

# Improved constraints on sterile neutrinos in the MeV to GeV mass range

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Improved upper bounds are presented on the coupling  $|U_{e4}|^2$  of an electron to a sterile neutrino  $\nu_4$  from analyses of data on nuclear and particle decays, including superallowed nuclear beta decays, the ratios  $R_{e/\mu}^{(\pi)} = \text{BR}(\pi^+ \rightarrow e^+ \nu_e) / \text{BR}(\pi^+ \rightarrow \mu^+ \nu_\mu)$ ,  $R_{e/\mu}^{(K)}$ ,  $R_{e/\tau}^{(D_s)}$ , and  $B_{e2}^+$  decay, covering the mass range from MeV to GeV.

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Neutrino oscillations and hence neutrino masses and lepton mixing have been established and are of great importance as physics beyond the original Standard Model (SM). Most oscillation experiments with solar, atmospheric, accelerator, and reactor (anti)neutrinos [1–10] can be explained within the minimal framework of three neutrino mass eigenstates with values of  $\Delta m_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$  given approximately by  $\Delta m_{21}^2 = 0.74 \times 10^{-4} \text{ eV}^2$  and  $|\Delta m_{32}^2| = 2.5 \times 10^{-3} \text{ eV}^2$ , with normal mass ordering  $m_{\nu_3} > m_{\nu_2}$  favored; furthermore, the lepton mixing angles  $\theta_{23}$ ,  $\theta_{12}$ , and  $\theta_{13}$  have been measured, with a tentative indication of a nonzero value of the  $CP$ -violating quantity  $\sin(\delta_{CP})$  (for compilations and fits, see [11–16]).

In addition to the three known neutrino mass eigenstates, there could be others, which would necessarily be primarily electroweak singlets (sterile) [17] (see, e.g., [18]). Indeed, sterile neutrinos are present in many ultraviolet (UV) extensions of the SM. Whether sterile neutrinos exist in nature is one of the most outstanding questions in particle physics, and therefore, improved constraints on their couplings are of fundamental and far-reaching importance. Taking account of the possibility of sterile neutrinos, the neutrino interaction eigenstates  $\nu_\ell$  would be given by

$$\nu_\ell = \sum_{i=1}^{3+n_s} U_{\ell i} \nu_i, \quad (1)$$

where  $\ell = e, \mu, \tau$ ;  $n_s$  denotes the number of sterile neutrinos; and  $U$  is the lepton mixing matrix [19].

Here we obtain improved upper limits on  $|U_{ei}|^2$  for a sterile neutrino  $\nu_i$  in a wide range of masses from the MeV to GeV scale and point out new experiments that would be worthwhile and could yield further improvements. For simplicity, we assume one heavy neutrino,  $n_s = 1$ , with  $i = 4$ ; it is straightforward to generalize to  $n_s \geq 2$ . Since a  $\nu_4$  in this mass range decays, it is not excluded by the cosmological upper limit on the sum of effectively stable neutrinos,  $\sum_i m_{\nu_i} \lesssim 0.12 \text{ eV}$  [20]. Such a  $\nu_4$  is subject to a number of constraints from cosmology (e.g., [21]); however, since these depend on assumptions about the early universe, we choose here to focus on direct laboratory bounds. Constraints from the nonobservation of neutrinoless double beta decay are satisfied by assuming that  $\nu_4$  is a Dirac neutrino [22]. Since sterile neutrinos violate the conditions for the diagonality of the weak neutral current [23,24],  $\nu_4$  has invisible tree-level decays of the form  $\nu_4 \rightarrow \nu_j \bar{\nu}_i \nu_i$  where  $1 \leq i, j \leq 3$  with model-dependent invisible branching ratios. Because our bounds are purely kinematic, they are complementary to bounds from searches for neutrino decays, which involve model-dependent assumptions on branching ratios into visible versus invisible final states.

