Heralded Interaction Control between Quantum Systems

Yiheng Duan,[1] Mahdi Hosseini,[2] Kristin M. Beck,[3] and Vladan Vuletić[1,*]

[1]*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

[2]*Birck Nanotechnology Center, School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

[3]*IonQ, College Park, Maryland 20740, USA*

(Received 27 September 2019; accepted 7 May 2020; published 4 June 2020)

Quantum mechanical expectation values for subsets can differ substantially from those for the whole ensemble. This implies that the effect of interactions between two systems can be altered substantially by conditioning. Here, we experimentally demonstrate that, for two light fields ψ_S (signal) and ψ_A (ancilla) that have only weakly interacted with one another, subsequent measurements on the ancilla can produce substantial conditional amplification, attenuation, or phase shift of ψ_S. We observe conditional signal power changes over a large range of 30, and phase shift up to $\pi/2$, induced by measurements in ancilla bases that differ only slightly from one another. The method is generically applicable to a variety of systems, and allows one to modify or boost a given interaction by trading in success probability for interaction strength.

DOI: 10.1103/PhysRevLett.124.223602

In quantum mechanics, rare measurement outcomes can have surprising consequences [1]. Here, we consider two quantum systems, S (signal) and A (ancilla), that are made to interact weakly, as characterized by an interaction-induced moderate average change $\langle \delta s \rangle$ of some quantity s associated with the signal system S (see Fig. 1). Now assume that there is some binary measurement basis for the ancilla system A, such that outcome a_1 with probability $p_1 \ll 1$ is observed rarely compared to outcome a_0 with probability $p_0 \approx 1$. If we also assume that a_0 is associated with no change in the system parameter s ($\delta s|_{a_0} = 0$), then in those rare occasions when outcome a_1 is observed for the ancilla system, there must be an associated very large signal change $\delta s|_{a_1} \propto 1/p_1$ to reproduce the average change $\langle \delta s \rangle = p_1 \delta s|_{a_1}$ when the ancilla system is not measured. A different measurement basis of the ancilla system can then give rise to different conditional changes in s, or induce large changes in an altogether different system parameter s'. Thus, one can think of the measurement basis of A and its corresponding measurement result as conditionally controlling the type and strength of the interaction outcome between S and A, respectively. Thus, at the expense of success probability, one can modify the quantum state of the signal system S and its observables far beyond the changes induced by the average (unconditional) interaction, and one can choose which observables are conditionally controlled.

Such heralded interaction control (HIC) can be viewed as an extension and generalization of weak-measurement [2–6] and noiseless-amplification schemes [7–12], and can be used for a variety of purposes in quantum engineering. Noiseless amplification of coherent optical states [7–12] can be viewed as HIC. By coupling light fields to other systems, HIC allows one to magnify and measure tiny physical quantities in the presence of technical noise (weak-measurement schemes) [2–6]. When applied to large systems such as the collective spin of an atomic ensemble, even a single photon can be used to control the atomic spin, and conditionally prepare it in a desired collective entangled spin state [13–15].

In this Letter, we report how a weak optical nonlinearity can be conditionally boosted to affect large amplitude or phase changes of a (weak) signal light field. We first

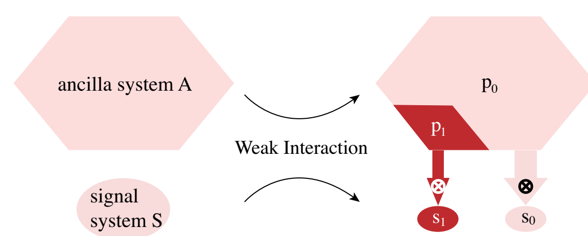

FIG. 1. Boosting a weak interaction conditionally. A weak interaction between a signal system S and an ancilla system A results in a small average shift $\langle \delta s \rangle$ of some signal quantity s. Assume that for binary measurement outcomes a_0, a_1 of A, if a_0 is observed with probability $p_0 \approx 1$, the system parameter s will maintain its value before the interaction. If, on the other hand, a_1 is measured (probability $p_1 \ll 1$), the associated signal $s = s_1$ can be much different from s_0, and the conditionally prepared state of S can differ substantially from the input state: at the expense of success probability, a strong heralded interaction is realized.

