Optimizing Item and Subgroup Configurations for Social-Aware VR Shopping

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ABSTRACT

Shopping in VR malls has been regarded as a paradigm shift for E-commerce, but most of the conventional VR shopping platforms are designed for a single user. In this paper, we envisage a scenario of VR group shopping, which brings major advantages over conventional group shopping in brick-and-mortar stores and Web shopping: 1) configure flexible display of items and partitioning of subgroups to address individual interests in the group, and 2) support social interactions in the subgroups to boost sales. Accordingly, we formulate the Social-aware VR Group-Item Configuration (SVGIC) problem to configure a set of displayed items for flexibly partitioned subgroups of users in VR group shopping. We prove SVGIC is APX-hard and also NP-hard to approximate within $\frac{32}{31} - \epsilon$. We design a 4-approximation algorithm based on the idea of Co-display Subgroup Formation (CSF) to configure proper items for display to different subgroups of friends. Experimental results on real VR datasets and a user study with hTC VIVE manifest that our algorithms outperform baseline approaches by at least 30.1% of solution quality.

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1. INTRODUCTION

Virtual Reality (VR) has emerged as a disruptive technology for social [16], travel [64], and E-commerce applications. Particularly, a marketing report about future retails from Oracle [7] manifests that 78% of online retailers already have implemented or are planning to implement VR and AI. Recently, International Data Corporation (IDC) forecasts the worldwide spending on VR/AR to reach 18.8 billion USD in 2020 [69], including \$1.5 billion in retails. It also foresees the VR/AR market to continue an annual growth rate of 77% through at least 2023. Moreover, shopping in VR malls is regarded as a paradigm shift for E-commerce stores, evident by emerging VR stores such as Amazon's VR kiosks [62], eBay and Myer's VR department store [60], Alibaba Buy+ [2], and IKEA VR Store [63]. Although these VR shopping platforms look promising, most of them are designed only for a single user instead of a group of friends, who often appear in brick-and-mortar stores. As a result, existing approaches for configuring the displayed items in VR shopping malls are based on personal preference (similar to online web shopping) without considering potential social discussions amongst friends on items of interests, which is nevertheless reported as beneficial for boosting sales in the marketing literature [73, 75, 76]. In this paper, with the support of *Customized Interactive Display* (CID), we envisage the scenario of group shopping with families and friends in the next-generation VR malls, where item placement is customized flexibly in accordance with both user preferences and potential social interactions during shopping.

The CID technology [25, 36] naturally enables VR group shopping systems with two unique strengths: 1) *Customization.* IKEA (see video [63] at 0:40) and Lowe's [37] respectively launch VR store applications where the displayed furniture may adapt to the preferences of their users, while Alibaba's [17] and eBay's VR stores [41] also devote themselves to provide personalized product offers. According to a marketing survey, 79% of surveyed US, UK, and China consumers are likely to visit a VR store displaying customized products [18]. Similar to group traveling and gaming in VR, the virtual environments (VEs) for individual users in VR group shopping need not be identical. While it is desirable to have consistent high-level layout and user interface for all users, the displayed items for each user can be customized based on her preferences. As CID allows different users to view different items at the same display slot, personalized recommendation is supported.

2) *Social Interaction.* While a specific slot no longer needs to display the same items to all users, users viewing a common item may engage a discussion on the item together, potentially driving up the engagement and purchase conversion [75, 76]. As a result, the displayed items could be tailored to maximize potential discussions during group shopping. 54% of 1,400 surveyed consumers in 2017 acknowledge social shopping as their ways to purchase products [43]. The L.E.K. consulting survey [26] shows that 70% of 1,000 consumers who had already experienced VR technology are

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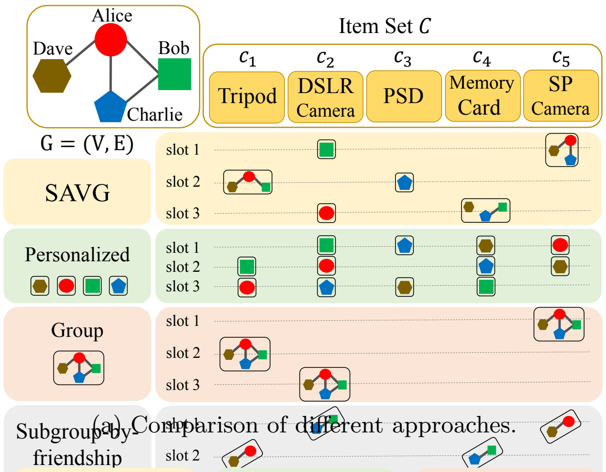


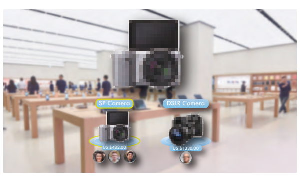
(a) Comparison of different approaches.

SAVG	slot 1	slot 2	slot 3
Alice	c_5	c_1	c_2
Bob	c_2	c_1	c_4
Charlie	c_5	c_3	c_4
Dave	c_5	c_1	c_4

Person.	slot 1	slot 2	slot 3
Alice	c_5	c_2	c_1
Bob	c_2	c_1	c_4
Charlie	c_3	c_4	c_2
Dave	c_4	c_5	c_3

Group	slot 1	slot 2	slot 3
Alice	c_5	c_1	c_2
Bob	c_5	c_1	c_2
Charlie	c_5	c_1	c_2
Dave	c_5	c_1	c_2

(b) Item assignments of different approaches.



(c) Alice's view at slot 1.

(d) Alice's view at slot 2.

Figure 1: Illustrative example.

strongly interested in virtual shopping with friends who are not physically present. Embracing the trend, Shopify [55] and its technical partner Qbit [44] build a social VR store supporting attendance of multiple users with customized display [1]. In summary, compared with brick-and-mortar shopping, VR group shopping can better address the preferences of individuals in the group due to the new-found flexibility in item placement, which can be configured not only for the group as a whole but also for individuals and subgroups. On the other hand, compared with conventional E-commerce shopping on the Web, VR group shopping can boost sales by facilitating social interactions and providing an immersive experience.

Encouraged by the above evidence, we make the first attempt to formulate the problem of configuring displayed items for *Social-Aware VR Group* (SAVG) shopping. Our strategy is to meet the item preferences of individual users via customization while enhancing potential discussions by displaying common items to a shopping group (and its subgroups). For customization, one possible approach is to use existing personalized recommendation techniques [10,23,24] to infer individual user preferences, and then retrieve the top-k favorite items for each user. However, this personalized approach fails to promote items of common interests that may trigger social interactions. To encourage social discussions, conventional group recommendation systems [27,45,51,52] may be leveraged to retrieve a bundled itemset for all users. Nevertheless, by presenting the same configuration for the whole group of all users, this approach may sacrifice the diverse individual interests. In the following example, we illustrate the difference amongst the aforementioned approaches, in contrast to the desirable SAVG configuration we target on.

Example 1 (Illustrative Example)**.** Figure 1(a) depicts a scenario of group shopping for a VR store of digital photography. At the upper left is a social network $G = (V, E)$ of four VR users, Alice, Bob, Charlie, and Dave (indicated by red circles, green squares, blue pentagons, and brown hexagons, respectively). On top is an item set C consisting of five items: tripod, DSLR camera, portable storage device (PSD), memory card, and self-portrait (SP) camera. Given three display slots, the shaded areas, corresponding to different configuration approaches, illustrate how items are displayed to individuals or subgroups (in black rectangles), respectively. For instance, in SAVG, the SP camera is displayed at slot 1 to Alice, Charlie, and Dave to stimulate their discussion. Figure 1(b) shows the same configuration as Figure 1(a) by depicting the individual item assignments for each user with different approaches.

A configuration based on the personalized approach is shown in the light-green shaded area. It displays the top-3 items of interests to individual users based on their preferences (which is consistent with the numerical example shown in Table 1 in Section 3). This configuration, aiming to increase the exposure of items of interests to customers, does not have users seeing the same item at the same slot. Next, the configuration shaded in light-orange, based on the group approach, displays exactly the same items to every user in the group. While encouraging discussions in the whole group, this configuration may sacrifice some individual interests and opportunities to sell some items, e.g., Dave may not find his favorite item (the memory card). Aiming to strike a balance between the factors of individual preferences and social interactions, the SAVG configuration forms subgroups flexibly across the displayed slots, having some users seeing the same items at the same slots (to encourage discussions), yet finding items of individual interests at the remaining slots. For example, the tripod is displayed at slot 2 to all users except for Charlie, who sees the PSD on his own. The DSLR camera is displayed to Bob at slot 1 and to Alice at slot 3, respectively, satisfying their individual interests. Figures 1(c) and 1(d) show Alice's view at slot 1 and slot 2, respectively, in this configuration. As shown, Alice is co-displayed the SP camera with Charlie and Dave at slot 1 (informed by the user icons below the primary view of the item), then co-displayed the Tripod with Bob and Dave at slot 2. As shown, SAVG shopping displays items of interests to individuals or different subgroups at each slot, and thereby is more flexible than other approaches. □

As illustrated above, in addition to identifying which items to be displayed in which slots, properly partitioning subgroups by balancing both factors of personal preferences and social discussions is critical for the above-depicted SAVG shopping. In this work, we define the notion of *SAVG k-Configuration*, which *specifies the partitioned subgroups (or individuals) and corresponding items for each of the allocated k slots.* We also introduce the notion of *co-display* that represents users sharing views on common items. Formally, we formulate a new optimization problem, namely *Social-aware VR Group-Item Configuration* (SVGIC), to find the optimal

SAVG k-Configuration that maximizes the overall 1) *preference utility* from users viewing their allocated items and 2) *social utility* from all pairs of friends having potential discussions on co-displayed items, where the basic preference utility of an individual user on a particular item and the basic social utility of two given users on a co-displayed item are provided as inputs. Meanwhile, we ensure that no duplicated items are displayed at different slots to a user. The problem is very challenging due to a complex trade-off between prioritizing personalized item configuration and facilitating social interactions. Indeed, we prove that SVGIC is NP-hard to approximate SVGIC within a ratio of $\frac{32}{31} - \epsilon$.