We first obtain improved upper bounds on  $|U_{e4}|^2$  from nuclear beta decay data. The emission of a  $\nu_4$  via lepton mixing in nuclear beta decay has several effects, including producing a kink in the Kurie plot and reducing the decay rate [25]. For the nuclear beta decays  $(Z, A) \rightarrow (Z+1, A) + e^- + \bar{\nu}_e$  or  $(Z, A) \rightarrow (Z-1, A) + e^+ + \nu_e$  into a set of neutrino mass eigenstates  $\nu_i \in \nu_e$ ,  $i = 1, 2, 3$  of negligibly small masses, plus a  $\nu_4$  of non-negligible mass, the differential decay rate is

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$$\begin{aligned} \frac{dN}{dE} = & C[(1 - |U_{e4}|^2)pE(E_0 - E)^2 \\ & + |U_{e4}|^2 pE(E_0 - E)[(E_0 - E)^2 - m_{\nu_4}^2]^{1/2} \\ & \times \theta(E_0 - E - m_{\nu_4})], \end{aligned} \quad (2)$$

where  $p \equiv |\mathbf{p}|$  and  $E$  denote the 3-momentum and (total) energy of the outgoing  $e^\pm$ ,  $E_0$  denotes its maximum energy for the SM case, the Heaviside  $\theta$  function is defined as  $\theta(x) = 1$  for  $x > 0$  and  $\theta(x) = 0$  for  $x \leq 0$ , and  $C = G_F^2 |V_{ud}|^2 F_F |\mathcal{M}|^2 / (2\pi^3)$ , where  $\mathcal{M}$  denotes the nuclear transition matrix element,  $V$  is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, and  $F_F$  is the Fermi function. Early bounds on  $|U_{e4}|^2$  were set from searches for kinks in Kurie plots in [25] and analyses of particle decays [26–28], and from dedicated experiments. For example, a search for kinks in the Kurie plot in  $^{20}\text{F}$  beta decay reported in Ref. [29] yielded an upper bound on  $|U_{e4}|^2$  decreasing from  $5.9 \times 10^{-3}$  for  $m_{\nu_4} = 0.4$  MeV to  $1.8 \times 10^{-3}$  for  $m_{\nu_4} = 2.8$  MeV. (Some recent reviews of searches for sterile neutrinos include [30–35].)

In addition to kink searches, a powerful method to set constraints on massive neutrino emission, via lepton mixing, in nuclear beta decays is to analyze the decay rates. Since, in general, the heavy neutrino would also be emitted in  $\mu$  decay, the measurement of the  $\mu$  lifetime performed assuming the SM would yield an apparent (app) value of the Fermi constant, denoted  $G_{F,\text{app}}$ , that would be smaller than the true value,  $G_F$ , given at tree level by  $G_F/\sqrt{2} = g^2/(8m_W^2)$ , where  $g$  is the SU(2) gauge coupling [26–28]. To avoid this complication, the ratios of rates of different nuclear beta decays are compared.

The integration of  $dN/dE$  over  $E$  gives the kinematic rate factor  $f$ . The combination of this with the half-life for the nuclear beta decay,  $t \equiv t_{1/2}$ , yields the product  $ft$ . Incorporation of nuclear and radiative corrections yields the corrected  $ft$  value for a given decay, denoted  $\mathcal{F}t$ . Conventionally, analyses of the  $\mathcal{F}t$  values for the most precisely measured superallowed  $0^+ \rightarrow 0^+$  nuclear beta decays have been used, in conjunction with the value of  $G_{F,\text{app}}$  from  $\mu$  decay, to infer a value of the weak mixing matrix element,  $|V_{ud}|$  [36–45]. A first step in these analyses has been to establish the mutual consistency of the  $\mathcal{F}t$  values for these superallowed  $0^+ \rightarrow 0^+$  decays. Since the emission of a  $\nu_4$  with mass of a few MeV would have a different effect on the kinematic functions and integrated rates for nuclear beta decays with different  $Q$  (energy release) values, it would upset this mutual consistency.