weakly entangle two optical modes (estimated upper bound on concurrence of 0.11 [16]) in a cavity quantum electrodynamics setup, and use HIC to coherently amplify, attenuate, and change the phase of the signal mode within a large parameter space. We succeed in modifying the average photon number $\langle n_s \rangle$ in the signal mode over a range of 30 ($\langle n_s \rangle$ changed by a factor between 0.1 to 3.2), and the phase between 0 and $\pi/2$ (in our previous work with a different scheme, the highest phase shift achieved was $\pi/3$ [17]). These conditional changes of the signal mode are accomplished under conditions of weak interaction with the ancilla mode, where the average unconditional photon number and phase change are as small as $\langle \delta n_s \rangle = -1.3\%$ and $\langle \delta \phi_s \rangle = \pi/80$, respectively. We further show that a small change in the (polarization) measurement basis of the ancilla mode by a few degrees can produce a large change in the signal state. Our scheme goes beyond the previous conditional phase shift experiments [17–19] since it realizes arbitrary phase and amplitude editing in a large range, and is no longer limited to a phase amplifier.

The experiment is performed with an ensemble of cold atoms in a cavity in the strong-coupling regime [20–23]. Previously, using a similar setup, we have shown that a measurement of the ancilla mode can project the input coherent state of the signal mode into a single-photon Fock state [24], and demonstrated that the phase of the signal light could be changed by about $\pi/3$ by a single ancilla photon transmitted through the cavity detuned from the atomic resonance [17]. In the current realization, we observe an anomalous and large conditional phase shift of the signal state in a near-resonant regime where the average phase shift is almost zero. The amplitude of the signal state can also be substantially changed by small changes to the conditioning polarization of the ancilla mode of a few degrees.

In each iteration of this experiment, we use an ensemble of laser-cooled ^{133}Cs atoms to create a two-mode weakly entangled state [Figs. 2(a) and 2(b)]. The atoms are held inside a high-finesse ($\mathcal{F} = 7.7 \times 10^4$) cavity by a far-off-resonant dipole trap [$\Delta/(2\pi) = 32$ THz], and prepared in the electronic ground state, $|g\rangle = |S_{1/2}, F = 3, m_F = 3\rangle$. ($F$ and m_F are the hyperfine and magnetic quantum numbers, respectively.) A weak optical coherent state with typical mean photon number $\langle n_s \rangle = 0.2$ (the signal light), resonant with the $|g\rangle \to |c\rangle = |P_{3/2}, 3, 3\rangle$ transition, is stored in the atoms through electromagnetically induced transparency by adiabatically reducing the power of a near-copropagating coupling laser which is resonant with the $|d\rangle = |S_{1/2}, 4, 4\rangle \to |c\rangle$ transition. The signal-mode input coherent state $|\alpha\rangle_S$ is thus mapped onto a collective atomic excitation in the $|d\rangle$ state [25]. The cavity is then probed with linearly polarized light (ancilla light) simultaneously resonant with the cavity and the $|d\rangle \to |e\rangle = |P_{3/2}, 5, 5\rangle$ cycling atomic transition. (The σ^--polarized component of the ancilla light interacts only weakly with the atoms on the $|d\rangle \to |f\rangle = |P_{3/2}, 5, 3\rangle$ transition.) Therefore the signal light stored in $|d\rangle$ blocks the transmission of σ^+ ancilla photons through the cavity due to the vacuum Rabi splitting [26], while σ^- light is transmitted. The joint state of the light transmitted through the cavity and the retrieved signal light is a two-mode (weakly) entangled state,

