



# Scaling exchange and correlation in the on-top density functional of multiconfiguration pair-density functional theory: effect on electronic excitation energies and bond energies

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## Abstract

Multiconfiguration pair-density functional (MC-PDFT) theory provides an economical way to calculate the ground-state and excited-state energetics of strongly correlated systems. The energy is calculated from the kinetic energy, density, and on-top pair-density of a multiconfiguration wave function as the sum of kinetic energy, classical Coulomb energy, and on-top density functional energy. We have usually found good results with the translated Perdew–Burke–Ernzerhof (tPBE) on-top density functional, and in this article, we examine whether the results can be systematically improved by introducing scaling constants into the exchange and correlation terms. We find that only a small improvement is possible for electronic excitation energies and that no improvement is possible for bond energies.

**Keywords** Excitation energies · Bond energies · Benchmark · MC-PDFT · Pair-density · On-top pair-density functionals

## 1 Introduction

Electronically excited states play a key role in many branches of chemistry, including photochemistry [1], photosynthesis [2], and optoelectronic material function [3–6]. Modeling electronically excited states remains a major challenge for modern electronic structure theories because they are usually strongly correlated due to near degeneracy effects. Kohn–Sham density functional theory (KS-DFT) is widely used for ground-state properties, and time-dependent Kohn–Sham density functional theory (TD-KS-DFT) is widely used to determine excitation energies. This approach

works more satisfactorily for valence excited states than for charge transfer and Rydberg excitations [7–12]. Moreover, even modern functionals do not always provide satisfactory performance in describing multiconfigurational systems.

Multiconfiguration pair-density functional theory (MC-PDFT) [13, 14] is an alternative to KS-DFT for excited states and multiconfigurational systems in general, and it has been shown to give promising results for atomic [13], organic [15–20], and inorganic systems [21], including transition metal species [22, 23] and heavy atoms [24, 25]. Unlike complete active space second-order perturbation theory (CASPT2) [26, 27] and multireference configuration interaction (MRCI), the cost of an MC-PDFT calculation is dominated by the cost of the parent multiconfigurational self-consistent field (MCSCF) calculation. MC-PDFT usually provides results comparable in accuracy to those of CASPT2, and therefore it is an appealing alternative to

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CASPT2 because of the significantly reduced computational costs, in terms of both memory and time requirements.

In MC-PDFT, the energy is computed from the kinetic energy and the classical Coulomb energy of an MCSCF wave function (third and fourth terms of Eq. 1) and from an on-top density functional (last term of Eq. 1) depending on the total electronic density  $\rho(\mathbf{r})$ , its gradient, and the on-top pair-density  $\Pi(\mathbf{r})$  [28, 29], where  $\mathbf{r}$  denotes a point in space

$$E^{MC-PDFT} = V_{NN} + V_{Ne}(\rho(\mathbf{r})) + \langle \Psi^{MCSCF} | T | \Psi^{MCSCF} \rangle + V_C(\rho(\mathbf{r})) + E_{ot}[\rho(\mathbf{r}), |\nabla\rho|, \Pi(\mathbf{r})] \quad (1)$$

where  $V_{NN}$  and  $V_{Ne}(\rho(\mathbf{r}))$  denote the nuclear repulsion and the electron–nucleus attraction, respectively.

The currently used on-top density functionals are translations of existing KS-DFT functionals. The most widely studied functional is the translated PBE (tPBE [13, 14]) functional, which has a translated exchange term and a translated correlation term (see Ref. [13] for details on “translation” of density functionals). In this study, we explore the effect of multiplying the exchange term by a constant  $S_X$  and multiplying the correlation term by a constant  $S_C$ . The tPBE functional with this scaling will be abbreviated tPBE ( $S_X, S_C$ ), where the standard tPBE energy corresponds to  $E^{MC-PDFT} = \text{tPBE}(1.0, 1.0)$ . The functional scaling is motivated by recent work in the context of KS-DFT, where it was shown that using  $S_X = 1.25$  and  $S_C = 0.5$  [30, 31] for PBE (this scaling being labeled “HLE”: high local exchange) provides an improved description of molecular electronic excitations, including those of Rydberg type, and of semiconductor band gaps. A main advantage of KS-DFT local functionals (such as PBE and HLE-type functionals) is the absence of static correlation error caused by the presence of HF exchange

[32–34]. On the other hand, the MCSCF wave function in MC-PDFT includes nonlocal MCSCF exchange; thus, only local on-top functionals are used (in their translated form), although these functionals have not been optimized for MC-PDFT. However, given the similarities between local KS-DFT functionals and local translated on-top functionals, the study of functional scaling in MC-PDFT helps to understand how electronic properties are affected by changes in the functional. For example, it was observed that increasing the exchange contribution improves the valence singlet and triplet  $\pi\pi^*$  excitation energies for benzene at the MC-PDFT level [20]. Calculations with these special values of  $S_X$  and  $S_C$  were previously labeled HLE in the context of KS-DFT, and so, we also use that label in the context of MC-PDFT, i.e., tPBE (1.25, 0.5)  $\equiv$  tPBE–HLE.