Hence, from this mutual agreement of  $\mathcal{F}t$  values, an upper limit on  $|U_{e4}|^2$  can be derived for values of  $m_{\nu_4}$  such that a  $\nu_4$  could be emitted in some of these superallowed decays. Reference [37] obtained upper bounds on  $|U_{e4}|^2$  ranging from  $10^{-2}$  down to  $2 \times 10^{-3}$  for  $m_{\nu_4}$  from 0.5 to 2 MeV, while Ref. [29] obtained the

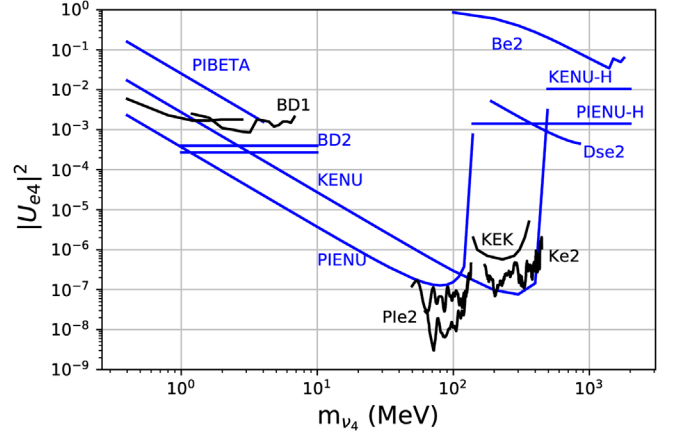


FIG. 1. The 90% C.L. upper limits on  $|U_{e4}|^2$  vs  $m_{\nu_4}$  from various sources: PIBETA, pion beta decay BD1, previous limits from beta decay [29]; BD2, beta decay with the two dashed horizontal lines based on our analysis using [42] and [43]; PIENU and PIENU-H, the ratio  $\frac{\text{BR}(\pi^+ \rightarrow e^+ \nu_e)}{\text{BR}(\pi^+ \rightarrow \mu^+ \nu_\mu)}$  in the kinematically allowed and forbidden regions for  $\nu_4$  emission; Pie2,  $\pi^+ \rightarrow e^+ \nu_{e4}$  peak search [47]; KENU and KENU-H, the ratio  $\frac{\text{BR}(K^+ \rightarrow e^+ \nu_e)}{\text{BR}(K^+ \rightarrow \mu^+ \nu_\mu)}$  in the kinematically allowed and forbidden regions for  $\nu_4$  emission; Ke2,  $K^+ \rightarrow e^+ \nu_{e4}$  peak search [48]; Dse2,  $D_s^+ \rightarrow e^+ \nu_{e4}$ ; and Be2,  $B^+ \rightarrow e^+ \nu_{e4}$ .

limits  $|U_{e4}|^2 < 1 \times 10^{-3}$  to  $|U_{e4}|^2 < 2 \times 10^{-3}$  depending nonmonotonically on  $m_{\nu_4}$  from 1 to 7 MeV. The maximum  $Q$  value in the current set of 14 superallowed  $0^+ \rightarrow 0^+$  beta decays used for the  $\mathcal{F}t$  fit in [41,42] is 9.4 MeV (for  $^{74}\text{Rb}$ ). A measure of the mutual agreement is the precision with which  $|V_{ud}|^2$  is determined, so a reduction in the fractional uncertainty of the value of  $|V_{ud}|^2$  results in an improved upper limit on  $|U_{e4}|^2$ . Reference [37] obtained  $|V_{ud}| = 0.9740 \pm 0.001$ . The recent analyses in [42] and [43] obtained  $|V_{ud}| = 0.97420(21)$  and  $|V_{ud}| = 0.97370(14)$ , respectively [46]. Applying these factors of improvement from [42] and [43] to the previous bounds in [37], improved upper bounds are obtained as

$$|U_{e4}|^2 < 4 \times 10^{-4} \quad (3)$$

and

$$|U_{e4}|^2 < 2.7 \times 10^{-4} \quad (4)$$

for  $\nu_4$  masses in the range  $1 \text{ MeV} \lesssim m_{\nu_4} < 9.4 \text{ MeV}$ , indicated in Fig. 1 as BD2. (These and other limits presented are at the 90% confidence level.)

