Simplified Equations for Shear-Modulus Degradation and Damping of Gravels

Kyle M. Rollins, M.ASCE[1]; Manali Singh[2]; and Jashod Roy[3]

Abstract: Two of the most important parameters in any dynamic analysis involving soils are the shear modulus and damping ratio. Based on lab tests on gravels from 18 investigations, simplified equations to define $G/G_{\max}$ and the damping ratio as a function of shear strain, γ, have been developed. The $G/G_{\max}$ versus γ equations rely on two parameters that can be defined in terms of confining pressure and uniformity coefficient. Increasing confining pressure leads to a more linear curve, while increasing the uniformity coefficient leads to a more nonlinear curve shape. $G/G_{\max}$ versus γ curves for gravels tend to plot somewhat below curves for sands under similar conditions. Estimates of the standard deviation in $G/G_{\max}$ versus γ curves are provided to consider scatter about the mean. The damping ratio versus γ equation employs the modified Masing approach with a minimum damping ratio of 1%. In addition, about 67% of the damping data points fall within an error band of $\pm 33\%$ from the computed value. The damping ratio of gravel specimens also decreases as the confining pressure increases, whereas it increases for higher uniformity coefficients. Other direct correlations between damping ratio and factors such as shear strain, uniformity coefficient, and confining pressure did not provide significant improvements in predictive capacity. **DOI: 10.1061/(ASCE)GT.1943-5606.0002300.**

Author keywords: Gravels; Shear modulus; Damping; Cyclic shear testing; Uniformity coefficient.

Introduction

Dynamic soil response is of considerable importance for loadings produced by earthquakes, machine foundations, wind, waves, and impacts. Two of the most important parameters in any dynamic analysis involving soils are the shear modulus and the damping ratio. Both the shear modulus, G, and damping ration, D, are dependent on the cyclic shear strain, γ. At very low shear strain levels (less than $10^{-4}\%$), which are typical of foundation vibrations problems, G and D remain essentially constant. However, for earthquake problems, the strain levels can be much higher and the variation of G and D with shear strain must be considered.

Rollins et al. (1998) summarized available test results involving gravel specimens and developed mean curves for $G/G_{\max}$ versus γ and D versus γ for gravels. Shortly thereafter, Stokoe et al. (1999) developed a modified hyperbolic formula that facilitates the definitions of G and D with shear strain. This approach also makes it much easier to evaluate the effect of various parameters on the curve shapes and incorporate them into the equation. This paper updates the data set collected by Rollins et al. (1998) with new test results completed since then. The paper then uses the data set to define the variation of normalized shear modulus and damping with shear strain and other parameters using the modified hyperbolic approach developed by Stokoe et al. (1999).

Characteristics of Gravel Test Data Set

A summary of the basic characteristics of the gravel data set and the investigators involved is provided in Table 1. The data are listed alphabetically by author. Relative densities of the gravels ranged from 27% to 95%, the maximum grain size varied from 10 to 150 mm, coefficients of uniformity ranged from 1.33 to 83.3, and the percentage of gravel size particles varied from 40% to 90%. The large range in these basic mechanical properties should facilitate the evaluation of the effects of these properties on G and D relationships. Shear modulus and damping measurements were typically performed with large diameter cyclic triaxial shear devices (30 cm diameter and 60 cm height) or cyclic torsional shear devices, and additional details regarding the data set and testing procedures are provided in Rollins et al. (1998), Menq (2003), Lo Presti et al. (2006), and Zou et al. (2012).

Particle size distribution curves for each of the specimens are summarized in Fig. 1. Generally, the specimens are classified as gravelly soil according to the percentage greater than 4.75 mm. However, some of the studies (Shamoto et al. 1986; Shibuya et al. 1990; Goto et al. 1994), which are classified as gravel specimens by their authors, use a 2-mm criterion that is quite widely used throughout the world. These data have been included in cases where the data set is relatively sparse and might otherwise be dominated by one investigator. Fines content for the overall data set ranged from 0% to 9% but was typically less than 5%. Therefore, this data set does not account for the potential effects of high fines content or plasticity on the dynamic behavior of gravel.

Approximately 58% of the test results in this study come from cyclic triaxial (CTX) shear tests, 20% come from torsional resonant column (RC) tests, and 22% come from cyclic torsional simple shear (CTSS) tests. Although the results have been collected from a variety of testing methods, no dramatic difference is observed

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among the G/G_{max} versus γ curves obtained from the various tests. This observation is consistent with results from previous investigations. For example, Yasuda and Matsumoto (1993) reported that the G/G_{max} relationships for gravels are essentially identical for both CTX and CTSS tests. Rollins et al. (1998) observed no appreciable difference between G/G_{max} relationships determined by CTX and CTSS tests in their study of gravels. In addition, Kokusho (1980) found that G/G_{max} curves from cyclic triaxial tests on sand were consistent with results from torsional simple shear tests obtained by Hardin and Drnevich (1972) and Iwasaki et al. (1978).

Table 1. Soil parameters for cyclic tests and back-calculated hyperbolic parameters