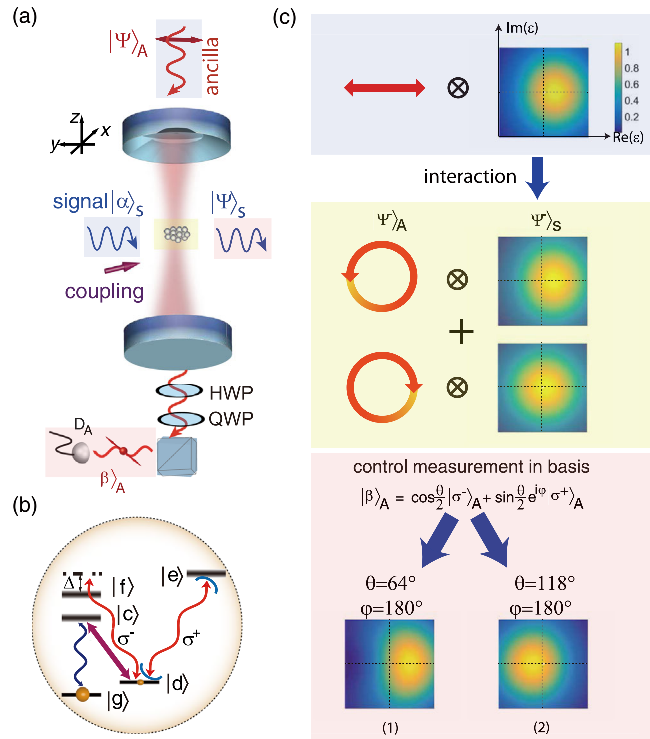

FIG. 2. (a) Schematic of the experimental setup and basic idea for HIC. An ensemble of cesium atoms is held in a high-finesse cavity. A weak signal coherent state is stored in the ensemble via electromagnetically induced transparency. For the cavity resonant with the atomic transition $|d\rangle \to |e\rangle$, weak ancilla light is sent through the cavity. The atomic excitation associated with a signal photon blocks the circularly polarized σ^+ component of the cavity light, but has little effect on the σ^- component. The transmission of the cavity light is measured in some chosen polarization basis $|\beta\rangle_A$, decided by the angle of the half- (HWP) and quarter- (QWP) wave plates and the polarizing beam splitter preceding the detector D_A. The signal state $|\Psi\rangle_S$ is then retrieved from the atom ensemble and measured. (b) Level diagram of the system. Atoms are prepared in state $|g\rangle = |S_{1/2}, F = 3, m_F = 3\rangle$. The signal light is stored as collective excitation on $|d\rangle = |S_{1/2}, 4, 4\rangle$ via resonant coupling to excited state $|c\rangle = |P_{3/2}, 3, 3\rangle$. The cavity is resonant with the $|d\rangle$ to $|e\rangle = |P_{3/2}, 5, 5\rangle$ transition. (c) Changes in the ancilla mode polarization $|\beta\rangle_A$ have a large effect on the signal mode, as illustrated in (1) for $\theta = 64°$, $\varphi = 180°$ and (2) for $\theta = 118°$, $\varphi = 180°$. The plot shows the Q representation for the signal state from numerical calculation with mean input photon number $\langle n_s \rangle = |\epsilon|^2 = 0.1$.

$$|\Psi\rangle = |\sigma^-\rangle_A(|0\rangle_S + \alpha|1\rangle_S) + |\sigma^+\rangle_A(|0\rangle_S + t\alpha|1\rangle_S), \qquad (1)$$

where the input weak coherent signal state is approximated as $|\alpha\rangle_S \approx |0\rangle_S + \alpha|1\rangle_S$ in terms of photon Fock states, and t is the transmission amplitude for σ^+-polarized light in the presence of a stored signal photon. We project the output cavity light onto a chosen polarization $|\beta\rangle_A = \cos(\theta/2)|\sigma^-\rangle_A + \sin(\theta/2)e^{i\varphi}|\sigma^+\rangle_A$, which we experimentally adjust by tuning the angles of half- (HWP) and quarter- (QWP) wave plates before the polarizing beam splitter in our detection path [Fig. 2(a)]. When this projection of the ancilla photon into state $|\beta\rangle_A$ succeeds, we measure a photon click on the detector D_A. Simultaneously, we measure the amplitude or phase of the signal mode [see Supplemental Material (SM) [27]]. When we operate on cavity and atomic resonance, t is given by $t = 1/(1+\eta)$, where $\eta = 8.6$ is the single-atom cooperativity [24].

