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Teacher educators' understanding of mathematical knowledge for teaching

Matthew Dahlgren[1], Reidar Mosvold[2] and Mark Hoover[3]

[1]University of Michigan, USA; mdahlg@umich.edu

[2]University of Stavanger, Stavanger, Norway; reidar.mosvold@uis.no

[3]University of Michigan, USA; mhoover@umich.edu

Researchers conceptualize mathematical knowledge for teaching in different ways, but a coherent approach to the mathematical education of teachers requires teacher educators' understanding to be robust and shared. At present, we know little about how teacher educators interpret and operationalize this important domain. Our analysis of interview data indicates two sites of divergence in teacher educators' understanding. Some view this knowledge as a resource for the mathematical work of teaching, treating it as distant from actual practice, whereas others view it as a slice of the dynamic and situational work. Also, some view the mathematical work of teaching, and its knowledge demands, as detached from particulars of students and schooling, while others view this work as inseparable from student identities and the larger environments within which instruction occurs, thus integrating regard for equity. Implications are discussed.

Keywords: Mathematical knowledge for teaching, Teaching, Teacher educators.

Introduction

Mathematics educators and researchers agree that the mathematical knowledge teachers need is not simply advanced mathematics — it is specialized, teaching-specific mathematical knowledge. Shulman's (1986) pedagogical content knowledge continues to stimulate the field. Ma's (1999) profound understanding of fundamental mathematics exposed subtle depth in the mathematical demands of teaching. Scholars continue to expand conceptual models (e.g., Ball, Thames, & Phelps, 2008; Carrillo-Yañez et al., 2018; Rowland, Huckstep, & Thwaites, 2005; Thompson, 2015) and measures (e.g., Baumert et al., 2010; Hill, Schilling, & Ball, 2004; Saderholm, Ronau, Brown, & Collins, 2010). Although scholars agree such knowledge is important, different constructs have been proposed, with underdeveloped theoretical grounding, and measures often operationalize a specific construct in different ways (Hoover, Mosvold, Ball, & Lai, 2016).

Awareness that mathematical knowledge for teaching is specialized and teaching-specific creates an imperative for the mathematical education of teachers. Unfortunately, shifting to teaching of mathematical knowledge for teaching is not a simple matter of introducing new content. Understanding the new content is a challenge and teaching it requires different instructional practices with new demands on teacher educators. A number of researchers have begun to investigate these demands. For example, some have used records of practice from teacher education courses or professional development programs to unpack the mathematical demands of such work (e.g., Chick & Beswick, 2018; Superfine & Li, 2014; Zopf, 2010). Others have examined teacher educators' collaborative work, by either drawing from reflections on their practice (Masingila, Olanoff, & Kimani, 2018), or interviewing teacher educators directly (Zazkis & Zazkis, 2011). Each of these efforts has focused on the connection between mathematical knowledge for teaching and the mathematical demands of teacher education.

While this investment in conceptualizing an analogous specialized knowledge for mathematics teacher education is valuable and timely work, a more basic question is whether teacher educators mean the same thing when referring to mathematical knowledge for teaching. Of course, individual interpretations vary, but meaning needs to be sufficiently shared for communication and programs to be effective. Teacher educators may believe they are each working on mathematical knowledge for teaching in their courses but could in fact be focusing on different issues. Alignment among instructional materials, courses, and instructors is crucial. As Cohen (2011) argues, absent a sufficiently shared notion of the content and aims of education (or in this case teacher education), efforts to assess and improve the quality of teaching will be much more difficult, if not impossible. In our own work with professionals concerned with the mathematical education of teachers, we have found that ambiguity often leads to individuals or groups talking past one another, even as they allegedly invest in the same content. In addition, if teacher educators' understandings of mathematical knowledge for teaching differ, then studies that seek to understand the mathematical demands of mathematically educating teachers could be scrutinizing arguably different aspects of professional practice and ignoring ambiguity that might lead to additional or different results. As the larger community of mathematics teacher educators (including mathematicians, teacher educators, and school-based personnel) learns about mathematical knowledge for teaching and becomes convinced of its importance, mixed understanding looms large.