References	Type of tests	Test designations	Uniformity coefficient (C_u)	Confining pressure (σ'_0)	Void ratio (e)	Relative density, (Dr) (%)	Reference shear strain (γ_{ref}) (%)	Exponent (a)	Damping provided?
Evans and Zhou (1995)	Undrained CTX	GC 40%	23.50	100.00	—	—	0.016	0.95	No
	Undrained CTX	GC 60%	30.00	100.00	—	—	0.018	0.95	No
Goto et al. (1992)	CTX	Gravel	25.00	186.00	—	27	0.064	0.95	Yes
	CTX	Gravel	37.50	128.00	—	69	0.037	0.88	Yes
Goto et al. (1994)	CTX	Site B	8.20	118.00	0.39	—	0.035	0.88	Yes
Hatanaka et al. (1988)	Undrained CTX	490.5	13.90	490.50	0.26	69	0.038	0.88	Yes
	Undrained CTX	294.3	60.00	294.30	0.33	57	0.068	0.76	Yes
Hatanaka and Uchida (1995)	CTX	A2(KFU)	64.85	98.00	0.24	—	0.032	0.95	Yes
	CTX	B2(KFL)	37.40	392.30	0.24	—	0.03	0.9	Yes
Hardin and Kalinski (2005)	RC	Crushed lime stone	1.67	10.00	0.73	—	0.01	0.9	No
	RC	Crushed lime stone	0.61	10.00	0.65	—	0.007	0.82	No
	RC	River gravel	1.58	10.00	0.52	—	0.007	0.82	No
	RC	River gravel	2.14	10.00	0.51	—	0.0075	0.82	No
Hynes (1988)	CTX	1	83.33	137.90	0.36	45	0.027	0.82	No
	CTX	2	83.33	206.80	0.37	44	0.024	0.82	No
	CTX	3	83.33	206.80	0.36	45	0.05	0.88	No
	CTX	4	83.33	413.69	0.37	43	0.02	0.98	No
	CTX	5	83.33	206.80	0.37	43	0.05	0.82	No
	CTX	6	83.33	137.90	0.38	40	0.02	0.82	No
	CTX	7	83.33	137.90	0.41	34	0.02	0.75	No
	CTX	8	83.33	137.90	0.45	25	0.02	0.82	No
Kokusho and Tanaka (1994)	CTX	Ksite	37.00	160.00	—	80	0.017	0.75	Yes
	CTX	Ksite	37.00	160.00	—	80	0.025	0.72	Yes
	CTX	A Site	37.00	75.00	—	80	0.02	0.74	Yes
	CTX	A Site	37.00	100.00	—	80	0.042	0.75	Yes
	CTX	A Site	37.00	200.00	—	80	0.058	0.75	Yes
	CTX	A Site	37.00	400.00	—	80	0.063	0.88	Yes
Iida et al. (1984)	CTX	CTX Dia 60	7.20	98.10	0.38	85	0.052	0.88	No
	CTX	CTX Dia 60	7.20	196.20	0.38	85	0.06	0.82	No
	CTX	CTX Dia 60	7.20	294.30	0.38	85	0.061	0.88	No
	CSS	CSS Dia=80	7.20	392.40	0.38	85	0.055	0.88	No
	CSS	CSS Dia=80	7.20	98.10	0.38	85	0.018	0.75	No
	CSS	CSS Dia=80	7.20	196.20	0.38	85	0.028	0.7	No
Lo presti et al. (2006)	RCT, CTX, CTSS	Holocene gravels	25.00	70.33	0.27	77	0.045	0.89	No
Menq (2003)	RC	C1D7	1.42	50.66	0.82	40	0.041	0.83	No
	RC	C1D7	1.42	50.66	0.70	60	0.046	0.84	No
	RC	C1D7	1.42	50.66	0.60	90	0.045	0.88	No
	RC	C1D17	1.10	50.66	0.80	55	0.053	0.72	No
	RC	C1D17	1.10	50.66	0.64	60	0.042	0.82	No
	RC	C1D17	1.10	50.66	0.60	90	0.048	0.92	No
	RC	C3D6	3.09	50.66	—	45	0.026	0.75	No
	RC	C3D6	3.09	50.66	—	45	0.038	0.81	No
	RC	C3D6	3.09	50.66	—	45	0.05	0.83	No
	RC	C8D2	8.70	50.66	—	—	0.04	0.82	No
	RC	C16D3	15.70	50.66	—	37	0.016	0.83	No
	RC	C16D3	15.70	50.66	—	37	0.017	0.83	No
	RC	C16D3	15.70	50.66	—	37	0.028	1.1	No
	RC	C50D3	49.70	50.66	—	—	0.012	0.7	No

Table 1. (*Continued.*)

References	Type of tests	Test designations	Uniformity coefficient (C_u)	Confining pressure (σ'_0)	Void ratio (e)	Relative density, (Dr) (%)	Reference shear strain (γ_{ref}) (%)	Exponent (a)	Damping provided?
Shamoto et al. (1986)	CTX	Gravel	8.50	118.00	—	96	0.038	1.25	No
Shibuya et al. (1990)	CTX	Loose hime gravel	1.33	29.40	0.59	39	0.034	1.4	Yes
	CTX		1.33	49.00	0.58	45	0.034	1.4	Yes
	CTX		1.33	78.50	0.57	50	0.07	1.1	Yes
	CTX	Dense hime gravel	1.33	29.40	0.51	89	0.0267	1.6	Yes
	CTX		1.33	49.00	0.51	89	0.04	1.4	Yes
	CTX		1.33	78.50	0.51	89	0.04	1.4	Yes
Souto et al. (1994)	CTX	Crushed gravel	25.50	100.00	—	—	0.022	0.92	Yes
Yasuda and Matsumoto (1994)	CTX and CTSS	Angular	7.60	100.00	0.33	80	0.041	0.89	Yes
	CTX and CTSS	Angular	7.60	200.00	0.33	80	0.06	0.89	Yes
	CTX and CTSS	Angular	7.60	400.00	0.33	80	0.08	0.89	Yes
	CTX and CTSS	Miho Dam rockfill	7.00	100.00	—	60	0.027	0.71	Yes
	CTX and CTSS	Miho Dam rockfill	7.00	200.00	—	60	0.032	0.7	Yes
	CTX and CTSS	Miho Dam rockfill	7.00	300.00	—	60	0.037	0.7	Yes
	CTX and CTSS	Miho Dam rockfill	7.00	400.00	—	60	0.046	0.7	Yes
	CTX and CTSS	Round	14.28	100.00	0.33	80	0.02	0.7	Yes
	CTX and CTSS	Round	14.28	200.00	0.33	80	0.04	0.7	Yes
	CTX and CTSS	Round	14.28	400.00	0.33	80	0.04	0.7	Yes
	CTX and CTSS	Angular	35.71	100.00	0.33	80	0.03	0.95	Yes
	CTX and CTSS	Angular	35.71	200.00	0.33	80	0.025	0.8	Yes
	CTX and CTSS	Angular	35.71	300.00	0.33	80	0.038	0.8	Yes
	CTX and CTSS	Miho Dam rockfill	36.19	100.00	0.31	—	0.046	0.81	Yes
	CTX and CTSS	Miho Dam rockfill	36.19	200.00	0.31	—	0.05	0.9	Yes
	CTX and CTSS	Miho Dam rockfill	36.19	300.00	0.31	—	0.056	0.83	Yes
	CTX and CTSS	Miho Dam rockfill	36.19	400.00	0.31	—	0.07	0.8	Yes
Zou et al. (2012)	CTX	Sinkiang Dam shell	58.30	200.00	0.20	—	0.032	0.84	No
	CTX	Sinkiang Dam cushion	68.20	300.00	0.23	—	0.052	0.84	No
	CTX	Tibet Dam shell	18.00	100.00	—	—	0.029	0.84	No
	CTX	Tibet Dam foundation	19.00	300.00	—	—	0.04	0.84	No
	CTX	Shuang Jiangkou	38.40	500.00	0.27	—	0.045	0.84	No

G/G_{max} versus Shear Strain Relationships

The variation of shear modulus with cyclic shear strain is customarily represented by dividing the shear modulus, G, at a given strain level by the maximum shear modulus, G_{max}, at very small strains (less than or equal to 10^{-4}%). This normalization process makes it possible to compare the relationships obtained by various investigators, and it also facilitates the use of the relationship in practice. Computer programs that employ the equivalent linear procedure, such as SHAKE (Schnabel et al. 1972), use these curves to ensure that the shear modulus for each soil layer is compatible with the average cyclic shear strain computed in the layer. Even nonlinear programs, such as DEEPSOIL (Park and Hashash 2004) use these curves to help define appropriate modulus values as a function of strain level.