To solve SVGIC, we first present an Integer Program (IP) as a baseline to find the exact solution which requires super-polynomial time. To address the efficiency issue while ensuring good solution quality, we then propose a novel approximation algorithm, namely *Alignment-aware VR Subgroup Formation* (AVG), to approach the optimal solution in polynomial time. Specifically, AVG first relaxes the IP to allow fragmented SAVG k-Configurations, and obtains the optimal (fractional) solution of the relaxed linear program. It then assigns the fractional decision variables derived from the optimal solution as the *utility factors* for various potential allocations of user-item-slot in the solution. Items of high utility factors are thus desirable as they are preferred to individuals or encouraging social discussions. Moreover, by leveraging an idea of *dependent rounding*, AVG introduces the notion of *Co-display Subgroup Formation* (CSF) to strike a balance between personal preferences and social interactions in forming subgroups. CSF forms a *target subgroup* of socially connected users with similar interests to display an item, according to a randomized *grouping threshold* on utility factors to determine the membership of the target subgroup. With CSF, AVG finds the subgroups (for all slots) and selects appropriate items simultaneously, and thereby is more effective than other approaches that complete these tasks separately in predetermined steps. Theoretically, we prove that AVG achieves 4-approximation in expectation. We then show that AVG can be derandomized into a deterministic 4-approximation algorithm for SVGIC. We design further LP transformation and sampling techniques to improve the efficiency of AVG.

Next, we enrich SVGIC by taking into account some practical VR operations and constraints. We define the notion of *indirect co-display* to capture the potential social utility obtained from friends displayed a common item at two different slots in their VEs, where discussion is facilitated via the *teleportation* [5] function in VR. We also consider a *subgroup size constraint* on the partitioned subgroups at each display slot due to practical limits in VR applications. Accordingly, we formulate the Social-aware VR Group-Item Configuration with Teleportation and Size constraint (SVGIC-ST) problem, and prove that SVGIC-ST is unlikely to be approximated within a constant ratio in polynomial time. Nevertheless, we extend AVG to support SVGIC-ST and guarantee feasibility of the solution. Moreover, we extend AVG to support a series of practical scenarios. Due to space limit, the details of SVGIC-ST and the above extensions are presented in the full-length version [31] of this paper.

The contributions of this work are summarized as follows:

- We coin the notion of SAVG k-Configuration under the context of VR group shopping and formulate the SVGIC problem, aiming to return an SAVG k-Configuration that facilitates social interactions while not sacrificing the members' individual preferences. We prove SVGIC is APX-hard and also NP-hard to approximate within $\frac{32}{31} - \epsilon$.
- We systematically tackle SVGIC by introducing an IP model and designing a 4-approximation algorithm, AVG, based on the idea of Co-display Subgroup Formation (CSF) that leverages the flexibility of CID to partition subgroups for each slot and display common items to subgroup members.
- A comprehensive evaluation on real VR datasets and a user study implemented in Unity and hTC VIVE manifest that our algorithms outperform the state-of-the-art recommendation schemes by at least 30.1% in terms of solution quality.

This paper is organized as follows. Section 2 reviews the related work. Section 3 formally defines the notion of SAVG k-Configuration, then formulates the SVGIC problem and an IP model. Then, Section 4 details the proposed AVG algorithm and the theoretical results. Section 5 discusses the directions for extensions. Section 6 reports the experimental results, and Section 7 concludes this paper.

2. RELATED WORK

Group Recommendation. Various schemes for estimating group preference have been proposed by aggregating features from different users in a group [45, 51]. Cao *et al.* [8] propose an attention network for finding the group consensus. However, sacrificing personal preferences, the above group recommenders assign a unified set of items for the entire group based only on the aggregate preference without considering social topologies. For advanced approaches, recommendation-aware group formation [49] forms subgroups according to item preferences. SDSSel [52] finds dense subgroups and diverse itemsets. Shuai *et al.* [56] customize the sequence of shops without considering CID. However, the above recommenders find *static* subgroups, i.e., a universal partition, and still assign a fixed configuration of items to each subgroup, where every subgroup member sees the same item in the same slot. In contrast, the SAVG approach considered in this paper allows the partitioned subgroups to vary across all display slots and thereby is more flexible than the above works.

Personalized Recommendation. Personalized recommendation, a cornerstone of E-commerce, has been widely investigated. A recent line of study integrates deep learning with Collaborative Filtering (CF) [10, 39] on heterogeneous applications [19], while Bayesian Personalized Ranking [9, 48] is proposed to learn the ranking of recommended items. However, the above works fail to consider social interactions. While social relations have been leveraged to infer preferences of products [79, 80], POIs [32], and social events [35], they do not take into account the social interactions among users in recommendation of items. Thus, they fail to consider the trade-off between social and personal factors. In this paper, we exploit the preferences obtained from such studies to serve as inputs for the tackled problems.

Social Group Search and Formation. Research on finding various groups from online social networks (OSNs) for different purposes has drawn increasing attention in recent years. Community search finds similar communities containing a given set of query nodes [11,33]. Group formation organizes a group of experts with low communication costs and specific skill combinations [4, 46]. In addition, organizing a group in social networks based on spatial factors [53, 71], team member skills [54], or privacy considerations [59] have also gained more attention. The problems tackled in the above studies are fundamentally different from the scenario in this paper, since they focus on retrieving only *parts* of the social network according to some criteria, whereas item selection across multiple slots is not addressed. Instead, SAVG group shopping aims to configure item display for all VR shopping users, whereas the subgroups partition the entire social network.

Related Combinatorial Optimization Problems. SVGIC is related to the *Multi-Labeling* (ML) [70] problem and its variations, including Multiway Partition [70,77], *Maximum Happy Vertices/Edges* (MHV/MHE) [74], and *Multiway Cut* [12] in graphs. In Section 3.3, we revisit the challenging combinatorial nature of the proposed SVGIC problem and its relation with the related problems, whereas detailed introduction of each problem, as well as a summary table, are presented in the full version [31].

3. PROBLEM FORMULATION AND HARDNESS RESULTS

In this section, we first define the notion of SAVG k-Configuration and then formally introduce the SVGIC problem. We also prove its hardness of approximation, and introduce an integer program for SVGIC as a cornerstone for the AVG algorithm.

3.1 Problem Formulation

Given a collection $\mathcal{C}$ of m items (called the *Universal Item Set*), a directed social network $G = (V, E)$ with a vertex set V of n users (i.e., shoppers to visit a VR store) and an edge set E specifying their social relationships. In the following, we use the terms *user set* and *shopping group* interchangeably to refer to V, and define SAVG k-Configuration to represent the partitioned subgroups and the corresponding items displayed at their allocated slots.

Definition 1. *Social-Aware VR Group k-Configuration* (SAVG k-Configuration). Given k display slots for displaying items to users in a VR shopping group, an SAVG k-Configuration is a function $\mathbf{A}(\cdot,\cdot) : (V \times [k]) \rightarrow \mathcal{C}$ mapping a tuple (u, s) of a user u and a slot s to an item c. $\mathbf{A}(u, s) = c$ means that the configuration has item c displayed at slot s to user u, and $\mathbf{A}(u, :) = \langle \mathbf{A}(u, 1), \mathbf{A}(u, 2), \dots, \mathbf{A}(u, k)\rangle$ are the k items displayed to u. Furthermore, the function is regulated by a *no-duplication constraint* which ensures the k items displayed to u are distinct, i.e., $\mathbf{A}(u, s) \neq \mathbf{A}(u, s'), \forall s \neq s'$.

For shopping with families and friends, previous research [73, 75, 76] demonstrates that discussions and interactions are inclined to trigger more purchases of an item. Specifically, the VR technology supporting social interactions has already been employed in existing VR social platforms, e.g., VRChat [65], AltspaceVR [3], and Facebook Horizon [15], where users, represented by customized avatars, interact with friends in various ways: 1) *Movement and gesture.* Commercial VR devices, e.g., Oculus Quest and HTC Vive Pro, support six-degree-of-freedom headsets and hand-held controllers to detect the user movements and gestures in real-time. These movements are immediately updated to the server and then perceived by other users. For example, a high-five gesture mechanism is used in Rec Room [47] to let users befriend others. 2) *Emotes and Emojis.* Similar to emoticons in social network platforms, VR users can present their emotions through basic expressions (happy face, sad face, etc.) or advanced motions (e.g., dance, backflip, and die in VRChat [66]). 3) *Real sound.* This allows users to communicate with friends verbally through their microphones. As such, current VR applications support immersive and seamless social interactions, rendering social interactions realistic in shopping systems.

With non-customized user environments in brick-and-mortar stores, discussing a specific item near its location is the default behavior. On the other hand, as users are displayed completely customized items in personalized e-commerce (e.g., the 20 items appearing on the main page of a shopping site can be completely disjoint for two friends), it often requires a "calibration" step (e.g., sharing the exact hyperlink of the product page) to share the view on the target item before e-commerce users can engage in discussions on some items. Since shopping users are used to having discussions on the common item they are viewing at the identical slot in their own Virtual Environments (VEs), we devote to enhance the possibility of this intuitive setting in our envisioned VR shopping VEs even though other settings are also possible in VR.

Therefore, for a pair of friends displayed a common item, it is most convenient to display the item at the same slot in the two users' respective VEs such that they can locate the exact position easily (e.g., "come to the second shelf; you should see this."). Accordingly, upon viewing any item, it is expected that the user interface indicates a list of friends being co-displayed the specific item with the current user, so that she is aware of the potential candidates to start a discussion with. To display an item at the same slot to a pair of friends, we define the notion of *co-display* as follows.

Definition 2. *Co-display* ($u \overset{c}{\underset{s}{\leftrightarrow}} v$). Let $u \overset{c}{\underset{s}{\leftrightarrow}} v$ represent that users u and v are *co-displayed* an identical item c at slot s, i.e., $\mathbf{A}(u, s) = \mathbf{A}(v, s) = c$. Let $u \overset{c}{\leftrightarrow} v$ denote that there exists at least one s such that $u \overset{c}{\underset{s}{\leftrightarrow}} v$.

Naturally, the original group of users are partitioned into subgroups in correspondence with the displayed items. That is, for each slot $s \in [k]$, the SAVG k-Configuration implicitly partitions the user set V into a collection of disjoint subsets $V^s = \{V^s_1, V^s_2, \dots, V^s_{\mathrm{N_p}(s)}\}$, where $\mathrm{N_p}(s)$ is the number of partitioned subgroups at slot s, such that $u \overset{c}{\underset{s}{\leftrightarrow}} v$ if and only if $u, v \in V^s_i, i = 1, \dots, \mathrm{N_p}(s)$.

Under the scenario of brick-and-mortar group shopping, previous research [38, 40, 50, 73, 75] indicates that the satisfaction of a group shopping user is affected by two factors: *personal preference* and *social interaction.* Furthermore, empirical research on dynamic group interaction in retails finds that shopping while discussing with others consistently boosts sales [73], while intra-group talk frequency has a significant impact on users' shopping tendencies and purchase frequencies [75, 76]. Accordingly, we capture the overall satisfaction by combining 1) the aggregated personal

Table 1: Preference and social utility values in Example 2.

