

# The importance of consumer multi-homing (joint purchases) for market performance: mergers and entry in media markets<sup>1</sup>

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**Abstract:** Consumer "multi-homing" (watching two TV channels, or buying two news magazines) has surprisingly important effects on market equilibrium and performance in (two-sided) media markets. We show this by introducing consumer multi-homing and advertising-finance into the classic circle model of product differentiation. When consumers multi-home (attend more than one platform), media platforms can charge only incremental-value prices to advertisers. Entry or merger leaves consumer prices unchanged under consumer multi-homing, but leaves advertiser prices unchanged under single-homing: multi-homing flips the side of the market on which platforms compete. In contrast to standard circle results, equilibrium product variety can be insufficient under multi-homing.

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# 1 Introduction

Readers who subscribe to more than one platform are called multi-homers. Multi-homers abound for streaming services like Netflix and HBO. Consumers with a multi-purpose tablet like Apple's iPad also frequently have Amazon's Kindle e-book reader. Facing multi-homers may dramatically change platforms' competitive strategies compared to when all consumers single-home. If consumers buy either an iPad or a Kindle, their reservation prices are the standalone value for iPad and Kindle. Amazon's CEO/founder Jeff Bezos illustrates (Amazon Press release, December 27th, 2010):

*"We're seeing that many of the people who are buying Kindles also own an LCD tablet (e.g. an iPad). (...) They report preferring Kindle for reading..."*

Bezos focuses on the value of having a Kindle in addition to an iPad, i.e. the value of a Kindle for (potential) multi-homers. The maximal price - the reservation price - these consumers are willing to pay is the incremental value of a having Kindle. Then the incremental-pricing principle applies to firms' price choices: equilibrium prices are those of the extra value (Anderson et al. 2017).

Incremental pricing is also important in ad-financed media platforms, which operate in two-sided markets. Such platforms sell eyeballs to advertisers, and the value of an ad increases with the audience size. But the value also depends on whether the audience can be reached elsewhere. If they cannot, each platform has exclusive market power in delivering its consumers to its advertisers. However, if a share of the audience visits two (or more) platforms, the overlapping consumers cannot be sold to the advertisers for a higher price than the extra value of reaching them more than once. Again, the incremental pricing principle applies (Anderson et al. 2012, 2018; Ambrus et al., 2016; Athey et al., 2018): each platform is able to price to advertisers only the incremental value of the platform over the rival platform. Other things equal, multi-homing consumers are thus less valuable than single-homing consumers for the media platforms.

This has surprising consequences for the performance and structure of competition in media markets. Various authors have pointed out this phenomenon recently, and have derived it in various different contexts.<sup>2</sup>

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<sup>2</sup>See Ambrus et al. (2016), Athey et al. (2018), Anderson et al. (2018). Anderson et al. (2012) give an overview and discuss various different directions through which the stark predictions of the single-homing model can be relaxed.

Our contribution in this paper is two-fold. First, we use a familiar setting to illustrate stark differences in outcomes between multi-homing and single-homing formulations. It is striking in our set-up how *competition flips completely from one side of the market to the other*. Second, we revisit the classic question of optimal and equilibrium product diversity in our setting, with an unexpected new finding.

Our model combines elements from Anderson et al. (2018) and Anderson et al. (2017). The former considers multi-homing consumers in a purely ad-financed two-sided market without a spatial structure, while the latter considers multi-homing consumers in a one-sided user-financed spatial duopoly market. We here allow for dual source financing; the platforms may charge consumers as well as advertisers.<sup>3</sup> Because we want to analyze entry and merger incentives, we need more than two platforms. Therefore, we employ a Vickrey-Salop circle model (Vickrey, 1964; Salop, 1979) rather than a traditional Hotelling (1929) duopoly model.<sup>4</sup> Despite the fact that our platforms charge both sides of the market, the model remains tractable and highlights the importance of incremental pricing.

The presence of multi-homing consumers yields some interesting pricing, entry, and merger incentives.<sup>5</sup> We show that equilibrium consumer prices are independent of the number of platforms when some (but not all) consumers multi-home, while advertising prices fall as the fraction of multi-homers increases. Similarly, a merger between two platforms does not affect consumer prices, but is nonetheless profitable since the merging platforms can charge advertisers more for jointly shared eyeballs. If two neighboring platforms merge, there is no incentive to deviate from symmetric locations. We also show that an incumbent might have less incentive than an entrant to set up an additional platform.

It is well known that there is excessive entry in the classic Vickrey-Salop circle model. The same is true in our two-sided market model with single-homing consumers. Explaining why there is excess entry (i.e., why the business-stealing effect on incumbents' profits dominates the consumer surplus non-appropriation impact of entry) can be elusive in general.

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<sup>3</sup>Foros et al. (2018) analogously allow for charging both sides of the market, but they restrict their model to a duopoly market. Foros et al. may be considered as an extension of Anderson and Gabszewicz (2006). Anderson and Gabszewicz allow for dual pricing, but make the conventional assumption of single-homing consumers in a Hotelling duopoly set-up.

<sup>4</sup>For surveys of the literature, see Anderson, Strömberg and Waldfogel (2015); in particular chapters by Anderson and Jullien (2015), Foros, Kind and Sørgard (2015), and Peitz and Reisinger (2015).

<sup>5</sup>See Anderson and Coate (2005) for a duopoly analysis with single-homing consumers.

One might attribute it to the localization of competition in the standard circle model, that each firm competes directly with only its two neighbors. Then equilibrium prices can be blamed not falling sufficiently fast with entry because each firm still only has two neighbors, with the upshot that too many firms enter, attracted by too high prices.