Upon projection of the two-mode entangled state $|\Psi\rangle$ onto the polarization state ${}_A\langle\beta|$, the unnormalized final state ${}_A\langle\beta|\Psi\rangle$ is given by [28]

$$\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}e^{i\varphi}\right)|0\rangle_S + \alpha\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}e^{i\varphi}t\right)|1\rangle_S$$
$$\propto |0\rangle_S + \alpha\frac{\cos(\theta/2) + \sin(\theta/2)e^{i\varphi}t}{\cos(\theta/2) + \sin(\theta/2)e^{i\varphi}}|1\rangle_S = |0\rangle_S + \alpha'|1\rangle_S. \tag{2}$$

We see that depending on the ancilla detection basis, as determined by the angles θ and φ on the Poincaré sphere describing ancilla light polarization, a weak coherent state $|\alpha\rangle_S$ is transformed into $|\alpha'\rangle_S$ with $\alpha' = \alpha(\cos(\theta/2) + \sin(\theta/2)e^{i\varphi}t/\cos(\theta/2) + \sin(\theta/2)e^{i\varphi})$ in the limit of $|\alpha|^2 \ll 1$ and $|\alpha'|^2 \ll 1$. In this weak-coherent-state limit, the power gain of the projected coherent state is then $G = |\alpha'/\alpha|^2$. Recalling that transmission amplitude $t \in (0,1)$ (as determined by the interaction strength, i.e., the single atom cooperativity), and θ and φ are angles chosen by the measurement basis, we see that the amplitude and phase of the projected coherent state can take on any value. If we project the ancilla photon's polarization onto $|\sigma^+\rangle_A$ ($\theta = 0$), the signal coherent state is unchanged. If instead we project the ancilla mode onto vertical polarization ($\theta = \pi/2, \varphi = \pi$), the signal state is maximally amplified, with the amplification attainable in the experiment set by a combination of signal-to-noise ratio and the higher-photon-number components that we have ignored in Eq. (2). In addition to modifying the amplitude, the choice of θ modifies the phase of the coherent state, changing it by up to π. In particular, when $\varphi = \pi$ and $t < \tan(\theta/2) < 1$, the phase of the projected signal state is changed by π. In the absence of technical noise sources, this method can prepare a photonic state with strongly modified amplitude and arbitrary phase. Even if there is no phase shift onto the ancilla-signal system after the resonant interaction, by detecting the ancilla photon in a basis with a relative phase shift between its two polarization components, a nonzero phase can be mapped onto the signal coherent state.

