Energy-Efficient Node Deployment in Wireless Ad-hoc Sensor Networks

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***Abstract*—We study a wireless ad-hoc sensor network (WASN) where N sensors gather data from the surrounding environment and transmit their sensed information to M fusion centers (FCs) via multi-hop wireless communications. This node deployment problem is formulated as an optimization problem to make a trade-off between the sensing uncertainty and energy consumption of the network. Our primary goal is to find an optimal deployment of sensors and FCs that minimizes a Lagrangian combination of sensing uncertainty and energy consumption. To support arbitrary routing protocols in WASNs, the routing-dependent necessary conditions for the optimal deployment are explored. Based on these necessary conditions, we propose a routing-aware Lloyd-like algorithm to optimize node deployment. Simulation results show that our proposed algorithm outperforms the existing deployment algorithms, on average.**

***Index Terms*—node deployment, wireless ad-hoc sensor networks, Lloyd algorithm, optimization.**

I. Introduction

Recent developments in wireless communications, digital electronics and computational power have enabled a large number of applications of wireless ad-hoc sensor networks (WASNs) in various fields such as agriculture and industry to name a few. In a general WASN, spatially dispersed sensors collect data, e.g. temperature, sound, pressure and radio signals from the physical environment, and then forward the gathered information to one or more fusion centers (FCs).

In order to collect accurate data from the physical surroundings, high sensing quality or sensitivity is required. In general, sensing quality diminishes as the distance between the sensor and its target point increases [1]–[6]. Thus, two distance-dependent measures, i.e., sensing coverage [1], [7]–[10] and sensing uncertainty [2], [11]–[16] are widely studied in the literature to evaluate the sensing quality. In the binary coverage model [1], [7]–[10], each sensor node can only detect the events within its sensing radius. Then, sensing coverage represents the percentage of events that is covered by at least one sensor [1], [7]–[9]. Another common model, centroidal Voronoi tessellation, formulates the sensing quality as a source coding problem with sensing uncertainty as its distortion [2], [11]–[16]. Sensing uncertainty reflects the distortion of a quantizer, and provides a distance-based measure of sensing quality [11], [13], [17], [26].

Energy efficiency is another key metric in WASNs as it is inconvenient or even infeasible to recharge the batteries of numerous and densely deployed sensors. In general, wireless communication, sensing and data processing are three primary energy consumption components of a static node. However, in many WASN applications, wireless communication dominates the node energy consumption [18], [19]. There are four primary energy saving methods for WASNs in the literature: (i) topology control [20], [21], in which unnecessary energy consumption is reduced by properly switching the nodes' states between sleeping and working; (ii) clustering [22], [23] which is used to balance the energy consumption among nodes in one-hop communication models by iteratively selecting cluster heads; (iii) energy-efficient routing [24], [25], [27], a widely used method that attempts to find the optimal routing paths to forward data to FCs while the communication cost between two nodes are held fixed; and (iv) deployment optimization that plays an important role in the energy consumption of WASNs since the communication cost between two nodes depends on their distance. Our previous works [26], [28] proposed Lloyd-like algorithms to save communication energy in homogeneous and heterogeneous WASNs by optimizing the node deployment. Nonetheless, a pre-existing network infrastructure, which only includes two-hop communications, is a basic assumption in [26], [28]. Compared to one-hop and two-hop communications, the generalized multi-hop communications can, on average, reduce the transmission distance and save more energy. However, to the best of our knowledge, the optimal node deployment with generalized multi-hop communications in WASNs is still an open problem.

In this paper, we study the node deployment problem in WASNs with arbitrary multi-hop routing algorithms. Our primary goal is to find the optimal FC and sensor deployment to minimize both sensing uncertainty and total energy consumption of the network. By deriving the routing-dependent necessary conditions of the optimal deployments in such WASNs, we design a Lloyd-like algorithm to deploy nodes.

The rest of this paper is organized as follows: In Section II, we introduce the system model and problem formulation. In Section III, we study the optimal FC and sensor deployment for a given routing algorithm. A numerical algorithm is proposed in Section IV to optimize the node deployment. Section V presents the experimental results and Section VI concludes the paper. Due to the page limit, proofs are provided in [30].