Now consider a multi-homing consumer regime. First, in our model, the consumer price does not change with entry, so that localized competition effects are removed as regards a price effect and the impact on consumer surplus of entry is less beneficial on this account, and hence the consumer surplus effect is muted, which effect mitigates over-entry. By the same token, though, the business-stealing effect is also weaker. However, there are additional effects in the multi-homing model: consumer surplus rises with entry because second choices become more attractive (in addition to higher likelihood of getting a better first choice platform), and this effect *per se* also points towards under-entry. In the two-sided model with multi-homing, advertiser surplus also rises with entry, which constitutes another force for under-entry. The upshot is that under-entry can also occur in our model, and we show that if the value of reaching consumers more than once with an ad is sufficiently large, the business-stealing effect becomes so weak that there can be under-entry. Owing to the positive effects on advertisers and consumers, it would be socially desirable to have a larger number of platforms than at a free-entry market equilibrium.

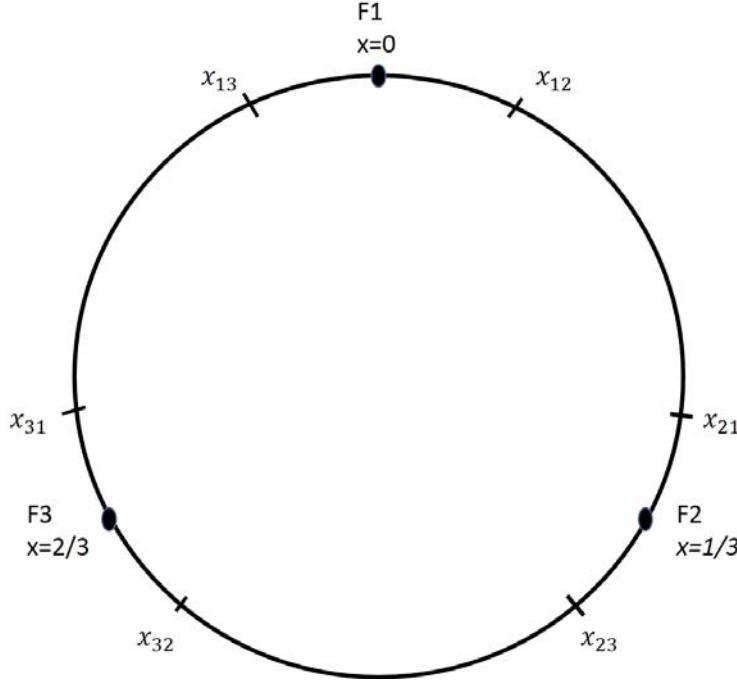
## 2 A model of multi-homing consumers

To make life interesting, we need more than two platforms before a merger (to avoid a complete monopoly after the merger). Let us allow for  $n \geq 2$  platforms, located on a circle of unit circumference (Vickrey, 1964; Salop, 1979). We restrict the analysis to outcomes with full market coverage. Consumers are uniformly distributed around the circle, and for simplicity we assume that they are ad neutral. Then, there is no direct effect on consumers' utility from the ad side of the market.

Platforms are located symmetrically around the circle. In Figure 1 we illustrate with  $n = 3$ ; platform 1 is located at  $x = 0$  ("noon", we hold this location fixed, independent of the number of firms), platform 2 at  $x = 1/3$  ("four o'clock"), and platform 3 at  $x = 2/3$  ("eight o'clock").<sup>6</sup>

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<sup>6</sup>Since the firms are located equidistantly from one another, platform 2 would be located at six o'clock



**Figure 1:** Platform locations and marginal consumer types.

Let the utility for a consumer located at  $x$  of buying from *only* platform  $i$  be  $u_i = v - t|x_i - x| - p_i$ , where  $x_i$  is the location of platform  $i$ . If the consumer is a multi-homer - meaning that she buys from more than one platform - the perceived value of good  $i$  might be lower than its stand-alone value. We follow Anderson et al. (2017) in assuming that the incremental value of buying from platform  $i$  in addition to platform  $j$  equals  $u_{ji} = \theta(v - t|x_i - x|) - p_i$ , where  $\theta \in [0, 1]$ . A utility maximizing consumer will multi-home as long as  $u_{ji} > 0$ .

Let  $x_{ji}$  denote the location of the consumer who is indifferent between buying only good  $j$  and buying both goods  $i$  and  $j$ . We find  $x_{ji}$  by solving  $u_{ji} = \theta(v - t|x_i - x_{ji}|) - p_i = 0$ . As an illustration, consider platform 1. The location of consumer  $x_{12}$ , who is indifferent between buying only from platform 1 and buying both from platform 1 and 2 (cf. Figure 1), is given by

$$u_{12} = 0 \Rightarrow x_{12} = \frac{1}{n} - \frac{v\theta - p_2}{t\theta}. \quad (1)$$

Solving  $u_{1n} = 0$  we likewise find the location of the consumer who is indifferent between if  $n = 2$ , four o'clock if  $n = 3$ , and three o'clock if  $n = 4$ , and so forth.

buying only from platform 1 and buying both from platform 1 and  $n$  as

$$u_{1n} = 0 \Rightarrow x_{1n} = 1 - \left( \frac{1}{n} - \frac{v\theta - p_n}{t\theta} \right). \quad (2)$$

The number of exclusive consumers for platform 1 is thus equal to  $x_{12} + (1 - x_{1n})$ . We shall focus on equilibria where platforms are symmetrically located, and where each platform has some multi-homers and some exclusive consumers (conditions for this to be the case are discussed below). We refer to this situation as an equilibrium with multi-homing consumers, even though multi-homing is partial.<sup>7</sup>

In such a multi-homing equilibrium, no consumer will buy more than two goods, and multi-homers buy one from each of the two platforms closest to her location. Thus, we can distinguish between two groups of consumers for platform  $i$ ; those who only buy from that platform, and those who buy from platform  $i$  as well as from *either* platform  $i-1$  *or*  $i+1$ . From (1) and (2) we deduce that the number of exclusive consumers for platform  $i$  equals (superscript  $e$ )

$$x_i^e = \frac{2}{n} - \frac{2\theta v - p_{i-1} - p_{i+1}}{\theta t}. \quad (3)$$

Other things being equal, the number of exclusive consumers is consequently decreasing in the number of platforms and increasing in the prices charged by the (two closest) rivals.

