

# Gravitational waves, CMB polarization, and the Hubble tension

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The discrepancy between the Hubble parameter inferred from local measurements and that from the cosmic microwave background (CMB) has motivated careful scrutiny of the assumptions that enter both analyses. Here we point out that the location of the recombination peak in the CMB B-mode power spectrum is determined by the light horizon at the surface of last scatter and thus provides an alternative early-Universe standard ruler. It can thus be used as a cross-check for the standard ruler inferred from the acoustic peaks in the CMB temperature power spectrum and to test various explanations for the Hubble tension. The measurement can potentially be carried out with a precision of  $\lesssim 2\%$  with stage-IV B-mode experiments. The measurement can also be used to measure the propagation speed of gravitational waves in the early Universe.

The tension [1–3] between the value of the Hubble parameter (the cosmic expansion rate) inferred from local measurements [4–7] and that [8, 9] inferred from the cosmic microwave background (CMB) has been lingering for a number of years. It is now established at the  $\gtrsim 4\sigma$  and should rightfully be promoted from a Hubble “tension” to a *bona fide* discrepancy. The discrepancy is not easily attributed to any obvious systematic error [10–13]. Several recent papers have shown that the local measurements, which are obtained by comparing the inferred distance to cosmological sources with their redshifts, are robust to new or alternative calibrations of the cosmic distance ladder [14–17]. Note, however, the recent debate [18, 19] with the calibration using the TRGB stars. The most recent local measurement is  $H_0 = 74.22 \pm 1.82$  km/sec/Mpc [14]. On the other hand, the Hubble parameter is inferred from the CMB from the angular scale of peaks in the CMB angular power spectrum. This angular scale is fixed by the ratio of the sound horizon (the distance a sound wave in the primordial baryon-photon fluid has traveled from big bang to the time the CMB decoupled) with the angular-diameter distance to the surface of last scatter [20, 21]. Both distances are obtained, within the standard cosmological model, by detailed modeling of the CMB peak structure. This procedure yields a value  $H_0 = 67.4 \pm 0.5$  km/sec/Mpc [9].

Solutions to the Hubble tension are not easily come by but generally involve modifications to cosmic evolution at early times (mechanisms that decrease the sound horizon) [4, 5, 22–30] or at late times (modifications to the cosmic expansion history that increase the angular-diameter distance to the surface of last scatter) [31–36]. However, the late-time resolutions are tightly constrained by other late-time observables [4, 23, 32, 34, 37–42], and the early-time solutions are tightly constrained by the acoustic oscillations in the CMB power spectrum. All of the proposed solutions require fairly exotic new physics.

Given the lack of any easy solutions to the Hubble

tension, as well as the increasing significance of the discrepancy, any possible cross-checks of the measurements and assumptions, as well as any possible complementary information that can be obtained, should be pursued vigorously. In particular, all the information we have about the Hubble parameter relies ultimately on distance measures in cosmology, and any new technique to obtain a cosmic distance will be valuable.

We propose that measurement of the B-mode polarization in the CMB [43, 44] induced by primordial gravitational waves [45–47] may be used to provide an independent cross-check of the early-Universe expansion history. These B modes have yet to be detected but are predicted in the canonical single-field slow-roll inflation models to be within the sensitivities of major experimental efforts—for example, CLASS [48], LiteBIRD [49], the Simons Observatory [50], CMB-S4 [51], or Probe Inflation and Cosmic Origins (PICO [52])—to be pursued within the next decade. If they exist and are detected, they may prove to be of value in efforts to understand the Hubble tension.

The primordial B-mode power spectrum exhibits oscillations that arise from the propagation of gravitational waves [54, 55]. These are analogous to the well-known acoustic oscillations in the CMB temperature power spectrum [56, 57] that arise from sound waves in the primordial baryon-photon fluid. The difference is that the propagation speed of sound waves in the photon-baryon fluid is roughly  $c/\sqrt{3}$ , while that of gravitational waves is the speed of light  $c$ .

If the Hubble tension is due to a late-time modification of the expansion history, both sets of peaks (those in the temperature power spectrum and those in the GW-induced B-mode power spectrum) should be affected in the same way. The peaks in the B-mode power spectrum should thus appear at the same multipole moment as predicted in the current best-fit cosmological model. If the discrepancy is resolved by new physics in the early Universe, the peak locations in the B-mode power spectrum

may differ. More precisely, the comoving sound horizon at decoupling is an integral  $r_s = \int^{t_{\text{ls}}} c_s(t) dt / a(t)$  of the sound speed  $c_s(t)$  until the time  $t_{\text{ls}}$  of CMB decoupling, while the comoving gravitational-wave horizon is  $r_{\text{gw}} = c \int^{t_{\text{ls}}} dt / a(t)$ . If the Hubble tension is resolved somehow by a reduction in the sound speed, then the B-mode peak location, relative to the acoustic peak, will change. Existing models generally involve some shift in the expansion history (which affects  $r_s$  and  $r_{\text{gw}}$  in a slightly different way) and some shift in the baryon and dark-matter densities (which can affect the two distances differently).

