Joint Optimization of Waveform Covariance Matrix and Antenna Selection for MIMO Radar

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***Abstract*—In this paper, we investigate the problem of jointly optimizing the waveform covariance matrix and the antenna position vector for multiple-input-multiple-output (MIMO) radar systems to approximate a desired transmit beampattern as well as to minimize the cross-correlation of the received signals reflected back from the targets. We formulate the problem as a non-convex program and then propose a novel cyclic optimization approach to efficiently tackle the problem. We further propose a variant of binary evolutionary optimization algorithm in order to efficiently design the corresponding antenna positions. Our numerical investigations demonstrate a good performance both in terms of accuracy and computational complexity, making the proposed framework a good candidate for usage in real-time radar signal processing applications.**

***Index Terms*—Antenna selection, evolutionary algorithms, MIMO radar, waveform design.**

I. Introduction

Multiple-input-multiple-output (MIMO) radar has been an emerging technology of the last two decades, attracting a great deal of interest from researchers in radar and signal processing communities [1]–[12]. One of the main advantages of MIMO radar systems compared with the traditional phased-array radars is their ability to transmit multiple probing waveforms which can be chosen freely. The waveform diversity provided by a MIMO radar system can increase the resolution and sensitivity to target movements, and specifically, paving the way for applying adaptive array techniques. An important task in MIMO radar systems is thus to design the probing waveforms to approximate a desired beampattern, and to further minimize the cross-correlation of the signals reflected from various targets, and from reflections of other waveforms. Alternatively, one can consider the design of the probing signal covariance matrix as it provides more degrees of freedom compared to designing the waveforms directly [13]–[20].

A large part of the existing research on covariance waveform design focuses mainly on the scenario with a uniform linear array (ULA) and half-wavelength inter-element spacing in order to match a desired beampattern. However, such designs are typically concerned with statistical properties of the transmitted waveforms rather than incorporating a design of the positions of the transmit antennas as well. Recently, it was shown in [19] that unlike a ULA configuration where the total number of antennas and their positions are fixed, one can achieve additional degrees of freedom by carefully designing the antenna positions on a grid point for approximating the transmit beampattern with the same number of antennas (distributed non-uniformly on a grid point). As a result, assuming the total number of transmit antennas is fixed, a joint optimization of the covariance matrix and the antenna selection vector can achieve superior results compared with methods operating on a ULA configuration.

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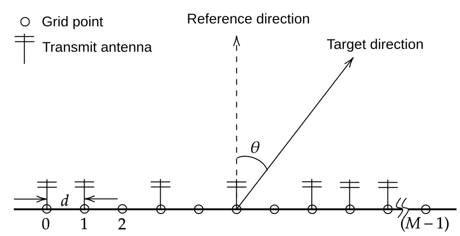

Fig. 1. Geometry of a colocated MIMO radar with M grid points.

In this paper, we propose a novel cyclic optimization approach to efficiently tackle the non-convex problem of joint optimization of the waveform covariance matrix and antenna positions, and further, we propose a variant of binary evolutionary optimization algorithm (e.g., see [21]) in order to efficiently design the corresponding antenna positions. In addition, our method is able to produce waveform covariance matrices with low cross-correlation properties.

II. Signal Model and Problem Formulation

We consider the problem of selecting N transmit antennas placed on a uniform linear array (ULA) positions with $M(\geq N)$ grid points with equal grid spacing d, in order to produce a desired beampattern as depicted in Fig. 1. We introduce a binary antenna position vector to represent the antenna configuration as

$$\boldsymbol{p} = [p_1, p_2, \cdots, p_m, \cdots, p_M]^T, \quad p_m \in \{0, 1\}, \tag{1}$$

where $p_m = 1$ represents the fact that the m-th grid point is selected for antenna placement; otherwise $p_m = 0$. Assuming a narrow-band signal model, the M-dimensional steering vector at the angle of interest θ is given as $\boldsymbol{a}(\theta) = [1, e^{j2\pi d \sin\theta/\lambda}, \cdots, e^{j2\pi(M-1)d\sin\theta/\lambda}]^T$, where λ is the wavelength of the transmitted signal. Our goal is to design the waveform covariance matrix $\boldsymbol{R}$ such that the transmitted beampattern $P(\theta)$ approximates a given beampattern $d(\theta)$ over the radial sectors of interest in a least squares (LS) sense, and also such that the cross-correlation of the reflected waveform from the targets is minimized. One can formulate this problem by defining a cost function as follows:

$$\begin{aligned} &J(\boldsymbol{p}, \boldsymbol{R}, \alpha) \qquad (2)\\ &= \frac{1}{K}\sum_{k=1}^{K} w_k \left|\boldsymbol{p}^T \Re\left\{\boldsymbol{R} \odot \left(\boldsymbol{a}(\theta_k)\boldsymbol{a}^H(\theta_k)\right)^*\right\}\boldsymbol{p} - \alpha d(\theta_k)\right|^2 \\ &+ \frac{2\omega_c}{\tilde{K}(\tilde{K}-1)}\sum_{p=1}^{\tilde{K}-1}\sum_{q=k+1}^{\tilde{K}} \left|\boldsymbol{p}^T \Re\left\{\boldsymbol{R} \odot \left(\boldsymbol{a}(\hat{\theta}_p)\boldsymbol{a}^H(\hat{\theta}_q)\right)^*\right\}\boldsymbol{p}\right|^2 \end{aligned}$$

where $\alpha > 0$ is a scaling factor to be optimized, $\omega_k \geq 0, k = 1, \cdots, K$, is the weight for the k-th radial sector and $\omega_c \geq 0$ is the weight for the cross-correlation term and α is a scaling parameter to be designed. Note that the first term on the right hand side of the above cost function captures the beampatten matching criteria, and the second term represents the cross-correlation between the reflected signal at different angles.

Finally, the joint optimization problem of designing the transmitted waveform covariance and the antenna position can be formulated as

$$\min_{\boldsymbol{p}, \boldsymbol{R}, \alpha} \quad J(\boldsymbol{p}, \boldsymbol{R}, \alpha) \qquad (3)$$

$$\text{s.t.} \ \ \boldsymbol{R} \succeq \boldsymbol{0}, \ \ R_{mm} = \frac{c}{M}, \ \text{for } m = 1, \cdots, M, \qquad (4)$$

$$\|\boldsymbol{p}\|_1 = N, \ \ \boldsymbol{p} \in \{0,1\}^M. \qquad (5)$$

Since $\boldsymbol{R}$ is a covariance matrix, it must be positive semidefinite matrix. We further impose the constraint that all the antennas are using the same transmit power, as reflected in (4). Furthermore, the constraint (5) guarantees that only N antennas are to be placed in M possible grid points, and that the vector $\boldsymbol{p}$ is binary. In order to tackle the non-convex program of (3), we propose a *cyclic optimization* approach with respect to the design variables $(\boldsymbol{R}, \alpha)$ and $\boldsymbol{p}$. Namely, for fixed $\boldsymbol{p}$, the minimization problem with respect to the design variables $(\boldsymbol{R}, \alpha)$ can be cast as

$$\begin{aligned} \min_{\boldsymbol{R}, \alpha} \quad & J(\boldsymbol{p}, \boldsymbol{R}, \alpha) \qquad (6)\\ \text{s.t.} \quad & \boldsymbol{R} \succeq \boldsymbol{0}, \ \ R_{mm} = \frac{c}{M}, \ \text{for } m = 1, \cdots, M. \end{aligned}$$

It can be shown that the above optimization problem can be reformulated as a constrained convex quadratic program, and hence, can be solved efficiently using off-the-shelf convex solvers (such as CVX [22]). On the other hand, for fixed $(\boldsymbol{R}, \alpha)$, the optimization of the antenna selection vector $\boldsymbol{p}$ can be written as follows

$$\min_{\boldsymbol{p}} \quad J(\boldsymbol{p}, \boldsymbol{R}, \alpha), \ \ \text{s.t.} \ \ \|\boldsymbol{p}\|_1 = N, \ \ \boldsymbol{p} \in \{0,1\}^M, \qquad (7)$$

which we solve using a variant of binary evolutionary optimization algorithm—details of which are omitted due to the lack of space.