We considered wide ranges of values for  $S_X$  and  $S_C$ , and we report the mean signed and unsigned errors (MSEs and MUEs) with respect to the experimental or accurate theoretical data available in the literature. For electronic excitations (Tables 1 and 2), we report the results obtained with tPBE (1.1, 0.5), which we found to perform best of all examined scalings, although no significant improvement on average is obtained with respect to tPBE. Note that, as compared to previous work, we updated many of the reference values to which we compare with the FCI/CBS estimates from the recent work of Loos et al. [35] that are reported for some of the electronic excitations. (The retained old reference values and the new ones are reported in Table S4 of ESM.) We also count the number of cases with an error below 0.28 eV—which was chosen (somewhat arbitrarily) as the borderline between reasonably successful and unsuccessful.

**Table 1** Spin-forbidden atomic excitation energies (eV). The reference values are taken from the experimental data

		AS <sup>a</sup>	CASSCF	CASPT2	tPBE	tPBE–HLE	tPBE (1.1,0.5)	References
Be	$^1S \rightarrow ^3P$	(2,4)	2.86	2.78	2.55	2.48	2.47	2.73 [55]
C	$^3P \rightarrow ^1D$	(4,4)	1.56	1.27	1.06	1.59	1.33	1.26 [56]
N <sup>+</sup>	$^3P \rightarrow ^1D$	(4,4)	2.19	1.91	1.51	2.18	1.85	1.89 [56]
N	$^4S \rightarrow ^2D$	(5,4)	2.79	2.51	2.05	2.99	2.53	2.38 [56]
O <sup>+</sup>	$^4S \rightarrow ^2D$	(5,4)	3.78	3.44	2.78	3.94	3.36	3.32 [57]
O	$^3P \rightarrow ^1D$	(6,4)	2.23	1.99	1.27	1.83	1.55	1.96 [56]
Sc <sup>+</sup>	$^3D \rightarrow ^1D$	(2,6)	0.37	0.32	0.47	0.55	0.53	0.30 [58]
Mn	$^6S \rightarrow ^8P$	(7,9)	2.19	2.38	2.15	1.76	1.92	2.15 [58]
Co	$^4F \rightarrow ^2F$	(9,6)	2.29	0.40	0.89	– 0.01 <sup>b</sup>	0.71	0.89 [58]
Mo	$^7S \rightarrow ^5S$	(6,6)	1.66	1.67	2.00	2.52	2.28	1.34 [59]
Ru	$^5F \rightarrow ^3F$	(8,6)	0.97	0.95	1.08	1.37	1.23	0.78 [60]
	MUE <sup>c</sup>		0.35	0.15	0.31	0.50	0.27	

<sup>a</sup>Active space (see pages S2–S3 of ESM to see how active spaces are defined). The numbers ( $m, n$ ) indicate  $m$  active electrons in  $n$  active orbitals