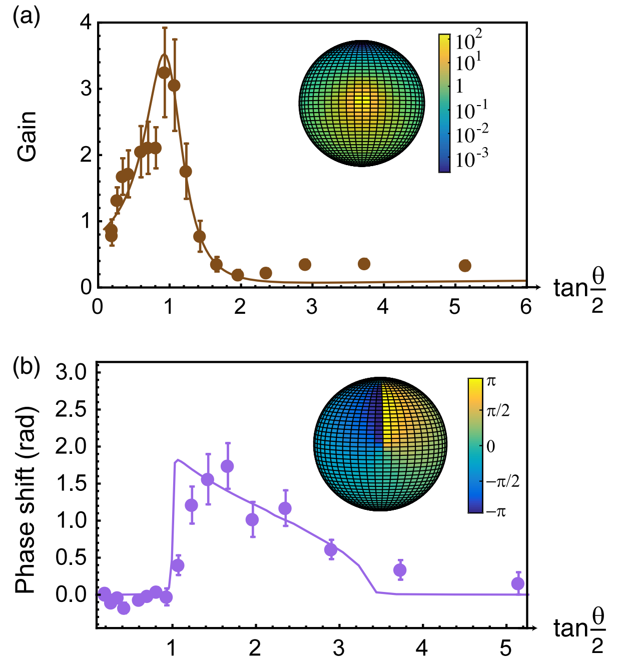

FIG. 3. Measured power gain (a) and reconstructed phase (b) of the final signal state as a function of conditioning angle θ of the ancilla mode for $\varphi = \pi$. The solid lines are predictions that include the effects of loss and nonuniform atomic coupling. Error bars in this and subsequent figures are ± 1 s.d. The insets show the predicted gain (a) and phase (b) as a function of the Poincaré sphere coordinates θ and φ, respectively, of the conditioning ancilla polarization for an ideal system.

The measured projected phase of the signal state is shown as a function of the conditioning angle θ in Fig. 3(a). In our experiment, the maximum observed phase is limited to $\pi/2$ due to inhomogeneous coupling of atoms to the cavity light as well as dark counts of the detector. In the low photon limit, the gain of the projected signal state approximates the cross-correlation function $g^{(2)}$ (see SM [27]), between the signal path and the cavity projection port shown in Fig. 3(b). Its maximal value is limited by background counts, which in turn limits the maximum gain in our system to $G = 3.2$. To account for the inhomogeneous coupling of atoms to the cavity light we model the spatial distribution of the atoms (see SM [27]). This model takes into account the fact that our atomic cloud extends beyond the cavity mode's Gaussian waist and that atoms are randomly distributed between the nodes and antinodes of the cavity standing wave. These imperfections reduce the purity of the initial weakly entangled state $|\Psi\rangle$, and limit both the phase and gain observed in the experiment. Moreover, background counts tend to decrease both the reconstructed phase and measured magnitude of the state [solid lines in Fig. 3(a) and 3(b)]. When these experimental imperfections are included in the theoretical

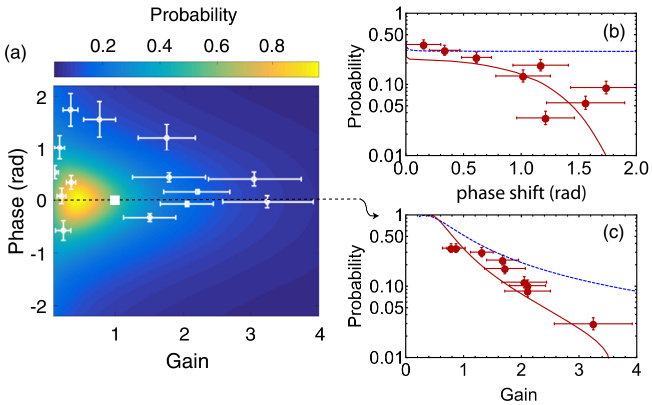

FIG. 4. Probability of a given interaction and signal modification. (a) The maximum theoretical probability for signal gain G and phase shift ϕ_s. The input state is indicated as a white square. A selection of the output states we produced are overlaid (circles); in particular, we produce states with both positive and negative phase shifts ϕ_s, and both gain $G > 1$ and attenuation $G < 1$. This probability of state creation in our experiment is shown for (b) constant amplification (where $\theta > \pi/2, \varphi = \pi$) as a function of the conditional phase shift ϕ_s, and (c) constant phase shift (where $\theta < \pi/2, \varphi = \pi$) as a function of the gain G of the signal state obtained from the measured correlation function, $g^{(2)}$ (see text). The solid lines in (b) and (c) represent a theoretical model taking into account the experimental imperfections. The blue dashed lines are predictions for an ideal system (i.e., one without background counts or inhomogeneous coupling) for an input state with $\langle n_s \rangle = 0.2$. In (b) and (c), the vertical statistical error bars are less than the symbol size. Systematic error bars from the possible imbalance between the fiber-coupling efficiencies of the two single-photon counting modules in the measurement (see SM [27]) are plotted.

description, the model agrees well with the experimental data.