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a wireless ad-hoc sensor network consisting of M homogeneous FCs and N homogeneous sensors over a target region $\Omega \in \mathbb{R}^2$. Let $\mathcal{I}_S = \{1, \ldots, N\}$ and $\mathcal{I}_F = \{N+1, \ldots, N+M\}$ denote the set of node indices for sensors and FCs, respectively. When $i \in \mathcal{I}_S$, Node i refers to Sensor i; however, when $i \in \mathcal{I}_F$, Node i refers to FC $(i - N)$. Let $\mathbf{P} = (p_1, \ldots, p_N, p_{N+1}, \ldots, p_{N+M})^T \in \mathbb{R}^{(N+M)\times 2}$ be the node deployment, where $p_i \in \Omega$ denotes the location of Node i. Throughout this paper, we assume that each event within the target region is sensed by only one sensor. Therefore, for each $i \in \mathcal{I}_S$, Sensor i monitors the events occurred in the cell $W_i \subseteq \Omega$, and $\mathbf{W} = (W_1, \ldots, W_N)$ provides a cell partitioning of Ω. According to [26], the frequency of random events taking place over Ω is modeled via a continuous and differentiable spatial density function $f(\omega) : \Omega \to \mathbb{R}^+$. Therefore, the amount of data generated at Sensor i during a unit of time is given by $\Gamma(W_i) = \kappa \int_{W_i} f(\omega) d\omega$ where κ is a positive constant, [26]. The data collected by each sensor node is forwarded to other nodes in the network until it eventually reaches to one or more fusion centers.

According to [27], this WASN can be modeled as a directed acyclic graph $\mathcal{G}(\mathcal{I}_S \bigcup \mathcal{I}_F, \mathcal{E})$ where $\mathcal{E}$ is the set of directed links (n, k) such that $n \in \mathcal{I}_S$ and $k \in \mathcal{I}_S \bigcup \mathcal{I}_F$. In particular, sensors and FCs are source nodes and sink nodes of this network, respectively, and there is no cycle in the flow network since each cycle can be eliminated by reducing the flows along the cycle without influencing the in-flow and out-flow links to that cycle. We define $\mathbf{F} = [F_{i,j}]_{N\times(N+M)}$ to be the flow matrix, where $F_{i,j}$ is the amount of data transmitted through the link (i, j) in the unit time. Since $\mathbf{F}$ depends on the cell partitioning $\mathbf{W}$, we can define the normalized flow matrix as follows:

$$\mathbf{S} = \left.\overbrace{\begin{bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,N+M} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,N+M} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N,1} & s_{N,2} & \cdots & s_{N,N+M} \end{bmatrix}}^{N+M}\right\}N, \tag{1}$$

where $s_{i,j} \triangleq \frac{F_{i,j}}{\sum_{j=1}^{N+M} F_{i,j}}$ is the ratio of in-flow data to Node i that is transmitted to Node j. The normalized flow matrix $\mathbf{S}$ satisfies the following properties:

(a) $s_{i,j} \in [0, 1]$;[1]

(b) $\sum_{j=1}^{N+M} s_{i,j} = 1, \forall i \in \{1, \ldots, N\}$;

(c) No cycle: if there exists a path $l_0 \to l_1 \to \cdots \to l_K$, i.e., $\prod_{k=1}^{K} s_{l_{k-1},l_k} > 0$, then we have $s_{l_K,l_0} = 0$. In particular, $s_{ii} = 0, \forall i \in \{1, \ldots, N\}$.

[1]For time-invariant routing algorithms, such as Bellman-Ford Algorithm [24], [25], the flows construct a tree-structured graph in which each node has only one successor. Under such a circumstance, the normalized flow from Node i to Node j is either 0 or 1, i.e., $s_{i,j} \in \{0, 1\}$. However, the time-variant routing algorithms, such as Flow Augmentation Algorithm [27], will generate different flows during different time periods. As a result, the overall normalized flow from Node i to Node j can be a real number between 0 and 1, i.e., $s_{i,j} \in [0, 1]$.

Since the flow $F_{i,j}$ can be determined by the cell partitioning $\mathbf{W}$ and normalized flow matrix $\mathbf{S}$, in the remaining of this paper we use $\mathbf{F}(\mathbf{W}, \mathbf{S})$ instead of $\mathbf{F}$. Let $F_i(\mathbf{W}, \mathbf{S}) \triangleq \sum_{j=1}^{N+M} F_{i,j}(\mathbf{W}, \mathbf{S})$ be the total flow originated from Node i. Since the in-flow to each sensor, say i, should be equal to the out-flow, we have $\sum_{j=1}^{N} F_{j,i}(\mathbf{W}, \mathbf{S}) + \Gamma(W_i) = \sum_{j=1}^{N+M} F_{i,j}(\mathbf{W}, \mathbf{S})$. In what follows, we provide an example to elucidate how to calculate $F(\mathbf{W}, \mathbf{S})$ in terms of $\mathbf{W}$ and $\mathbf{S}$.

Example 1. We consider a WASN with three sensor nodes and one FC, i.e., $N = 3$ and $M = 1$. The parameter κ is set to 4. For a cell partitioning $\mathbf{W}$ with cell volumes $v(W_1) = v(W_2) = 0.25$, $v(W_3) = 0.5$, and the normalized flow matrix $\mathbf{S} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, the corresponding flow network is illustrated in Fig. 1.