<sup>b</sup>The tPBE–HLE functional gives the wrong sign

<sup>c</sup>Mean unsigned error

**Table 2** Spin-conserving molecular excitation energies (eV) of the EE27/19 database)

		$N^a$	AS <sup>b</sup>	CASSCF	CASPT2	tPBE	tPBE–HLE	tPBE (1.1,0.5)	References
<i>n</i> → <i>π</i> * valence									
Acetaldehyde	<sup>1</sup> A''	2	(12,12)	4.56	4.27	4.28	4.21	4.22	<b>4.31</b> [35] <sup>c</sup>
Acetone	<sup>1</sup> A <sub>2</sub>	2	(12,12)	4.71	4.44	4.47	4.38	4.40	4.43 [61]
Formaldehyde	<sup>1</sup> A <sub>2</sub>	2	(12,10)	4.73	3.89	3.89	3.85	3.86	<b>3.97</b> [35]
Pyrazine	<sup>1</sup> B <sub>3u</sub>	3	(10,10)	5.08	4.04	3.96	4.16	4.06	3.97 [62, 63]
Pyridazine	<sup>1</sup> B <sub>1</sub>	2	(10,10)	4.63	3.67	3.50	3.71	3.60	3.60 [61]
Pyridine	<sup>1</sup> B <sub>1</sub>	3	(8,8)	5.64	5.06	5.06	4.95	5.00	4.74 [64, 65]
Pyrimidine	<sup>1</sup> B <sub>1</sub>	2	(10,10)	4.97	4.39	4.34	4.40	4.36	4.18 [66]
s-tetrazine	<sup>1</sup> B <sub>3u</sub>	2	(14,14)	3.65	2.56	2.54	2.82	2.68	2.25 [61]
<i>π</i> → <i>π</i> * valence									
N <sub>2</sub>	<sup>1</sup> Σ <sub>u</sub> <sup>−</sup>	1	(6,6)	10.90	9.88	9.58	9.91	9.73	<b>9.91</b> [35]
Ethylene	<sup>1</sup> B <sub>1u</sub>	5	(4,10)	7.72	8.16	6.93	8.12	7.19	<b>7.91</b> [35]
Butadiene	<sup>1</sup> B <sub>u</sub>	4	(10,15)	6.38	6.51	6.18	6.58	6.22	6.21 [67]
Cyclopentadiene	<sup>1</sup> B <sub>2</sub>	1	(4,4)	7.35	5.51	4.12	4.68	4.37	5.26 [68, 69]
Benzene	<sup>1</sup> B <sub>2u</sub>	2	(6,13)	4.99	4.83	5.04	4.72	4.88	4.90 [70]
"	<sup>3</sup> B <sub>1u</sub>	1	(6,13)	3.94	3.99	4.28	3.91	4.09	4.12 [71]
Naphthalene	<sup>1</sup> B <sub>3u</sub>	2	(10,10)	4.24	4.21	4.38	4.07	4.23	4.00 [72]
"	<sup>3</sup> B <sub>2u</sub>	1	(10,10)	3.04	3.18	3.32	3.05	3.18	3.11 [71]
Furan	<sup>1</sup> B <sub>2</sub>	2	(6,10)	6.69	6.32	6.47	6.26	6.33	6.06 [73]
"	<sup>3</sup> B <sub>2</sub>	1	(6,10)	3.93	4.07	4.21	3.78	3.99	4.17 [71]
Hexatriene	<sup>1</sup> B <sub>u</sub>	2	(6,12)	5.78	5.31	5.46	5.25	5.34	4.93 [74]
"	<sup>3</sup> B <sub>u</sub>	1	(6,12)	2.88	2.66	2.74	2.64	2.69	2.69 [71]
<i>Other valence</i>									
Be	<sup>1</sup> P	1	(2,4)	6.19	5.71	4.36	5.20	4.75	5.28 [55]
N <sub>2</sub>	<sup>1</sup> Π <sub>g</sub>	1	(6,6)	11.87	9.40	8.64	9.13	8.86	<b>9.33</b> [35]
Rydberg									
Water	<sup>1</sup> B <sub>1</sub>	2	(8,9)	7.69	7.61	7.45	8.44	7.62	<b>7.70</b> [35]
"	<sup>3</sup> B <sub>1</sub>	1	(8,9)	7.16	7.19	7.09	7.86	7.18	<b>7.33</b> [35]
<i>π</i> → <i>π</i> * charge transfer									
pNA <sup>d</sup>	<sup>1</sup> A <sub>1</sub>	3	(12,12)	6.09	4.64	3.95	3.89	3.94	4.30 <sup>e</sup>
DMABN <sup>f</sup>	<sup>1</sup> A <sub>1</sub>	3	(12,12)	6.18	4.87	4.28	4.44	4.36	4.57 [75]
B-TCNE <sup>g</sup>	<sup>1</sup> A <sub>1</sub>	2	(4,4)	4.48	3.84	3.63	3.48	3.58	3.59 [76]
<b>Valence MUE</b>				0.72	0.17	0.32	0.20	0.24	
<b>Rydberg MUE</b>				0.09	0.12	0.24	0.64	0.11	
<b>Charge transfer MUE</b>				1.43	0.30	0.22	0.22	0.19	
<b>Total MUE</b>				<b>0.75</b>	<b>0.18</b>	<b>0.30</b>	<b>0.24</b>	<b>0.22</b>	

<sup>a</sup> $N$  is the number of states whose energies are averaged in the CASSCF and is also the number of states in the model space in MS-CASPT2 calculations

