, including the voltages at every node, state of each voltage regulation device, and value of the cost function. It is also assumed that no constraints are broken in the initial state.

A. Practical Algorithm (PA)

The idea is to check if there exists a change in a certain VRD which will not break any constraints while also lowering the value of the cost function—switching a CB on/off or changing the tap changer position of a LTC/RT by one position. If so, apply it, note it into the SOL and try again. If multiple such changes exits, utilize the one which lowers the cost function the most. This is done until no such change exists. As a result, both the practical optimal solution and the SOL are attained.

Next, the algorithm implementation is described in detail, where the employed notation is as follows:

$\boldsymbol{x}_{\text{VRD}}^{\text{initial}}$ – vector of initial VRD states;

Algorithm 1 Practical Algorithm (PA)

initialization:
  - Load the examined network parameters;
  - $\hat{\text{C}} = \text{C}(\boldsymbol{x}_{\text{VRD}}^{\text{initial}})$; $\boldsymbol{x}^*_{\text{VRD}} = \boldsymbol{x}_{\text{VRD}}^{\text{initial}}$; k=size of $\boldsymbol{x}^*_{\text{VRD}}$;
  repeat
    SOL_step = 0;
    for $l = 1{:}k$
      if $\boldsymbol{x}^*_{\text{VRD}} = \boldsymbol{x}^*_{\text{VRD}} + \boldsymbol{e}_l$ is possible (3)-(5) **then**
        Calculate $\text{C}(\hat{\boldsymbol{x}}_{\text{VRD}} + \boldsymbol{e}_l)$ (1);
        if $\text{C}(\boldsymbol{x}^*_{\text{VRD}} + \boldsymbol{e}_l) < \hat{\text{C}}$ **then**
          $\hat{\text{C}} = \text{C}(\boldsymbol{x}^*_{\text{VRD}} + \boldsymbol{e}_l)$; SOL_step= $+\boldsymbol{e}_l$;
        end
      end
      if $\boldsymbol{x}^*_{\text{VRD}} = \boldsymbol{x}^*_{\text{VRD}} - \boldsymbol{e}_l$ is possible (3)-(5) **then**
        Calculate $\text{C}(\boldsymbol{x}^*_{\text{VRD}} - \boldsymbol{e}_l)$ (1);
        if $\text{C}(\boldsymbol{x}^*_{\text{VRD}} - \boldsymbol{e}_l) < \hat{\text{C}}$ **then**
          $\hat{\text{C}} = \text{C}(\boldsymbol{x}^*_{\text{VRD}} - \boldsymbol{e}_l)$; SOL_step= $-\boldsymbol{e}_l$;
        end
      end
    end
    $\boldsymbol{x}^*_{\text{VRD}} = \boldsymbol{x}^*_{\text{VRD}} + \text{SOL\_step}$;
    Denote SOL_step as the next step in SOL;
    Stopping criterion: SOL_step=0;
  until *stopping criterion achieved*;

$\boldsymbol{e}_l$ – lth unit vector;

$\hat{C}$ – cost function evaluated at $\hat{\boldsymbol{x}}_{\text{VRD}}$.

B. Global Optimal Algorithm (GOA)

The second proposed algorithm tries to solve this problem as a convex optimization [9]. As a result, the global optimal solution is attained but not the SOL.

To design an algorithm solving an optimization problem, derivatives are needed, in our case the Hessian matrices and gradient vectors [9]. Since there are no closed form functions for node voltages (as discussed in Section II), such derivative values cannot be calculated analytically. Rather, numerical differentiation has to be used for corresponding gradient vectors and Hessian matrices, respectively [10]:

$$\nabla f(\boldsymbol{x}) = \begin{bmatrix} \dfrac{f(\boldsymbol{x}+\varepsilon_1\boldsymbol{e}_1) - f(\boldsymbol{x}-\varepsilon_1\boldsymbol{e}_1)}{2\varepsilon_1} \\ \vdots \\ \dfrac{f(\boldsymbol{x}+\varepsilon_k\boldsymbol{e}_k) - f(\boldsymbol{x}-\varepsilon_k\boldsymbol{e}_k)}{2\varepsilon_k} \end{bmatrix}; \quad (6)$$