Let  $D_i$  denote total demand faced by platform  $i$ . For platform 1, this is equal to the shorter arc distance between  $x_{31}$  and  $x_{21}$  in Figure 1, where  $x_{31}$  and  $x_{21}$  solve  $u_{31} = 0$  and  $u_{21} = 0$ , respectively. This yields  $D_1 = \frac{2}{t} \left( \frac{\theta v - p_1}{\theta} \right)$ . For an arbitrary platform  $i$  we consequently have

$$D_i = \frac{2}{t} \left( \frac{\theta v - p_i}{\theta} \right), \quad (4)$$

so that we have a downward-sloping demand curve. Interestingly, since the *incremental* value of platform  $i$  is unaffected both by the number of platforms in the market and of the prices charged by the rivals,  $D_i$  is independent of  $n$  and  $p_j$  ( $j \neq i$ ).

Subtracting (3) from (4) we find that the number of multi-homers (superscript  $mh$ ) on platform  $i$  equals

$$x_i^{mh} = D_i - x_i^e = \frac{4\theta v - 2p_i - p_j - p_k}{\theta t} - \frac{2}{n}. \quad (5)$$

With partial multi-homing, we have  $x_i^e > 0$  and  $x_i^{mh} > 0$ .

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<sup>7</sup>We later point out situations with full multi-homing.

Summing up, platform  $i$ 's own pricing behavior does not affect its number of exclusive consumers, only how many multi-homers it will have. This is directly observed from Figure 1. A reduction in  $p_1$  moves  $x_{21}$  clockwise and  $x_{31}$  counter-clockwise, thereby increasing the total demand and the number of multi-homers for platform 1, while  $x_{12}$  and  $x_{13}$ , determining the number of exclusive consumers, are not affected.

We allow platforms to place ads on their platforms. To analyze two-sided pricing, we follow Anderson et al. (2018) and Foros et al. (2018).

The platforms decide a price per ad. Advertisers only place one advert per platform. Demand for ads is perfectly elastic, with a mass  $A$  of homogenous advertisers. We set  $A \equiv 1$ , so we need not to make a distinction between price per ad and total ad revenue. (We later allow for a general downward-sloping advertiser demand in Section 6.)

We follow Anderson et al. (2018) and assume that each advertiser is willing to pay  $b$  per ad per exclusive consumer reached, and  $\sigma b$  per multi-homing consumer. A third impression is worth nothing to advertisers. We let  $0 \leq \sigma \leq 1$ , so that the value of re-reaching the same consumer is (weakly) lower than the value of a first impression. Shi (2016) estimates for US magazines that an exclusive reader is worth twice as much as a multi-homing one; this translates to  $\sigma = 1/2$ .

We set all costs to zero, and write profit for platform  $i$  as  $\pi_i = p_i D_i + bx_i^e + \sigma bx_i^{mh}$ . Inserting for demand from equations (3) - (5) and maximizing with respect to  $p_i$  we find that the equilibrium price and total sales per platform equal

$$p = \frac{\theta v - \sigma b}{2} \text{ and } D = \frac{1}{t} \left( \frac{\theta v + \sigma b}{\theta} \right). \quad (6)$$

Throughout the analysis the second order conditions hold locally so that we have at least a local equilibrium, which is what we consider henceforth.<sup>8</sup> The consumer price in (6) is independent of the number of platforms. Consequently, if an entrant sets up another

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<sup>8</sup>It is beyond our current scope to investigate whether there are incentives for global deviations from the local equilibria that we consider. Based on insight from the exhaustive deviation analysis in Anderson et al. (2017), who provide a comprehensive analysis of deviation incentives in a spatial duopoly when platforms are purely user-financed, we conjecture that deviation at least will not arise under some parameter values when the ad side of the market is not "too important" (i.e.,  $b$  is not too high). Indeed, Anderson et al. (2017) show for a linear spatial market that there are parameters for which there is always a price equilibrium; this can be with single-homing consumers, or multi-homing consumers, or both such equilibria can exist. The linear model differs from the circle because consumers can be picked up "behind" a rival. This feature should not impact the deviation analysis in our current context if we assume that third

platform, the consumer price is not affected. Consumers are nonetheless better off, since total industry volume increases (from  $nD$  to  $(n+1)D$ ) and the price is below the consumer reservation price. Notice also that the consumer price is decreasing in the incremental value of selling ads, as measured by  $\sigma b$ . Consistent with Armstrong (2006), the extra profit per mhc,  $\sigma b$ , is akin to a negative marginal cost.<sup>9</sup>

With  $n$  platforms, it follows from (3) - (5) and (6) that the number of multi-homers and exclusive consumers on each of them is

$$x_n^{mh} = 2 \left( D - \frac{1}{n} \right) \text{ and } x_n^e = \frac{2}{n} - D. \quad (7)$$

Clearly, all consumers will multi-home if the number of platforms is sufficiently large. Note also that  $dx_n^e/d(\sigma b) < 0$  and  $dx_n^e/d\theta > 0$ , which follows from (6) and (7). An increase in  $\sigma b$  means that it becomes more profitable to sell ads, so the value of capturing an extra (multi-homing) consumer increases. This makes it optimal to reduce the subscriber price in order to increase demand ( $D$ ). An increase in  $\theta$ , on the other hand, reflects a higher consumer willingness to pay for the second platform. This increases the relative importance of the consumer market compared to the advertising market, and makes it optimal to increase  $p$  so much that total sales per platform ( $D$ ) actually falls.

We restrict attention to outcomes  $D \in (D^{\min}, D^{\max})$ , where  $D^{\min} \equiv \frac{1}{n}$  and  $D^{\max} \equiv \frac{2}{n}$ . That is, we presuppose that

$$D = \frac{1}{t} \left( \frac{\theta v + \sigma b}{\theta} \right) \in \left( \frac{1}{n}, \frac{2}{n} \right). \quad (8)$$

This implies partial multi-homing.