To be relevant for the  $\Delta H_0 / H_0 \sim 10\%$  tension, the angular scale of the peaks in the B-mode power spectrum must be determined to better than 10% (the magnitude of the discrepancy). As the calculation below indicates, this is conceivable with measurements to be carried out on a decade timescale. The measurement is, however, by no means guaranteed, even if the experiments perform as expected, as the determination requires that primordial gravitational waves (which are hypothesized but have yet to be detected) have an amplitude  $r \gtrsim 0.001$  (see Fig. 1). Here,  $r$  is the tensor-to-scalar ratio of the primordial power spectra.

In this paper, we study the possibility to determine the light-horizon scale from future B-mode polarization experiments such as LiteBIRD [49], a CMB stage-IV experiment (e.g., the Simons Observatory [50] or CMB-S4 [51]), or Probe Inflation and Cosmic Origins (PICO [52]). These efforts aim to detect the primordial B-mode polarization with sensitivity better than  $\sigma_r \sim 0.001$ .

We first begin with some rough estimates of the precision with which the light horizon can be determined and some scalings. We then follow with a more detailed calculation, taking into account possible degeneracies with parameters that affect the B-mode power spectrum.

To begin with, consider an idealized full-sky (or nearly full sky) experiment and assume that the B-mode power spectrum is measured with a detector-noise contribution  $C_\ell^{\text{n}}$ ; ignore for now any lensing-induced [53] B modes. Consider the shift  $C_\ell^{\text{BB}} \rightarrow C_{\ell(1-\alpha)}^{\text{BB}}$  in the B-mode power spectrum induced by a change  $\delta r_{\text{gw}} = \alpha r_{\text{gw}}$  in the light horizon. We then estimate the  $1\sigma$  (68 % C.L.) uncertainty with which the parameter  $\alpha$  can be determined, for an experiment that surveys a fraction  $f_{\text{sky}}$  of the sky with noise power spectrum  $C_\ell^{\text{n}}$  as

$$\sigma_\alpha = \left[ \sum_\ell \frac{(2\ell+1)f_{\text{sky}}}{2} \left( \frac{\partial C_\ell^{\text{BB}} / \partial \alpha}{C_\ell^{\text{BB}} + C_\ell^{\text{n}}} \right)^2 \right]^{-1/2}. \quad (1)$$

The partial derivatives can be evaluated by  $(\partial C_\ell^{\text{BB}} / \partial \alpha) = -dC_\ell^{\text{BB}} / d \ln \ell$ . For this estimation, we take the B-mode power spectrum  $C_\ell^{\text{BB}}$  obtained for a scale-invariant gravitational-wave power spectrum as expected from inflation. Given that the signal we seek is the location of the peaks in  $C_\ell^{\text{BB}}$ , we take the