$$\nabla^2 f^{\mathrm{T}}(\boldsymbol{x}) = \begin{bmatrix} \dfrac{\nabla f(\boldsymbol{x}+\varepsilon_1\boldsymbol{e}_1) - \nabla f(\boldsymbol{x}-\varepsilon_1\boldsymbol{e}_1)}{2\varepsilon_1} \\ \vdots \\ \dfrac{\nabla f(\boldsymbol{x}+\varepsilon_k\boldsymbol{e}_k) - \nabla f(\boldsymbol{x}-\varepsilon_k\boldsymbol{e}_k)}{2\varepsilon_k} \end{bmatrix}, \quad (7)$$

where

$f(\cdot)$ and $\boldsymbol{x}$ – arbitrary function and variables, respectively;

$\varepsilon_i = \left|\boldsymbol{x}_i\right| \cdot 10^{-8}$ – small deviation from $\boldsymbol{x}_i$;

k – size of $\boldsymbol{x}$.

Furthermore, for the problem defined in Section II to be convex, the following conditions must be true [9]:

- Hessian matrix of the cost function (1) must be positive definite. Note that a formal proof is not provided, but the requirement is verified numerically to be met for all examined networks (see Section IV).
- All constraints must be convex. Thus, all practical constraints (2)-(5) are redefined as follows.

$$U^{\min} \le U_i \le U^{\max}, \quad \rightarrow \quad \begin{matrix} U_i \ge U^{\min}, \forall i \\ U_i \le U^{\max}, \forall i \end{matrix}; \quad (8)$$

$$T_{\text{LTC}i} \in \left(p_1^{\text{LTC}}, \ldots, p_{ki}^{\text{LTC}}\right), \forall i; \quad \rightarrow \quad \begin{matrix} T_{\text{LTC}i} \ge p_1^{\text{LTC}}, \forall i \\ T_{\text{LTC}i} \le p_{ki}^{\text{LTC}}, \forall i \end{matrix}; \quad (9)$$

$$T_{\text{RT}i} \in \left(p_1^{\text{RT}}, \ldots, p_{ki}^{\text{RT}}\right), \forall i; \quad \rightarrow \quad \begin{matrix} T_{\text{RT}i} \ge p_1^{\text{RT}}, \forall i \\ T_{\text{RT}i} \le p_{ki}^{\text{RT}}, \forall i \end{matrix}; \quad (10)$$

$$S_{\text{CB}i} \in \left(0, 1\right), \forall i; \quad \rightarrow \quad \begin{matrix} S_{\text{CB}i} \ge 0, \forall i \\ S_{\text{CB}i} \le 1, \forall i \end{matrix}, \quad (11)$$

where the set of all these inequality functions will be denoted $g(\cdot)$. Note that a relaxation of the integer constraints (3)-(5) is used here, which will be compensated in the *Rounding up* step.

Taking all this into account, the GOM can be formulated as a convex optimization problem, for which we design an algorithm consisting of:

- *Outer loop*—solved by the Barrier interior point method [9].
- *Inner loop*—used to execute the centering step of the Barrier method (*Outer loop*). Solved by the Newton's method [9].
- *Backtracking line search*—used to calculate the step size t_{IL} for the Newton's method (*Inner loop*).
- *Rounding up*—since the voltage regulation devices can only take certain integer values, which was relaxed by (9)-(11), the calculated optimal value must be "rounded". That is, every value in $\boldsymbol{x}_{\text{VRD}}^{*}$ is rounded to the nearest integer permitted by (3)-(5). Note that this rounding might lead us away from the optimal solution, which is discussed in Section IV.A.