III. Numerical Examples

In this section, we provide several numerical examples in order to assess the performance of our proposed algorithm. We compare our method with the ADMM-based algorithm proposed in [19]. In the following experiments, we assume a colocated narrow-band MIMO radar with a non-uniform linear array with $M = 15$ grid points with half-wavelength inter-grid interval i.e., $d = \lambda/2$, unless stated otherwise, and $N = 10$ antennas. The range of angle is $(-90^\circ, 90^\circ)$ with 1° resolution. We set the weights for the k-th angular direction as $w_k = 1$, for $k = 1, \cdots, K$; and the weight of the cross-correlation term as $w_c = 1$.

In Fig. 2 we compare the resulting beampattern with the desired one for the two scenarios of $\omega_c = 0$ and $\omega_c = 1$. In addition we provide the simulation results of [19] for three mainlobes at $\theta = \{-50^\circ, 0^\circ, 50^\circ\}$. In Fig. 3, we consider approximating the beampatterns with one mainlobe at $\theta = 0^\circ$, and a beamwidth of 60°. Furthermore, in Fig. 4, we consider approximating the beampattern with $\theta = \{-60^\circ, -30^\circ, 0^\circ, 30^\circ, 60^\circ\}$ and a beamwidth of 10°. As it can be seen from Figs 2–4, our proposed method can accurately match the desired beampattern. Also, note that our propose algorithm outperforms the one proposed in [19] in terms of accuracy, and moreover, is capable of designing waveform covariance matrix with low cross-correlation, unlike [19]. Further note that the designed beampatterns obtained with $\omega_c = 0$ and with $\omega_c = 1$ are similar to one another. However, the cross-correlation behavior of the former is much better than that of the latter in that the reflected signal waveforms corresponding to using $\omega_c = 1$ are almost uncorrelated with each other. This can be further verified from Fig. 6 in which we provide the comparison of the normalized magnitudes of the cross-correlation coefficients (as formulated in the second term of the right hand side of (2)) for three targets of interest at directions $\theta = \{-50^\circ, 0^\circ, 50^\circ\}$, as functions of ω_c.

In Fig. 7, we demonstrate the final antenna position vectors suggested by the proposed algorithm for the two cases of $\omega_c = 0$ and $\omega_c = 1$. Finally, Fig. 5 demonstrates the computational cost of our proposed algorithm and that of proposed in [19]. Note that our proposed algorithm significantly reduces the computational cost of the ADMM-based method in [19] by a factor of more than 100, making our algorithm particularly suitable for real-time applications.

IV. Conclusion

In this paper, the problem of jointly designing the probing signal covariance matrix as well as the antenna positions to approximate a given beampattern was studied. In order to tackle the problem, we proposed a novel cyclic optimization method based on the non-convex formulation of the problem. In addition, we used a variant of evolutionary optimization algorithm to tackle the non-convex problem of designing antenna positions. Several numerical examples were provided which demonstrates the superiority of the proposed method over the existing ADMM-based method in terms of accuracy and computational complexity.

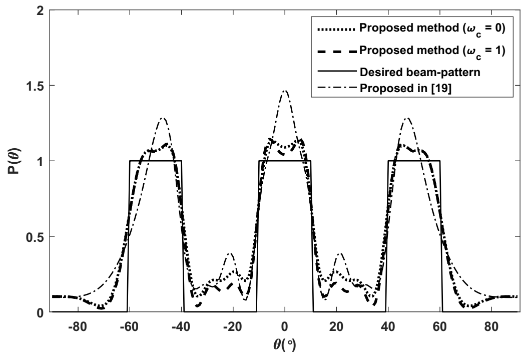

Fig. 2. The transmit beampattern design for $M = 15, N = 10$ with and without the cross-correlation suppression with three mainlobes at $\theta = \{-50°, 0°, 50°\}$ with a beamwidth 20°. The proposed algorithm can accurately approximate the desired beampattern for both cases of $\omega_c = 0$ (without cross-correlation) and $\omega_c = 1$ (with cross-correlation). Note that the designed beampatterns obtained with and without considering the cross-correlation term are similar to one another. However, the cross-correlation behavior of the former is much better than that of the latter in that the reflected signal waveforms corresponding to using $\omega_c = 1$ are almost uncorrelated with each other. The proposed algorithm outperforms the method in [19] in terms of accuracy, and additionally is capable of designing waveform covariance matrices with low cross-correlation.