	$\mathrm{p}(u_A,\cdot)$	$\mathrm{p}(u_B,\cdot)$	$\mathrm{p}(u_C,\cdot)$	$\mathrm{p}(u_D,\cdot)$	$\tau(u_A, u_B,\cdot)$	$\tau(u_A, u_C,\cdot)$	$\tau(u_A, u_D,\cdot)$	$\tau(u_B, u_A,\cdot)$	$\tau(u_B, u_C,\cdot)$	$\tau(u_C, u_A,\cdot)$	$\tau(u_C, u_B,\cdot)$	$\tau(u_D, u_A,\cdot)$
c_1	0.8	0.7	0	0.1	0.2	0	0.2	0.2	0	0	0.1	0.3
c_2	0.85	1.0	0.15	0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
c_3	0.1	0.15	0.7	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05
c_4	0.05	0.2	0.6	1.0	0	0	0.05	0.05	0.2	0.05	0.2	0
c_5	1.0	0.1	0.1	0.95	0.05	0.3	0.2	0.05	0	0.3	0.05	0.25

preferences and 2) the retailing benefits from facilitating social interactions and discussions. We note that, from an optimization perspective, these two goals form a trade-off because close friends/families may have diverse personal preferences over items. One possible way to simultaneously consider both factors is to use an end-to-end machine learning-based approach, which generates the *user embeddings* and *item embeddings* [8] and designs a neural network aggregator to derive the total user satisfaction. However, this approach requires an algorithm to generate candidate configurations for ranking and relies on a huge amount of data to train the parameters in the aggregator. On the other hand, previous research [57, 67] has demonstrated that a *weighted combination* of the preferential and social factors is effective in measuring user satisfaction, where the weights can be directly set by a user or implicitly learned from existing models [35,78]. Therefore, we follow [57,67] to define the SAVG utility as a combination of *aggregated preference utility* and *aggregated social utility*, weighted by a parameter λ.

Specifically, for a given pair of friends u and v, i.e., $(u, v) \in E$, and an item $c \in \mathcal{C}$, let $\mathrm{p}(u, c) \geq 0$ denote the *preference utility* of user u for item c, and let $\tau(u, v, c) \geq 0$ denote the *social utility* of user u from viewing item c together with user v, where $\tau(u, v, c)$ can be different from $\tau(v, u, c)$. Following [57, 67], the *SAVG utility* is defined as follows.

Definition 3. *SAVG utility* ($\mathbf{w_A}(u, c)$). Given an SAVG k-Configuration $\mathbf{A}$, the SAVG utility of user u on item c in $\mathbf{A}$ represents a combination of the preference and social utilities, where $\lambda \in [0, 1]$ represents their relative weight.

$$\mathrm{w}_{\mathbf{A}}(u, c) = (1 - \lambda) \cdot \mathrm{p}(u, c) + \lambda \cdot \sum_{v | u \overset{c}{\longleftrightarrow} v} \tau(u, v, c)$$

The preference and social utility values, as well as the weight λ, can be directly given by the users or obtained from social-aware personalized recommendation learning models [35,78]. Those models, learning from purchase history, are able to infer (u, c) and (u, v, c) tuples to relieve users from filling those utilities manually. To validate the effectiveness of this objective model, we have conducted a user study in real CID VR shopping system which shows high correlations between user satisfaction and the proposed problem formulation (Please see Section 6.5 for the results).

Example 2. Revisit Example 1 where $\mathcal{C} = \{c_1, c_2, \ldots, c_5\}$ and let $V = \{u_A, u_B, u_C, u_D\}$ denote Alice, Bob, Charlie, and Dave, respectively. Table 1 summarizes the given preference and social utility values. For example, the preference utility of Alice to the tripod is $\mathrm{p}(u_A, c_1) = 0.8$. In the SAVG 3-configuration described at the top of Figure 1, $\mathbf{A}(u_A, \cdot) = \langle c_5, c_1, c_2 \rangle$, meaning that the SP camera, the tripod, and the DSLR camera are displayed to Alice at slots 1, 2, and 3, respectively. Let $\lambda = 0.4$. At slot 2, Alice is *co-displayed* the tripod with Bob and Dave. Therefore, $\mathbf{w_A}(u_A, c_1) = 0.6 \cdot 0.8 + 0.4 \cdot (0.2 + 0.2) = 0.64$. $\square$

We then formally introduce the SVGIC problem as follows.
Problem: Social-aware VR Group-Item Configuration (SVGIC).
Given: A social network $G = (V, E)$, a universal item set $\mathcal{C}$, preference utility $\mathrm{p}(u, c)$ for all $u \in V$ and $c \in \mathcal{C}$, social utility $\tau(u, v, c)$ for all $(u, v) \in E$ and $c \in \mathcal{C}$, the weight λ, and the number of slots k.
Find: An SAVG k-Configuration $\mathbf{A}^*$ to maximize the total SAVG utility

$$\sum_{u \in V} \sum_{c \in \mathbf{A}^*(u,\cdot)} \mathrm{w}_{\mathbf{A}^*}(u, c),$$

where $\mathrm{w}_{\mathbf{A}^*}(u, c)$ is the SAVG utility defined in Definition 3, and the no-duplication constraint is ensured. $\square$

Finally, we remark here that the above additive objective function in SVGIC can be viewed as a generalization of more limited variations of objectives. For instance, some objectives in personal/group recommendation systems without considering social discussions, e.g., the AV semantics in [49], are special cases of SVGIC where $\lambda = 0$. We detail this in Section 3.1 in the full version [31].

3.2 Indirect Co-display and SVGIC-ST

In SVGIC, we characterize the merit of social discussions by the social utilities to a pair of users when they see the same item at the *same slot*. On the other hand, if a common item is displayed at different slots in two users' VEs, a prompt discussion is more difficult to initiate since the users need to locate and move to the exact position of the item first. *Teleportation* [5], widely used in VR tourism and gaming applications, allows VR users to directly transport between different positions in the VE. Thus, for a pair of users Alice and Bob co-displayed an item at different slots, as long as they are aware of where the item is displayed, one (or both) of them can teleport to the respective display slot of the item to trigger a discussion. To model this event that requires more efforts from users, we further propose a generalized notion of *indirect co-display* to consider the above-described scenario.

Definition 4. *Indirect Co-display.* ($u \overset{c}{\underset{s,s'}{\longleftrightarrow}} v$). Let $u \overset{c}{\underset{s,s'}{\longleftrightarrow}} v$ denote that users u and v are *indirectly* co-displayed an item c at slots s and s' respectively in their VEs, i.e., $\mathbf{A}(u, s) = \mathbf{A}(v, s') = c$. We use $u \overset{c}{\underset{\text{ind}}{\longleftrightarrow}} v$ to denote that there exist different slots $s \neq s'$ such that $u \overset{c}{\underset{s,s'}{\longleftrightarrow}} v$.

As social discussions on indirectly co-displayed items are less immersive and require intentional user effort, we introduce a discount factor $d_{\text{tel}} < 1$ on teleportation to downgrade the social utility obtained via indirect co-display. Therefore, the total SAVG utility incorporating indirect co-display is as follows.

In the full version of this paper [31], we explicitly formulate a more generalized SVGIC-ST problem with the above notions of indirect display and a modified SAVG utility, where the problem is associated with an additional *subgroup size constraint* to accommodate practical limits in existing VR applications. Tables of all used notations in both SVGIC and SVGIC-ST are also provided in the full version [31].

3.3 Hardness Result and Integer Program

In the following, we state theoretical hardness results for SVGIC and briefly discuss its relation with other combinatorial problems.

Theorem 1. SVGIC is APX-hard. Moreover, for any $\epsilon > 0$, it is NP-hard to approximate SVGIC within a ratio of $\frac{32}{31} - \epsilon$.

Proof. We prove the theorem with a gap-preserving reduction from the MAX − E3SAT problem [22]. Due to space constraint, please see the full version [31] for the proof. □

Comparison with Related Combinatorial Problems. Among the related *Multi-Labeling* (ML)-type problems detailed in Section 2, SVGIC is particularly related to the graph coloring problem MHE [74], where, different from traditional graph coloring, vertices are encouraged to share the same color with neighbors. The optimization goal of MHE is to maximize the number of *happy edges*, i.e., edges with same-color vertices. Regarding the displayed items in SVGIC as colors (labels) in MHE, the social utility achieved in SVGIC is closely related to the number of preserved happy edges in MHE, and SVGIC thereby encourages partitioning all users into dense subgroups to preserve the most social utility. However, SVGIC is more difficult than the ML-type problems due to the following reasons. 1) The ML-type problems find a strict partition that maps each entity/vertex to only one color (label), while SVGIC assigns k items to each user, implying that any direct reduction from the above problems can only map to the $k = 1$ special case in SVGIC. 2) Most of the ML-type problems do not discriminate among different labels, i.e., switching the assigned labels of two different subgroups does not change their objective functions. This corresponds to the special case of SVGIC where all preference utility $\mathrm{p}(u, c)$ and social utility $\tau(u, v, c)$ do not depend on the item c. 3) Most of the ML-type problems admit a partial labeling (pre-labeling) in the input such that some entities have predefined fixed labels (otherwise labeling every entity with the same label is optimal, rendering the problem trivial), while SVGIC does not specify any item to be displayed to specific users. However, SVGIC requires k items for each user; moreover, even in the $k = 1$ special case, simply displaying the same item to all users in SVGIC is not optimal due to the item-dependent preference and social utility.