Next, the algorithm implementation is described in detail, where the employed notation is as follows:

t_{OL} – *Outer loop* parameter;
ξ_{outer} – parameter for *Outer loop* convergence testing;
t_{IL} – *Inner loop* parameter;
ξ_{inner} – parameter for *Inner loop* convergence testing;
μ – step parameter for increasing t_{OL};
α, β – constants needed for Backtracking line search;
$f_{\text{CS}}(\cdot)$ – cost function for the centering step;
m – number of constraints;

Algorithm 2 Global Optimal Algorithm (GOA)

initialization:
  - Load the examined network parameters;
  - $t_{\text{OL}} = 1$; $\xi_{\text{outer}} = 10^{-3}$; $\xi_{\text{inner}} = 10^{-3}$; $\mu = 10$;
  - $\boldsymbol{x}_{\text{VRD}}^{*} = \boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}}) = \boldsymbol{x}_{\text{VRD}}^{\text{initial}}$; $\boldsymbol{x}_{\text{VRD}}^{*}$
  - α=0.1; β=0.8;
repeat (*Outer loop*)
  Formulate the centering step equation
  $f_{\text{CS}} = t_{\text{OL}} \cdot \text{C}(\boldsymbol{x}_{\text{VRD}}) - \sum_{i=1}^{m} \log(-g_i(\boldsymbol{x}_{\text{VRD}}))$;
  repeat *(Inner loop*)
    Calculate ∇f_{CS} (6) and $\nabla^2 f_{\text{CS}}$ (7);
    Compute Newton step $\Delta\boldsymbol{x}_{\text{nt}} = -\nabla^2 f_{\text{CS}}^{-1} \cdot \nabla f_{\text{CS}}$;
    Compute Newton decrement
    $\lambda^2 = \nabla f_{\text{CS}}^{\mathrm{T}} \cdot \nabla^2 f_{\text{CS}}^{-1} \cdot \nabla f_{\text{CS}}$;
    Stopping criterion 1: $\lambda^2 / 2 \le \xi_{\text{inner}}$;
  until *stopping criterion 1 achieved*
  $t_{\text{IL}} = 1$;
  repeat (*Backtracking line search*)
    Stopping criterion 2:
    $f_{\text{CS}}(\boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}}) + t_{\text{IL}}\Delta\boldsymbol{x}_{\text{nt}}) \le$
    $\le f_{\text{CS}}(\boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}})) + \alpha t_{\text{IL}} \nabla f_{\text{CS}}^{\mathrm{T}}(\boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}}))\Delta\boldsymbol{x}_{\text{nt}}$;
    $t_{\text{IL}} = t_{\text{IL}} \cdot \beta$;
  until *stopping criterion 2 achieved*;
  Update $\boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}}) = \boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}}) + r \cdot \Delta\boldsymbol{x}_{\text{nt}}$;
  Update $\boldsymbol{x}_{\text{VRD}}^{*} = \boldsymbol{x}_{\text{VRD}}^{\text{OL}}(t_{\text{OL}})$;
  Stopping criterion 3: $m / t_{\text{OL}} \le \xi_{\text{outer}}$;
  Increase $t_{\text{OL}} = \mu \cdot t_{\text{OL}}$;
until: *stopping criterion 3 achieved*;

$g_i(\cdot)$ – constraint function i (8)-(11);
$x_{\text{VRD}}^{\text{OL}}(t_{\text{OL}})$ – optimal point for t_{OL} after the centering step;
Δx_{nt} – Newton step;
$\nabla^2 f_{\text{CS}}$ – Hessian matrix of f_{CS};
∇f_{CS} – gradient vector of f_{CS};
λ – Newton decrement.