<sup>b</sup>The numbers ( $m$ ,  $n$ ) indicate  $m$  active electrons in  $n$  active orbitals (see pages S2–S3 of ESM to see how active spaces are defined)

<sup>c</sup>Theoretical FCI/CBS estimates from Loos et al. (see Table 6 of Ref. [35]) are highlighted in bold

<sup>d</sup>Intramolecular charge transfer in pNA

<sup>e</sup> $\delta$ -CR-EOMCC(2,3), D, see Refs. [77, 78]

<sup>f</sup>DMABN is a case with intramolecular charge transfer

<sup>g</sup>There is intermolecular charge transfer in the B-TCNE complex

Although the present work is concerned only with MC-PDFT, we note that some previous work [36–38] employed range-separated on-top density functionals in other kinds of hybrid multiconfigurational theories.

## 2 Databases

We consider three databases: SFAEE11, EE27/19, and DBE6.

SFAEE11 contains eleven spin-changing excitations of atomic systems (investigated previously with MC-PDFT in Ref. [13]), including both light atoms and transition metals. We do not include spin–orbit coupling in any of the calculations. For atomic excitation energies, we removed spin–orbit coupling from the experimental values by considering for each state the weighted average of the multiplet’s energy.

EE27 includes the 27 spin-conserving excitation energies: the 23 in the previously defined EE23 database [16, 39] containing valence, Rydberg, and intramolecular and intermolecular charge transfer transitions of organic molecules plus the Be valence  $s \rightarrow p$  excitation, the  $N_2$  valence  $\sigma \rightarrow \pi^*$  and  $\pi \rightarrow \pi^*$  excitations, and the cyclopentadiene  $^1B_2$  valence  $\pi \rightarrow \pi^*$  excitation. We updated the database from the values used previously by using the more accurate data of Loos et al. [35] where available. The updated database is called EE27/19, and full details of the update are given in electronic supplementary material (ESM).

DBE6 contains six bond energies (investigated previously with MC-PDFT in Ref. [13]) that are examples of strongly correlated systems.

The values we report for MUE in the article are based on deviations from the reference data. Electronic supplementary material contains MSEs.

### 3 Methods

#### 3.1 General

We report results for CASSCF, CASPT2, and MC-PDFT. All MC-PDFT [13] calculations employed the tPBE, tPBE–HLE, or tPBE ( $S_x$ ,  $S_c$ ) on-top density functionals. All calculations are gas-phase calculations performed using *Molcas* 8.2 [40].

CASSCF [41] calculations employed a level shift of  $1.0 E_h$  ( $1 E_h \equiv 1 \text{ hartree} = 27.211 \text{ eV}$ ). The Cholesky decomposition

[42] algorithm is used throughout with a threshold of  $10^{-8} E_h$ . The numbers of active electrons and active orbitals in each active space (AS) are given in the tables of the main text, and further details are given in Table S1. (Tables and figures with the prefix S are in ESM.)

CASPT2 calculations were performed for comparison, using an imaginary shift of  $0.2 E_h$  and the default value ( $0.25 E_h$  [43]) for the IPEA shift. The state average (SA) method [44] was applied to organic molecules’ excitations as indicated in Table 2 of the manuscript considering  $N$  states. For these systems, the multistate (MS) method [45] was used for CASPT2 [26, 27] calculations.

The keywords “Atom” and “Linear” were employed to impose spherical and linear symmetry in the case of atoms and linear molecules, respectively. The highest spatial symmetry operator ( $D_{2h}$ ) was employed for the excitation energies of Be, N,  $N_2$ ,  $O^+$ , Co, Mn, and Ru (see Table 1). For all the organic molecules and diatomic molecules reported in Tables 2 and 3, no spatial symmetry was imposed, except for cyclopentadiene and NiCl. (The  $C_{2v}$  symmetry operator was applied in both of those cases.)

For  $Cr_2$ , NiCl, and the transition metal atoms, the second-order Douglas–Kroll–Hess Hamiltonian [46, 47] was used; all the other calculations are nonrelativistic.

For calculating bond energies, in every case the geometry is optimized with the method under consideration. (Sometimes, this is called using “consistently optimized geometries.”)

#### 3.2 Basis sets

The cc-pVTZ basis set [48] was used for main group atoms, for CaO, and for the singlet-to-singlet Be excitation energy. The aug-cc-pVTZ basis set [49] was used for the excitation energies of  $N_2$ .