We next discuss upper bounds from two-body leptonic decays of charged pseudoscalar mesons (generically denoted as  $M^+$ ) [25,26]. This method is quite powerful because the signal is a monochromatic peak in  $dN/dp_\ell$ , and for  $M^+ \rightarrow e^+ \nu_e$  (denoted  $M_{e2}^+$ ) decays, the strong helicity suppression in the SM case is removed when a heavy

neutrino is emitted. The presence of a massive  $\nu_4$  also changes the ratio of branching ratios  $\text{BR}(M^+ \rightarrow e^+ \nu_e) / \text{BR}(M^+ \rightarrow \mu^+ \nu_\mu)$  from its SM value, and this was used to set further bounds [25,26,49]. A number of dedicated experiments have been performed to search for a peak due to heavy neutrino emission and also to measure  $\text{BR}(M^+ \rightarrow e^+ \nu_e) / \text{BR}(M^+ \rightarrow \mu^+ \nu_\mu)$  with  $\pi_{\ell^2}^+$ ,  $K_{\ell^2}^+$ , and  $B_{\ell^2}^+$ , where  $\ell = e, \mu$  [47–60].

In the SM with only the three known neutrinos with negligibly small masses, the ratio

$$R_{\ell/\ell'}^{(M)} \equiv \frac{\text{BR}(M^+ \rightarrow \ell^+ \nu_\ell)}{\text{BR}(M^+ \rightarrow \ell'^+ \nu_{\ell'})} \quad (5)$$

is given by

$$R_{\ell/\ell',\text{SM}}^{(M)} = \frac{m_\ell^2}{m_{\ell'}^2} \left[ \frac{1 - \delta_\ell^{(M)}}{1 - \delta_{\ell'}^{(M)}} \right]^2 (1 + \delta_{\text{RC}}), \quad (6)$$

where  $\delta_\ell^{(M)} = m_\ell^2 / m_M^2$  and  $\delta_{\text{RC}}$  is the radiative correction (RC) [61–66].

We denote the ratio of the experimental measurement of  $R_{\ell/\ell'}^{(M)}$  to the SM prediction as

$$\bar{R}_{\ell/\ell'}^{(M)} \equiv \frac{R_{\ell/\ell'}^{(M)}}{R_{\ell/\ell',\text{SM}}^{(M)}}. \quad (7)$$

The most precise measurement of  $R_{e/\mu}^{(\pi)}$  is from the PIENU experiment at TRIUMF, with the result  $R_{e/\mu}^{(\pi)} = (1.2344 \pm 0.0023_{\text{stat}} \pm 0.0019_{\text{syst}}) \times 10^{-4}$  [58]. Including [67–69], the resultant PDG world average is  $R_{e/\mu}^{(\pi)} = (1.2327 \pm 0.0023) \times 10^{-4}$  [11], in agreement with the SM prediction with RC,  $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0002) \times 10^{-4}$  [62,63,65], resulting in

$$\bar{R}_{e/\mu}^{(\pi)} = 0.9980 \pm 0.0019. \quad (8)$$

The ratio  $R_{e/\mu}^{(K)}$  has recently been measured by the NA62 experiment at CERN [56], dominating the world average [11]

$$R_{e/\mu}^{(K)} = (2.488 \pm 0.009) \times 10^{-5}. \quad (9)$$

The SM prediction with RC [63,66] is

$$R_{e/\mu,\text{SM}}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}, \quad (10)$$

resulting in

$$\bar{R}_{e/\mu}^{(K)} = 1.0044 \pm 0.0037. \quad (11)$$

With emission of a heavy neutrino  $\nu_4$ , the ratio  $R_{\ell/\ell',\text{SM}}^{(M)}$  for general  $\ell, \ell'$  changes to

$$R_{\ell/\ell'}^{(M)} = \frac{[(1 - |U_{\ell 4}|^2) \rho(\delta_\ell^{(M)}, 0) + |U_{\ell 4}|^2 \rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})]}{[(1 - |U_{\ell' 4}|^2) \rho(\delta_{\ell'}^{(M)}, 0) + |U_{\ell' 4}|^2 \rho(\delta_{\ell'}^{(M)}, \delta_{\nu_4}^{(M)})]} \times (1 + \delta_{\text{RC}}), \quad (12)$$