IV. Numerical Results and Analysis

Application of both PA and GOA is shown on a single feeder distribution test network (Section IV.A) and on a distribution test network with laterals (Section IV.B). Both networks are modeled in MATLAB. For all examples, the following should be noted:

- voltage is limited by ±10% of the nominal value (2);
- $T_{\text{LTC}} \in (-16, \ldots, +16)$ for all LTCs (3);
- $T_{\text{RT}} \in (-16, \ldots, +16)$ for all RTs (4);
- by utilizing MATLABs' built-in function for Cholesky decomposition ("chol"), Hessian matrices of all cost functions have been verified to be positive definite (as required for convex problems, Section III.B).

A. *Single feeder test network*

The test network is shown in Fig. 1. It consists of 30 equal branches (R=1.077 Ω, X=0.737 Ω, B=118.5 µS). At the end of each branch there is a consumer, modeled as constant current load (I=const.; I=3.5 A, cosφ=0.95 ind.). A LTC transformer (110/23±16×0.625% kV/kV) is set at the beginning of the network, 2 voltage regulators (23/23±16×0.625% kV/kV) are set between nodes 9 and 10 and nodes 19 and 20, and 6 equal CB (b_{CB}=1 mS) are set at nodes 3, 7, 13, 17, 23 and 27.

Taking this into account, the vector of VRD statuses is:

$$x_{\text{VRD}} = \left[S_{\text{CB}}, T_{\text{LTC1}}, T_{\text{RT}} \right], \tag{12}$$

where:

$$S_{\text{CB}} = [S_{\text{CB1}}, S_{\text{CB2}}, S_{\text{CB3}}, S_{\text{CB4}}, S_{\text{CB5}}, S_{\text{CB6}}]; \tag{13}$$

$$T_{\text{RT}} = [T_{\text{RT1}}, T_{\text{RT2}}]. \tag{14}$$

Four examples are considered here: 1) Main example; 2) Slight change in initial VRD statuses; 3) Same results from PA and GOA; 4) Introducing high priority nodes.

Example 1: Main example

Initial conditions for this example are:

- $x_{\text{VRD}}^{\text{initial}} = [0,0,0,0,0,0,-2,-5,-4]$;
- node priority $w_i = 1, \forall i$.

The following voltage profiles are shown on Fig. 2: initial, after using PA and after using GOA. Note that on average a more flat voltage profile is achieved with GOA, which is expected as it gives the global optimal solution. To better observe GOA convergence, the duality gap versus iterations is shown in Fig. 3, where a near linear convergence can be seen.

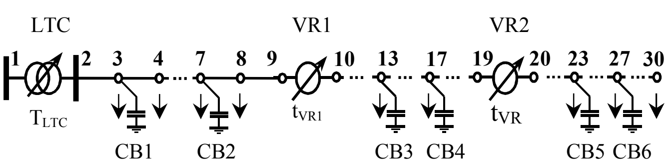

Figure 1. Single feeder test network

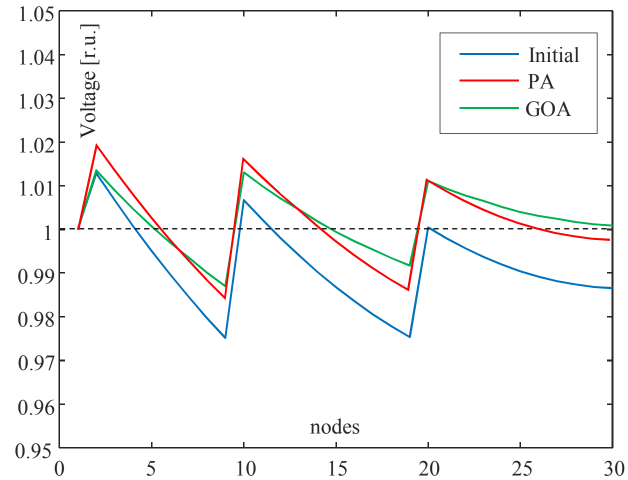

Figure 2. Voltage profiles – Initial and after using GOA

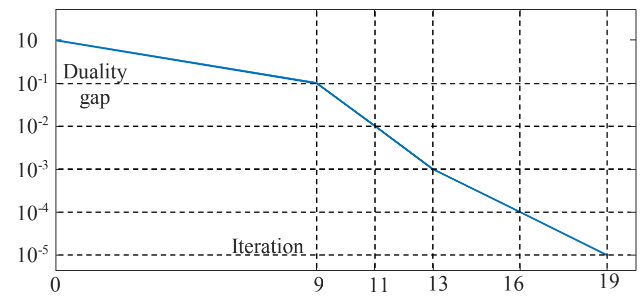

Figure 3. Duality gap for GOA

Numerical results are as follows:

- Initial cost function $C(x_{\text{VRD}}^{\text{initial}}) = 0.0572$.
- PA: optimal VRD is $\hat{x}_{\text{VRD}}^{\text{PA}} = [0,0,1,0,0,0,-3,-5,-4]$. Cost function is $C(\hat{x}_{\text{VRD}}^{\text{PA}}) = 0.0332$, which is 41.9% better than the initial value.
- GOA: optimal VRD is $\hat{x}_{\text{VRD}}^{\text{GOA}} = [0,1,1,1,1,1,-2,-4,-3]$. Cost function $C(\hat{x}_{\text{VRD}}^{\text{GOA}}) = 0.0272$, which is 52.4% better than the initial value and 18.1% than the PA solution.

Example 2: Slight change in initial VRD statuses

Initial conditions for this example are:

- $x_{\text{VRD}}^{\text{initial}} = [0,0,0,0,0,1,-2,-5,-4]$;
- node priority $w_i = 1, \forall i$.

Note that the only difference is in S_{CB6}. Values of the cost function [$C(x_{\text{VRD}}^{*})$] and how much that value is lower than the initial one [$\Delta C(x_{\text{VRD}}^{*})$] is given in Table I.

TABLE I. Proposed Algorithms Results, Example 2

	Initial	PA	GOA
$C(x_{\text{VRD}}^{*})$	0.0392 [r.u.]	0.0291 [r.u.]	0.0272 [r.u.]
$\Delta C(x_{\text{VRD}}^{*})$	0.00 %	25.7 %	30.5 %

Comparing results from Table I to the previous example, we can conclude the following:

- A significant difference in PA results exist. This is due to the possibility of a different SOL, and thus final solution, based on initial VRD statuses.

- GOA results are the same as the global optimal solution does not depend on initial VRD statuses.

Example 3: Same results for PA and GOA

Initial conditions for this example are:

- $\boldsymbol{x}_{\text{VRD}}^{\text{initial}} = [0,1,1,1,0,0,-2,-2,-2]$;
- node priority $w_i = 1, \forall i$.

Values of the cost function and how much that value is lower than the initial cost function value are given in Table II.

TABLE II. PROPOSED ALGORITHMS RESULTS, EXAMPLE 3

	Initial	PA	GOA
$\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0114 [r.u.]	0.0272 [r.u.]	0.0272 [r.u.]
$\Delta\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	76.1 [r.u.]	76.1 %

Due to specific initial VRD statuses, PA is able to form such a SOL which will lead it to the same solution as GOA—global optimal solution (Table II). Or rather, $\hat{\boldsymbol{x}}_{\text{VRD}}^{\text{POA}} = \hat{\boldsymbol{x}}_{\text{VRD}}^{\text{GOA}} = [0,1,1,1,1,1,-2,-4,-3]$.

Example 4: Introducing high priority nodes

Initial VRD are set to $\boldsymbol{x}_{\text{VRD}}^{\text{initial}} = [0,0,0,0,0,0,-2,-5,-4]$. Two case will be examined. First, we set node priorities as:

- node priority $w_i = 1, \forall i$ except $w_5 = w_6 = w_{15} = 5$.

Values of the cost function and how much that value is lower than the PA cost function value are given in Table III.