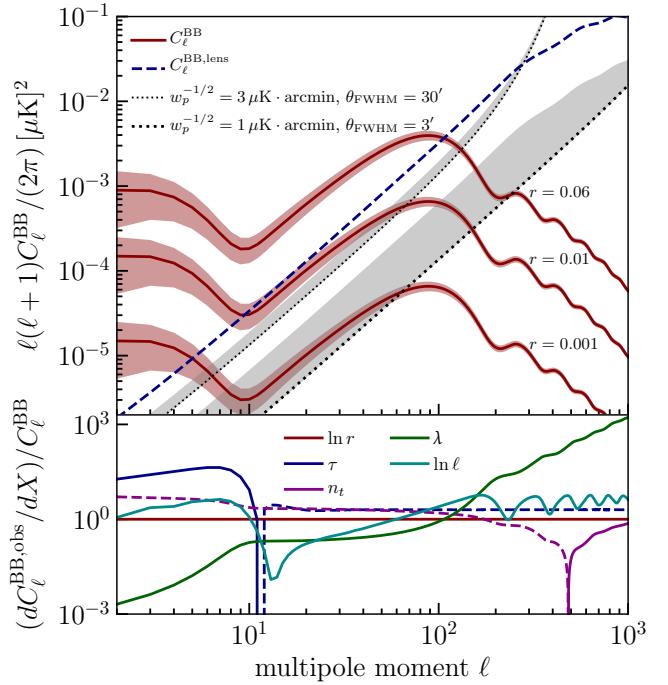


FIG. 1. *Top:* The B-mode polarization power spectrum (red lines) for  $r = 0.06, 0.01$  and  $0.001$  (from top to bottom), along with the cosmic-variance uncertainty (red shaded regions) and instrumental noise (black lines) similar to LiteBIRD (dotted) and CMB stage-IV or PICO (dashed). The gray shaded regions show the extra noise contribution from the gravitationally lensed B-mode (blue line) between perfect removal case (bottom edge) and 15% delensing residual (upper edge). *Bottom:* The response of the B-mode polarization power spectrum  $d \ln C_\ell / dX$  to the change of parameters:  $X = \ln r$  (tensor amplitude),  $\tau$  (optical depth),  $n_t$  (tensor spectra index),  $\lambda$  (gravitational lensing amplitude), and  $\ln \ell$  (distance scales). For the Fisher analysis, we set  $dC_\ell / d \ln \ell = 0$  for  $\ell < 15$  in order to exclude the reionization bump.

reionization optical depth  $\tau = 0$ . We take the sum from  $\ell = 20$  to  $\ell = 500$  (well within the target range of a stage-IV CMB experiment; see Fig. 1). We choose the lower limit,  $\ell = 20$  as the recombination peak at  $\ell \lesssim 10$  will not be shifted by a change to the light horizon at the surface of last scatter. In practice, the results are insensitive to changes in either the lower or upper bounds, as the signal peaks near  $\ell \sim 100$ .

We next determine  $C_\ell^{\text{n}}$  in terms of  $\sigma_r$ , the smallest detectable (at  $1\sigma$ ) tensor-to-scalar ratio, as it is a commonly discussed figure of merit for B-mode searches. We thus estimate smallest detectable tensor-to-scalar ratio  $r$

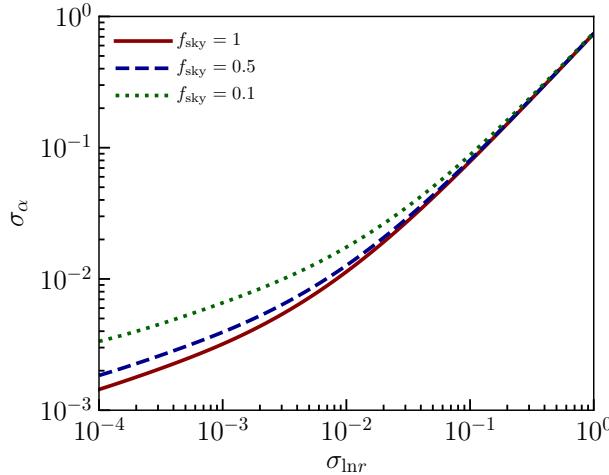


FIG. 2. Projected 1- $\sigma$  (68 % C.L.) accuracy of measuring the light horizon from the B-mode polarization power spectrum as a function of the inverse,  $\sigma_r/r = \sigma_{\ln r}$ , of the signal-to-noise ratio of the tensor-to-scalar ratio. Here, we do not marginalize over the other parameters.

to be [58]

$$\sigma_r = \left[ \sum_{\ell} \frac{(2\ell+1)f_{\text{sky}}}{2} \left( \frac{\partial C_{\ell}^{\text{BB}} / \partial r}{C_{\ell}^{\text{n}}} \right)^2 \right]^{-1/2}. \quad (2)$$

Note that the signal power spectrum  $C_{\ell}^{\text{BB}}$  does not appear in the denominator here, as this expression is for the error with which  $r$  is measured under the null hypothesis  $r = 0$ . We then use Eq. (2) to fix the noise power spectrum  $C_{\ell}^{\text{n}}$  in terms of  $\sigma_{\ln r} \equiv \sigma_r/r$ , which has been carefully forecast in several detailed studies of hypothetical or specific experimental designs. In so doing, we circumvent issues involving imperfectly subtracted foregrounds and lensing-induced B modes (which act effectively as a contribution to  $C_{\ell}^{\text{n}}$ ) by using results from these prior studies.

We show in Fig. 2 the error, inferred from Eqs. (1) and (2), with which the B-mode peak location can be determined (at 1 $\sigma$ ) for three different values of  $f_{\text{sky}} = 1$  (red solid line), 0.5 (blue dashed line) and 0.1 (green dotted line). In the most optimistic case that  $r = 0.06$ , and with  $\sigma_r = 0.001$ , the calculation indicates a  $\lesssim 2\%$  (at 1 $\sigma$ ) measurement of the light-horizon distance at decoupling. This is encouraging.