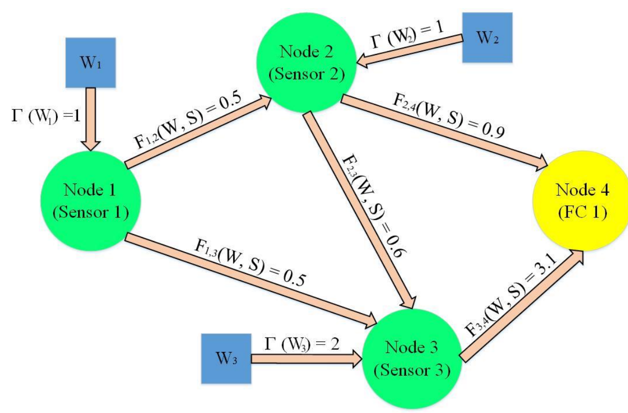

Fig. 1. Example 1

The amount of data generated from each sensor node can be calculated as: $\Gamma(W_1) = \kappa v(W_1) = 1$, $\Gamma(W_2) = \kappa v(W_2) = 1$, and $\Gamma(W_3) = \kappa v(W_3) = 2$. As a leaf node, Sensor 1 does not receive data from any other sensor nodes, and only transmits its sensed data; thus, $F_1(\mathbf{W}, \mathbf{S}) = \Gamma(W_1) = 1$. The flows from Sensor 1 are then $F_{1,2}(\mathbf{W}, \mathbf{S}) = s_{1,2} \times F_1(\mathbf{W}, \mathbf{S}) = 0.5$ and $F_{1,3}(\mathbf{W}, \mathbf{S}) = s_{1,3} \times F_1(\mathbf{W}, \mathbf{S}) = 0.5$, respectively. Sensor 2's flows come from $F_{1,2}(\mathbf{W}, \mathbf{S})$ and the data gathered from the region W_2. Hence, $F_2(\mathbf{W}, \mathbf{S}) = \Gamma(W_2) + F_{1,2}(\mathbf{W}, \mathbf{S}) = 1.5$. Therefore, the flows from Sensor 2 are $F_{2,3}(\mathbf{W}, \mathbf{S}) = s_{2,3} \times F_2(\mathbf{W}, \mathbf{S}) = 0.6$ and $F_{2,4}(\mathbf{W}, \mathbf{S}) = s_{2,4} \times F_2(\mathbf{W}, \mathbf{S}) = 0.9$. Similarly, for Sensor 3, we have $F_3(\mathbf{W}, \mathbf{S}) = \Gamma(W_3) + F_{1,3}(\mathbf{W}, \mathbf{S}) + F_{2,3}(\mathbf{W}, \mathbf{S}) = 3.1$; hence, the unique flow from Sensor 3 is $F_{3,4}(\mathbf{W}, \mathbf{S}) = s_{3,4} \times F_3(\mathbf{W}, \mathbf{S}) = 3.1$.

We focus on power consumption of sensors since FCs are usually equipped with reliable energy sources and their power consumption is not the main concern. The average power consumption through link (i, j) consists of two components: (i) average transmitter power, $\overline{\mathcal{P}}_{i,j}^{T}$; and (ii) average receiver power, $\overline{\mathcal{P}}_{i,j}^{R}$. As shown in [26], because of the free-space path-loss, the instant transmission power is proportional to the square of distance between nodes i and j. Therefore, Sensor i's average transmitter power through link (i, j) is modeled as $\overline{\mathcal{P}}_{i,j}^{T} = \beta\|p_i - p_j\|^2 F_{i,j}(\mathbf{W},\mathbf{S})$ where the coefficient β depends on the characteristics of nodes i and j [26]. In homogeneous WASNs, all nodes share the same characteristics; thus, the coefficient β is the same for all links (i,j). According to [31], Sensor j's average receiver power through link (i,j) can be modeled as $\overline{\mathcal{P}}_{i,j}^{R} = \rho F_{i,j}(\mathbf{W},\mathbf{S})$, where ρ is a constant coefficient for receiving data. In sum, the average power consumption over link (i,j) can be written as

$$\begin{aligned}\overline{\mathcal{P}}_{i,j}(\mathbf{P},\mathbf{W},\mathbf{S}) &= \overline{\mathcal{P}}_{i,j}^{T} + \overline{\mathcal{P}}_{i,j}^{R}\\ &= \begin{cases}\left(\beta\|p_i - p_j\|^2 + \rho\right) F_{i,j}(\mathbf{W},\mathbf{S}), & j \in \mathcal{I}_S\\ \left(\beta\|p_i - p_j\|^2\right) F_{i,j}(\mathbf{W},\mathbf{S}), & j \in \mathcal{I}_F\end{cases}\end{aligned} \tag{2}$$

and the total power consumption can be written as

$$\begin{aligned}\overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S}) &= \sum_{i=1}^{N}\sum_{j=1}^{N+M}\overline{\mathcal{P}}_{i,j}(\mathbf{P},\mathbf{W},\mathbf{S})\\ &= \sum_{i=1}^{N}\left[\sum_{j=1}^{N+M}\beta\|p_i - p_j\|^2 F_{i,j}(\mathbf{W},\mathbf{S}) + \rho\sum_{j=1}^{N}F_{i,j}(\mathbf{W},\mathbf{S})\right].\end{aligned} \tag{3}$$