TABLE III. PROPOSED ALGORITHMS RESULTS, EXAMPLE 4, CASE 1

	Initial	PA	GOA
$\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.1831 [r.u.]	0.0508 [r.u.]	0.0414 [r.u.]
$\Delta\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	72.2 %	77.4 %

Note the following (Table III):

- Even though high priority nodes are introduced, both algorithms improve the value of the cost function, with GOA being more effective.
- GOA results (global optimal solution) differ than the previous example, as high priority nodes are introduced.

Another case is where we set node priorities as follows: $w_i = 1, \forall i$ except $w_3 = w_{12} = w_{28} = 10$ .Values of the cost function and how much that value is lower than the PA cost function value are given in Table III. Note that for this example, results of GOA are given both before (no RU) and after (RU) the *Rounding up* step.

TABLE IV. PROPOSED ALGORITHMS RESULTS, EXAMPLE 4, CASE 2

	Initial	PA	GOA (no RU)	GOA (RU)
$\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.1546 [r.u.]	0.0870 [r.u.]	0.0427 [r.u.]	0.1151 [r.u.]
$\Delta\text{C}(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	43.7 %	72.4 %	25.6 %

Note that again both algorithms improve the cost function value, but this time PA being more effective. This is due to the *Rounding up* step not taking into account node priorities (Section III.B)—note that before rounding up, GOA gives significantly better results (Table IV).

The effect of rounding up on the cost function is shown in Fig. 4. This is done by moving VRD values derived by GOA (no RU) towards the rounded up values in small increments (100% means VRD values have been rounded up). Or rather, the vertical axis presents $\text{C}(\boldsymbol{x}_{\text{VRD}})$, while the horizontal axis presents parameter *a*, where:

$$\boldsymbol{x}_{\text{VRD}} = \boldsymbol{x}_{\text{VRD}}^{*\,\text{no RU}} + (\boldsymbol{x}_{\text{VRD}}^{*\,\text{RU}} - \boldsymbol{x}_{\text{VRD}}^{*\,\text{no RU}}) \cdot a \tag{15}$$

Note that even though best results are achieved before rounding up, due to technical constraint (3)-(5), final results must be rounded up to integers. The effect of this rounding is an increase in the cost function which is captured in Fig. 4 in relation to distance of VRD statuses from $\boldsymbol{x}_{\text{VRD}}^{*\,\text{no RU}}$.

B. *Test network with laterals*

Verification of the proposed algorithms is also shown on a distribution test network with laterals shown in Fig. 5, whose details are given in [7].

Notice one main feeder (nodes 1-16) and 5 laterals in the test network. Also, the vector of VRD statuses is:

$$\boldsymbol{x}_{\text{VRD}} = \left[S_{\text{CB1}}, S_{\text{CB2}}, S_{\text{CB3}}, S_{\text{CB4}}, S_{\text{CB5}}, T_{\text{LTC1}}, t_{\text{VR1}}\right] \tag{15}$$

Two examples are considered here: 1) Main example; and 2) Consumer model type influence. For both examples, the initial conditions are the same:

- $\boldsymbol{x}_{\text{VRD}}^{\text{initial}} = [0,0,0,0,0,0,-12]$;
- node priority $w_i = 1, \forall i$.

Example 1: Main example

The following voltage profiles are shown on Fig. 6: initial, after using PA and after using GOA. To better observe GOA convergence, the duality gap versus iterations is shown in Fig. 7, where a near linear convergence can be seen.