Despite these implications, we know surprisingly little about teacher educators' conceptualizations of mathematical knowledge for teaching. In this study, we analyze interviews with teacher educators to better understand their thinking, and in so doing, we contribute to the growing literature on the teaching of mathematical knowledge for teaching.

Conceptual and contextual background

Before laying out the particulars of our study, we briefly describe the study context and the perspectives we bring to our analysis. Our team relies on a conception of mathematical knowledge for teaching that understands knowledge to be embedded in teaching and considers specialized content knowledge to be mathematical knowledge unique to the work of teaching (Ball, 2017; Ball et al., 2008). We conceptualize teaching as the management of interactions of instruction in environments (Brousseau, 1997; Cohen, Raudenbush, & Ball, 2003; Jaworski, 1994; Wickman, 2012). In this, we understand attention to equity to be inherent to teaching that is educative and consequently to be inherent in the dynamic mathematical work of teaching. We understand equity in the sense of both "reasonableness and moderation in the exercise of one's rights, and the disposition to avoid insisting on them too rigorously" as well as "recourse to general principles of justice (the *naturalis æquitas* of Roman jurists) to correct or supplement the provisions of the law" (Equity, 2018). For this study, we are interested in the many different ways that equity might be considered in teaching and the mathematical dimensions and demands of this work.

As a means of building capacity among teacher educators concerning specialized content knowledge, our team has run a series of workshops that bring together different professional communities with the purpose of collectively creating tasks for teachers that address specialized content knowledge. Workshops are organized around a cycle of constructing, discussing, and reviewing tasks. Whereas some of our previous projects have focused on developing multiple-choice items for assessment or measurement purposes, we have adopted a more inclusive interpretation of task type in the present workshops with the intention of using task development as a tool for building understanding and instructional materials.

In the first year of the project, four workshops were held. Each consisted of roughly 30 to 50 participants. Some participated in more than one workshop, but over 150 professionals participated in at least one. They came from over 25 states of the United States, as well as Brazil, Canada, Norway, Turkey, and Iran. Their professional roles varied. Some were higher education faculty from mathematics departments and schools of education. Some were professional developers, teacher leaders, curriculum specialists, or other school-based personnel. Some were state leaders. All were involved in the mathematical education of teachers. Consistent with discussions above, our framing of specialized content knowledge at the workshop emphasized that the mathematical work of teaching requires coordination of pedagogical and mathematical entailments while simultaneously attending to, and acting against, patterns of marginalization and inequity.

Our aim is to understand how those responsible for the mathematical education of teachers understand mathematical knowledge for teaching and the extent to which they shared a coherent vision of it. The design of our study is to examine differences in understanding among our participants. We acknowledge that our workshop participants constitute a limited sample of professionals engaged in the mathematical education of teachers. They have likely read some of the same research, self-selected to attend our workshop, and listened to our workshop framing of mathematical knowledge for teaching and task development. This suggests that they may hold more similar views than those held in the broader community. We argue that, because we are examining differences among participants, our sample may actually strengthen our claims, revealing patterns that are likely alive and well in the broader professional community.

The study

To investigate teacher educators' understanding of specialized content knowledge, we interviewed 13 teacher educators after the fourth workshop, held in July 2017. These interviewees varied in terms of demographics, number of workshops attended, professional affiliation, perceived fluency with the ideas, etc. Interviews were conducted via video conferencing and recorded. In addition to probing their experiences at the workshops, we asked explicitly about their views of specialized content knowledge, teaching, and equity. To elicit interviewees' understanding of specialized content knowledge, we asked them to comment on a recording of a discussion that occurred at the workshop. The focus of the workshop discussion was a video of teaching where a Black girl named Aniyah is called to the board to show 1/3 on the number line (see Ball, 2017 for analysis). The aim of the workshop discussion was to use the classroom video as a seed for developing specialized content knowledge tasks. The aim of viewing the workshop discussion in the interview was to provide a context for expressing understanding of mathematical knowledge for teaching.

After the interviews were completed, a larger team of seven researchers processed the data, created collective summaries of the interviews, and decided on four cases to highlight in this paper. The four chosen, Paula, Alyssa, Ranesh, and Daniele (names are pseudonyms), provided a spectrum of different roles and of different perspectives visible in the 13 interviews (Table 1). The larger team also piloted preliminary frames and coding schemes and developed an approach using the instructional triangle to categorize components of the mathematical work of teaching evident in participants' responses. The first two authors then created detailed characterizations (individually, then jointly reconciling differences) and identified common patterns. It should be noted that our goal here is not to say something about the effects of the workshop but rather to unpack how participants understand the construct.