The numerical results in Fig. 2 are insensitive to the highest multipole moment  $\ell_{\text{max}}$  used in the sums in Eqs. (1) and (2), and remain more or less the same for any  $\ell_{\text{max}} \gtrsim 150$ . In other words, the measurement comes almost entirely from the first peak, the ‘‘recombination peak’’ (which occurs at  $l \simeq 86$ ), in the B-mode power spectrum. This also implies that the measurement requires the B modes to be mapped with an angular reso-

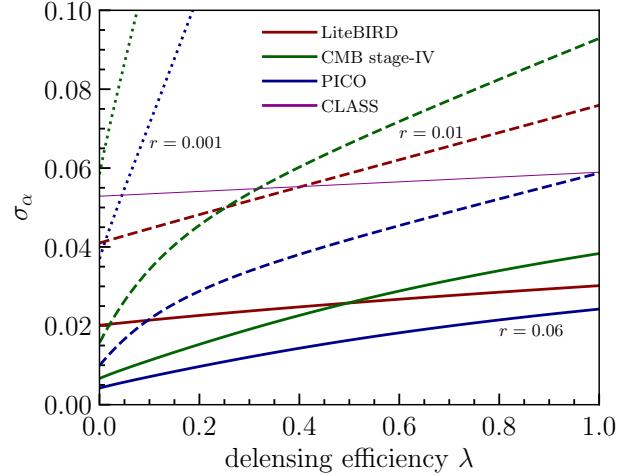


FIG. 3. Marginalized 1- $\sigma$  (68 % C.L.) accuracy of measuring the light horizon from the B-mode polarization power spectrum from three future experiments: LiteBIRD, CMB stage-IV and PICO, as a function of delensing efficiency  $\lambda$  for  $r = 0.001$  (dotted),  $r = 0.01$  (dashed),  $r = 0.06$  (solid), after marginalizing over  $r$ ,  $n_t$ ,  $\tau$  and  $\lambda$ . For comparison, we also show the same uncertainty for the CLASS mission when fixing  $r = 0.06$ ,  $n_t = 0$  with the thin magenta line.

lution no better than 1°.

We now follow up with a more careful Fisher forecast which takes into account the possibility of imperfect subtraction of lensing-induced B modes, possible shifts in the first-peak location from uncertainties in the spectral index  $n_t$  of the gravitational-wave power spectrum, and covariances between the different parameters. We further include the effects of reionization, as for some experiments (e.g., LiteBIRD), the sensitivity to B modes may be dominated by the low- $\ell$  reionization peak, rather than the recombination peak assumed in the simple estimates above. We provide results for experimental specifications that correspond roughly to those for several projects being pursued or under consideration. The Fisher matrix for the B-mode polarization power spectrum is given as [60, 61]

$$F_{ij} = \sum_{\ell} \frac{f_{\text{sky}}(2\ell+1)}{2} \frac{1}{N_{\ell}^2} \left( \frac{\partial C_{\ell}^{\text{BB,obs}}}{\partial \theta_i} \right) \left( \frac{\partial C_{\ell}^{\text{BB,obs}}}{\partial \theta_j} \right), \quad (3)$$

with  $\theta = \{\alpha, r, n_t, \tau, \lambda\}$  the vector of parameters being determined by the measurement of the B-mode power spectrum. Here  $\tau$  is the reionization optical depth and  $\lambda \in [0, 1]$  is the fraction of the lensing-induced B modes that remain after de-lensing [62, 63].

The observed power spectrum that appears in Eq. (3) is then  $C_{\ell}^{\text{BB,obs}} = C_{\ell}^{\text{BB}}(r, n_t, \tau) + \lambda C_{\ell}^{\text{BB,lens}}$ , including a contribution from imperfectly subtracted lensed-induced B modes. We define the noise power per each harmonic

mode as  $\mathcal{N}_\ell = C_\ell^{\text{BB,obs}} + C_\ell^{\text{n}} \exp [\ell(\ell+1)\sigma_b^2]$ , with the instrumental noise,

$$C_\ell^{\text{n}} = \left( \frac{\pi}{10800} \frac{w_p^{-1/2}}{\mu\text{K arcmin}} \right)^2 \mu\text{K}^2 \text{str}, \quad (4)$$

and

$$\sigma_b = \left( \frac{\pi}{180} \right) \frac{\theta_{\text{fwhm}}}{\sqrt{8 \ln 2}}, \quad (5)$$