Next, we propose an Integer Programming (IP) model for SVGIC to serve as the cornerstone for the approximation algorithm proposed later in Section 4. Let binary variable $x^c_{u,s}$ denote whether user u is displayed item c at slot s, i.e., $x^c_{u,s} = 1$ if and only if $\mathbf{A}(u, s) = c$. Let x^c_u indicate whether u is displayed c at any slot in the SAVG k-Configuration. Moreover, for each pair of friends $e = (u, v) \in E$, let binary variable $y^c_{e,s}$ denote whether u and v are co-displayed item c at slot s, i.e., $y^c_{e,s} = 1$ if and only if $u \overset{c}{\underset{s}{\leftrightarrow}} v$. Similarly, variable $y^c_e = 1$ if and only if $u \overset{c}{\leftrightarrow} v$. The objective of SVGIC is specified as follows.

$$\max \sum_{u\in V}\sum_{c\in\mathcal{C}}[(1-\lambda)\cdot \mathrm{p}(u,c)\cdot x^c_u + \lambda \cdot \sum_{e=(u,v)\in E}(\tau(u,v,c)\cdot y^c_e)]$$

subject to the following constraints,

$$\sum_{s=1}^{k} x^c_{u,s} \leq 1, \quad \forall u \in V, c \in \mathcal{C} \tag{1}$$

$$\sum_{c\in\mathcal{C}} x^c_{u,s} = 1, \quad \forall u \in V, s \in [k] \tag{2}$$

$$x^c_u = \sum_{s=1}^{k} x^c_{u,s}, \quad \forall u \in V, c \in \mathcal{C} \tag{3}$$

$$y^c_e = \sum_{s=1}^{k} y^c_{e,s}, \quad \forall e = (u,v) \in E, c \in \mathcal{C} \tag{4}$$

$$y^c_{e,s} \leq x^c_{u,s}, \quad \forall e = (u,v) \in E, s \in [k], c \in \mathcal{C} \tag{5}$$

$$y^c_{e,s} \leq x^c_{v,s}, \quad \forall e = (u,v) \in E, s \in [k], c \in \mathcal{C} \tag{6}$$

$$x^c_{u,s}, x^c_u, y^c_{e,s}, y^c_e \in \{0,1\}, \quad \forall u \in V, e \in E, s \in [k], c \in \mathcal{C}. \tag{7}$$

Constraint (1) states that each item c can only be displayed at most once to a user u (i.e., the no-duplication constraint). Constraint (2) guarantees that each user u is displayed exactly one item at each slot s. Constraint (3) ensures that $x^c_u = 1$ (u is displayed c in the configuration) if and only if there is exactly one slot s with $x^c_{u,s} = 1$. Similarly, constraint (4) ensures $y^c_e = 1$ if and only if $y^c_{e,s} = 1$ for exactly one s. Constraints (5) and (6) specify the co-display, where $y^c_{e,s}$ is allowed to be 1 only if c is displayed to both u and v at slot s, i.e., $x^c_{u,s} = x^c_{v,s} = 1$. Finally, constraint (7) ensures all decision variables are binary. Note that the x variables in the IP model are sufficient to represent the solution of SVGIC (i.e., $x^c_{u,s}$ denotes whether an item c is displayed at slot s for user u), whereas the y variables are auxiliary to enable formulating SVGIC as an IP.

4. ALGORITHM DESIGN

In this section, we introduce the Alignment-aware VR Subgroup Formation (AVG) algorithm to tackle SVGIC. As shown in Example 1, the personalized and group approaches do not solve SVGIC effectively, as the former misses on social utility from co-display while the latter fails to leverage the flexibility of CID to preserve personal preference. An alternative idea, called the *subgroup* approach, is to first pre-partition the shopping group (i.e., the whole user set) into some smaller social-aware subgroups (e.g., using traditional community detection techniques), and then determine the displayed items based on preferences of the subgroups. While this idea is effective for social event organization [67] where each user is assigned to exactly one social activity, it renders the partitioning of subgroups static across all display slots in SVGIC, i.e., a user is always co-displayed common items only with other users in the same subgroup. Therefore, this approach does not fully exploit the CID flexibility, leaving some room for better results.

Instead of using a universal partition of subgroups as in the aforementioned subgroup approach, we aim to devise a more sophisticated approach that allows varied co-display subgroups across the display slots to maximize the user experience. Accordingly, we leverage Linear Programming (LP) relaxation strategies that build on the solution of the Integer Program formulated in Section 3.3 because it naturally facilitates different subgroup partitions across all slots while allocating proper items for those subgroups with CID. In other words, our framework partitions the subgroups (for each slot) and selects the items simultaneously, thus avoiding any possible shortcomings of two-phased approaches that finish these two tasks sequentially. By relaxing all the integrality constraints in the IP, we obtain a relaxed linear program whose *fractional* optimal solution can be explicitly solved in polynomial time. For an item c to be displayed to a user u at a certain slot s, the fractional decision variable ${x^*}^c_{u,s}$ obtained from the optimal solution of the LP relaxation problem can be assigned as its *utility factor*. Items with larger utility factors are thus inclined to contribute more SAVG utility (i.e., the objective value), since they are preferred by the users or more capable of triggering social interactions.

Next, it is important to design an effective rounding procedure to construct a promising SAVG k-Configuration according to the utility factors. We observe that simple *independent* rounding schemes may perform egregiously in SVGIC because they do not facilitate effective co-displaying of common items, thereby losing a huge amount of potential social utility upon constructing the SAVG k-Configuration, especially in the cases where the item preferences are not diverse. Indeed, we prove that independent rounding schemes may achieve an expected total objective of only $O(1/m)$ of the optimal amount. Motivated by the incompetence of independent rounding, our idea is to leverage *dependent rounding* schemes that encourage co-display of items of common interests, i.e., with high utility factors to multiple users in the optimal LP solution.

Based on the idea of dependent rounding schemes in [30], we introduce the idea of *Co-display Subgroup Formation* (CSF) that co-displays a *focal item* c at a specific *focal slot* s to every user u with a utility factor $x^c_{u,s}$ greater than a *grouping threshold* α. In other words, CSF clusters the users with high utility factors to a focal item c to form a *target subgroup* in order to co-display c to the subgroup at a specific display slot s. Depending on the randomly chosen set of *focal parameters*, including the focal item, the focal slot, and the grouping threshold, the size of the created target subgroups can span a wide spectrum, i.e., as small as a single user and as large as the whole user set V, to effectively avoid the pitfalls of personalized and group approaches. The randomness naturally makes the algorithm less vulnerable to extreme-case inputs, therefore resulting in a good approximation guarantee. Moreover, CSF allows the partitions of subgroups to vary across all slots in the returned SAVG k-Configuration, exploiting the flexibility provided by CID. However, different from the dependent rounding schemes in [30], the construction of SAVG k-Configurations in SVGIC faces an additional challenge – it is necessary to carefully choose the displayed items at multiple slots to ensure the no-duplication constraint.

We prove that AVG is a 4-approximation algorithm for SVGIC and also show that AVG can be derandomized into a deterministic approximation algorithm. In the following, we first deal with the case with $\lambda = \frac{1}{2}$. We observe that all other cases with $\lambda \neq 0$ can be reduced to this case by proper scaling of the inputs, i.e., $\mathrm{p}'(u,c) = \frac{1-\lambda}{\lambda}\mathrm{p}(u,c)$, whereas $\lambda = 0$ makes the problem become trivial. We explicitly prove this property in Section 4.4 in the full version [31]. Moreover, for brevity, the total SAVG utility is scaled up by 2 so that it is a direct sum of the preference and social utility. A table of all notations used in AVG is also provided in [31].

Table 2: Optimal fractional solution for slot 1 in Example 2.

	${x^*}^{c_1}_{\cdot,1}$	${x^*}^{c_2}_{\cdot,1}$	${x^*}^{c_3}_{\cdot,1}$	${x^*}^{c_4}_{\cdot,1}$	${x^*}^{c_5}_{\cdot,1}$
u_A	0.33	0.33	0	0	0.33
u_B	0.33	0.33	0	0.33	0
u_C	0	0	0.33	0.33	0.33
u_D	0.33	0	0	0.33	0.33

4.1 LP Relaxation and an Independent Rounding Scheme

Following the standard linear relaxation technique [68], the LP relaxation of SVGIC is formulated by replacing the integrality constraint (constraint (7)) in the IP model, i.e., $x^c_{u,s}, x^c_u, y^c_{e,s}, y^c_e \in \{0,1\}$, with linear upper/lower bound constraints, i.e., $0 \leq x^c_{u,s}, x^c_u, y^c_{e,s}, y^c_e \leq 1$. The optimal fractional solution of the relaxed problem can be acquired in polynomial time with commercial solvers, e.g., CPLEX [28] or Gurobi [21]. Recall that the x-variables are sufficient to represent the solution of SVGIC (i.e., $x^c_{u,s}$ denotes whether an item c is displayed at slot s for user u), whereas the y-variables are auxiliary. Therefore, the optimal solution can be fully represented by X^* (the set of optimal x variables). The fractional decision variable ${x^*}^c_{u,s}$ in X^* is then taken as the *utility factor* of item c at slot s for user u. Note that the optimal objective in the relaxed LP is an upper bound of the optimal total SAVG utility in SVGIC, because the optimal solution in SVGIC is guaranteed to be a feasible solution of the LP relaxation problem.

Example 3. Table 2 shows the utility factors in Example 2, where the fractional solution is identical for all slots 1-3 (thereby only slot 1 is shown). For example, the utility factor of c_1 (the tripod) to Alice at each slot is ${x^*}^{c_1}_{u,1} = {x^*}^{c_1}_{u,2} = {x^*}^{c_1}_{u,3} = 0.33$.

Note that three items (c_1, c_2, and c_5) have nonzero utility factors to Alice at slot 1 in Example 3, which manifests that the optimal LP solution may not construct a valid SAVG k-Configuration because each user is allowed to display exactly one item at each display slot in SVGIC. Therefore, a rounding scheme is needed to construct appropriate SAVG k-Configurations from the utility factors. Given X^*, a simple rounding scheme is to randomly (and independently) assign item c to user u at slot s with probability ${x^*}^c_{u,s}$, i.e., the utility factor of c to u at s, so that more favorable items are more inclined to be actually displayed to the users.

However, as this strategy selects the displayed items independently, for a pair of friends u and v, the chance that the algorithm obtains high social utility by facilitating co-display is small, since it requires the randomized rounding process to hit on the same item for both u and v simultaneously. Furthermore, this strategy could not ensure the final SAVG k-Configuration to follow the no-duplication constraint, as an item c can be displayed to a user u at any slot with a nonzero utility factor. The following lemma demonstrates the ineffectiveness of this rounding scheme.

Lemma 1. There exists an SVGIC instance I on which the above rounding scheme achieves only a total SAVG utility of $O(\frac{1}{m})$ of the optimal value in expectation.

Proof. Due to space constraints, please see Section 4.1 in the full version [31] for the proof. □

4.2 Alignment-aware Algorithm

To address the above issues, we devise the *Co-display Subgroup Formation* (CSF) rounding scheme, inspired by the dependent rounding scheme for labeling problems [30], as the cornerstone of AVG to find a target subgroup U according to a set of focal parameters for co-display of the focal item to all users in U. Given the optimal fractional solution X^* to the LP relaxation problem, AVG iteratively 1) samples a set of focal parameters (c, s, α) with $c \in \mathcal{C}, s \in \{1, 2, \ldots, k\}$, and $\alpha \in [0, 1]$ uniformly at random; it then 2) conducts CSF according to the selected set of parameters (c, s, α) until a complete SAVG k-Configuration is constructed. It is summarized in Algorithm 1.