We label the profit level of each of the  $n$  platforms as  $\pi_n$ :

$$\pi_n = pD + bx_n^e + \sigma b x_n^{mh}. \quad (9)$$

Entry does not affect the incumbents' profit on the consumer side of the market, but reduces their profit from the ad side by increasing the fraction of multi-homing consumers.

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impressions are not valued at all: as price drops below the value at which a firm captures all consumers between itself and its nearest neighbor, it then picks up multi-homers from the next platform over but at a lower rate (so that its demand curve kinks down, which does not jeopardize equilibrium).

<sup>9</sup>This could result in negative consumer prices through this two-sided market effect, although one might typically want to impose a non-negativity constraint to reflect the issue that negative prices might induce consumers to pick up multiple units of the media product to enjoy the subsidy, without any extra benefit to advertisers.

Instead of entry from an outside firm, suppose that one of the incumbents sets up an additional platform, and that it places the new platform next to itself on the Vickrey-Salop circle (we maintain the assumption that all platforms are located equidistantly from each other: we show this is an equilibrium in Section 5). The two-platform company may then charge  $b(1 + \sigma)$  for the eyeballs they have in common, because they control access completely to these consumers. As before, they charge  $b$  for exclusive eyeballs and  $b\sigma$  for eyeballs they share with rival platforms.

With no loss of generality, let us assume that platform 1 establishes platform 2. If we assume that there were  $n - 1$  platforms at the outset, there will be  $n$  platforms after platform 2 is established. The number of shared consumers between platform 1 and 2 is given by

$$x_{21} - x_{12} = \left( \frac{2v\theta - p_1 - p_2}{t\theta} - \frac{1}{n} \right).$$

A two-platform company may thus capture  $b(1 + \sigma)(x_{21} - x_{12})$  when selling these eyeballs to advertisers. However, this does not affect the consumer price. The reason is easily seen from Figure 1. A slight reduction of  $p_1$  will turn a previously exclusive consumer on platform  $n$ , located at  $x_{n1}$ , into a multi-homer ( $n = 3$  in Figure 1). The consumer at  $x_{n1}$  may be sold to advertisers for  $\sigma b$  regardless of the number of rivals. By the same token, at  $x_{21}$ , a consumer previously exclusive to platform 2, is turned into a multi-homer. The two-platform company can now charge advertisers  $b(1 + \sigma)$  for the marginal consumer at  $x_{21}$ . However, the incremental value is still given by  $\sigma b$ , since this consumer without the slight price reduction in  $p_1$  was sold as an exclusive consumer to platform 2 for  $b$ . Consequently, a multi-platform company sets the same consumer price as a single-platform firm. The price is given by (6). By the same token, a merger between two platforms does not affect the consumer price.

Since the consumer price is identical for a two-platform company and a single-platform company, the number of jointly shared consumers is given by  $x_n^{mh}/2$ . Consequently, joint profit for a two-platform company, consisting of platform  $i$  and  $j$ , is given by:

$$\pi_{i+j} = 2pD + 2bx_n^e + 2\sigma bx_n^{mh} + b(1 - \sigma) \frac{x_n^{mh}}{2} = 2\pi_n + b(1 - \sigma) \frac{x_n^{mh}}{2}. \quad (10)$$

Let us now ask whether an incumbent or a potential newcomer has the greater incentive to set up a new platform. To answer that question, let us for the moment assume that the incumbent cannot charge more than  $\sigma b$  for any shared consumers. The gain from setting

up a new platform for this firm would then be equal to

$$G_I \equiv b(2x_n^e - x_{n-1}^e) + b\sigma(2x_n^{mh} - x_{n-1}^{mh}), \quad (11)$$

where the two terms are the extra profit the incumbent earns on its larger numbers of exclusive consumers and multi-homers, respectively.

The gain from setting up a new platform for a newcomer, which, by definition does not have any existing consumers, is likewise equal to

$$G_N \equiv bx_n^e + b\sigma x_n^{mh}. \quad (12)$$

Since the number of exclusive consumers is decreasing in the number of platforms, it follows that the first term in (11) is smaller than the first term in (12). We thus have  $(2x_n^e - x_{n-1}^e) < x_n^e$ . The difference between  $(2x_n^{mh} - x_{n-1}^{mh})$  and  $x_n^{mh}$  is correspondingly larger, but it must nonetheless be true that  $G_I < G_N$  as long as multi-homers are worth less than exclusive consumers (i.e., when  $\sigma < 1$ ). Formally, using equation (7), we find

$$G_I - G_N = -2b \frac{1-\sigma}{n(n-1)} < 0 \text{ (for } \sigma < 1\text{).}$$

We thus have an indication that a potential newcomer has a greater incentive to set up a new platform than an incumbent. However, this is only half the story: if the incumbent sets up a new platform it will internalize advertising competition between its two platforms. Compared to a newcomer, this gives the incumbent an extra advertising profit ( $A_I$ ) from setting up a new platform (which is the value of internalizing competition for its jointly shared consumers):

$$A_I = b(1-\sigma) \frac{x_n^{mh}}{2} = b(nD-1) \frac{1-\sigma}{n}. \quad (13)$$

Note that  $dA_I/dn > 0$ : this is because the number of multi-homers per platform is increasing in  $n$ .

From this analysis it follows that the incumbent will have more incentive than the potential newcomer to set up a new platform if  $A_I > (G_N - G_I)$ , or

$$\Delta \equiv b(1-\sigma) \left( D - \frac{n+1}{n(n-1)} \right) > 0,$$

where  $d\Delta/dn > 0$ . To see that this indicates that a newcomer might have more incentive than the incumbent to set up a new platform if  $n$  is low, and vice versa if  $n$  is large, note

that in the limit  $n = 1/D$  (approaching single-homing) we have  $\Delta = b(1 - \sigma)2\frac{D^2}{1-D} < 0$  while in the limit  $n = 2/D$  (full multi-homing) we have  $\Delta = b(1 - \sigma)D\frac{2-3D}{2(2-D)} > 0$  if  $D < 2/3$ .