According to [2], [11]–[16], for a given node deployment $\mathbf{P}$ and cell partitioning $\mathbf{W}$, the sensing uncertainty can be formulated as:

$$\mathcal{H}(\mathbf{P},\mathbf{W}) = \sum_{n=1}^{N}\int_{W_n}\|p_n - \omega\|^2 f(\omega)d\omega. \tag{4}$$

Our main goal is to minimize the power consumption and sensing uncertainty defined in (3) and (4), respectively. However, as will be shown in Section V, there is a trade-off between sensing uncertainty and power consumption. Intuitively, sensing uncertainty is minimized when sensors are located on the centroid of their corresponding regions; however, this will usually increase the pair-wise distance between nodes which leads to an increase in power consumption. Therefore, one objective is to minimize the sensing uncertainty given a constraint on the total power consumption, or vice versa. This constrained optimization can equivalently be formulated as the following Lagrangian cost function:

$$\begin{aligned}D(\mathbf{P},\mathbf{W},\mathbf{S}) &= \mathcal{H}(\mathbf{P},\mathbf{W}) + \lambda\overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S})\\ &= \sum_{i=1}^{N}\int_{W_i}\|p_i - \omega\|^2 f(\omega)d\omega + \lambda\rho\sum_{i=1}^{N}\sum_{j=1}^{N}F_{i,j}(\mathbf{W},\mathbf{S})\\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N+M}\left(\lambda\beta\|p_i - p_j\|^2\right) F_{i,j}(\mathbf{W},\mathbf{S}),\end{aligned} \tag{5}$$

where $\lambda \geq 0$, i.e. the Lagrangian multiplier, makes a trade-off between sensing uncertainty and total power consumption. Our main goal in this paper is to minimize the cost function defined in (5) over the node deployment $\mathbf{P}$, cell partitioning $\mathbf{W}$, and the normalized flow matrix $\mathbf{S}$.

III. Optimal node deployment in WASNs

As it is shown in (5), the cost function depends on three variables $\mathbf{P}$, $\mathbf{W}$ and $\mathbf{S}$. Therefore, our goal is to find the optimal node deployment, cell partitioning and the normalized flow matrix, denoted by $\mathbf{P}^* = \left(p_1^*,\ldots,p_{N+M}^*\right)$, $\mathbf{W}^* = \left(W_1^*,\ldots,W_N^*\right)$ and $\mathbf{S}^* = \left[s_{i,j}^*\right]_{N\times(N+M)}$, respectively, that minimizes the cost function. Note that not only the optimal values of these variables depend on each other, but also the optimization problem is NP-hard. We aim to design an iterative algorithm that optimizes the value of one variable while the other two variables are held fixed. To accomplish this goal, we study the necessary conditions for optimal deployment at each step. Let

$$e_{i,j}(\mathbf{P}) \triangleq \frac{\overline{\mathcal{P}}_{i,j}(\mathbf{P},\mathbf{W},\mathbf{S})}{F_{i,j}(\mathbf{W},\mathbf{S})} = \begin{cases}\beta\|p_i - p_j\|^2 + \rho, & j \in \mathcal{I}_S\\ \beta\|p_i - p_j\|^2, & j \in \mathcal{I}_F\end{cases} \tag{6}$$

be the Link (i,j)'s energy cost (Joules/bit). Without loss of generality, we assume that Sensor i's collected data goes through K_i paths $\{L_k^{(i)}(\mathbf{S})\}_{k\in\{1,\ldots,K_i\}}$, where $L_k^{(i)}(\mathbf{S}) = l_{k,0}^{(i)} \to l_{k,1}^{(i)} \to \cdots \to l_{k,J_k^{(i)}}^{(i)}$, $l_{k,0}^{(i)} = i$, $l_{k,J_k^{(i)}}^{(i)} \in \mathcal{I}_F$, and $J_k^{(i)}$ is the number of nodes on the k-th path except Node i. Then, the data rate (bits/s) and the path cost (Joules/bit) corresponding to the k-th path can be written as

$$\mu_k^{(i)}(\mathbf{W},\mathbf{S}) = F_i(\mathbf{W},\mathbf{S})\prod_{j=1}^{J_k^{(i)}} s_{l_{k,j-1}^{(i)},l_{k,j}^{(i)}}, \tag{7}$$

and

$$\overline{e}_k^{(i)}(\mathbf{P},\mathbf{S}) = \sum_{j=1}^{J_k^{(i)}} e_{l_{k,j-1}^{(i)},l_{k,j}^{(i)}}(\mathbf{P}), \tag{8}$$