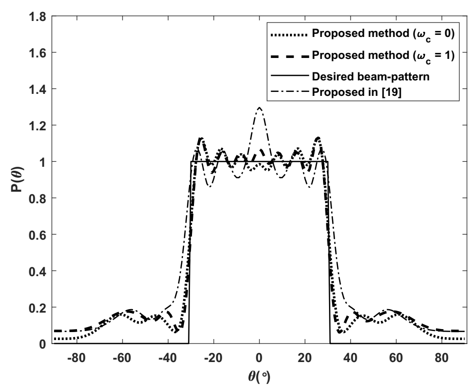

Fig. 3. The transmit beampattern design for $M = 15, N = 10$ with and without the cross-correlation suppression with one mainlobe at $\theta = 0°$ with a beam-width of 60°. Note that in both cases of $\omega_c = 0$ and $\omega_c = 1$ our proposed method can accurately approximate the desired beampattern.

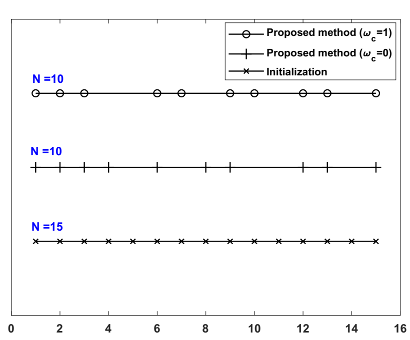

Fig. 7. The antenna position $\boldsymbol{s}$ for $M = 15, N = 10$ with and without the cross-correlation suppression. y-axis is used only for representation purposes.

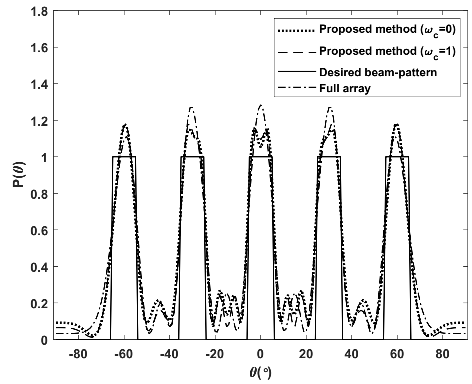

Fig. 4. The transmit beampattern design for $M = 15, N = 10$ with and without the cross-correlation suppression with five mainlobes at $\theta = \{-60°, -30°, 0°, 30°, 60°\}$ with a beamwidth of 10°. The transmitted power values are almost the same in all mainlobes.

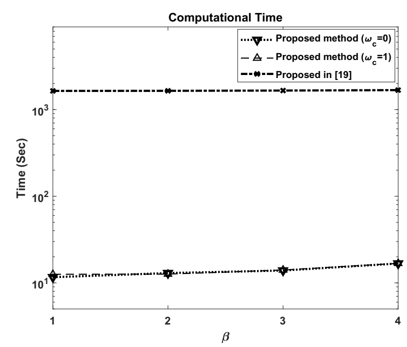

Fig. 5. Comparison of the computational cost of the proposed algorithm and that of the method in [19] for different number of grid points and that of antennas. We consider $M = 4$ and $N = 3$ as initialization, and then linearly scale M and N by the factor of $\beta \in \{1, 2, 3, 4\}$. The proposed algorithm significantly outperforms the ADMM-based method proposed in [19] by a factor of more than 100, resulting our algorithm particularly suitable for real-time applications.

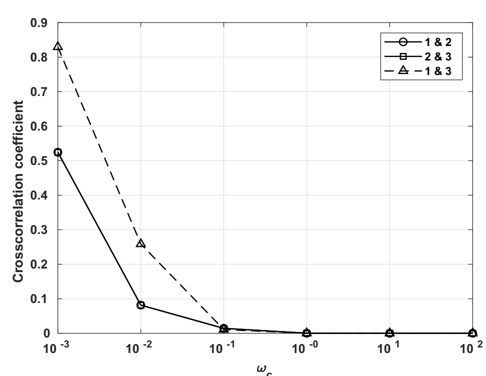

Fig. 6. The comparison of the normalized magnitudes of the cross-correlation coefficients (as formulated in the second term of the right hand side of (2)) for three targets of interest at directions $\theta = \{-50°, 0°, 50°\}$, as functions of ω_c. Note that when ω_c is very small (close to zero), the first and third reflected signals are highly correlated. On the other hand, for $\omega_c > 0.1$ all cross-correlation coefficients are approximately zero.

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