### 3 Comparison with consumer single-homing

Results are quite different when all consumers are assumed to single-home. Then the model behaves much like the standard (one-sided) circle model. Anderson and Gabszewicz (2006) analyze the circle model with only advertising finance, and Choi (2006) compares that with pure subscription pricing; here we look at joint advertiser and subscriber finance.

We illustrate with 3 platforms. Given linear transport costs, the location of the consumer who is indifferent between buying from platform  $i$  and platform  $j$  is given by  $tx + p_i = t\left(\frac{1}{3} - x\right) + p_j$ , where  $i, j, k = 1, 2, 3$  and  $i \neq j \neq k$ . Consumer demand is

$$D_i(\mathbf{p}) = \frac{1}{3} - \frac{2p_i - (p_j + p_k)}{2t},$$

so there is direct competition for consumers in each direction with a different neighbor.

Because each platform holds the sole conduit for reaching the consumers on its platform and consumers are indifferent to ads, every platform will charge  $b$  per ad per viewer it delivers. Each advertiser will buy an ad on every platform. Profit to platform  $i$  is thus

$$\pi_i = p_i D_i + b D_i, \quad i = 1, \dots, 3$$

where  $p_i$  is the subscription price that platform  $i$  sets to consumers and  $b$  the ad price per consumer. Notice that again *advertisers multi-home*, while now consumers single-home.

The first-order conditions deliver the symmetric equilibrium price:

$$p^* = \frac{1}{3}t - b$$

The ad-price thus acts as a "negative" marginal cost (see Armstrong, 2006, and Anderson and Gabszewicz, 2006). Profit is  $\pi^* = t/9$  per platform, which is independent of  $b$  because the market has been assumed to be fully covered (with the consumer indifferent between any neighboring pair of platforms strictly preferring to participate: this condition

is guaranteed as long as the consumer reservation price is high enough). More generally, with  $n$  platforms, the subscriber price is  $p^* = \frac{1}{n}t - b$  and profit is  $\pi^* = t/n^2$ .

The striking difference with the multi-homing case is that the price per ad is now constant and the advertisers earn no surplus, but the subscribers pay less for their preferred platform the more platforms there are. Under consumer multi-homing, these results were reversed: more platform competition played out as lower prices per ad, but consumer prices stayed the same. That is, competition flips completely from one side of the market to the other.

Now consider a merger between two platforms: suppose that platforms 1 and 2 merge. They still want to charge the maximal price to each advertiser per viewer delivered,  $b$ . Hence, the profit function of the merged company becomes:

$$\pi_{1+2} = (p_1 + b) D_1 + (p_2 + b) D_2$$

(the profit function of platform 3 is as before). The first-order conditions now imply<sup>10</sup>

$$p_1 = p_2 = \frac{5}{9}t - b; \quad p_3 = \frac{4}{9}t - b.$$

Comparing with the pre-merger price  $p^* = \frac{1}{3}t - b$  shows that the price on the merged platform goes up, with the outsider price rising (by strategic complementarity of prices), but by less. (We eschew the case of merger when there are more than 3 firms, for the post-entry sub-game is asymmetric, but the same qualitative results hold.)

By contrast, for the multi-homing consumer case, the consumer price remained unchanged, but the advertising price went up. Again, the impact is on the opposite side of the market. With single-homing, each platform has a monopoly position when selling eyeballs to advertisers both before and after the merger, and so platforms retain full power to extract all advertiser surplus. Instead, it is prices to consumers that increase due to the merger. The profit result resembles the standard outcome from one-sided theory: both the merging platforms and the non-merging (outsider) platform benefit from a merger under price competition with differentiated products, and the non-merging platform benefits most (see Davidson and Deneckere, 1985). A merger distorts the allocation of consumers. The latter effect decreases social welfare compared to the no-merger case (while the direct effect of increased prices is just transfer of surplus from consumers to platforms due to the full market coverage assumption).

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<sup>10</sup>Second-order conditions for maximization are again readily verified.

## 4 Optimum and equilibrium firm numbers

The classic Vickrey-Salop circle model, with single-homing consumers and zero-profits for firms, delivers the result that the equilibrium has twice the optimum number of firms when transport costs are linear (Vickrey, 1964, Salop, 1979). More generally, oligopoly results tend to point to excessive entry, while under-entry is less common. Two exceptions (in quite different contexts) are Ghosh and Morita (2007) and Nocke, Peitz, Stahl (2007). The latter show that network effects might generate socially insufficient entry.

We now address the question of whether there is excessive entry in the two-sided market context with (partially) multi-homing consumers (mhc forthwith). Notice that a mhc regime requires some parameter restrictions. Namely, as noted above (see (8)), that

$$D = \frac{1}{t} \left( \frac{\theta v + \sigma b}{\theta} \right) \in \left( \frac{1}{n}, \frac{2}{n} \right).$$

When we come to free-entry, we shall require that the fixed cost,  $K$ , delivers such an outcome for  $n$ .

We first determine the free-entry (symmetric) equilibrium number of platforms, and then we find the social welfare function (the sum of consumer, producer, and advertiser surplus) to determine the welfare derivative when evaluated at the equilibrium number of firms.

For profits, the key ingredients on the consumer side (from (6)) are the equilibrium subscriber price,  $p = \frac{\theta v - \sigma b}{2}$ , and equilibrium demand as given above. The product represents subscription revenues. There are also the advertising revenues. These are  $b$  on the exclusive consumers, and  $\sigma b$  on the mhc for each platform. Using (7), the fractions of each type are  $x_n^{mh} = 2(D - \frac{1}{n})$  and  $x_n^e = \frac{2}{n} - D$  (so if  $D \rightarrow \frac{1}{n}$  all consumers are exclusives, and if  $D \rightarrow \frac{2}{n}$  they are all shared two ways).