respectively. Note that $\sum_k \mu_k^{(i)}(\mathbf{W},\mathbf{S}) = F_i(\mathbf{W},\mathbf{S})$ which means the data from Node i eventually reaches one or more FCs. Sensor i's power coefficient, denoted by $g_i(\mathbf{P},\mathbf{S})$, is then defined to be the energy consumption (Joules/bit) for transmitting 1 bit data from Sensor i to the FCs, i.e, we have[2]:

$$\begin{aligned}g_i(\mathbf{P},\mathbf{S}) &= \frac{\sum_{k=1}^{K_i}\mu_k^{(i)}(\mathbf{W},\mathbf{S})\overline{e}_k^{(i)}(\mathbf{P},\mathbf{S})}{F_i(\mathbf{W},\mathbf{S})}\\ &= \sum_{k=1}^{K_i}\left[\prod_{j=1}^{J_k^{(i)}} s_{l_{k,j-1}^{(i)},l_{k,j}^{(i)}}\left(\sum_{j=1}^{J_k^{(i)}}\beta\left\|p_{l_{k,j-1}^{(i)}} - p_{l_{k,j}^{(i)}}\right\|^2 + \rho\left(J_k^{(i)} - 1\right)\right)\right].\end{aligned} \tag{9}$$

In what follows, we provide an example to clarify how to calculate the sensor power coefficients.

Example 2. Consider the WASN described in Fig. 1, and let $\mathbf{P} = ((0,0),(0,1),(1,0),(1,1))$, $\beta = 1$ and $\rho = 1$. We aim to find Sensor 1's power coefficient $g_1(\mathbf{P},\mathbf{S})$. The link energy costs for this network can be calculated as $e_{1,2}(\mathbf{P}) = e_{1,3}(\mathbf{P}) = 2$, $e_{2,3}(\mathbf{P}) = 3$, and $e_{2,4}(\mathbf{P}) = e_{3,4}(\mathbf{P}) = 1$. Note that Sensor 1's data goes through the following 3 paths: $L_1^{(1)}(\mathbf{S}) = 1 \to 2 \to 4$, $L_2^{(1)}(\mathbf{S}) = 1 \to 3 \to 4$, and $L_3^{(1)}(\mathbf{S}) = 1 \to 2 \to 3 \to 4$. The data rate through the above paths are, respectively, $\mu_1^{(1)}(\mathbf{W},\mathbf{S}) = F_1(\mathbf{W},\mathbf{S}) \times s_{1,2} \times s_{2,4} = 0.3F_1(\mathbf{W},\mathbf{S})$, $\mu_2^{(1)}(\mathbf{W},\mathbf{S}) = F_1(\mathbf{W},\mathbf{S}) \times s_{1,3} \times s_{3,4} = 0.5F_1(\mathbf{W},\mathbf{S})$, and

[2]The term $F_i(\mathbf{W},\mathbf{S})$ is canceled in (9), indicating that power coefficient $g_i(\mathbf{P},\mathbf{S})$ is independent of $\mathbf{W}$.

$\mu_3^{(1)}(\mathbf{W},\mathbf{S}) = F_1(\mathbf{W},\mathbf{S}) \times s_{1,2} \times s_{2,3} \times s_{3,4} = 0.2F_1(\mathbf{W},\mathbf{S})$. Moreover, we can calculate the path costs using (8) as follows: $\overline{e}_1^{(1)}(\mathbf{P}) = e_{1,2}(\mathbf{P}) + e_{2,4}(\mathbf{P}) = 3$, $\overline{e}_2^{(1)}(\mathbf{P}) = e_{1,3}(\mathbf{P}) + e_{3,4}(\mathbf{P}) = 3$, and $\overline{e}_3^{(1)}(\mathbf{P}) = e_{1,2}(\mathbf{P}) + e_{2,3}(\mathbf{P}) + e_{3,4}(\mathbf{P}) = 6$. Then, Sensor 1's power coefficient is $g_1(\mathbf{P},\mathbf{S}) = 0.3 \times 3 + 0.5 \times 3 + 0.2 \times 6 = 3.6$.

Note that the average power consumption for transmitting Sensor i's data is $g_i(\mathbf{P},\mathbf{S})\Gamma(W_i) = g_i(\mathbf{P},\mathbf{S})\kappa \int_{W_i} f(\omega)d\omega$. Thus, the total power consumption (3) can be rewritten as:

$$\overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S}) = \sum_{i=1}^{N} g_i(\mathbf{P},\mathbf{S})\kappa \int_{W_i} f(\omega)d\omega. \tag{10}$$

Therefore, the cost function in (5) can be rewritten as:

$$\begin{aligned} D(\mathbf{P},\mathbf{W},\mathbf{S}) &= \mathcal{H}(\mathbf{P},\mathbf{W}) + \lambda\overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S}) \\ &= \sum_{i=1}^{N} \int_{W_i} \left(\|p_i - \omega\|^2 + \lambda\kappa g_i(\mathbf{P},\mathbf{S})\right) f(\omega)d\omega. \end{aligned} \tag{11}$$