**Table 3** Bond energies (in eV)

	AS <sup>a</sup>	CASSCF	CASPT2	tPBE	tPBE–HLE	References
H <sub>2</sub>	(2,2)	4.13	4.59	4.68	4.74	4.75 [79]
N <sub>2</sub>	(6,6)	8.84	9.39	9.78	9.36	9.74 [80]
F <sub>2</sub>	(2,2)	0.71	1.49	2.12	2.14	1.66 [81]
Cr <sub>2</sub>	(12,12)	0.00 <sup>b</sup>	0.99 <sup>c</sup>	0.59	– 1.78 <sup>d</sup>	1.47 [82]
CaO	(8,8)	3.99	3.59	4.44	5.99	4.22 [83]
NiCl	(11,12)	2.71	3.94	4.10	5.67	3.97 [84, 85]
	<b>MUE</b>	<b>0.91</b>	<b>0.30</b>	<b>0.30</b>	<b>1.27</b>	

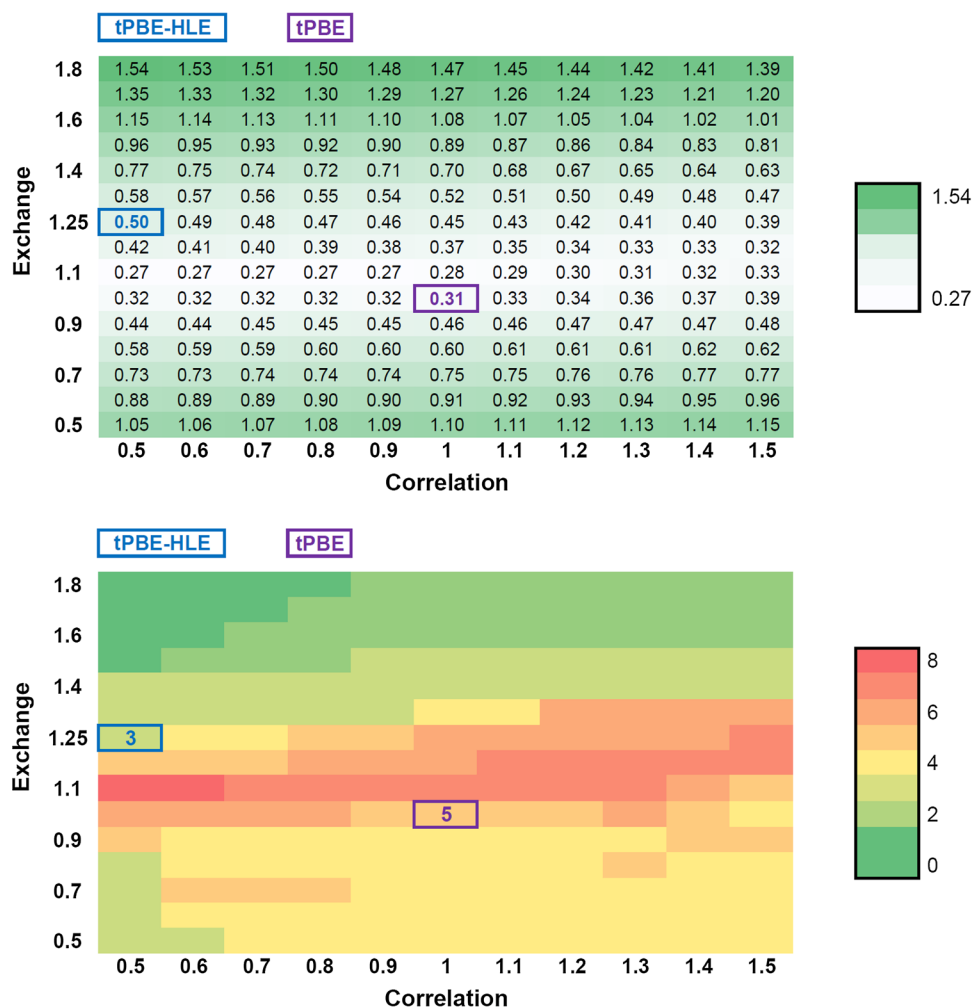
<sup>a</sup>Active space (please refer to pages S2–S3 of ESM to see how active spaces are defined)

<sup>b</sup>The CASSCF dissociation curve of  $Cr_2$  (Figure S7 of ESM) does not have a minimum at  $R_e$  (Ref. 1.68 Å [82])

<sup>c</sup>The CASPT2 minimum energy is erroneously located at 2.65 Å

<sup>d</sup>The tPBE–HLE functional gives the wrong sign

**Fig. 1** (Top) Mean unsigned error (MUE in eV) for the SFAEE11 database (see Table 1) with respect to the amount of exchange and correlation for the tPBE functional. The tPBE and tPBE–HLE MUEs are highlighted. (Bottom) Number of systems that have, for each combination of exchange and correlation, a maximum absolute deviation (from the benchmark) less than or equal to 0.28 eV, out of a total of 11 spin-forbidden atomic excitations. The tPBE and tPBE–HLE results are highlighted with numerical values



The cc-pVTZ-DK basis set [50] was used for  $\text{Cr}_2$ . The ANO-RCC-VTZP basis set [51] was used for NiCl and transition metals in SFAEE11.