Algorithm 1 Alignment-aware VR Subgroup Formation (AVG)

Input: X^*
Output: An SAVG k-Configuration $\mathbf{A}$

$\mathbf{A}(\hat{u}, \hat{s}) \leftarrow$ NULL for all $\hat{u}, \hat{s}$
$X^* \leftarrow X^*_{\text{LP}}$
while some entry in $\mathbf{A}$ is NULL **do**
  Sample $c \in \mathcal{C}$, $s \in [k]$, $\alpha \in [0, 1]$ randomly
  for $\hat{u} \in V$ **do**
    if $\mathbf{A}(\hat{u}, s) =$ NULL and $\mathbf{A}(\hat{u}, t) \neq c \, \forall t \neq s$ **then**
      ($\hat{u}$ eligible for (c, s))
      if ${x^*}^{c}_{\hat{u},s} \geq \alpha$ **then**
        $\mathbf{A}(\hat{u}, s) \leftarrow c$
return A

Co-display Subgroup Formation. Given the randomly sampled set of parameters (c, s, α), CSF finds the target subgroup as follows. With the focal item c and the focal slot s, a user $\hat{u}$ is *eligible* for (c, s) in CSF if and only if 1) $\hat{u}$ has not been displayed any item at slot s, and 2) c has not been displayed to $\hat{u}$ at any slot. Users not eligible for (c, s) are not displayed any item in CSF to ensure the no-duplication constraint. For each eligible user $\hat{u}$, CSF selects c for $\hat{u}$ at slot s (i.e., $\mathbf{A}(\hat{u}, s) \leftarrow c$) if and only if ${x^*}^{c}_{\hat{u},s}$ is no smaller than the grouping threshold α. In other words, given (c, s, α), CSF co-displays c to a target subgroup U that consists of every eligible user $\hat{u}$ with ${x^*}^{c}_{\hat{u},s} \geq \alpha$. Therefore, the grouping threshold α plays a key role to the performance bound in the formation of subgroups. Later we prove that with the above strategy, for any pair of users u, v and any item c, $\Pr(u \overset{c}{\leftrightarrow} v) \geq \frac{{y^*}^{c}_{e}}{4}$; or equivalently, the expected social utility of u from viewing c with v obtained in the final SAVG k-Configuration is at least a constant factor within that in the optimal LP solution.

AVG repeats the process of parameter sampling and CSF until a *feasible* SAVG k-Configuration is fully constructed, i.e., each user is displayed exactly one item at each slot, and the no-duplication constraint is satisfied.

Example 4. For Example 2 with the utility factors shown in Table 2, assume that the set of focal parameters are sampled as $(c, s, \alpha) = (c_1, 3, 0.06)$. Since ${x^*}^{c_1}_{u_A,3} = {x^*}^{c_1}_{u_B,3} = {x^*}^{c_1}_{u_D,3} = 0.33 > 0.06 > {x^*}^{c_1}_{u_C,3} = 0$, CSF co-displays the tripod to the subgroup {Alice, Bob, Dave} at slot 3. Next, for the second set of parameters $(c, s, \alpha) = (c_4, 2, 0.22)$, {Bob, Charlie, Dave} is formed and co-displayed the memory card at slot 2, since ${x^*}^{c_4}_{u_B,2} = {x^*}^{c_4}_{u_C,2} = {x^*}^{c_4}_{u_D,2} = 0.33 > 0.22 > {x^*}^{c_4}_{u_A,2} = 0$. The subsequent sets of parameters (c, s, α) are respectively $(c_3, 1, 0.04)$, $(c_5, 3, 0.2)$, $(c_5, 1, 0.31)$, $(c_2, 1, 0.01)$, and $(c_2, 2, 0.19)$ in the next five iterations, achieving a total SAVG utility of 9.75. Please see Section 4.2 in the full version [31] for more details. □

The theoretical guarantee of the AVG algorithm is given in the following results, which we explicitly prove in Section 4.2 in the full version [31] due to the space constraint.

Theorem 2. Given the optimal fractional solution X^*, AVG returns an expected 4-approximate SAVG k-Configuration in $O(n^2 \cdot k)$-time.

Corollary 2.1. Repeating AVG and selecting the best output returns a $(4 + \epsilon)$-approximate SAVG k-Configuration in $O(n^2 \cdot k \cdot \log_\epsilon n)$-time with high probability, i.e., with a probability $1 - \frac{1}{n^{O(1)}}$.

Corollary 2.2. Given a (non-optimal) fractional solution $\tilde{X}^*$ as a β-approximation of the LP relaxation problem, AVG returns an expected $(4 \cdot \beta)$-approximate SAVG k-Configuration.

The second corollary is particularly useful in practice for improving the efficiency of AVG since state-of-the-art LP solvers often reach a close-to-optimal solution in a short time but need a relatively long time to return the optimal solution, especially for large inputs. Therefore, it allows for a quality-efficiency trade-off in solving SVGIC.

4.3 Derandomizing AVG

From the investigation of AVG, we observe that the grouping threshold α plays a key role in forming effective target subgroups in CSF. If α is close to 0, CSF easily forms a large subgroup consisting of all users and co-displays the focal item to them, similar to the ineffective group approach. On the other hand, large α values lead to small subgroups, not good for exploiting social interactions. Due to the randomness involved in AVG, these caveats cannot be completely avoided. To address these issues, we aim to further strengthen the performance guarantee of AVG by derandomizing the selection of focal parameters to obtain a stronger version of the algorithm, namely Deterministic Alignment-aware VR Subgroup Formation (AVG-D), which is a *deterministic* 4-approximation algorithm. First, we observe that α can be assigned in a discrete manner.

Observation 1. There are $O(knm)$ distinct possible outcomes in CSF, each corresponding to a grouping threshold $\alpha = {x^*}^{c}_{u,s}$, i.e., the utility factor of an item c to a user u at a slot s.

The above observation can be verified as follows. Given c and s, the outcome of CSF with grouping threshold $\alpha = x$, for any $x \in [0, 1]$, is equivalent to that with α set to the smallest ${x^*}^c_{u,s} \geq x$. It enables us to derandomize AVG effectively. Instead of randomly sampling (c, s, α), we carefully evaluate the outcomes (of CSF) from setting α to every possible ${x^*}^c_{u,s}$ in the optimal fractional solution. Intuitively, it is desirable to select an α that results in the largest increment in the total SAVG utility. However, this short-sighted approach ignores the potentially significant increase in total SAVG utility in the future from the remaining users and slots that have not been processed. In fact, it always selects an outcome with $\alpha = 0$ to maximize the current utility increment. Therefore, it is necessary for AVG-D to carefully evaluate all potential future allocations of items.

Specifically, AVG-D selects the set of focal parameters (c, s, α) to maximize a weighted sum of 1) the increment of SAVG utility in the current iteration via CSF with (c, s, α), and 2) the expected SAVG utility in the future from item assignments at slots that are *left unfilled* in CSF with (c, s, α). Due to the space constraint, please see Section 4.3 in the full version [31] for the details of AVG-D, and the proof of the following theoretical result.

Theorem 3. Given the optimal fractional solution X^*, AVG-D returns a worst-case 4-approximate SAVG k-Configuration in $O(n \cdot m \cdot k \cdot |E|)$-time.

4.4 Enhancements of AVG

In the following, we detail enhancements of the AVG and AVG-D algorithms. First, we show that AVG and AVG-D support values of $\lambda \neq \frac{1}{2}$ via a simple scaling on the inputs. We then design two advanced strategies, including an *advanced LP transformation technique* and a *new focal parameter sampling scheme*, to improve the efficiency of AVG and AVG-D. The LP transformation technique derives a new LP formulation to reduce the number of decision variables and constraints from $O((n + |E|)mk)$ to $O((n + |E|)m)$ by condensing the $x^c_{u,s}$ variables (k is the number of slots). We prove that the optimal objective in the new formulation is exactly that in the original one. The focal parameter sampling scheme maintains a *maximum utility factor* $\bar{x^*}^c_s$ (detailed later) for each pair of item c and slot s to avoid unnecessary sampling of focal parameters (c, s, α) when $\alpha \geq \max_{u \in V} {x^*}^c_{u,s}$, especially for a large k. We prove that the sampling results of the new sampling technique and the original one are the same. Therefore, the efficiency of AVG can be improved without sacrificing the solution quality. Finally, we extend AVG and AVG-D to support SVGIC-ST (and also the Social Event Organization (SEO)-type problems) by tailoring CSF with consideration of the additional VR-related constraints. Due to the space constraint, we present the details in Section 4.4 in the full version [31].

5. EXTENSIONS FOR PRACTICAL SCENARIOS

In this section, we extend SVGIC and AVG to support a series of practical scenarios. 1) *Commodity values.* Each item is associated with a commodity value to maximize the total profit. 2) *Layout slot significance.* Each slot location is associated with a different significance weight (e.g., center is better) according to retailing research [13, 58]. 3) *Multi-View Display*, where a user can be displayed multiple items in a slot, including one default, personally preferred item in the primary view and multiple items to view with friends in group views, and the primary and group views can be freely switched. 4) *Generalized social benefits*, where social utility can be measured not only pairwise (each pair of friends) but also group-wise (any group of friends). 5) *Subgroup change*, where the fluctuations (i.e., change of members) between the partitioned subgroups at consecutive slots are limited to ensure smooth social interactions as the elapse of time. 6) *Dynamic scenario*, where users dynamically join and leave the system with different moving speeds. Due to the space constraint, we present the details in Section 5 in the full version of the paper [31].

6. EXPERIMENTS

In this section, we evaluate the proposed AVG and AVG-D along with various baseline algorithms on three real datasets. We also build a prototype of a VR store with Unity and SteamVR to conduct a user study.

6.1 Experiment Setup and Evaluation Plan

Datasets. To evaluate the proposed algorithms, three real datasets are tested in the experiment. The first dataset *Timik* [29] is a 3D VR social network containing 850k vertices (users) and 12M edges (friendships) with 12M check-in histories of 849k virtual Point-of-Interests (POIs). The second dataset, *Epinions* [14], is a website containing the reviews of 139K products and a social trust network with 132K users and 841K edges. The third dataset, *Yelp* [72], is a location-based social network (LBSN) containing 1.5M vertices, 10M edges, and 6M check-ins. For Timik and Yelp, we follow the settings in [6,20,52] to treat POIs in the above datasets as the candidate items in SVGIC. The preference utility and social utility values are learned by the PIERT learning framework [35] which jointly models the social influence between users and the latent topics of items. Following the scales of the experiments in previous research [49,51], the default number of slots k, number of items m, and size of user set n selected from the social networks to visit a VR store are set as 50, 10000, and 125, respectively.