Now, given the node deployment $\mathbf{P}$ and normalized flow matrix $\mathbf{S}$, the optimal cell partitioning is equal to:

$$\begin{aligned} \mathcal{V}_i(\mathbf{P},\mathbf{S}) =& \{\omega | \|p_i - \omega\|^2 + \lambda\kappa g_i(\mathbf{P},\mathbf{S}) \leq \\ & \|p_j - \omega\|^2 + \lambda\kappa g_j(\mathbf{P},\mathbf{S}), \forall j \neq i\}, i \in \mathcal{I}_S. \end{aligned} \tag{12}$$

Moreover, given the link costs $\{e_{ij}(\mathbf{P})\}$s and generated sensing data rates $\{\Gamma(W_i)\}$s, the total power consumption can be minimized by Bellman-Ford Algorithm [24], [25]. For convenience, we represent the functionality of Bellman-Ford Algorithm by $\mathcal{R}(\mathbf{P},\mathbf{W})$, where $\mathbf{P}$ and $\mathbf{W}$ are inputs and $\mathbf{S}$ is the output, i.e., $\mathcal{R}(\mathbf{P},\mathbf{W}) = \arg\min_{\mathbf{S}} \overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S})$. Since sensing uncertainty $\mathcal{H}(\mathbf{P},\mathbf{W})$ is independent of $\mathbf{S}$, we have:

$$\begin{aligned} \mathcal{R}(\mathbf{P},\mathbf{W}) &= \arg\min_{\mathbf{S}} \mathcal{H}(\mathbf{P},\mathbf{W}) + \lambda\overline{\mathcal{P}}(\mathbf{P},\mathbf{W},\mathbf{S}) \\ &= \arg\min_{\mathbf{S}} D(\mathbf{P},\mathbf{W},\mathbf{S}). \end{aligned} \tag{13}$$

The optimal flow matrix for a given $\mathbf{P}$ and $\mathbf{W}$ is then $\mathbf{F}(\mathbf{W},\mathcal{R}(\mathbf{P},\mathbf{W}))$. The following theorem provides the necessary conditions for the optimal deployment.

Theorem 1. The necessary conditions for the optimal deployments in the WASNs with the cost defined in (5) are

$$p_i^* = \frac{c_i^* v_i^* + \lambda\beta\left(\sum\limits_{j=1}^{N+M} F_{i,j}^* p_j^* + \sum\limits_{j=1}^{N} F_{j,i}^* p_j^*\right)}{v_i^* + \lambda\beta\left(\sum\limits_{j=1}^{N+M} F_{i,j}^* + \sum\limits_{j=1}^{N} F_{j,i}^*\right)}, \quad \forall i \in \mathcal{I}_S \tag{14}$$

$$p_i^* = \frac{\sum\limits_{j=1}^{N} F_{j,i}^* p_j^*}{\sum\limits_{j=1}^{N} F_{j,i}^*}, \quad \forall i \in \mathcal{I}_F \tag{15}$$

$$\mathbf{W}^* = \mathcal{V}(\mathbf{P}^*,\mathbf{S}^*), \tag{16}$$

$$\mathbf{S}^* = \mathcal{R}(\mathbf{P}^*,\mathbf{W}^*), \tag{17}$$

where $v_i^* = \int_{\mathcal{V}_i(\mathbf{P}^*,\mathbf{S}^*)} f(\omega)d\omega$ and $c_i^* = \frac{\int_{\mathcal{V}_i(\mathbf{P}^*,\mathbf{S}^*)} \omega f(\omega)d\omega}{\int_{\mathcal{V}_i(\mathbf{P}^*,\mathbf{S}^*)} f(\omega)d\omega}$ are the Lebesgue measure (volume) and geometric centroid of the region $\mathcal{V}_i(\mathbf{P}^*,\mathbf{S}^*)$, respectively, and $F_{i,j}^* = F_{i,j}(\mathbf{W}^*,\mathbf{S}^*)$ is the optimal flow from Node i to Node j.

The proof of Theorem 1 is provided in [30]. Let $\mathcal{N}_i^P(\mathbf{S}) \triangleq \{j | F_{j,i}(\mathbf{W},\mathbf{S}) > 0\}$ be the set of Node i's predecessors, and $\mathcal{N}_i^S(\mathbf{S}) \triangleq \{j | F_{i,j}(\mathbf{W},\mathbf{S}) > 0\}$ be the set of Node i's successors. Hence, (14) and (15) can be simplified as

$$p_i^* = \frac{c_i^* v_i^* + \lambda\beta\left(\sum\limits_{j \in \mathcal{N}_i^S(\mathbf{S}^*)} F_{i,j}^* p_j^* + \sum\limits_{j \in \mathcal{N}_i^P(\mathbf{S}^*)} F_{j,i}^* p_j^*\right)}{v_i^* + \lambda\beta\left(\sum\limits_{j \in \mathcal{N}_i^S(\mathbf{S}^*)} F_{i,j}^* + \sum\limits_{j \in \mathcal{N}_i^P(\mathbf{S}^*)} F_{j,i}^*\right)} \tag{18}$$

for each $i \in \mathcal{I}_S$, and

$$p_i^* = \frac{\sum\limits_{j \in \mathcal{N}_i^P(\mathbf{S}^*)} F_{j,i}^* p_j^*}{\sum\limits_{j \in \mathcal{N}_i^P(\mathbf{S}^*)} F_{j,i}^*} \tag{19}$$

for each $i \in \mathcal{I}_F$, respectively. In other words, Sensor i's optimal location is a linear combination of its geometric centroid, predecessors, and successors while FC i's optimal location is a linear combination of its predecessors.