where  $\delta_{\nu_4}^{(M)} = m_{\nu_4}^2 / m_M^2$ , and the kinematic function  $\rho(x, y)$  is [25,26]

$$\rho(x, y) = [x + y - (x - y)^2][\lambda(1, x, y)]^{1/2}, \quad (13)$$

with

$$\lambda(z, x, y) = x^2 + y^2 + z^2 - 2(xy + yz + zx). \quad (14)$$

Thus, in the SM case,  $\rho(x, 0) = x(1 - x)^2$ . Here and below, it is implicitly understood that  $\rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) = 0$  if  $m_{\nu_4} \geq m_M - m_\ell$ , where the decay  $M^+ \rightarrow \ell^+ \nu_4$  is kinematically forbidden. We define

$$\bar{\rho}(x, y) = \frac{\rho(x, y)}{\rho(x, 0)} = \frac{\rho(x, y)}{x(1 - x)^2} \quad (15)$$

so

$$\bar{R}_{\ell/\ell'}^{(M)} = \frac{1 - |U_{\ell 4}|^2 + |U_{\ell 4}|^2 \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}{1 - |U_{\ell' 4}|^2 + |U_{\ell' 4}|^2 \bar{\rho}(\delta_{\ell'}^{(M)}, \delta_{\nu_4}^{(M)})}. \quad (16)$$

With no loss of generality, we order  $\ell$  and  $\ell'$  such that  $m_{\ell'} > m_\ell$  and define the mass intervals (i)  $I_1^{(M)}: m_{\nu_4} < m_M - m_{\ell'}$ ; (ii)  $I_2^{(M)}: m_M - m_{\ell'} < m_{\nu_4} < m_M - m_\ell$ ; and (iii)  $I_3^{(M)}: m_{\nu_4} > m_M - m_\ell$ . Thus, a  $\nu_4$  with  $m_{\nu_4} \in I_1^{(M)}$  contributes to both  $M_{\ell^2}^+$  and  $M_{\ell'^2}^+$  decays, while if  $m_{\nu_4} \in I_2^{(M)}$ , then  $\nu_4$  contributes to  $M_{\ell^2}^+$ , but not to  $M_{\ell'^2}^+$  decay, and if  $m_{\nu_4} \in I_3^{(M)}$ , then  $\nu_4$  cannot be emitted in either  $M_{\ell^2}^+$  or  $M_{\ell'^2}^+$  decay.

If for a given  $m_{\nu_4}$  one knows, e.g., from peak-search experiments, that  $|U_{\ell' 4}|^2$  is sufficiently small that the denominator of (16) can be approximated well by 1, then an upper bound on the deviation of  $\bar{R}_{\ell/\ell'}^{(M)}$  from 1 yields an upper bound on  $|U_{\ell 4}|^2$ . Thus, one has the bound

$$|U_{\ell 4}|^2 < \frac{\bar{R}_{\ell/\ell'}^{(M)} - 1}{\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) - 1} \quad \text{for } m_{\nu_4} \in I_2^{(M)}. \quad (17)$$

This gives very stringent upper limits on  $|U_{\ell 4}|^2$  because  $\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) \gg 1$  over much of the interval  $I_2^{(M)}$

(see Figs. 3–5 in [26]). If  $m_{\nu_4} \in I_3^{(M)}$ , then (16) reduces to  $\bar{R}_{\ell/\ell'}^{(M)} = (1 - |U_{\ell 4}|^2)/(1 - |U_{\ell' 4}|^2)$ , so if  $|U_{\ell' 4}|^2 \ll 1$  in this interval, then the upper limit is

$$|U_{\ell 4}|^2 < 1 - \bar{R}_{\ell/\ell'}^{(M)} \quad \text{for } m_{\nu_4} \in I_3^{(M)}. \quad (18)$$