Baseline Methods. We compare AVG and AVG-D with five baseline algorithms: Personalized Top-k (PER), Fairness Maximization in Group recommendation (FMG) [51], Social-aware Diverse and Preference selection (SDP) [52], Group Recommendation and Formation (GRF) [49], and Integer Programming (IP). PER and FMG correspond to the two baseline approaches outlined in Section 1. Specifically, PER retrieves the top-k preferred items for each user (the personalized approach), while FMG selects a bundled itemset for all users as a group (the group approach) with considerations of fairness of preference among the users. SDP selects socially-tight subgroups to display their preferred items, which corresponds to the subgroup approach outlined in Section 4. GRF splits the input users into subgroups with similar item preferences without considering the social network topology, which can be viewed as a variation of the subgroup approach where the subgroups are partitioned based on preferences instead of social connections. Finally, IP is the integer program formulated in Section 3.3 that finds the optimal solutions of small SVGIC instances by Gurobi [21]. All algorithms are implemented in an HP DL580 Gen 9 server with four 3.0 GHz Intel CPUs and 1TB RAM. Each result is averaged over 50 samples.

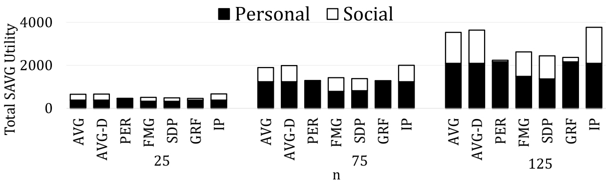


Figure 2: Total SAVG utility vs. size of user set (n).

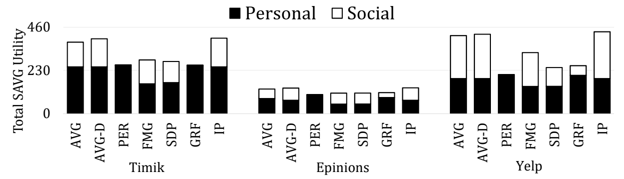


Figure 3: Total SAVG utility in diff. datasets.

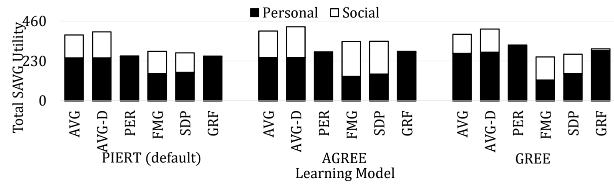


Figure 4: Total SAVG utility vs. different input.

Evaluation Metrics. To evaluate the algorithms considered for SVGIC and analyze the returned SAVG k-Configurations, we introduce the following metrics: 1) total SAVG utility achieved, 2) total execution time (in seconds), 3) the percentages of personal preference utility (*Personal%*) and social utility (*Social%*) in total SAVG utility, 4) the percentage of *Inter*-subgroup edges (*Inter%*) and *Intra*-subgroup edges (*Intra%*) in the returned partition of subgroups, 5) the average network density among partitioned subgroups, normalized by the average density of the original social network, 6) the percentage of friend pairs viewing common items together (*Co-display%*), 7) the percentage of users viewing items alone (*Alone%*), and 8) *regret ratio* (a fairness measure detailed later in Section 6.4.)

Evaluation Plan. To evaluate the performance of the above algorithms, we use IP to derive the optimal total SAVG utility on small datasets. The social networks and items in the small datasets are respectively sampled by random walk and uniform sampling from Timik according to the setting of [42]. Due to the space constraint, please see Section 6.2 in the full version [31] for the experimental results on small datasets. Next, in Section 6.2, we evaluate the efficacy of AVG and AVG-D in large datasets with input scales following previous research [49,51], while conducting experiments on different inputs (the p and τ values) generated by PIERT [35] (default), AGREE, and GREE [8]. We examine the efficiency of all algorithms, including various configurations of mixed integer programming (MIP) algorithms, in Section 6.3. We then compare the algorithms on the aforementioned group formation-related performance metrics in Section 6.4. A case study on a 2-hop ego network in Yelp is shown in Section 6.6 in [31] to examine the subgroup partition patterns in different algorithms. Result of a sensitivity test on r, an important algorithmic parameter of the deterministic AVG-D algorithm and experimental results on the SVGIC-ST problem are reported in Sections 6.7 and 6.8 in [31], respectively. Finally, we build a prototype of VR store with Unity 2017.1.1.1 (64bit), Photon Unity Network free 1.86, SteamVR Plugin 1.2.2, VRTK, and 3ds Max 2016 for hTC VIVE HMD to validate the proposed objective in modelling SVGIC. We detail the user study settings and results in Section 6.5.

6.2 Sensitivity Tests on Large Datasets

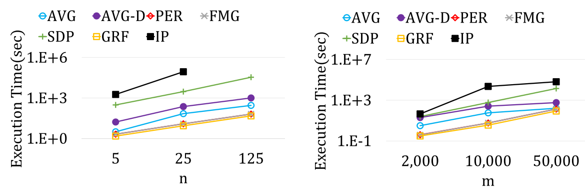
We evaluate the efficiency and efficacy of AVG and AVG-D in large datasets with the scales of the three dimensions following previous research [49,51], i.e., $m = 10000$, $k = 50$, and $n = 125$. Figure 2 presents the total SAVG utility by varying the sizes of user set in Timik. The results manifest that AVG and AVG-D outperform all baselines by at least 30.1%, while AVG-D is slightly better than AVG since it selects better pivot parameters for CSF. Moreover, the returned objective values of AVG and AVG-D respectively achieve at least 93.7% and 96.4% of the objective value of IP, manifesting that our algorithms are effective. Compared with GRF, the improvement of AVG-D grows from 43.6% to 54.6% as n increases, since GRF splits the users into subgroups without considering social relations, but social interactions among *close friends* become more important for a larger group. By striking a balance between preference and social utility, AVG and AVG-D achieve a greater total SAVG utility. Compared with PER and GRF, FMG achieves a higher social utility but a lower preference utility because it displays a universal configuration to all users.

Figure 3 compares the results on Timik, Yelp, and Epinions. The social utility obtained in Epinions is lower than in Yelp due to the sparser social relations in the review network, and group consensus thereby plays a more important role in Yelp. Despite the different characteristics of datasets, AVG and AVG-D prevail in all datasets and outperform all baselines since CSF operates on the utility factors from the optimal LP solution, which does strike a good balance among all factors. FMG and SDP benefit from the high social utility in Yelp and outperform PER. By contrast, PER performs nearly as good as FMG and SDP in Epinions since the social utility is lower.

Next, to examine the influence of input models on the tackled problem, Figure 4 shows the experimental results on different inputs generated by PIERT [35] (default), AGREE and GREE [8]. PIERT jointly learns the preference and social utilities by modeling the social influence between users and the latent topics of items. For AGREE and GREE [8], the former assumes the social influence between users is equal, and the later learns sophisticated weights of the triple (user, user, item). AVG and AVG-D outperform the baselines with regards to all different input models, manifesting that our method is generic to different distributions of inputs. Note that the social utilities returned by AVG and AVG-D with PIERT and AGREE are slightly greater than the ones with GREE. The result manifests that AVG and AVG-D can select better items for users to enjoy if social utilities are different across items.

6.3 Scalability Tests on Large Datasets

Figure 5(a) presents the execution time in Yelp with different n. IP cannot terminate within 24 hours when $n \geq 25$,



(a) Execution time vs. size of user set (n). (b) Execution time vs. size of item set (m).

Figure 5: Execution time in Ye

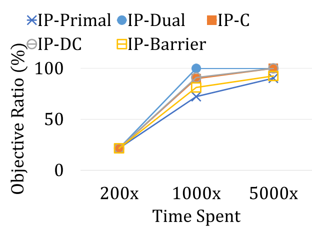
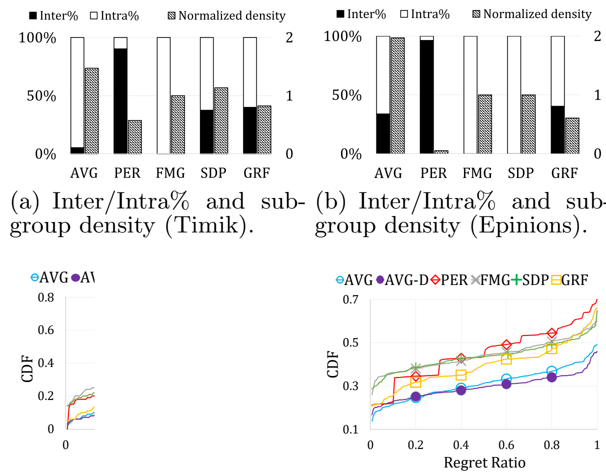


Figure 6: Results of different IP heuristics.



(a) Inter/Intra% and subgroup density (Timik). (b) Inter/Intra% and subgroup density (Epinions).

(c) CDF of Regret ratio (Timik). (d) CDF of Regret ratio (Epinions).

Figure 7: Comparisons on subgroup metrics.

and SDP needs 300 seconds to return a solution even when $n = 5$. Figure 5(b) shows the execution time with different m. Note that AVG and AVG-D are both more scalable to m than the baseline approaches because CSF exploits the fractional solution without m in the complexity. Although AVG-D provides a stronger theoretical guarantee, the scalability of AVG on n is better than AVG-D because AVG samples the target subgroups randomly.

To examine the performance of different mixed integer programming (MIP) algorithms, we further conducted experiments with Primal-first Mixed Integer Programming (IP-Primal), Dual-first Mixed Integer Programming (IP-Dual), Concurrent Mixed Integer Programming (IP-C), Deterministic Concurrent Mixed Integer Programming (IP-DC), and the Barrier Algorithm (IP-Barrier) in the Gurobi [21] package. Figure 6 shows the trade-off between efficiency and efficacy of five different IP algorithms on the Timik dataset with the default parameters ($(k, m, n) = (50, 10000, 125)$). For every MIP algorithm, we evaluate the solution quality of different algorithms with the running time constrained by 200, 1000 and 5000 times the running time of our proposed AVG-D algorithm for the same instance to compare the obtained solutions in different time limits. The y-axis shows the objective value normalized by the solution of AVG-D, which manifests none of the 5 baselines achieves any solution better than that of AVG-D in 5000X of the running time of AVG-D. As such, although there is some subtle difference in performance across different MIP algorithms, none of the examined algorithms shows a reasonable scalability. Please see the full version [31] for more discussions on the scalability.