The jul-cc-pVTZ basis set [52] was used for molecules in EE27/19 with valence excitations, except for cyclopentadiene, for which the cc-pVTZ basis set was used. The aug-cc-pVTZ basis set was employed for water, the 6-31 + G\*\* basis set [53, 54] was employed for *para*-nitroaniline (*p*NA) and 4-(dimethylamino)-benzonitrile (DMABN), and the aug-cc-pVTZ basis set was used for the benzene–tetracyanoethylene (B-TCNE) complex.

## 4 Results and discussion for excitation energies

The results for the SFAEE11 database of spin-forbidden atomic excitation energies are reported in Table 1. The table shows that CASSCF overestimates the atomic excitation energies by an average of 0.35 eV. CASPT2 provides the smallest errors with respect to the reference data, with an

MUE of 0.15 eV. The tPBE functional performs fairly well for atomic excitations (MUE of 0.31 eV), though still giving larger errors than CASPT2. The tPBE–HLE functional gives a larger MUE (0.50 eV) than tPBE and in general overestimates the excitation energies; the error is especially large for Mo and Ru atoms, whereas the  $\text{Co } ^4\text{F} \rightarrow ^2\text{F}$  excitation is underestimated by tPBE–HLE by 0.9 eV, resulting in an erroneous negative sign.

Figure 1 shows that errors can be reduced if the exchange is scaled by 1.1 and correlation by any factor in the range 0.5–0.9. Figure 1 also shows that one obtains the most accurate predictions with  $S_X = 1.1$  and  $S_C = 0.5$ –0.6, but this does not markedly improve the results obtained with the unscaled tPBE pair-density functional (MUE of 0.27 eV for tPBE (1.1, 0.5) vs. 0.31 eV for tPBE). It is perhaps significant though that the tPBE (1.1, 0.5) functional, which is much more similar to tPBE–HLE than to tPBE, gives a reasonable  $\text{Co } ^4\text{F} \rightarrow ^2\text{F}$  excitation energy.

The results for the EE27/19 database of spin-conserving excitations energies are reported in Table 2. The EE27/19 database contains 22 valence excitations, two Rydberg



transitions, and three charge transfer transitions. CASSCF gives the largest deviations with respect to the reference data, especially for valence excitations (MUE 0.72 eV) and charge transfer excitations (MUE 1.43 eV). CASPT2 give better results, although charge transfer excitations are still overestimated (MUE 0.30 eV).

The cyclopentadiene valence  $\pi \rightarrow \pi^*$  excitation, which is well predicted by CASPT2 (5.51 eV) with respect to experiment (5.26 eV), remains problematic for both tPBE and tPBE–HLE (underestimated by more than 0.5 eV). Overall, tPBE–HLE is better than tPBE for valence excitations, but less accurate for Rydberg excitations. The top panel of Fig. 2 shows that again one gets the best results with tPBE (1.1, 0.5), with the MUE being reduced from 0.30 to 0.22 eV. The bottom panel of Fig. 2 shows the number of systems, out of 27, that have an absolute deviation lower than or equal to 0.28 eV; this value is the largest for  $S_X = 1.1$ –1.2 and  $S_C = 0.5$ –0.7 (for tPBE (1.1, 0.5) there are 20 systems with this accuracy), whereas it corresponds to 15 and 18 systems for tPBE and tPBE–HLE, respectively. However, the improvement given by tPBE (1.1, 0.5) over tPBE and