Although reasonable relationships defining the variation of G/G_{max} with γ for sands have been available for nearly 40 years (Seed and Idriss 1970), Stokoe et al. (1999) developed a procedure which facilitates the definition of the curve shape and easily accounts for confining pressure and other effects. Based on the Stokoe et al. (1999) approach, the G/G_{max} curve shape is defined by the equation

$$G/G_{max} = \frac{1}{1+\left(\frac{\gamma}{\gamma_{ref}}\right)^{a}} \tag{1}$$

where the reference strain, γ_{ref} = strain where $G/G_{max} = 0.50$; and a = curvature parameter that varies the shape of the G/G_{max} curve, as shown in Fig. 2. As a increases, the G/G_{max} values increase at small strains and decrease at high strains. Stokoe et al. (1999) found that a was relatively independent of void ratio and confining pressure and that a reasonable average value is 0.87 for sands. In contrast, γ_{ref} increases as the confining pressure increases, shifting the G/G_{max} versus γ curve to the right. Based on tests on sand conducted by Darendeli (2001), the reference strain can be given by the equation

$$\gamma_{ref} = 0.0063(\sigma'_o)^{0.38} \tag{2}$$

where σ'_0 = confining pressure in kPa.

G/G_{max} versus γ curves for gravels were first published by Seed et al. (1986) based on large diameter (approximately 300 mm) cyclic triaxial shear tests on four rockfill dam materials. Later, Rollins et al. (1998) used results from 16 investigations involving large diameter shear tests on gravels to develop the equation

$$G/G_{max} = \frac{1}{\left\{1+20\gamma\left[1+10^{(-10\gamma)}\right]\right\}} \tag{3}$$

to define a mean G/G_{max} versus γ curve for gravels. However, using the formulation defined by E, the mean G/G_{max} versus γ curve for gravels determined by Rollins et al. (1998) can be simplified to

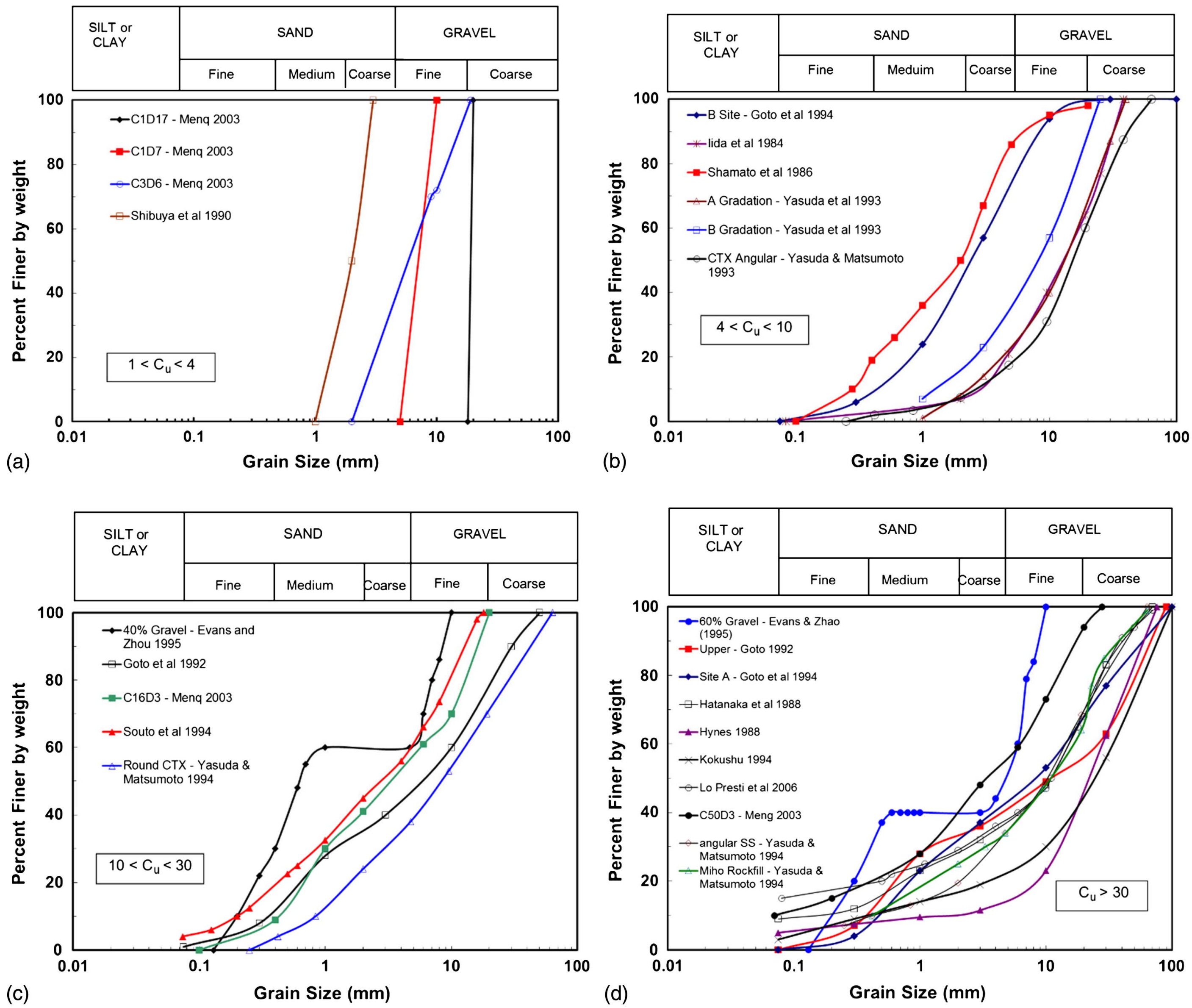

Fig. 1. Particle size distribution curves for test specimens grouped by uniformity coefficient, C_u.