We now apply this analysis to  $R_{e/\mu}^{(\pi)}$ , using (17) and (18) with  $M^+ = \pi^+$ ,  $\ell = e$ , and  $\ell' = \mu$ . From previous  $\pi_{\mu 2}^+$  peak search experiments [47,48,50–60] and the calculation of  $\bar{\rho}(\delta_\mu^{(\pi)}, \delta_{\nu_4}^{(\pi)})$ , it follows that  $|U_{\mu 4}|^2$  is sufficiently small for  $m_{\nu_4} \in I_2^{(\pi)}$  that we can approximate the denominator of Eq. (16) by 1. From  $\bar{R}_{e/\mu}^{(\pi)}$  in Eq. (8), using the procedure from [70], we obtain the limit  $\bar{R}_{e/\mu}^{(\pi)} < 1.0014$ . Then, for  $\nu_4 \in I_2^{(\pi)}$ , we find

$$|U_{\ell 4}|^2 < \frac{\bar{R}_{e/\mu}^{(\pi)} - 1}{\bar{\rho}(\delta_e^{(\pi)}, \delta_{\nu_4}^{(\pi)}) - 1} < \frac{0.0014}{\bar{\rho}(\delta_e^{(\pi)}, \delta_{\nu_4}^{(\pi)}) - 1}. \quad (19)$$

This bound is labeled as PIENU in Fig. 1. If  $m_{\nu_4} \in I_3^{(\pi)}$ , i.e.,  $m_{\nu_4} > 139$  MeV, then, using (18), we obtain the upper bound on  $|U_{e 4}|^2$  given by the flat line labeled PIENU-H in Fig. 1.

We next obtain a bound on  $|U_{e 4}|^2$  by applying the same type of analysis to  $R_{e/\mu}^{(K)}$ . From  $K_{\mu 2}$  peak search experiments [48,51,57] and the calculation of  $\bar{\rho}(\delta_\mu^{(K)}, \delta_{\nu_4}^{(K)})$ ,  $|U_{\mu 4}|^2$  is sufficiently small that we can approximate the denominator of Eq. (16) well by 1. Using Eq. (11) for  $\nu_4 \in I_2^{(K)}$ , we find

$$|U_{\ell 4}|^2 < \frac{\bar{R}_{e/\mu}^{(K)} - 1}{\bar{\rho}(\delta_e^{(K)}, \delta_{\nu_4}^{(K)}) - 1} < \frac{0.010}{\bar{\rho}(\delta_e^{(K)}, \delta_{\nu_4}^{(K)}) - 1}. \quad (20)$$

This upper limit on  $|U_{e 4}|^2$  is labeled KENU in Fig. 1. For  $m_{\nu_4} \in I_3^{(K)}$ , i.e.,  $m_{\nu_4} > 493$  MeV, using ((18), we obtain the flat upper bound labeled KENU-H in Fig. 1.

One can also apply these methods to two-body leptonic decays of heavy-quark hadrons. We first consider  $D_s^+ \rightarrow \ell^+ \nu_\ell$  decays [71], using (17) and (18) with  $M^+ = D_s^+$ ,  $\ell = e$ , and  $\ell' = \tau$ . Experimental data from CLEO, BABAR, Belle, and BES have determined  $\text{BR}(D_s^+ \rightarrow \mu^+ \nu_\mu) = (5.49 \pm 0.17) \times 10^{-3}$  and  $\text{BR}(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.48 \pm 0.23) \times 10^{-2}$  [72–76]. Furthermore, searches by CLEO [72], BABAR [73], and Belle [74] have yielded the limit  $\text{BR}(D_s^+ \rightarrow e^+ \nu_e) < 0.83 \times 10^{-4}$ . Hence,  $R_{e/\tau}^{(D_s)} < 1.6 \times 10^{-3}$ .