IV. Routing-aware Lloyd Algorithm

First, we quickly review Lloyd Algorithm [29]. Lloyd Algorithm iterates between two steps: (i) Voronoi partitioning and (ii) Moving each node to the geometric centroid of its corresponding Voronoi region. Although the conventional Lloyd Algorithm can be used for one-tier quantizers or one-tier node deployment tasks, it cannot be applied to WASNs with multi-hop wireless communications. Based on the properties explored in Section III, we design a Routing-aware Lloyd (RL) Algorithm to optimize the node deployment in WASNs and minimize the cost function in (5). To avoid a poor initial deployment, first, we design a quantizer with N (M) points for the spatial density function $f(\omega)$ and place the sensors (FCs) on the corresponding centroids. This results in an even distribution of sensors among FCs as the initial deployment. RL Algorithm then iterates between three steps: (i) Update the node deployment $\mathbf{P}$ according to (14) and (15); (ii) run Bellman-Ford Algorithm to update the normalized flow matrix $\mathbf{S}$ and obtain the sensor power coefficients $g_i(\mathbf{P},\mathbf{S})$ and the flow matrix $\mathbf{F}(\mathbf{W},\mathbf{S})$; and (iii) update the cell partitioning $\mathbf{W}$ according to (16) and update the value of volumes v_n and centroids c_n. The algorithm continues until the stop criterion, $\frac{D_{\text{old}} - D_{\text{new}}}{D_{\text{old}}} \geq \epsilon$ is satisfied (D_{old} and D_{new} are the cost functions in the previous and current iterations, respectively).

Theorem 2. RL Algorithm is an iterative improvement algorithm, i.e., the cost function is non-increasing and the algorithm converges.

The proof of Theorem 2 is provided in [30].

V. PERFORMANCE EVALUATION

We provide the experimental results for a WASN including 4 FCs and 40 sensors. To make a fair comparison, we use the same target region and density function as in [26], [28], i.e., $\Omega = [0, 10]^2$ and $f(\omega) = \frac{1}{\int_\Omega d\omega} = 0.01$. Other parameters are set as follows: $\beta = 1$, $\rho = 0.1$, $\kappa = 1$, $\epsilon = 10^{-6}$.

To the best of our knowledge, this is the first work to consider both sensing uncertainty and power consumption in WASNs. Bellman-Ford Algorithm [24], [25] is the best routing algorithm to minimize the total energy consumption, but it does not take node deployment into account. To compare with Bellman-Ford Algorithm, we apply random deployment and Lloyd Algorithm [29] for the node deployment part. Random deployment + Bellman-Ford (RBF) employs Bellman-Ford Algorithm on 100 random node deployments and selects the best one. Similarly, Lloyd + Bellman-Ford (LBF) first applies Lloyd Algorithm to both FCs and Sensors to obtain a node deployment with small cost, and then employs Bellman-Ford Algorithm to reduce the average power. Furthermore, we compare RL with Combining Lloyd (CL) [26] which combines two Lloyd-like algorithms to optimize the node deployment with one-hop communications.

Performance results[3] for different values of $\lambda \in \{0, 0.05, 0.15, 0.25, 0.5, 1, 1.5, 2, 3, 4, 5, 7, 10, 16\}$ are provided in Fig. 2. Note that the trade-off between sensing uncertainty and power consumption, represented by the constant parameter λ, is taken into account in both CL and RL algorithms. However, RBF and LBF algorithms are independent of λ. In particular, since LBF determines the node deployment by Lloyd Algorithm before employing Bellman-Ford Algorithm, LBF's performance is almost independent of the initial deployments, and its experimental results in Fig. 2 converge to a point with small sensing uncertainty but large power consumption. For small values of λ, the cost function in (5) favors the sensing uncertainty over power consumption, which leads to the points on the left-hand side of the RL curve in Fig. 2. Similarly, large values of λ results in points on the right-hand side of the RL curve. Overall, the proposed RL algorithm outperforms other algorithms by saving more power and reducing more sensing uncertainty, in addition to providing a trade-off.