$$G/G_{\max} = \frac{1}{\left[1 + \left(\frac{\gamma}{0.04}\right)^{0.84}\right]} \tag{4}$$

where $\gamma_{\text{ref}} = 0.04$; and $a = 0.84$. Rollins et al. (1998) also found that the $G/G_{\max}$ versus γ curves for gravels became more linear (shifted to the right) as the confining pressure increased, but they did not provide equations to account for this variation. Based on the tests on gravel summarized by Rollins et al. (1998) and the formulation suggested by Stokoe et al. (1999), this dependence on confining pressure can be simply accounted for using the equation

$$\gamma_{\text{ref}} = 0.0039(\sigma_o')^{0.42} \tag{5}$$

where σ_0' = confining pressure in kPa. The relationships for γ_{ref} versus σ_0' for sand and gravel defined by Eqs. (2) and (5), respectively are plotted in Fig. 3. For a given confining pressure, the reference strain is higher for sand than for gravel, although the difference is generally quite small. This observation is consistent with the findings by Kokusho et al. (2005).

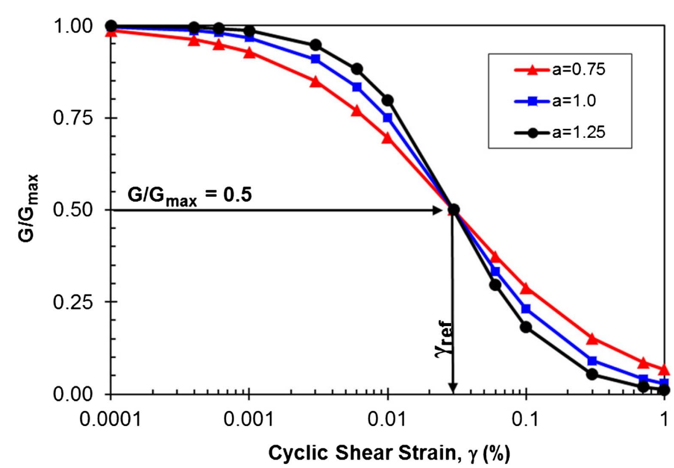

Fig. 2. Description of a and γ_{ref} parameters in the modified hyperbolic model developed by Stokoe et al. (1999).

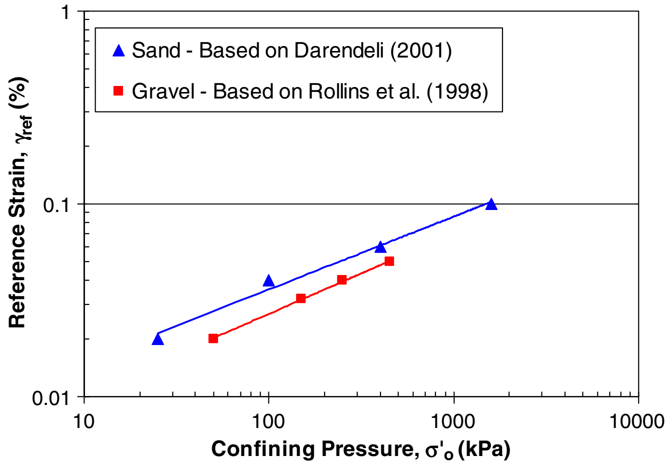

Fig. 3. Comparison of reference strain versus confining pressure relationships for sand based on tests by Darendeli (2001) and gravel based on tests summarized by Rollins et al. (1998).

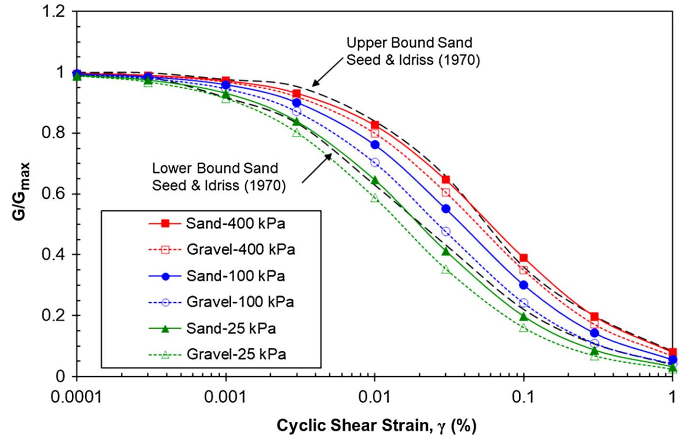

Fig. 4. Comparison of $G/G_{\max}$ versus γ curves for sand (Darendeli 2001) and gravel (Rollins et al. 1998) relative to upper and lower bound curves for sand developed by Seed and Idriss (1970).

Combining Eqs. (1) and (5), yields the equation

$$G/G_{\max} = \frac{1}{\left[1 + \left(\frac{\gamma}{0.0039(\sigma'_o)^{0.42}}\right)^{0.84}\right]} \tag{6}$$

which provides $G/G_{\max}$ versus γ as a function of confining pressure for gravels. A comparison of the $G/G_{\max}$ versus γ curves for sand and gravel over a range of confining pressures is provided in Fig. 4. As indicated previously, for a given confining pressure, the $G/G_{\max}$ versus γ curve for gravel is offset slightly to the left of the corresponding curve for sand defined by Darendeli (2001). The upper and lower bound $G/G_{\max}$ versus γ curves for sand defined by Seed and Idriss (1970) are also shown in Fig. 4 for comparison. It may be observed that the $G/G_{\max}$ versus γ curves as a function of confining pressure (25–400 kPa) for both sand and gravel typically fall within the range of data for sand originally defined by Seed and Idriss (1970).

Besides the model given by Rollins et al. (1998), Menq (2003) also proposed an equation of γ_{ref} as a function of confining pressure (σ'_o) and uniformity coefficient (C_u) based on a series of laboratory experiments. The relationship proposed by Menq (2003) is given by the equation

$$\gamma_{ref} = 0.12(C_u)^{0.6}\left(\frac{\sigma'_o}{P_a}\right)^{0.5C_u^{-0.15}} \tag{7}$$

Hardin and Kalinski (2005) proposed another approach to obtain the reference strain that requires additional independent variables e.g., void ratio, Poisson's ratio, overconsolidation ratio, peak friction angle, the D_{50} size from the particle size distribution curve, gravel particle shape factor, and the range of grain size, along with correlation factors a and b. Thus, the selection of these parameters would require additional testing, engineering judgment, and effort and was not typically available for the data set.