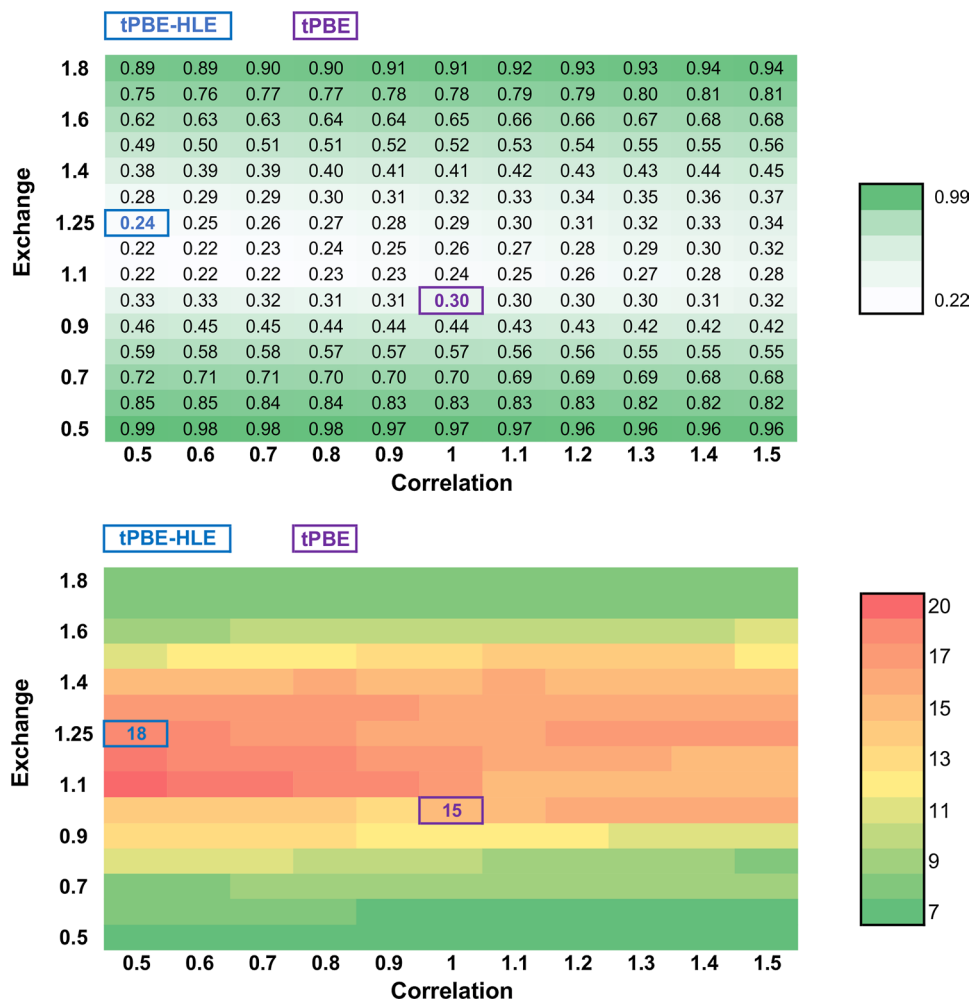
tPBE–HLE is modest and involves mostly the two Rydberg excitations.