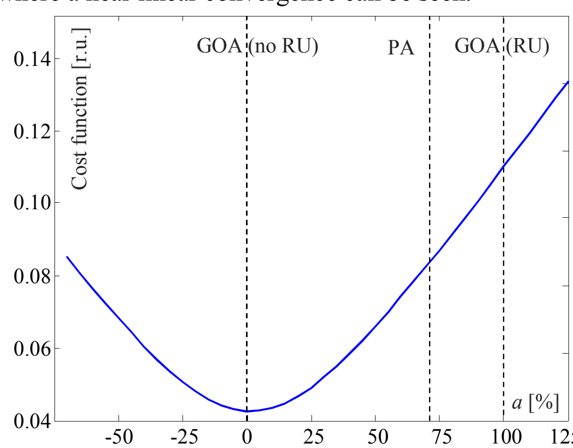

Figure 4. Duality gap for GOM

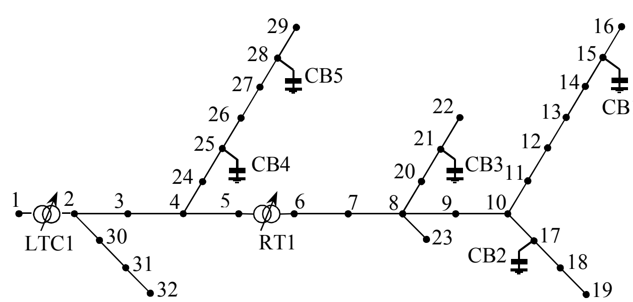

Figure 5. Test network with laterals

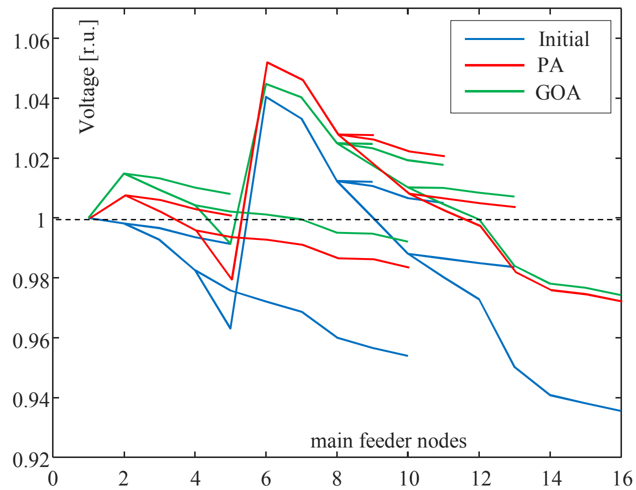

Figure 6. Voltage profiles – Initial, after using PA and after using GOA

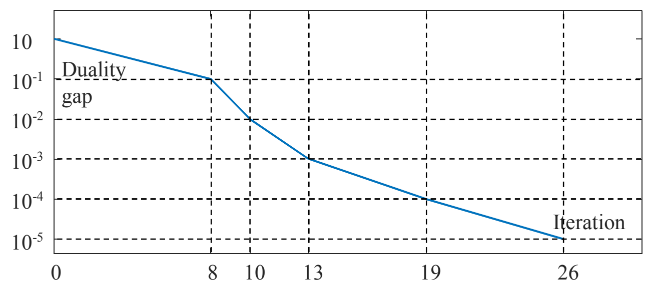

Figure 7. Duality gap for GOA

Numerical results are as follows:

- Initial cost function $C(\boldsymbol{x}_{\text{VRD}}^{\text{initial}}) = 0.1544$.
- PA: optimal VRD is $\boldsymbol{x}_{\text{VRD}}^{*\,\text{PA}} = [1, 0, 0, 1, 1, -1, -1 1]$. Cost function is $C(\boldsymbol{x}_{\text{VRD}}^{*\,\text{PA}}) = 0.0774$, which is 49.8% better than the initial value.
- GOA: optimal VRD is $\boldsymbol{x}_{\text{VRD}}^{*\,\text{GOA}} = [1, 1, 0, 1, 1, -2, -8]$. Cost function $C(\boldsymbol{x}_{\text{VRD}}^{*\,\text{GOA}}) = 0.0675$, which is 56.3% better than the initial value and 12.8% than the PA solution.

Example 2: Consumer model type influence

The following consumer types are examined here:

- Constant power (S=const.), whose load consumption does not depend on corresponding node voltage;
- Constant current magnitude and power factor (I=const.), whose load consumption depends linearly on corresponding node voltage;
- Constant admittance (Y=const.), whose load consumption depends quadratically on corresponding node voltage.