Because the $G/G_{\max}$ versus γ curve shape can be defined by two parameters using the Stokoe et al. (1999) formulation, it is important to determine the influence of various soil parameters on the shape of the curve as attempted by Menq (2003) by investigating several other data sets reported by many researchers across the world. Therefore, as part of this study, the γ_{ref} and a parameters were determined for all the tests summarized by Rollins et al. (1998) along with some additional tests that have been completed since that time. Basic mechanical properties for each of the gravels are summarized in Table 1 along with the parameters γ_{ref} and a. Statistical analyses indicated that γ_{ref} increased with confining pressure, as expected, but analyses also suggested that γ_{ref} decreased as the coefficient of uniformity, C_u increased, as reported by Menq (2003). Variation in γ_{ref} with soil parameters such as void ratio (e) and relative density (D_r), as listed in Table 1, was not found to be statistically significant. The mean a value was found to be 0.84 and was not correlated with σ'_o, C_u, D_r, or e for this data set.

Based on a regression analysis of the entire data set, the overall equation to predict γ_{ref} as a function of both the uniformity coefficient (C_u) and confining pressure (σ'_o) is given by

$$\gamma_{ref} = 0.0046(C_u)^{-0.197}(\sigma'_o)^{0.52} \tag{8}$$

where confining pressure (σ'_o) is in units of kPa. Using Eq. (8) for γ_{ref} in Eq. (1), the $G/G_{\max}$ versus γ curve can be given by the equation

$$G/G_{\max} = \frac{1}{\left\{1 + \left[\frac{\gamma}{0.0046(C_u)^{-0.197}(\sigma'_o)^{0.52}}\right]^{0.84}\right\}} \tag{9}$$

Fig. 5 shows that the $G/G_{\max}$ versus γ curves shift upwards and to the right as the confining pressure increases because of the higher reference strain. Owing to the increase in confining pressure, the stiffness of the specimen is increased, and consequently, the degradation of shear modulus is also reduced. However, as the uniformity coefficient (C_u) increases, the $G/G_{\max}$ versus γ curves shift downwards and to the left owing to the lower reference strain value.

The reference strain reflects the transition from linear elastic to nonelastic behavior (Ishihara 1996). When soil behaves elastically, deformations are concentrated at particle contacts, with little particle rotation or sliding between particles. Nonlinear behavior occurs when particle rotation initiates, while nonelastic behavior develops with sliding between particles (Mitchell and Soga 2005). In the case of higher uniformity coefficients, the soil matrix becomes more complex with large particle distribution having more contacts and linkages between particles. Hence, during cyclic loading, the particles may tend to initiate nonelastic deformations at lower reference strain and lose their interlocking stability,

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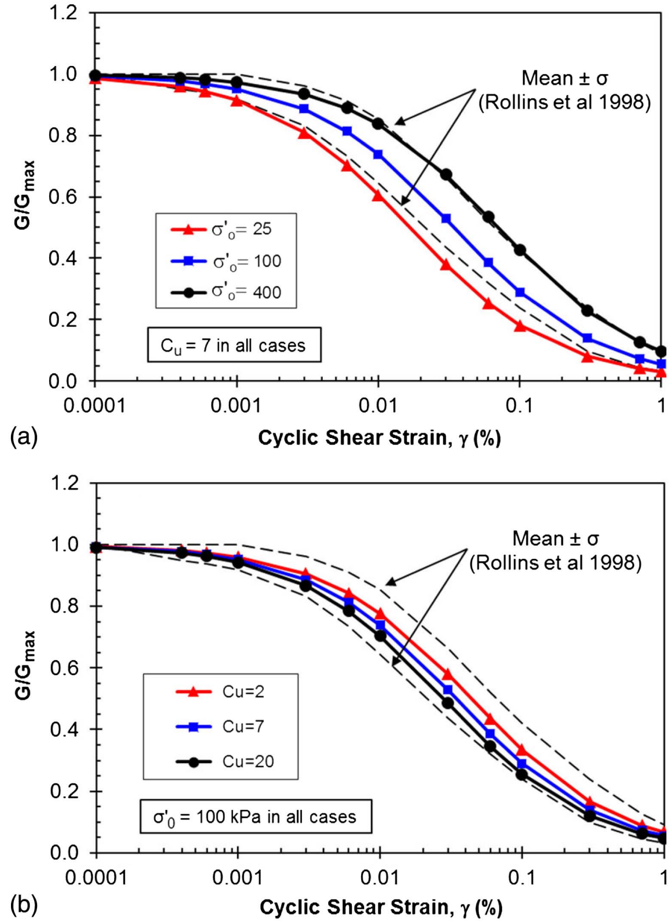

Fig. 5. Effect of (a) confining pressure, σ_o'; and (b) uniformity coefficient, C_u, on the shape of the $G/G_{\max}$ versus cyclic shear strain, γ curve.

causing a greater reduction in shear modulus. However, in the case of poorly graded soil, the loss of interlocking stability requires higher strain to be mobilized that causes less degradation of shear modulus.

Fig. 6 shows the comparison between measured and computed γ_{ref} values using the proposed model for all the investigators given in Table 1. The figure indicates that the data from all the investigators are quite symmetrically distributed around the perfect agreement line and the other error bands. No particular data set from any investigator is such that they should be treated as outliers.

To compare the proposed model with the other existing models given by Rollins et al. (1998) and Menq (2003), the γ_{ref} values computed using Eqs. (5), (7), and (8) are plotted in Fig. 7 corresponding to the measured γ_{ref} values. The results show that Eq. (8) gives better agreement between measured and computed values than Eqs. (5) and (7). The statistics indicate that approximately 36% of the computed γ_{ref} values fall within ±25% of the measured value, while 79% of the computed values fall within ±50% of measured values for the proposed model. However, the same 25% and 50% error bounds contain 32% and 69% of the data for the Rollins et al. (1998) model and 29% and 70% of the data for the Menq (2003) model. Alternatively, the results show that 67% of the data falls within a 39% error bound using the proposed Eq. (8), whereas the error bound for 67% of the data expands to 48% and 64% in the case of the Rollins et al. (1998) and Menq (2003) equations, respectively. Hence, the comparison shows a clear improvement