with the full width at half maximum size  $\theta_{\text{fwhm}}$  of the beam. We adopt the following values for the four types of experiments that we consider here:  $(w_p^{-1/2}, \theta_{\text{fwhm}}, f_{\text{sky}}) = (10\mu\text{K arcmin}, 60 \text{ arcmin}, 0.4)$  for the CLASS mission,  $(3\mu\text{K arcmin}, 30 \text{ arcmin}, 1)$  for the LiteBIRD satellite,  $(1\mu\text{K arcmin}, 3 \text{ arcmin}, 0.4)$  for the ground-based CMB stage-IV experiments (Simons Observatory and CMB-S4), and  $(1\mu\text{K arcmin}, 3 \text{ arcmin}, 1)$  for the PICO satellite.

For the fiducial cosmology, we use the best-fitting cosmological parameters from Planck 2018 [9] and calculate primordial and lensing B-mode polarization power spectra by using CAMB [64]. We set  $\partial C_\ell^{\text{BB,obs}} / d\ln \ell = 0$  for  $\ell < 15$ , as a shift in the light horizon in the early Universe will not affect (by the model assumptions we are making here) the light horizon at reionization. Because  $C_\ell^{\text{BB}}$  is almost flat at large angular scales, the Fisher-matrix results are insensitive to the exact value of  $\ell$  we use for this cutoff.

As shown in the bottom panel of Fig. 1, the derivative with respect to the distance-scale (cyan line,  $d\ln \ell$ ) has characteristic wiggles due to the acoustic peaks. It turns out that the light-horizon measurement is almost independent of the optical depth ( $\tau$ ) and lensing ( $\lambda$ ), but moderately degenerate with the amplitude ( $r$ ) and slope ( $n_t$ ) of the primordial gravitational-wave power spectrum. Note that within the context of specific inflationary models, or classes of inflationary models, further information on  $n_t$  might be inferred from the precise measurement of the scalar amplitude and spectral index from the CMB temperature and E-mode power spectra. If so, the results we present here may err on the pessimistic side.

As expected, the results depend quite sensitively on  $r$ , and de-lensing becomes increasingly important at lower values of  $r$ . As shown in Fig. 1, for  $r = 0.001$ , the B-mode power spectrum is barely above the noise curve even for perfect delensing ( $\lambda = 0$ ) and drops below the noise curve when using the moderate delensing efficiency ( $\lambda = 0.15$ ) expected from combining various galaxy surveys [65]. Indeed, we can see that in Fig. 3, the projected uncertainties of measuring  $\alpha = \delta r_{\text{gw}}/r_{\text{gw}}$  for  $r = 0.001$  (dotted lines) sharply rise beyond  $\sigma_\alpha = 10\%$  at  $\lambda \simeq 0.1$ . There, we show the projected uncertainties on  $\delta r_{\text{gw}}/r_{\text{gw}}$  for the three experiments (LiteBIRD, CMB stage-IV, PICO), after marginalizing over the other four parameters ( $r, n_t$ ,

$\lambda, \tau$ ), as a function of the delensing efficiency  $\lambda$ . As we have estimated earlier, for  $r \lesssim 0.06$  and  $\sigma_r \simeq 0.001$ , we can measure the light-horizon scale to a few-percent level. We have also verified that for experiments like PICO and stage-IV, which target primarily the recombination bump, the scalings in Fig. 2 are valid. The scalings are not quite as effective, however, for an experiment like LiteBird that targets primarily the low- $\ell$  reionization bump.

To summarize, a  $\lesssim 2\%$  determination of the angular scale subtended by the light horizon at the surface of last scatter is conceivable through measurement of the B-mode power spectrum. The CMB-polarization experiments are similar to those being pursued already to detect the B-mode signal, and could in the best-case scenario provide results of the precision relevant for the Hubble tension on a  $\sim$ decade timescale. The measurement does require that inflationary gravitational waves exist with a tensor-to-scalar ratio  $r$  not too much smaller than the current upper bound, and there is no way of telling, until the measurement is done, whether Nature will cooperate in this regard. If B modes are detected and the peak location determined, it will narrow the range of possible resolutions to the Hubble tension. If the result disagrees with the canonical expectation, it will rule out late-time solutions to the Hubble tension. If it agrees, it will constrain (though not rule out categorically) early-time solutions.

We also note, before closing, that the predictions assume that gravitational waves propagate at the speed of light in the early Universe. This measurement can thus be used to test this general-relativistic prediction at the  $\sim 2\%$  level, which may be relevant for some alternative-gravity models.

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