We can then write the profit per platform (see (9)) as

$$\pi_n = \frac{(\theta v - \sigma b) D}{2} + b \left( \frac{2}{n} - D \right) + 2\sigma b \left( D - \frac{1}{n} \right).$$

Setting this equal to entry cost,  $K$ , yields the free-entry equilibrium number of platforms as

$$n = \frac{2b(1 - \sigma)}{K - \frac{(\theta v - \sigma b)D}{2} + bD(1 - 2\sigma)}.$$

Consider next the social optimum. We decompose total welfare into its constituent parts to look at profits (for the platforms and the advertisers) and consumer surplus separately.

The sum of profits is as follows. Total profits with  $n$  platforms are  $n$  times the profit expression above, minus the  $nK$  in entry costs. Thus the total profit derivative with respect to  $n$  is:

$$\frac{(\theta v - \sigma b) D}{2} - Db + 2\sigma b D - K;$$

evaluating this where profits are zero yields

$$\frac{d(n(\pi_n - K))}{dn} \Big|_{\pi_n = K} = \frac{2b}{n}(\sigma - 1) < 0$$

as the profit externality on other platforms from entry (where the relevant  $n$  solves  $\pi_n = K$ ). This is the business stealing effect in this two-sided market context. The other externalities are the consumer surplus one and the advertiser surplus one. The former is more intricate when consumers multi-home, while the latter is particular to the two-sided market.

First, advertiser surplus is only earned on the multi-homing consumers, because platforms can extract full value on the exclusives (single-homers). The multi-homers are worth  $b(1 + \sigma)$  but the advertisers pay only  $2\sigma b$  for them ( $\sigma b$  to each firm providing a particular platform multi-homing pair), for a surplus of  $b(1 - \sigma)$  each. Notice that each firm has  $2(D - \frac{1}{n})$  multi-homers, so the total number of them is  $n/2$  times this amount since each one is delivered by 2 platforms. Hence the total advertiser surplus,  $AS$ , is

$$nb(1 - \sigma) \left( D - \frac{1}{n} \right)$$

which is increasing in  $n$  at rate  $b(1 - \sigma)D$ . Combining with the profit externality, we have so far the total producer surplus externality as

$$\frac{d(n(\pi_n - K) + AS)}{dn} \Big|_{\pi_n = K} = b(1 - \sigma) \left( D - \frac{2}{n} \right) < 0, \quad (14)$$

where we can sign the expression under the restriction (see (8)) that  $D \in (\frac{1}{n}, \frac{2}{n})$ . As far as the full producer side is concerned, entry is excessive, despite the benefit to advertisers.

Therefore we need to turn to the consumer side to resolve whether entry can be insufficient or excessive. We next determine consumer surplus.

The first, traditional, component of consumer surplus accrues on "first"-purchases, that is, the more preferred product. To this we must add the extra surplus accruing on the second-preference product. The first choice is the closest product, the second is the second closest one. For first choices, the average "distance" travelled is  $1/4n$  (to the closer product

of the two bought, from the two neighboring firms). Because transport costs are linear, at rate  $t$ , the average distance disutility suffered on the first choices is then  $t/4n$ ; the market is covered by first choice products (all consumers buy at least their best choice). Hence the consumer surplus on first choices is

$$v - \frac{t}{4n} - \frac{\theta v - \sigma b}{2}.$$

For the second choice products, the minimal distance travelled is  $1/2n$  and the maximal one is  $D/2$  (for the consumer indifferent between adding the second product). So the average distance travelled, conditional on multi-purchase, is  $\frac{1}{2}(\frac{1}{2n} + \frac{D}{2})$ . Such products are valued by their buyers at a gross surplus of  $\theta v$  minus  $t\theta$  times the distance cost, so that the average surplus per multi-homer's second purchase is

$$\theta v - \frac{t\theta}{2} \left( \frac{1}{2n} + \frac{D}{2} \right) - \frac{\theta v - \sigma b}{2}$$

(where the last term is again the price).

Now, the mass of multi-homing consumers is  $(nD - 1) \in (0, 1)$  (recall that each such consumer is shared twice). Therefore the total consumer surplus is

$$\begin{aligned} CS &= v - \frac{t}{4n} - \frac{\theta v - \sigma b}{2} + (nD - 1) \left( \theta v - \frac{t\theta}{2} \left( \frac{1}{2n} + \frac{D}{2} \right) - \frac{\theta v - \sigma b}{2} \right) \\ &= v - \frac{t}{4n} + nD \left( \frac{\theta v + \sigma b}{2} \right) + (nD - 1) \left( -\frac{t\theta}{2} \left( \frac{1}{2n} + \frac{D}{2} \right) \right) - \theta v. \end{aligned}$$

The derivative is

$$\frac{dCS}{dn} = \frac{t}{4n^2} + D \frac{\theta v + \sigma b}{2} - D \frac{t\theta}{2} \left( \frac{1}{2n} + \frac{D}{2} \right) + (nD - 1) \frac{t\theta}{2} \left( \frac{1}{2n^2} \right),$$

which is positive by its construction.

Letting  $W = n(\pi_n - K) + AS + CS$  denote welfare, there is excessive entry if  $\frac{dW}{dn}|_{\pi_n=K} < 0$  and under-entry if  $\frac{dW}{dn}|_{\pi_n=K} > 0$ . The leading question is whether the classic result of excessive entry still holds. Or could there be under-entry? Recalling from (14) that  $\frac{d(n(\pi_n - K) + AS)}{dn}|_{\pi_n=K} = b(1 - \sigma)(D - \frac{2}{n})$ , with  $D = \frac{1}{t}(\frac{\theta v + \sigma b}{\theta})$ , we see that the producer-side externality vanishes if  $\sigma \rightarrow 1$  (given that all parameter restrictions are satisfied). Since consumer surplus is increasing in  $n$ , this indicates that we might have under-entry if the second impression value is sufficiently large. Note that the producer-side externality also vanishes as we approach full multi-homing, i.e.,  $D \rightarrow \frac{2}{n}$ , or the advertising value goes towards zero,  $b \rightarrow 0$ ).