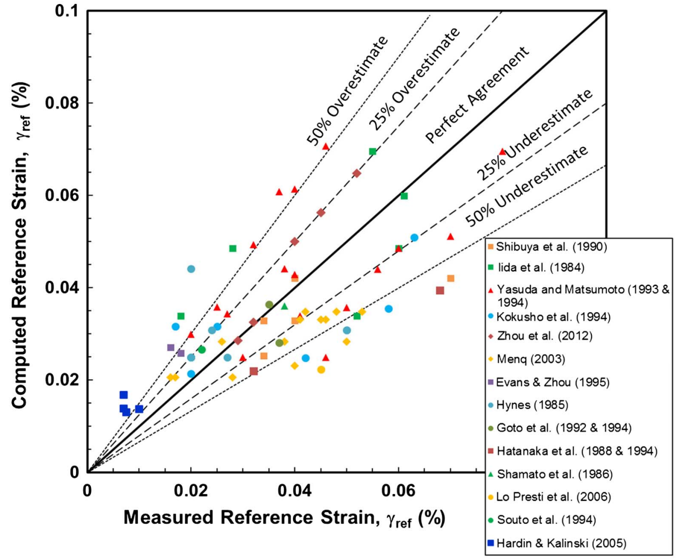

Fig. 6. Comparison of measured and computed reference strain (γ_{ref}) for all the investigators using Eq. (8) based on confining pressure (σ_o') and uniformity coefficient (C_u).

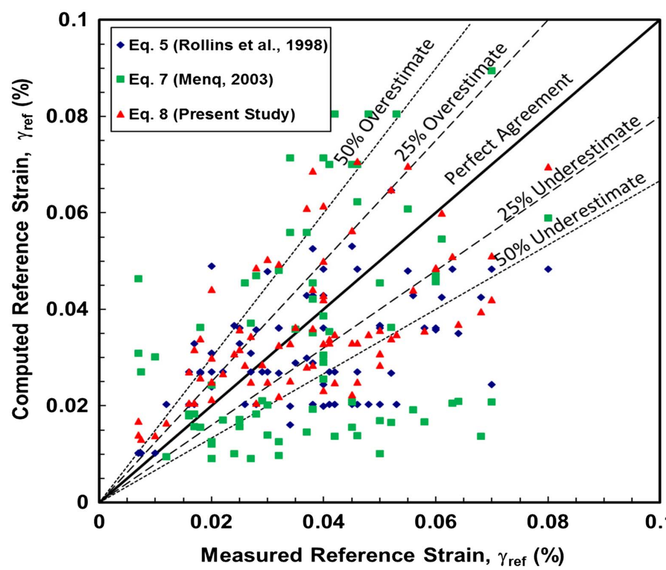

Fig. 7. Comparison of measured and computed reference strain (γ_{ref}) using Eq. (5) (Rollins et al. 1998), Eq. (7) (Menq 2003), and Eq. (8) (present study).

in predicting γ_{ref} values and corresponding $G/G_{\max}$ versus γ curves using the proposed equation in relation to previously existing models for the current data set.

To provide a better indication of the effect of error in the computed γ_{ref} on the error in the resulting $G/G_{\max}$ versus γ curve, the $G/G_{\max}$ value for each data point in the data set was computed using Eq. (8) with the appropriate C_u and σ_0', but replacing γ_{ref} with the measured γ_{ref}, rather than the computed γ_{ref}. Assuming perfect agreement, the computed $G/G_{\max}$ would be 0.5 in each case. However, the computed $G/G_{\max}$ was between 0.57 and 0.41 (error of ±16.4%) for 67% of the data points, providing a rough estimate of one standard deviation. The mean computed reference strain was obtained as 0.036 using Eq. (7) over the whole data set.

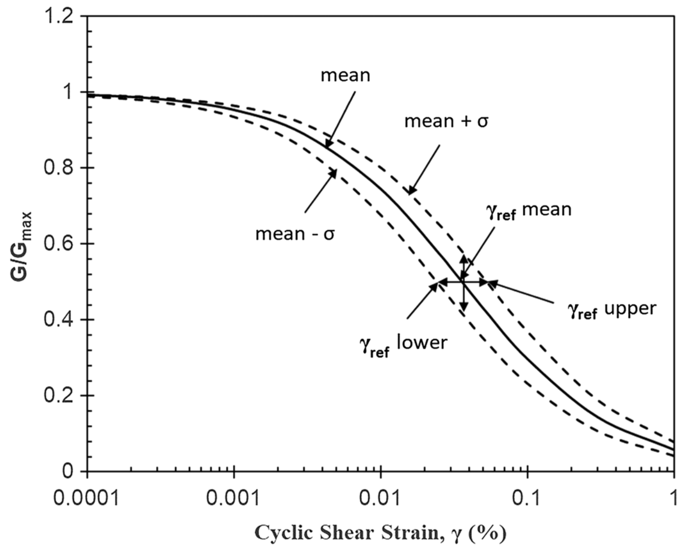

Fig. 8. Best-fit curve ± one standard deviation bounds for gravelly soils based on all test results after consideration of confining pressure and uniformity coefficient.

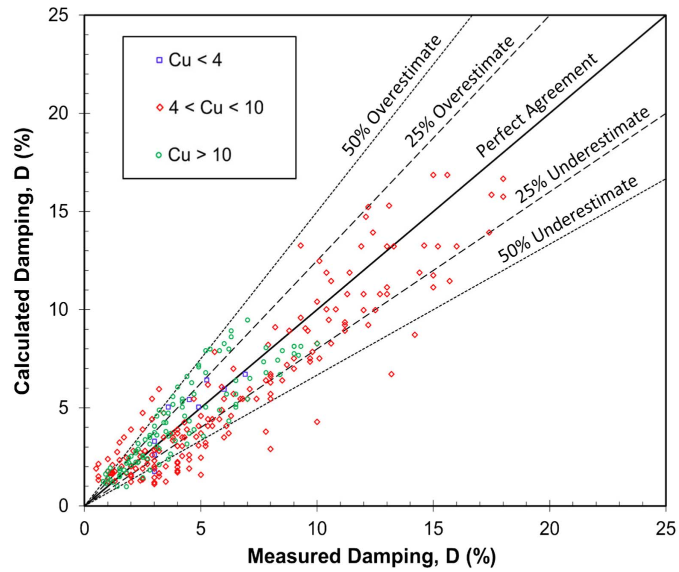

Fig. 9. Comparison of measured and calculated damping ratio (%) for gravel using Eq. (10) based on Rollins et al. (1998) which computes damping ratio based only on shear strain.