The node deployments of the four algorithms (RBF, LBF, CL, and RL) in the WASN with $\lambda = 0.25$ are illustrated in Figs. 3a, 3b, 3c, and 3d. FCs and sensors are denoted by five-pointed stars and circles, respectively. Flows are denoted by black dotted lines. As shown in Fig. 3, cell partitions in LBF, CL and RL algorithms tend to have similar shapes; however, RBF does not result in a similar pattern. Moreover, sensors in Fig. 3b are placed on top of their corresponding centroids while sensors in Fig. 3c are placed between their corresponding FC and centroid. However, in Fig. 3d, location of each sensor depends on its centroid, predecessors, and successors, as provided in Theorem 1. Note that in Figs. 3b, 3c and 3d, sensors inside each cluster tend to be close to each other with their FC in the middle; however, the same relationship does not appear in Fig. 3a. Besides, CL only uses one-hop communications, i.e., sensors are directly connected to the FC while other algorithms utilize multi-hop communications to reduce the average power. The corresponding cost function given $\lambda = 0.25$ for RBF, LBF, CL, and RL are, respectively, 1.87, 1.25, 1.17, 1.01; thus, our RL Algorithm achieves a lower cost function and outperforms other algorithms.

[3]To better exhibit the performance of LBF, CL, RL, we do not show the results of RBF with excessive powers ($\overline{\mathcal{P}} > 6$) in Fig. 2.

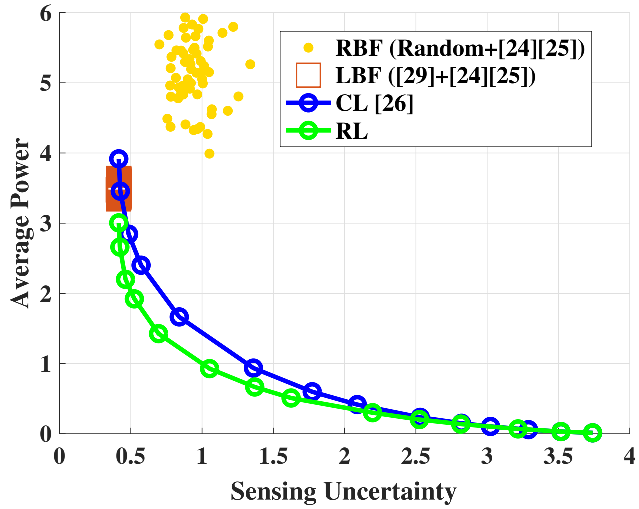

Fig. 2. Performance comparison for RBF, LBF, CL and RL algorithms.

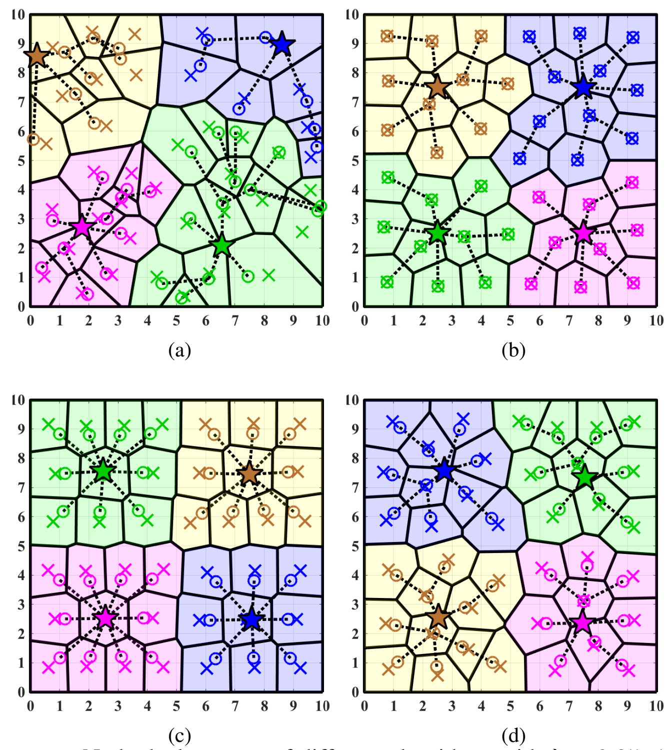

Fig. 3. Node deployments of different algorithms with $\lambda = 0.25$: (a) RBF (b) LBF (c) CL (d) RL.

VI. Conclusions and Discussion

In this paper, we formulated the node deployment in WASNs as an optimization problem to make a trade-off between sensing uncertainty and energy consumption. The necessary conditions for the optimal deployment imply that each sensor location should be a linear combination of its centroid, predecessors and successors. Based on these necessary conditions, we proposed a Lloyd-like algorithm to minimize the total cost. Our experimental results show that the proposed algorithm significantly reduces both sensing uncertainty and energy consumption. Although we only considered Bellman-Ford Algorithm as the routing algorithm in this paper, the proposed system model in Section II can be applied to arbitrary routing algorithms, such as Flow Augmentation Algorithm [27] (a network lifetime maximization routing algorithm). The optimal deployment with maximum network lifetime will be our future work.

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