Results and discussion

Our preliminary analysis of interview data revealed two divergences in teacher educators' understanding of specialized content knowledge. The first relates to the nature of such knowledge and its connection to the mathematical work of teaching. The second relates to the relationship between such knowledge and regard for equity, in particular regard for the development of students' identities and the larger environments in which instruction occurs.

Interviewee (Role)	*Nature of knowledge*	*Regard for equity*
Paula (Math education faculty)	Static, stable, prerequisite resource	Not integrated
Alyssa (K-12 school-based)	Dynamic, situational slice of work	Integrated
Ranesh (Math faculty)	Dynamic, situational slice of work	Partly integrated
Daniele (Math education faculty)	Partly dynamic	Not integrated

Table 1: Summary of interviewees

Nature of knowledge

The first site of divergence lies in how teacher educators understand the nature of specialized content knowledge. For example, Paula often collapsed real-time decision making with particular students and particular settings with the kind of work that one might do outside the classroom, independent of those students and settings. Although she insisted that teachers need to be able to see what each student knows and understands, at times, her focus was squarely on knowledge that is *required for* teaching. In particular, when Paula spoke about her attempts to assess such knowledge, she referred to it as something teachers "need to know" in order to be an effective teacher, rather than speaking about specialized content knowledge as something present in (and arising from) particular demands in teaching. It seems as though, for Paula, having particular knowledge outside the classroom ensures success inside the classroom. This kind of conceptualization of the mathematical work of teaching is consistent with a didactical or instructional triangle that ignores certain bidirectional interactions between the vertices. Thus, students, environments, and content often did not seem to affect the kinds of mathematical work that Paula envisioned for the teacher.

Several other participants drew clear lines between knowledge and knowledge-in-use but viewed both as essential to teaching well. For example, while Alyssa contended that “teachers need broad knowledge of content” as well as “a really specific knowledge of mathematics” in order to “see what kids are doing, to see how kids are accessing problems”, she distinguished this knowledge from that which is embedded in the mathematical work of teaching. For her, the mathematical work was “what [teachers] are doing with that knowledge, and what they’re doing with the data they are gathering in the moment…like how they are facilitating a classroom environment with that understanding.” She also referred to how “content knowledge empowers [teachers] to make … strategic real-time decisions with the information they have.” Similarly, Ranesh also considered specialized content knowledge to be related to, but distinct from, the mathematical work of teaching. In his interview, he spoke about how the mathematical work of teaching was about “knowing *what* to do, and *how* you would do that in the classroom.” In his view: “SCK is the noun, and the mathematical work as a teacher … that would be the verb.” In their interviews, Ranesh and Alyssa each seemed to pay more attention to the dynamic interactions present in instruction and this focus seemed to help them make a distinction between knowledge and knowledge-in-use. From their perspective, specialized content knowledge is necessary, but not sufficient for successful execution of the mathematical work.

Our analytic framing of teaching as management of interactions among teachers, students, content, and environments suggests that teacher educators’ views of mathematical knowledge for teaching differ in sophistication. In particular, Paula’s view of it as static, stable, prerequisite knowledge overlooks certain interactional pairings central to teaching. In contrast, Alyssa’s and Ranesh’s view of it as a dynamic, situational slice of work reflects their fuller understanding and skill in thinking about the full range of interactions central to teaching.