Hence, satisfying Eq. (1) with the mean computed reference strain and the limiting values of $G/G_{\max}$ for one standard deviation range (0.57 and 0.43), the upper and lower bounds of γ_{ref} are obtained as 0.052 and 0.026. Based on these mean and limiting γ_{ref} values, the mean $G/G_{\max}$ versus γ curve along with the one standard deviation bounds are produced for a range of shear strain values, as depicted in Fig. 8. The mean and mean ± one standard deviation values of $G/G_{\max}$ and γ_{ref} are also shown in Fig. 8. These curves show a typical range of $G/G_{\max}$ versus γ curves for gravelly soils (±16.4%) that should be considered when conducting ground response analyses using these curves. Considering the variability in testing methods, results from 18 investigators, and variability in gravel specimen properties involved, an error of about 16% appears to be a positive outcome.

The data set used to develop Eqs. (8) and (9) primarily consists of gravels or sandy gravels with relatively low fines contents. Thus, the extrapolation to conditions with high fines contents or plastic fines should be undertaken with caution as variation from the mean values could occur.

Damping Relationships

Three methods for computing the nonlinear damping ratio versus shear strain relationship were evaluated during this study. Rollins et al. (1998) developed the best-fit hyperbolic equation

$$D = 0.8 + 18\left(1 + 0.15\gamma - 0.9\right)^{-0.75} \tag{10}$$

where D = damping ratio in percent; and γ = cyclic shear strain in percent. A comparison between the measured and calculated damping ratio is presented in Fig. 9 for three curvenges. About 57% of the calculated values fall within ±25% of the measured values, while 86% of the calculated values lie within ±50% of the measured values. Additionally, 67% of the damping data fall within an error band of 31% relative to the computed values, giving a range of standard deviation. The variation appears to be somewhat larger for damping ratios below 5% than at the higher ratios.

Stokoe et al. (1999) proposed that the damping ratio be computed using the nonlinear $G/G_{\max}$ curve with a modified Masing approach and an equation having the form

$$D = \mathrm{FD_{Masing}} + \mathrm{D_{min}} \tag{11}$$

$$\text{where } \mathrm{F} = \mathrm{b}(G/G_{\max})^{0.1} \tag{12}$$

$$\text{and } \mathrm{b} = 0.6329 - 0.0057\ \ln(N) \tag{13}$$

where N = number of cycles.

The Masing approach makes it unnecessary to separately reconsider effects produced by confining pressure and uniformity coefficients because they are accounted for through the shape of the $G/G_{\max}$ versus γ curves. However, this approach also has two deficiencies. Several researchers have found that the Masing approach overestimates the material damping ratio at higher strain levels (Hardin and Drnevich 1972; Seed et al. 1986; Vucetic and Dobry 1991). The F factor is designed to reduce this error and produce an improved agreement with measured response. In addition, the Masing approach leads to zero damping in the small strain range, while testing indicates that soils have a relatively constant minimum damping ratio, $D_{\min}$, in this range (Stokoe et al. 1999). This weakness can be corrected by simply adding $D_{\min}$ to the damping ratio. Additional details regarding the calculation of damping using the Masing approach are provided by Darendeli (2001). For the gravels in this data set, the $D_{\min}$ value was typically about 1%. The best agreement with the measured damping for the gravels in this study was obtained when b was computed using the equation

$$b = 0.53 - 0.0057\ \ln(N) \tag{14}$$

that leads to a greater reduction in damping than suggested by Stokoe et al. (1999) for sands.

Using the modified Masing approach, Fig. 10 shows that the damping ratio of gravel decreases as the confining pressure increases, whereas the damping ratio of gravel increases as the uniformity coefficient increases. Hence, the damping behavior is just

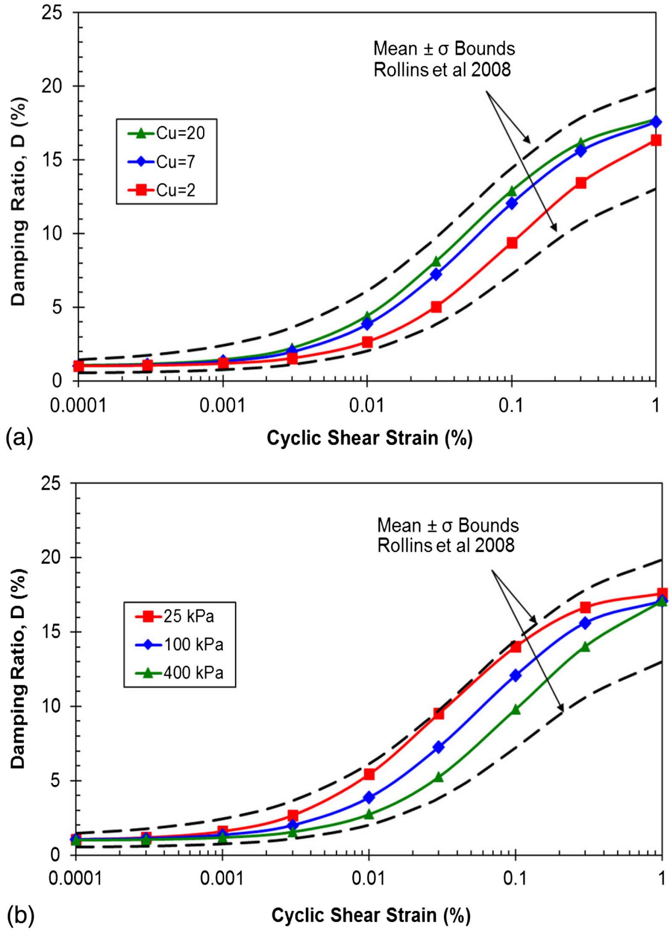

Fig. 10. Effect of (a) uniformity coefficient, C_u; and (b) confining pressure, σ_o' on the shape of the damping ratio, D, versus cyclic shear strain, γ, curves.

the reverse of the modulus reduction behavior explained previously and demonstrates the consistency of the results.

A comparison of the measured and calculated damping values using the modified Masing approach is shown in Fig. 11. For comparison purposes, the data points have again been grouped into three categories based on the uniformity coefficient. Overall, the agreement is relatively good and the data points appear to be reasonably distributed about the line representing perfect agreement for the three C_u groupings. This suggests that the variations in damping due to σ_0' and C_u are being adequately considered in this formulation. About 53% of the computed damping ratios fall within $\pm 25\%$ of the measured damping ratios, while 87% fall within $\pm 50\%$. This is nearly identical to the agreement obtained with Eq. (10). About 67% of the damping data points fall within an error band of $\pm 33\%$ from the computed value that provides a basis for establishing standard deviation bounds. Once again, the scatter in the data appears to be more pronounced for damping ratios less than about 5%.