For  $R_{e/\tau}^{(D_s)}$ , using the results of [62], we calculate  $1 + \delta_{\text{RC}} = 0.948$ . Substituting this in Eq. (6) with  $M = D_s$ ,  $\ell = e$ ,  $\ell' = \tau$ , we find

$$R_{e/\tau, \text{SM}}^{(D_s)} = 2.29 \times 10^{-6}. \quad (21)$$

Therefore,  $\bar{R}_{e/\tau}^{(D_s)} < 7.0 \times 10^2$ . For  $R_{e/\tau}^{(D_s)}$ , the interval  $I_2^{(D_s)}$  is 191 MeV  $< m_{\nu_4} < 1.457$  GeV. Actually, we restrict  $m_{\nu_4}$  to a lower-mass subset of this interval because for sufficiently great  $m_{\nu_4}$ , even though the  $D_s^+ \rightarrow e^+ \nu_4$  decay is kinematically allowed to occur, the momentum  $p_e$  (in the  $D_s$  rest frame) would be below the minimal value set by experimental cuts in the BES III event reconstruction. With  $p_{e, \text{cut}} \simeq 0.8$  GeV [77], this means that  $m_{\nu_4}$  must be less than 0.85 GeV for the event to be accepted. Thus, we consider  $0.191 \text{ GeV} < m_{\nu_4} < 0.85 \text{ GeV}$ . Substituting the experimental limit on  $\bar{R}_{e/\tau}^{(D_s)}$  in the special case of (16) with  $M = D_s$ ,  $\ell = e$ ,  $\ell' = \tau$  and using the fact that  $|U_{\tau 4}|^2 \ll 1$  for this  $m_{\nu_4}$  mass range [11], we obtain a resultant limit from (17). For  $m_{\nu_4} = 0.191$  GeV,  $\bar{\rho}(\delta_e^{(D_s)}, \delta_{\nu_4}^{(D_s)}) = 1.37 \times 10^5$ , increasing to  $\bar{\rho}(\delta_e^{(D_s)}, \delta_{\nu_4}^{(D_s)}) = 1.83 \times 10^6$  for  $m_{\nu_4} = 0.85$  GeV. We thus obtain the upper bound on  $|U_{e 4}|^2$  labeled  $D_{se2}$  in Fig. 1.

A dedicated peak-search experiment to search for the heavy-neutrino decay  $D_s^+ \rightarrow e^+ \nu_4$  would be worthwhile and could improve the upper bound on  $|U_{e 4}|^2$ . Similarly, a search for leptonic  $D$  decays like  $D^+ \rightarrow e^+ \nu_4$  would be valuable and will be discussed elsewhere. The very large values of  $\bar{\rho}(\delta_e^{(D_s)}, \delta_{\nu_4}^{(D_s)})$  and  $\bar{\rho}(\delta_e^{(D)}, \delta_{\nu_4}^{(D)})$  over a large portion of the kinematically allowed ranges of  $m_{\nu_4}$  in  $D_s^+ \rightarrow e^+ \nu_4$  and  $D \rightarrow e^+ \nu_4$  mean that there would be quite strong kinematic enhancement of the heavy neutrino decay relative to the corresponding  $(D_s^+)_{e2}$  and  $D_{e2}^+$  decays. In particular, these searches could be performed by the BES III experiment, which recently reported results from a data sample of  $3.19 \text{ fb}^{-1}$  and expects to collect considerably higher statistics.

Finally, we consider  $B^+ \rightarrow \ell^+ \nu_\ell$  decays. There is an upper limit  $\text{BR}(B^+ \rightarrow e^+ \nu_e) < 0.98 \times 10^{-6}$  from Belle [78] and BABAR [79]. For the other two leptonic decay modes,  $\text{BR}(B^+ \rightarrow \mu^+ \nu_\mu) = (6.46 \pm 2.22_{\text{stat}} \pm 1.60_{\text{syst}}) \times 10^{-7}$  from Belle [80], with a recent update  $\text{BR}(B^+ \rightarrow \mu^+ \nu_\mu) = (5.3 \pm 2.0_{\text{stat}} \pm 0.9_{\text{syst}}) \times 10^{-7}$  [81,82], and  $\text{BR}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.09 \pm 0.24) \times 10^{-4}$  from BABAR [83] and Belle [84,85]. The measured values of  $\text{BR}(B^+ \rightarrow \mu^+ \nu_\mu)$  are in agreement with the SM prediction  $\text{BR}(B^+ \rightarrow \mu^+ \nu_\mu)_{\text{SM}} = (3.80 \pm 0.31) \times 10^{-7}$  [80]. The measured value of  $\text{BR}(B^+ \rightarrow \tau^+ \nu_\tau)$  is also in agreement with the SM prediction  $\text{BR}(B^+ \rightarrow \tau^+ \nu_\tau)_{\text{SM}} = (0.75_{-0.05}^{+0.10}) \times 10^{-4}$  [85,86].