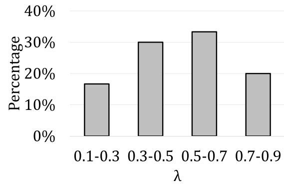
6.4 Comparisons on Subgroup Metrics

In the following, we analyze the subgroups in the SAVG k-Configurations returned from all algorithms in terms of subgroup-related metrics. Figures 7(a) and 7(b) compare the ratios of Inter-/Intra-subgroup edges averaged across all slots in the SAVG k-Configurations in the Timik and Epinions datasets, respectively. It also shows the average network density among the partitioned subgroups normalized by the original density of the input social network. The results from all datasets indicate that the majority of preserved edges by AVG are within the same subgroups (large Intra%). FMG achieves 100% in Intra%, 0% in Inter%, and 1 in normalized density because it consistently views the whole network as a large subgroup. In contrast, PER has a high Inter% as it separates most users into independent subgroups to display their favorite items. There exist a small subset of widely liked or adopted items in Epinions that appear as most users' favorite (hence the small nonzero Intra% of PER), while famous VR locations (e.g., transportation hubs in Timik) are inclined to be associated with high preference utilities by the exploited recommendation learning models as they generate a lot of check-ins among all users. Therefore, they have a higher chance to be co-displayed by PER. Among all methods, AVG achieves the largest normalized density as CSF carefully considers the utility factors to partition the network into dense communities.

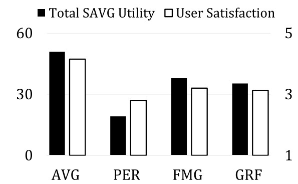
Figures 7(c) and 7(d) report the Cumulative Distribution Function (CDF) of *regret ratios* of all algorithms in the Timik and Epinions datasets. The regret ratio $\mathrm{reg}(u)$ [34] is a game-theory based measurement for the satisfaction of individual users and the overall *fairness* of the solution. For each user u, the regret ratio $\mathrm{reg}(u)$, and its converse, *happiness* ratio $\mathrm{hap}(u)$, are defined as follows.

$$\mathrm{reg}(u) \equiv 1 - \mathrm{hap}(u); \quad \mathrm{hap}(u) \equiv \frac{\sum\limits_{c \in \mathbf{A}^*(u,\cdot)} \mathrm{w}_{\mathbf{A}}(u,c)}{\max\limits_{\mathcal{C}_u} \sum\limits_{c \in \mathcal{C}_u} \bar{\mathrm{w}}_{\mathbf{A}}(u,c)}$$

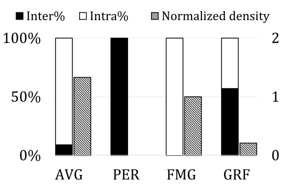
where the numerator in $\mathrm{hap}(u)$ is the achieved SAVG utility, the denominator with $\bar{\mathrm{w}}_{\mathbf{A}}(u,c) = (1-\lambda) \cdot \mathrm{p}(u,c) + \lambda \cdot \sum_{v \in V} \tau(u,v,c)$ is an *upper bound* of possible SAVG utility when all users view c together with user u, and $\mathcal{C}_u$ is a k-itemset, corresponding to a very optimistic scenario favoring u the most. Note that the second term of $\bar{\mathrm{w}}_{\mathbf{A}}(u,c)$ is different from $\mathrm{w}_{\mathbf{A}}(u,c)$ in Definition 3. In other words, the upper bound is the SAVG utility of u if she dictates



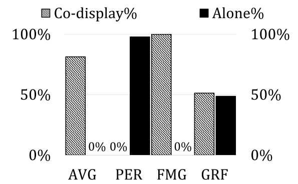
(a) Percentage of diff. λ.



(b) Total SAVG utility and user satisfaction.



(c) Inter/Intra% and subgroup density.



(d) Co-display rate and alone rate.

Figure 8: Comparisons on user study.

the whole SAVG k-Configuration selfishly. Therefore, a high $\text{hap}(u)$ (equivalently, a low $\text{reg}(u)$) implies that user u is relatively satisfied with the SAVG k-Configuration, and *fairness* among the users can be observed by inspecting the distribution of regret ratio in the final configurations.

AVG and AVG-D consistently have the lowest regret ratios. Among the other approaches, PER incurs the highest regret ratios for users in all datasets, since it does not foster social interactions in shared views. Consistent with the performances on SAVG utility, FMG and SDP outperform PER and GRF in the Timik dataset, but their performances are comparable in Epinions because the sparser social relations in the review network generates lower social utility. Interestingly, in Timik, some users in GRF are highly satisfied (as the CDF of GRF matches well with that of AVG and AVG-D from the beginning) but some others have very high regret ratios (as the CDF dramatically rises near the end). This indicates that a portion of the users in GRF may actually have their preferences sacrificed, i.e., they are forced to share views on uninterested items with other subgroup members. In contrast, FMG and SDP show flatter CDFs, implying the user preferences are more balanced. However, users in FMG and SDP consistently have regret ratios over 20%, while the regret ratios seldom exceed 20% in AVG/AVG-D; this is because the randomly chosen pivot parameters (in AVG) and the deterministically optimized one (in AVG-D) can effectively form dense subgroups with similar item preferences. More experimental results on subgroup metrics can be found in Section 6.5 in the full version [31].

6.5 User Study

For the user study, we recruit 44 participants to visit our VR store. Their social networks, preference utility, and λ are pre-collected with questionnaires, which follows the setting of [49], and the social utility is learned by PIERT [35]. A Likert scale questionnaire [61] is used to find the preference utility of items, where users are allowed to discuss the products so that the social utility can be learned. Finally, they are asked to provide λ in [0,1]. We investigate the following research question: After experiencing the VR store, are the participants satisfied with the SAVG k-Configurations generated by AVG, PER, FMG, and GRF? User feedbacks are collected in Likert score [61] from 1 to 5 (very unsatisfactory, unsatisfactory, average, satisfactory, and very satisfactory). Each group of participants visits the VR stores twice via hTC VIVE with the items selected by each scheme in randomized order.

Figure 8(a) reports that λ values specified by the users range from 0.15 to 0.85 with the average as 0.53, indicating that both personal preferences and social interactions are essential in VR group shopping. Figure 8(b) compares the total SAVG utility as well as the recorded user satisfaction of each method. AVG outperforms the baselines by at least 34.2% and 29.6% in terms of the average total SAVG utility and average user satisfaction, respectively. The difference of AVG is statistically significant (p-value $\leq 0.019 < 0.05$). It is worth noting that the correlation between the SAVG utility and user satisfaction is high (Spearman correlation 0.835; Pearson correlation 0.814), which manifest that the SAVG utility is a good estimation of user satisfaction.

Figures 8(c) and 8(d) report the subgroup metrics in the user study datasets. GRF, which separates users into subgroups according to preference similarities, returns a low normalized density (0.21), i.e., users in the same subgroup tends to be strangers. Compared with the results in large-scale datasets (Figure 7), GRF performs worse here since the normalized density is more sensitive when the user set is relatively small. In contrast, AVG flexibly assigns proper items to different subgroups of friends such that the normalized density is greater than 1 and the alone rate is 0%.

7. CONCLUSION

To the best of our knowledge, there exists no prior work tackling flexible configurations under the envisaged scenario of VR group shopping. In this paper, we formulate the SVGIC problem to retrieve the optimal SAVG k-Configuration that jointly maximizes the preference and the social utility, and prove SVGIC is NP-hard to approximate within $\frac{32}{31} - \epsilon$. We introduce an IP model and design a novel 4-approximation algorithm, AVG, and its deterministic version, AVG-D, by exploring the idea of Co-display Subgroup Formation (CSF) that forms subgroups of friends to display them the same items. Experimental results on real VR datasets manifest that our algorithms outperform baseline approaches by at least 30.1% in terms of solution quality.

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9. REFERENCES

[1] 3D virtual stores - the new virtual reality social shopping in 3D. https://www.qbittech.com/index.php/component/k2/item/41-virtual-reality-ecommerce-3d-stores. Accessed: 2020-02-03.

[2] Alibaba offers VR shopping. https://edition.cnn.com/videos/world/2016/11/28/alibaba-vr-shopping-stevens-pkg.cnn. Accessed: 2020-02-03.

[3] AltspaceVR. https://altvr.com/. Accessed: 2020-02-03.

[4] A. An, M. Kargar, and M. Zihayat. Finding affordable and collaborative teams from a network of experts. In *SDM*, pages 587–595. SIAM, 2013.

[5] E. Bozgeyikli, A. Raij, S. Katkoori, and R. V. Dubey. Point & teleport locomotion technique for virtual reality. In *CHI PLAY*, pages 205–216. ACM, 2016.

[6] C. Cai, R. He, and J. J. McAuley. SPMC: socially-aware personalized Markov chains for sparse sequential recommendation. In *IJCAI*, pages 1476–1482, 2017.

[7] Can virtual experiences replace reality? The future role for humans in delivering customer experience. https://www.oracle.com/webfolder/s/delivery_production/docs/FY16h1/doc35/CXResearchVirtualExperiences.pdf. Accessed: 2020-02-03.

[8] D. Cao, X. He, L. Miao, Y. An, C. Yang, and R. Hong. Attentive group recommendation. In *SIGIR*, pages 645–654. ACM, 2018.

[9] H. Chen, D. Niu, K. Lai, Y. Xu, and M. Ardakani. Separating-plane factorization models: Scalable recommendation from one-class implicit feedback. In *CIKM*, pages 669–678. ACM, 2016.

[10] J. Chen, H. Zhang, X. He, L. Nie, W. Liu, and T. Chua. Attentive collaborative filtering: Multimedia recommendation with item- and component-level attention. In *SIGIR*, pages 335–344. ACM, 2017.

[11] L. Chen, C. Liu, R. Zhou, J. Li, X. Yang, and B. Wang. Maximum co-located community search in large scale social networks. *PVLDB*, 11(10):1233–1246, 2018.

[12] E. Dahlhaus, D. S. Johnson, C. H. Papadimitriou, P. D. Seymour, and M. Yannakakis. The complexity of multiterminal cuts. *Journal on Computing*, 23(4):864–894, 1994.

[13] X. Dreze, S. J. Hoch, and M. E. Purk. Shelf management and space elasticity. *Journal of retailing*, 70(4):301–326, 1994.

[14] Epinions dataset. http://www.trustlet.org/extended_epinions.html.

[15] Facebook Horizon. https://www.oculus.com/facebookhorizon/. Accessed: 2020-02-03.

[16] Facebook social VR demo - Oculus connect 2016. https://youtu.be/YuIgyKLPt3s. Accessed: 2020-02-03.

[17] From virtual reality to personalized experiences: Alibaba is bringing us the future of retail this singles day. https://www.forbes.com/sites/helenwang/2016/11/06/how-alibaba-will-use-the-worlds-biggest-shopping-day-to-transform-retail/#41d6340c6d4e. Accessed: 2020-02-03.

[18] Future of retail: Artificial intelligence and virtual reality have big roles to play. https://www.forbes.com/sites/rachelarthur/2016/06/15/future-of-retail-artificial-intelligence-and-virtual-reality-have-big-roles-to-play/#b87b8ae7f9df. Accessed: 2020-02-03.

[19] R. Guerraoui, A. Kermarrec, T. Lin, and R. Patra. Heterogeneous recommendations: What you might like to read after watching interstellar. *PVLDB*, 10(10):1070–1081, 2017.