## 5 Results and discussion for bond energies

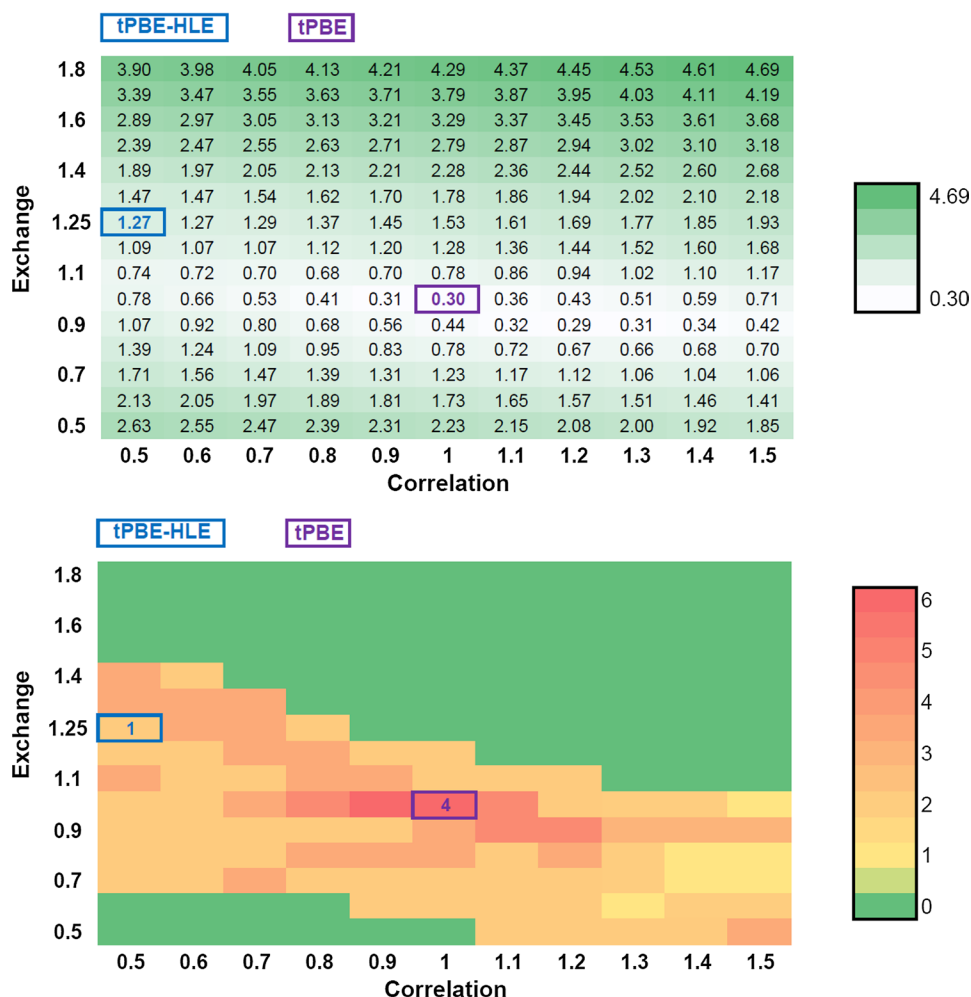
Table 3 presents the results for six bond energies. These include the  $\text{Cr}_2$  molecule, whose ground-state dissociation is particularly difficult to describe. The tPBE–HLE bond energy has the wrong sign, and its MUE is therefore increased to 1.27 eV, higher even than for CASSCF (MUE 0.90 eV). Figure 3 shows that the original tPBE exchange–correlation scaling provides the most reliable results for bond energies. It also shows, somewhat amazingly, that no improvement can be gained on average by scaling exchange and/or correlation. The original tPBE functional slightly outperforms CASPT2 for both energy values and the qualitatively correct behavior of dissociation curves, which are shown in ESM.

Bond energies appear to be very sensitive to parameter scaling, especially the exchange term. In contrast to

**Fig. 2** Results for EE27/19 database statistics of spin-conserving atomic excitation energies. (Top) MUEs (in eV) reported for each combination of exchange and correlation. (Bottom) Heat map showing the number of systems (out of 27) with maximum absolute deviation less than or equal to 0.28 eV. The tPBE and tPBE–HLE values are highlighted



**Fig. 3** (Top) Mean unsigned error (MUE) in eV for six bond energies (see Table 3) with respect to the amount of exchange and correlation for the tPBE functional. The tPBE and tPBE–HLE MUEs are highlighted. (Bottom) Number of systems that have, for each combination of exchange and correlation, a maximum absolute deviation (from the benchmark) less than or equal to 0.28 eV, out of a total of six bond energies (see Table 3). The tPBE and tPBE–HLE results are highlighted with numerical values



excitation energies, which are discussed above, the best combination is given by  $S_X = 1.0$  and  $S_C = 0.9$ – $1.1$ , yet this suffices to suitably describe only 4 systems out of 6. This suggests that a simple functional scaling is not sufficient to obtain on-top density functionals that perform broadly and significantly better than the ones we have been using.

## 6 Conclusions

We investigated how the scaling of exchange and correlation contributions of on-top density functionals in MC-PDFT affects properties such as atomic and molecular excitation energies and bond dissociation energies. A new combination of excitation energies for spin-conserving excitation energies including valence, Rydberg and CT excitation was introduced as database EE27/19 for this study, and we also compared to spin-forbidden atomic excitation energies and to bond energies.

The results are that only a small improvement in excitation energies over those obtained with the original tPBE

functional can be obtained by scaling the exchange and correlation components of the tPBE on-top density functional (with  $S_X = 1.1$  and  $S_C = 0.5$ ), and no improvement at all can be obtained for the mean unsigned error in bond energies. We thus conclude that the development of significantly improved on-top density functionals will require either new or more flexible functional forms rather than just scaling parameters.

## 7 Electronic supplementary information

Further information about the computational details, basis sets, active spaces and dissociation curves; Cartesian coordinates for the EE27/19 database. (PDF) CASSCF orbital files in Molcas format (.RasOrb) in compressed archive (RAR).

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## Compliance with ethical standards

**Conflict of interest** The authors declare that they have no conflict of interest.

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