On the other side of the coin, when can we expect over-entry? Taking the welfare derivative around  $\pi_n = K$  at the limit  $D \rightarrow \frac{1}{n}$  (i.e., when we approach single-homing) the derivative reduces to

$$\frac{dW}{dn}|_{\pi_n=K} = \frac{t}{4n^2} - \frac{b(1-\sigma)}{n},$$

so that this is negative for  $\frac{t}{4n} < b(1-\sigma)$ .<sup>11</sup> This indicates that over-entry is associated with low second impression value ( $\sigma$  small), high advertiser value, and low loyalty cost,  $t$ .

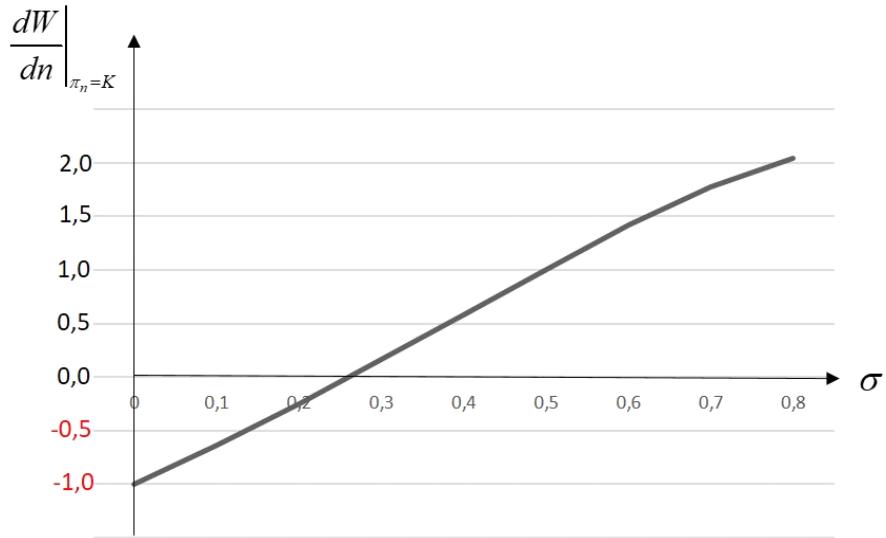
To verify that both too much and too little entry are possible once we take parameter restrictions into account, we have to solve the model numerically.<sup>12</sup> Figure 2 provides a corroborating example.<sup>13</sup> Here we measure the value of second impressions on the horizontal axis, and the welfare derivative on the vertical axis. Consistent with the reasoning above, we see that there is excessive entry ( $\frac{dW}{dn}|_{\pi_n=K} < 0$ ) for low values of  $\sigma$ , and under-entry for high values of  $\sigma$ . This reflects the fact that the business-stealing effect is smaller the greater is  $\sigma$ .

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<sup>11</sup>Salop (1979) noted the inverse demand curve facing an individual firm kinks down as competition moves from facing the outside good to facing a competitor. This feature can lead to multiple (asymmetric) price equilibria. What is different in the present context is that there is an upward kink in the demand curve as firms transition from the single-homing to the mhc regime. To see this, note that the (inverse) demand curve slope under single-homing is  $-1/2t$ . However, under a mhc regime it is  $-1/\theta t$ . Such upward kinks imply a jump up in marginal revenue, and so potentially two local maxima in profit. This feature may jeopardize the existence of the mhc equilibrium candidate price, and is more contentious the smaller the number of multi-homing consumers.

<sup>12</sup>In particular, the parameter values must ensure partial multi-homing;  $D \in (1/n, 2/n)$ . They must also ensure that  $p > 0$ , since we implicitly have assumed that the consumer price is positive in the calculations above.

<sup>13</sup>Parameter values for Figure 2 are  $t = 1$ ,  $\theta = 1/2$ ,  $K = 3$ ,  $v = 2$  and  $b = 1$ .



**Figure 2:** Possible under-entry.

## 5 Location analysis

We have chosen above a symmetric set of locations for firms. However, each firm in a mhc regime is locally indifferent to moving. To see this, note that any small move does not change its price as long as it faces mhc on both sides (see Figure 1). A small move then loses a consumer on one side, with advertising loss  $\sigma b$ , but gains one on the other side, with benefit  $\sigma b$ . Such a change is profit neutral. Hence any set of locations with firms facing mhc on both sides constitutes an equilibrium. That is, each pair of locations around the circle must be more than  $D$  apart, but less than  $2D$ . We selected the symmetric set of locations.

More interesting is the result that location incentives do not change if a firm merges with its neighbor. It has no strict incentive to move, and nor do its neighbors, and so the equi-spaced configuration prevails. To see this, suppose that Firms 1 and 2 are merged and move closer to Firm 3, such that one more consumer on each side of Firm 3 will now also read paper 1/paper 2 in addition to newspaper 3. The merged unit will gain  $2\sigma b$ . However, two consumers who previously bought both 1 and 2 will now buy only one of them. The loss from this is  $2(b + \sigma b) - 2b = 2\sigma b$ . So there is no change in profit if the merged firm moves its outlets.

## 6 Generalizations

The advertiser demand curve was set deliberately simple. However, the results still hold with a more general advertiser demand curve, with the only difference being that the equilibrium ad level is just the monopoly one on the advertiser demand curve. Assume that each advertiser has a value  $r(a)$  per unique impression, with advertisers ranked from high to low willingness to pay (as is standard). Assume that they value 2 impressions at  $(1 + \sigma)br(a)$ . Then the advertiser demand curve just pivots up when there are multiple types of viewer in the basket offered by a platform. The equilibrium ad level is then the monopoly one against the demand curve,  $a_m$ , independently of the composition of exclusive and mhc. The price of the bundle does depend on the composition: it is  $b(x^e r(a_m) + \sigma x^{mh} r(a_m))$ .