[20] A. Gulati and M. Eirinaki. With a little help from my friends (and their friends): Influence neighborhoods for social recommendations. In *WWW*, pages 2778–2784. ACM, 2019.

[21] Gurobi optimizer. http://www.gurobi.com. Accessed: 2020-02-03.

[22] J. Håstad. Some optimal inapproximability results. *Journal of the ACM*, 48(4):798–859, 2001.

[23] R. He and J. J. McAuley. VBPR: visual Bayesian personalized ranking from implicit feedback. In *AAAI*, pages 144–150. AAAI, 2016.

[24] X. He, L. Liao, H. Zhang, L. Nie, X. Hu, and T. Chua. Neural collaborative filtering. In *WWW*, pages 173–182. ACM, 2017.

[25] L. Homaeian, N. Goyal, J. R. Wallace, and S. D. Scott. Group vs individual: Impact of TOUCH and TILT cross-device interactions on mixed-focus collaboration. In *CHI*, page 73. ACM, 2018.

[26] How will V-Commerce arrive? consumers show the way. https://www.lek.com/insights/ei/how-will-v-commerce-arrive-consumers-show-way. Accessed: 2020-02-03.

[27] L. Hu, J. Cao, G. Xu, L. Cao, Z. Gu, and W. Cao. Deep modeling of group preferences for group-based recommendation. In *AAAI*, pages 1861–1867, 2014.

[28] IBM CPLEX optimizer. https://www.ibm.com/analytics/cplex-optimizer. Accessed: 2020-02-03.

[29] J. Jankowski, R. Michalski, and P. Bródka. A multilayer network dataset of interaction and influence spreading in a virtual world. *Scientific Data*, 4:170144, 2017.

[30] J. M. Kleinberg and É. Tardos. Approximation algorithms for classification problems with pairwise relationships: metric labeling and Markov random fields. *Journal of the ACM*, 49(5):616–639, 2002.

[31] S. Ko, H. Lai, H. Shuai, D. Yang, W. Lee, and P. S. Yu. Optimizing item and subgroup configurations for social-aware VR shopping (full version). *CoRR*, abs/2002.04338, 2020.

[32] H. Li, Y. Ge, R. Hong, and H. Zhu. Point-of-interest recommendations: Learning potential check-ins from friends. In *KDD*, pages 975–984. ACM, 2016.

[33] J. Li, X. Wang, K. Deng, X. Yang, T. Sellis, and J. X. Yu. Most influential community search over large social networks. In *ICDE*, pages 871–882. IEEE, 2017.

[34] K. Li, W. Lu, S. Bhagat, L. V. S. Lakshmanan, and C. Yu. On social event organization. In *KDD*, pages 1206–1215. ACM, 2014.

[35] Y. Liao, W. Lam, L. Bing, and X. Shen. Joint modeling of participant influence and latent topics for recommendation in event-based social networks. *ACM Transactions on Information Systems*, 36(3):29:1–29:31, 2018.
[36] R. Lissermann, J. Huber, M. Schmitz, J. Steimle, and M. Mühlhäuser. Permulin: mixed-focus collaboration on multi-view tabletops. In *CHI*, pages 3191–3200. ACM, 2014.
[37] Lowes Innovation Labs: The next-generation Lowes holoroom. https://youtu.be/DVsEb9vla-I. Accessed: 2020-02-03.
[38] B. Lu, W. Fan, and M. Zhou. Social presence, trust, and social commerce purchase intention: An empirical research. *Computers in Human Behavior*, 56:225–237, 2016.
[39] Z. Lu, Z. Dou, J. Lian, X. Xie, and Q. Yang. Content-based collaborative filtering for news topic recommendation. In *AAAI*, pages 217–223, 2015.
[40] X. Luo. How does shopping with others influence impulsive purchasing? *Journal of Consumer Psychology*, 15(4):288–294, 2005.
[41] Macys ramps up tech push with virtual reality and personalized online shopping. https://fortune.com/2018/03/19/macys-tech-2/. Accessed: 2020-02-03.
[42] A. Nazi, Z. Zhou, S. Thirumuruganathan, N. Zhang, and G. Das. Walk, not wait: Faster sampling over online social networks. *PVLDB*, 8(6):678–689, 2015.
[43] New Walker Sands study: Drones and virtual reality as a solution for the modern retail experience. https://www.walkersands.com/new-walker-sands-study-drones-and-virtual-reality-as-a-solution-for-the-modern-retail-experience/. Accessed: 2020-02-03.
[44] Qbit. https://www.qbittech.com/index.php. Accessed: 2020-02-03.
[45] S. Qi, N. Mamoulis, E. Pitoura, and P. Tsaparas. Recommending packages to groups. In *ICDM*, pages 449–458. IEEE, 2016.
[46] S. S. Rangapuram, T. Bühler, and M. Hein. Towards realistic team formation in social networks based on densest subgraphs. In *WWW*, pages 1077–1088. ACM, 2013.
[47] Rec Room. https://recroom.com/rec-room. Accessed: 2020-02-03.
[48] S. Rendle, C. Freudenthaler, Z. Gantner, and L. Schmidt-Thieme. BPR: Bayesian personalized ranking from implicit feedback. In *UAI*, pages 452–461, 2009.
[49] S. B. Roy, L. V. S. Lakshmanan, and R. Liu. From group recommendations to group formation. In *SIGMOD*, pages 1603–1616. ACM, 2015.
[50] L. Q. Sánchez, J. A. Recio-García, B. Díaz-Agudo, and G. Jiménez-Díaz. Social factors in group recommender systems. *ACM Transactions on Intelligent Systems and Technology*, 4(1):8:1–8:30, 2013.
[51] D. Serbos, S. Qi, N. Mamoulis, E. Pitoura, and P. Tsaparas. Fairness in package-to-group recommendations. In *WWW*, pages 371–379. ACM, 2017.
[52] C. Shen, C. P. K. Fotsing, D. Yang, Y. Chen, and W. Lee. On organizing online soirees with live multi-streaming. In *AAAI*, pages 151–159, 2018.
[53] C. Shen, D. Yang, L. Huang, W. Lee, and M. Chen. Socio-spatial group queries for impromptu activity planning. *IEEE Transactions on Knowledge and Data Engineering*, 28(1):196–210, 2016.
[54] C. Shen, D. Yang, W. Lee, and M. Chen. Spatial-proximity optimization for rapid task group deployment. *ACM TKDD*, 10(4):47:1–47:36, 2016.
[55] Shopify. https://www.shopify.com/. Accessed: 2020-02-03.
[56] H. Shuai, Y. Li, C. Feng, and W. Peng. Four-dimensional shopping mall: Sequential group willingness optimization under VR environments. In *WWW*, pages 131–134. ACM, 2018.
[57] H. Shuai, D. Yang, P. S. Yu, and M. Chen. Willingness optimization for social group activity. *PVLDB*, 7(4):253–264, 2013.
[58] V. Sigurdsson et al. Brand placement and consumer choice: an in-store experiment. *Journal of Applied Behavior Analysis*, 2009.
[59] C. Tai, P. S. Yu, D. Yang, and M. Chen. Privacy-preserving social network publication against friendship attacks. In *KDD*, pages 1262–1270. ACM, 2011.
[60] The world's first virtual reality department store - brought to you by eBay and Myer. https://youtu.be/yAuiXhJPnr8. Accessed: 2020-02-03.
[61] W. M. Trochim and J. P. Donnelly. *Research methods knowledge base*, volume 2. Atomic Dog Publishing Cincinnati, OH, 2001.
[62] VentureBeat: Amazon virtual reality VR mall kiosks - the future of retail. https://youtu.be/J5NviNVdOsc. Accessed: 2020-02-03.
[63] Virtual reality store IKEA. https://youtu.be/5_znFPj5Lis. Accessed: 2020-02-03.
[64] Virtual reality: Tourism firms use VR to attract visitors. https://bbc.in/2MExzZx. Accessed: 2020-02-03.
[65] VRChat. https://vrchat.net/. Accessed: 2020-02-03.
[66] VRChat guides. https://vrchat.com/guides. Accessed: 2020-02-03.
[67] X. Wang, W. Zhu, C. Chen, and M. Ester. Joint user- and event- driven stable social event organization. In *WWW*, pages 1513–1522. ACM, 2018.
[68] D. P. Williamson and D. B. Shmoys. *The Design of Approximation Algorithms*. Cambridge University Press, 2011.
[69] Worldwide augmented and virtual reality spending guide. https://www.idc.com/getdoc.jsp?containerId=prUS45679219. Accessed: 2020-02-03.
[70] Y. Xu, R. Goebel, and G. Lin. Submodular and supermodular multi-labeling, and vertex happiness. *CoRR*, abs/1606.03185, 2016.
[71] D. Yang, C. Shen, W. Lee, and M. Chen. On socio-spatial group query for location-based social networks. In *KDD*, pages 949–957. ACM, 2012.

[72] Yelp dataset. https://www.yelp.com/dataset. Accessed: 2020-02-03.

[73] M. Y.-C. Yim, S.-C. Yoo, P. L. Sauer, and J. H. Seo. Hedonic shopping motivation and co-shopper influence on utilitarian grocery shopping in superstores. *Journal of the Academy of Marketing Science*, 42(5):528–544, 2014.

[74] P. Zhang and A. Li. Algorithmic aspects of homophyly of networks. *Theoretical Computuer Science*, 593:117–131, 2015.

[75] X. Zhang, S. Li, and R. R. Burke. Modeling the effects of dynamic group influence on shopper zone choice, purchase conversion, and spending. *Journal of the Academy of Marketing Science*, 46(6):1089–1107, 2018.

[76] X. Zhang, S. Li, R. R. Burke, and A. Leykin. An examination of social influence on shopper behavior using video tracking data. *Journal of Marketing*, 78(5):24–41, 2014.

[77] L. Zhao, H. Nagamochi, and T. Ibaraki. Greedy splitting algorithms for approximating multiway partition problems. *Mathematical Programming*, 102(1):167–183, 2005.

[78] T. Zhao, J. J. McAuley, and I. King. Leveraging social connections to improve personalized ranking for collaborative filtering. In *CIKM*, pages 261–270. ACM, 2014.

[79] W. X. Zhao, Y. Guo, Y. He, H. Jiang, Y. Wu, and X. Li. We know what you want to buy: a demographic-based system for product recommendation on microblogs. In *KDD*, pages 1935–1944. ACM, 2014.

[80] W. X. Zhao, S. Li, Y. He, E. Y. Chang, J. Wen, and X. Li. Connecting social media to E-Commerce: Cold-start product recommendation using microblogging information. *IEEE TKDE*, 28(5):1147–1159, 2016.