Note that the load values are chosen such that the initial network state is the same regardless of consumer type. Values of the cost function $[C(\boldsymbol{x}_{\text{VRD}}^{*})]$ and how much that value is lower than the PA cost function value $[\Delta C(\boldsymbol{x}_{\text{VRD}}^{*})]$, for different types of consumers, are given in Table V. What should be concluded from this example is that even though the initial states are the same (voltage values and VRD states) the final results depend significantly on how the consumers have been modeled. Such results are due to the noted different load consumption dependency of different load model types.

TABLE V. CONSUMER TYPE INFLUENCE

		PA	GOA (no rounding up)	GOA (rounded up)
S=const	$C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0679 [r.u.]	0.0582 [r.u.]	0.0603 [r.u.]
	$\Delta C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	16.5 %	13.5 %
I=const	$C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0775 [r.u.]	0.0675 [r.u.]	0.0699 [r.u.]
	$\Delta C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	12.9 %	9.8 %
Y=const	$C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0725 [r.u.]	0.0648 [r.u.]	0.0675 [r.u.]
	$\Delta C(\boldsymbol{x}_{\text{VRD}}^{*})$	0.0 %	9.4 %	5.6 %

V. CONCLUSION

We presented two reactive power optimization based algorithms for flat voltage profiles in a distribution network, formulated as optimizing the usage of available voltage regulation devices. The first algorithm is shown to find a practical solution, which might not be the best one but can practically be achieved. The second algorithm is shown to find the global optimal solution, by reformulating the problem as a convex one; however, this solution might not be practically achievable due to the lack of operation steps to get there. Numerical results and the advantages and disadvantages of both algorithms are illustrated and discussed on two distribution test networks. To better improve the second algorithm, further work should involve forming a Switching Order List and taking weights into account when rounding results.

REFERENCES

[1] S. X. Chen, Y. S. Foo. Eddy, H. B. Gooi, M. Q. Wang and S. F. Lu, "A Centralized Reactive Power Compensation System for LV Distribution Networks," in *IEEE Trans. on Power Systems*, vol. 30, no. 1, pp. 274-284, Jan. 2015.

[2] T. Ding, S. Liu, W. Yuan, Z. Bie and B. Zeng, "A Two-Stage Robust Reactive Power Optimization Considering Uncertain Wind Power Integration in Active Distribution Networks," in *IEEE Transactions on Sustainable Energy*, vol. 7, no. 1, pp. 301-311, Jan. 2016.

[3] K. Zhou, H. Tang, T. Liu, F. Xiong and L. Wang, "Reactive power optimization in smart distribution network under power market," *2015 5th Int. Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT)*, Changsha, 2015, pp. 663-666.

[4] Z. Tang, D. J. Hill and T. Liu, "Fast Distributed Reactive Power Control for Voltage Regulation in Distribution Networks," in *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 802-805, Jan. 2019.

[5] A. H. Zaidi, K. Sunderland and M. Conlon, "Role of reactive power (STATCOM) in the planning of distribution network with higher EV charging level," in *IET Generation, Transmission & Distribution*, vol. 13, no. 7, pp. 951-959, 9 4 2019.

[6] J.J. Grainger, W.D. Stevenson Jr., *Power System Analysis*, McGraw-Hill Education, 1994.

[7] V.Svenda: "Practical Load Reduction in Distribution Network"; *2016 IEEE PES General Meeting*, 17-21 July 2016, Boston, MA, USA.

[8] Z.Wang, J.Wang, "Rewiew on Implementation and Assessment of Coservation Voltage Reduction", *IEEE Trans. on Power Systems*, vol. 29, No.3, May 2014, pp. 1306-1315

[9] S. Boyd and L. Vandenberghe, *Convex Optimization,* Cambridge University Press, March 2004.

[10] R.L. Burden and J.D. Faires "Numerical Differentiation and Integration," in *Numerical Analysis*, 7th edition, Brooks Cole, 2000, pp. 166-251.