We focus on data from a  $B_{\ell 2}^+$  peak search experiment by Belle [59]. In general [25],

$$\frac{\text{BR}(M^+ \rightarrow \ell^+ \nu_\ell)}{\text{BR}(M^+ \rightarrow \ell^+ \nu_\ell)_{\text{SM}}} = \frac{|U_{\ell 4}|^2 \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}{1 - |U_{\ell 4}|^2}. \quad (22)$$



For  $m_{\nu_4}$  in the range from 0.1 GeV to 1.4 GeV, the Belle experiment obtained an upper limit on  $\text{BR}(B^+ \rightarrow e^+\nu_4)$  of  $2.5 \times 10^{-6}$ , while in the interval of  $m_{\nu_4}$  from 1.4 GeV to 1.8 GeV, this upper limit increased to  $7 \times 10^{-6}$ . In the range of  $m_{\nu_4}$  from 0.1 to 1.3 GeV, the Belle experiment obtained (nonmonotonic) upper limits on  $\text{BR}(B^+ \rightarrow \mu^+\nu_4)$  of approximately  $2\text{--}4 \times 10^{-6}$ , and in the interval of  $m_{\nu_4}$  from 1.3 GeV to 1.8 GeV, it obtained upper limits varying from  $2 \times 10^{-6}$  to  $1.1 \times 10^{-5}$ . Substituting the  $\text{BR}(B^+ \rightarrow e^+\nu_4)$  limits in Eq. (22) with  $M = B$  and  $\ell = e$ , we obtain the upper limits on  $|U_{e4}|^2$  shown as the curve  $B_{e2}$  in Fig. 1 [87]. From the  $\text{BR}(B^+ \rightarrow e^+\nu_4)$  limits we infer upper limits on  $|U_{\mu 4}|^2$  that decrease from 0.83 to  $3.4 \times 10^{-2}$  as  $m_{\nu_4}$  increases from 0.1 GeV to 1.2 GeV. Further peak searches for  $B^+ \rightarrow \ell^+\nu_4$  with  $\ell = e, \mu$  at Belle II would be worthwhile as a higher-statistics extension of [59].

We briefly remark on other constraints on a Dirac  $\nu_4$  in the mass range considered here. From the results of [23,88], it follows that there is a negligibly small contribution to decays such as  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow ee\bar{e}$ . Similarly, there is no conflict with bounds on neutrino magnetic moments [11,89], and contributions to invisible Higgs decays [90] are well below the current upper limit of  $\text{BR}(H \rightarrow \text{invis}) < 19\%$  [91].

In this work, improved upper limits on  $|U_{e4}|^2$  have been presented covering most of the range from  $m_{\nu_4} \simeq 1$  MeV to

$m_{\nu_4} \simeq 1$  GeV, representing the best available laboratory bounds for a Dirac neutrino  $\nu_4$  that do not make model-dependent assumptions concerning visible neutrino decay modes. Over parts of this range, the bounds obtained are competitive with those that assume specific visible  $\nu_4$  decays. For example, for  $m_{\nu_4} = 30$  MeV, our upper bound is  $|U_{e4}|^2 < 0.8 \times 10^{-6}$ , while the best bound for this value of  $m_{\nu_4}$  from experiments searching for neutrino decays is  $|U_{e4}|^2 < 1 \times 10^{-6}$  [92]. New peak search experiments to search for  $D_s^+ \rightarrow e^+\nu_4$  and  $D^+ \rightarrow e^+\nu_4$  as well as a continued search for  $B^+ \rightarrow e^+\nu_4$  and continued searches for  $\pi^+ \rightarrow e^+\nu_4$  [47] and  $K^+ \rightarrow e^+\nu_4$  [48] would be valuable; these could improve the bounds further. Other constraints on sterile neutrinos, such as from  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$  decay, and a detailed report of the results presented here will be published elsewhere.

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