As a third approach, a completely independent regression analysis has been performed based on the available data to define the damping ratio (D) as a function of shear strain (γ), uniformity coefficient (C_u), and confining pressure (σ_0'). The correlation is given by the equation

$$D = 26.05\left(\frac{\gamma}{1+\gamma}\right)^{0.375} C_u^{0.08}\sigma_0'^{-0.07} \tag{15}$$

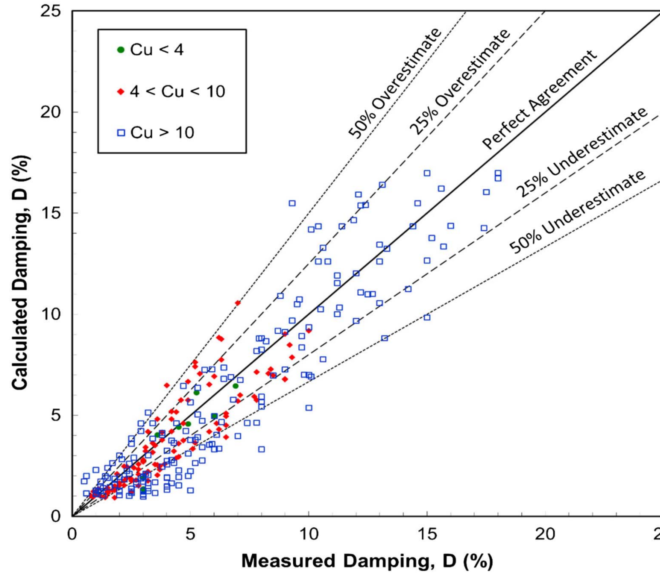

Fig. 11. Comparison of measured and calculated damping ratio (%) using the modified Masing approach (Stokoe et al. 1999) using Eq. (11). Damping ratio is a function of shear strain, confining pressure, and uniformity coefficient.

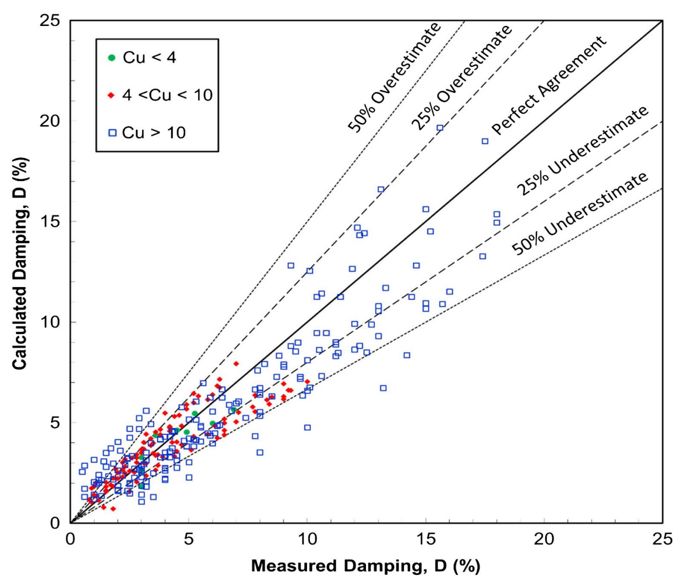

Fig. 12. Comparison of measured and calculated damping ratio (%) using the newly developed correlation equation in the present study, Eq. (5).

Using this newly developed equation, another comparison has been drawn between the computed and measured damping values for all available test data, as shown in Fig. 12. Similar to Fig. 11, the whole data set has been divided into the same three categories based on the uniformity coefficient. Fig. 12 shows that about 59% of the computed damping ratios fall within $\pm 25\%$ of measured damping ratios and 89% data fall within $\pm 50\%$. From another viewpoint, about 67% of the damping data points fall within a 28% error band of the computed value, giving a rough idea about the standard deviation bounds. Eq. (15) also accounts for the variation of damping ratio values with the variation of uniformity coefficient and confining pressure reasonably well.

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However, comparing the statistical measures for all the damping correlations shows that the newly developed regression equation, which is independent of G/G_{max} curves unlike the Masing approach, only provides a slightly better correlation than the existing models. Therefore, statistically, the modified Masing approach predicts the damping ratio with about the same error as direct correlation equations and represents a reasonable approach for estimating the damping ratio.

Conclusions

1. A simple two-parameter hyperbolic curve shape (Stokoe et al. 1999) can be used to define the normalized shear modulus, G/G_{max}, versus cyclic shear strain, γ, curves for gravels based on data from 18 investigators. For similar conditions, the curves for gravel are slightly lower than those for sands.
2. G/G_{max} versus γ curves are a function of both the confining pressure and the coefficient of uniformity. As confining pressure increases, the curves become more linear (shift upward), and as the uniformity coefficient increases, the curves become more nonlinear (shift downward). These influences on the curve shape can be easily accounted for using the hyperbolic equation.
3. The error in the computed G/G_{max} at the reference strain is $\pm 16.4\%$ or less for 67% of the points in the data set. Rough standard deviation curves can thus be obtained by adjusting the reference strains to produce this error in the G/G_{max} value at the reference strain.
4. A modified Masing approach (Stokoe et al. 1999) with a minimum damping ratio of 1% can be used to define the damping ratio, D, versus cyclic shear strain, γ, relationship based on data from ten investigators. This approach also accounts for variations resulting from confining pressure and uniformity coefficient. About 67% of the damping data points fall within an error band of $\pm 33\%$ from the computed value, providing a basis for standard deviation bounds.
5. In this study, a new direct correlation equation for D was also developed as a function of shear strain (γ), uniformity coefficient (C_u), and confining pressure (σ_0'). Using this new equation, 67% of the measured damping ratios fall within a 28% error band of the computed values. This model provides somewhat better predictions than the existing models, but the improvement is not dramatically different from the modified Masing approach.
6. The void ratio and relative density of the specimen do not have any statistically significant effects on the variation of γ_{ref} and G/G_{max} versus γ curves; hence, they have not been included in the proposed equation of reference strain. This finding is consistent with previous investigations by Rollins et al. (1998) and Menq (2003).
7. In this study, the data set primarily consists of relatively clean gravel or sandy gravel material. Hence, potential variations in G/G_{max} degradation and damping behavior due to higher fines contents or soil plasticity may not be fully captured by the correlation equations.

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