On the demand side we have used the traditional linear transport cost formulation. The key property for the pricing equilibrium to be independent of  $n$  is that competition among firms should be with the outside good at the margin of the indifferent mhc. This continues to hold with other transport cost functions. For example, under the often-used quadratic transport cost assumption, we have the marginal consumer in the mhc regime given (in inverse demand form) by  $p = \theta v - tx^2$ . Qualitative results are unaffected.

A more complex extension is to consider i.i.d. preferences across goods (non-localized competition). Then each platform faces competition with both the outside good and all other products for the second choice good. In the case of a merger, this suggests that a merged firm's price will be higher than when unmerged because it internalizes the effect on its sibling product when merged. Results are then more nuanced.

We have analyzed a partial mhc regime above. However, if there are enough firms, there will be full multi-homing. That is, each consumer in each inter-firm interval will buy from its two closest firms. We can readily determine the corresponding equilibrium price candidate. Competition, at the margin, now moves to the marginal multi-homing consumer, who is located atop the next rival's location, and this competition is with the outlet two firms over. Thus for the case of a one-sided market, the traditional Vickrey-Salop price,  $t/n$  is modified by replacing  $n$  by  $n/2$  and  $t$  by  $\theta t$ . So then the price becomes  $2\theta t/n$ . Whether prices are higher or lower in the market then depend on the size of  $\theta$ . Indeed, if  $\theta > 1/2$ , prices are higher with multi-homing. This idea extends clearly to when consumers may buy more than one extra product.

For a two-sided market, the single-homing consumer price is  $\frac{t}{n} - b$ , because the value of advertisers is competed away (recall that the Vickrey-Salop price can be interpreted as the mark-up, so the claimed result follows). With a full mhc regime, we make the changes above, with the additional change that the value of an advertiser goes from  $b$  to  $\sigma b$ . So the price under full mhc becomes  $\frac{2\theta t}{n} - \sigma b$ . The lower value of advertisers to firms provides an additional boost to raising price.

The models have deliberately closed down ad nuisance in order to retain simplicity, as well as to focus on markets where nuisance does not have a first-order effect (newspapers perhaps, as opposed to television). Ad nuisance intertwines effects, but in a rather interesting way. The case of single-homing consumers is quite straightforward: merger does not change ad levels. This is because ad levels are determined by the condition  $R'(a) = \gamma$  to get the ad level (marginal revenue per consumer in the ad market equals nuisance cost per ad: see e.g. Anderson and Coate, 2005, and Anderson and Jullien, 2017 for further discussion). There are then only subscriber price effects to consider, but then it is a standard circle analysis (albeit asymmetric due to the merger), so the insights noted above apply that advertising benefits in the analysis accrue as if they were negative marginal costs, and the same for all firms. Things are more involved when there are mhc.<sup>14</sup> Then the composition effects (of the viewer basket) impinge, so that the  $R'(a) = \gamma$  condition becomes a weighted average condition  $x^e R'(a) + x^{mh} \sigma R'(a) = D\gamma$ . This condition defines the relation between  $a$  and  $p$  as the firm varies the latter.

## 7 Conclusions

Two key properties are at play in the result that merger under multi-homing consumers impacts only advertisers, and not consumers or other platforms (and the ancillary result that there is no relocation incentive). The first stems from incremental pricing to the marginal consumer: a platform does not change a rival's (or sibling's) consumer base when it changes its subscription price. This property implies there is no spill-over on the consumer side from merger. Secondly, incremental value pricing in advertising implies that switching a rival's customer from being that rival's exclusive to a shared customer gets the firm

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<sup>14</sup>With full mhc, the condition determining the ad level is simply  $\sigma R'(a) = \gamma$ , and the equilibrium price is the straightforward application of that given at the end of the preceding Section.

the incremental value (from the pricing of ads)  $\sigma b$ ; while converting a sibling's exclusive customer to a mutually shared one delivers the firm  $b(1 + \sigma)$  in place of  $b$ . So the economic incentive is the same for conversions.

Any model with the two properties would give the consumer-price neutrality result, and have the merger impact fall just on advertisers. It would, for example, hold with a downward-sloping ad demand as described in the previous section. On the consumer side of the tally, any model where the monopoly incremental pricing result holds would have the same result (i.e., not just a circle).

A single-homing consumer two-sided model gives the impact of a merger only on consumers, and not on advertisers. The same is true for entry of a new firm. For a merger (in a context with three outlets, and two merging), the merged firm charges higher consumer prices and earns more than pre-merger (though the remaining firm's profit rises by more). Advertiser prices remain at  $b$  per consumer, because all consumers are reached only once under single-homing. This is an example of a (weak) see-saw effect (see Anderson and Peitz, 2017, for see-saw effects in media markets with single-homing consumers): a change in circumstances that affects market participants on one side in the opposite direction to those on the other side. Another (again weak) see-saw effect occurs *in the opposite direction* under multi-homing consumers. Then the full brunt (for merger) or benefit (for entry) is borne by advertisers.

In order to deliver some continuity between these extreme cases, we could ask what happens if we have some fraction of consumers who are potential multi-homers. That is,  $\theta v$  is high for a fraction  $\kappa$  and prohibitively low for the rest, and we vary  $\kappa$  to put more or less weight on the segment of multi-homing consumers. Then the impact of a change (merger or entry) will fall more on the advertisers the larger is  $\kappa$ . That is, a larger fraction of single-homing consumers will cause a bigger consumer price response. There is no see-saw effect because both sides of the market are affected the same way, with the incidence depending on  $\kappa$ .

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