Mathematical knowledge for teaching and integration of student identities and environments

A second site of divergence is the extent of integration of equity concerns, in particular student identities and environmental factors, into mathematical knowledge for teaching. Alyssa, for example, when shown specific instances of teaching, referenced the larger environments in which instruction occurs and how the mathematical work in those moments is shaped by such considerations. For instance, when commenting on the record of practice referenced in our interview, Alyssa highlighted how the teacher calls upon “a student like Dante” at a particularly crucial moment in a class discussion about naming fractions on the number line. In the video, several other students had each put forward correct pieces of the final answer (one solution drew attention to the need for equal partitioning of the whole, while another correctly identified the whole as the unit interval from zero to one) but each student had a different incorrect answer on the board. Dante also did not have the correct answer in his notebook, but when he is called on, he attempts to articulate his thinking and goes on to ask a question about how the previously presented solutions relate to each other. Alyssa described this moment as an act of empowerment for Dante (a Black student in the classroom video being discussed in the workshop video) and declared that the teaching move allows Dante to be a “bearer of mathematical knowledge”, a position not typically offered to students “like him” (perhaps referencing how non-standard responses of Black boys are often interpreted). She also asserted that calling on Dante at this moment does more than just empower a particular student, it also disrupts systemic patterns of injustice and racism. Alyssa explicitly connected this idea to the mathematical work of teaching, maintaining that, "part of the work of teaching is knowing your students and knowing how to strategically call on students at set times." Even though it is apparent that Dante is "a student who understands quickly and is good at synthesizing other people's thoughts", she believes many teachers would not choose to call on him in that moment, especially since he does not have the correct answer. However, Alyssa contended that the teacher in the video intentionally "*creates* the situation where a student like Dante could then say, 'what is the connection'?" and she saw this mathematical and pedagogical work as intertwined with issues of equity. Through her comments, we see concerns for each of the components of the instructional triangle — mathematics, teachers, students, and environments — as well as their interactions. Her regard for equity in relation to the mathematical work of teaching is coincident with her mutual consideration of the intertwined interactions of teaching.

By contrast, in her interview, Daniele seemed to acknowledge the general possibility that student identity and patterns of systemic inequity can interact in ways that shape the mathematical trajectory of a class, but admitted that she does not think much about equity in her own work. When asked about the video of the workshop discussion, Daniele remained focused on mathematical content and failed to consider comments about how Dante's race/ethnicity shapes the kind of mathematical work the teacher must do in that moment.

Daniele never mentioned Dante's identity as a Black boy as something that might relate to the mathematical work of teaching. Instead, Daniele suggested that calling on Dante was a way of including other students in the class discussion and gaining access to their thinking. She explained the teacher's choice to call on Dante in the following way:

> We think about the students that are maybe up in front talking, but that's not the whole class. That's only two students in the class and it's a large class. And so, what are the other students doing and thinking about? And we think about the behavior issue, and that stands in the forefront, but if we really actually listen to kids that have behavior issues potentially … then we can think about well what is Dante saying and how is *he* thinking about this task?

Viewed in this light, the teacher might have called on any particular student at that moment rather than Dante specifically. For Daniele, the only things about Dante that seem to be relevant for the work of teaching are his particular classroom behaviors and the mathematical concerns he raises. Daniele's comments are fundamentally about the content of Dante's questions rather than Dante as an individual or about the environments that shape both his identity and the work of teaching. Daniele seemed to be articulating that issues of equity are not embedded in the work of teaching, but are an optional add-on — something that is not central. This conceptualization of the mathematical work of teaching, is consistent with an instructional triangle would have little or no interaction with particulars of the environment.

Again, our analytic framing of teaching as management of interactions among teachers, students, content, and environments suggests that teacher educators' integration of concerns for student identities and environmental factors with mathematical knowledge for teaching differ in sophistication. In particular, Daniele's disregard for student identities and environmental factors in her consideration of mathematical issues contrasts with Alyssa's nuanced consideration of their import for the mathematics at hand.

Conclusions

As the instructional triangle emphasizes dynamic interactions among teachers, students, content, and environments, it seems natural that such a framing would draw out the absence or presence of dynamic interactions. But it is clear that certain interactions are more uniformly understood than others (at least within our data). None of the teacher educators we interviewed could be characterized as not attending to the dynamic interaction between teacher and mathematical content. This is significant given observed divergences. The two primary divergences we found thus say something important about the ways in which teacher educators understand mathematical knowledge for teaching. They suggest sites where a shared conceptualization breaks down or areas where there is considerable variation in understanding. These may be fundamentally different conceptualizations, built on different foundations, or they may be developmental issues related to their understanding of teaching. Regardless, we suggest that appreciating differences in these conceptualizations and being able to name and talk about constituents of teacher educators' understandings are useful building blocks for the field.

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