Although we can prepare photonic states with different amplitudes and phases, we do not prepare every such state with equal probability, and states corresponding to a large change of the signal mode, or a large associated ancilla-probe interaction, are prepared more rarely. If we normalize the states ${}_A\langle \beta|$ and $|\Psi\rangle$, the preparation probability is simply the magnitude $|{}_A\langle \beta|\Psi\rangle|^2$, or the probability of observing a conditioning event before path and detector efficiency losses.

The ideal probability of projecting into a signal state with coherent amplitude α' is plotted in Fig. 4(a) as a function of phase and relative amplitude of the final coherent state $|\alpha'\rangle_S$. Overall, the state preparation probability decreases as the projected state is displaced further from the original state (shown with a square symbol). We note that due to the blocking nature of the interaction, causing a reduction in signal transmission when averaging over the ancilla mode, the gain that maximizes the success probability is smaller than unity. Several experimentally projected states are shown in this figure to illustrate that we are able to produce states with different amplitudes and phases. Figures 4(b) and 4(c) represent experimental success probability along with the theoretical prediction for a noiseless system (dashed line), and our system with experimental imperfections (solid line), as a function of phase and gain of the final signal state, respectively. A gain $G = 3.2$ is achieved with a success probability of 3%. Because of limited quantum efficiency of 0.3 of the detector, for an input signal state with a mean photon number $\langle n_s \rangle = 0.2$, the detected amplified state is still within the weak coherent state limit. Thus, the main deviation of the data from the theory is from higher-order excitations of the atom ensemble caused by the signal light. Provided weak enough input signal state ($G\langle n_s \rangle < 1$), by confining atoms in the antinodes of the cavity standing wave and minimizing the background counts, it should be possible to achieve a conditional phase shift of π and gain of 40 with success probabilities of 25% and 1%, respectively.

In summary, we have demonstrated HIC for modes of light: the coherent transformation of photonic states by measurements on an ancilla mode that had previously been weakly entangled with the signal mode. The demonstrated scheme provides a powerful tool to engineer quantum states of light by, in principle, arbitrary manipulation of their phase and amplitude. Such coherent transformation of optical states has potentially important applications in quantum communication, computations, and sensing. For example, the scheme can be used for remote state preparation [29,30] in quantum communication, which relies on entanglement preparation of a distant qubit conditional on the measurement outcome of another qubit without the need for Bell state measurement. The coherent amplification of optical coherent states observed here may be used to develop an optimum nondeterministic noiseless amplification [31,32] for applications in quantum key distribution [33], state discrimination [34], and entanglement distillation [35–37]. The anomalous phase shift observed on atomic resonance can also be explained in terms of weak-value measurements [38,39] that further our understanding of fundamental concepts in quantum mechanics [40,41], and have found applications in metrology [42]. In the latter realm, weak-value methods have been demonstrated as a means of overcoming technical noise [43–47]. The scheme can be also generalized to systems of massive particles and spin systems [13,15,48,49]. Finally, this experiment illustrates a general paradigm that enables the heralded transformation of a quantum state that could otherwise only be accomplished by strong interactions.

K. M. B. thanks Julian Martinez for insightful discussions. This work was supported by the NSF, the NSF CUA, and a Multidisciplinary University Research Initiative (MURI) grant through AFOSR.

*vuletic@mit.edu

[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).
[2] G. J. Pryde, J. L. O'Brien, A. G. White, T. C. Ralph, and H. M. Wiseman, Phys. Rev. Lett. **94**, 220405 (2005).
[3] O. Hosten and P. Kwiat, Science **319**, 787 (2008).
[4] X.-Y. Xu, Y. Kedem, K. Sun, L. Vaidman, C.-F. Li, and G.-C. Guo, Phys. Rev. Lett. **111**, 033604 (2013).
[5] M. Merano, G. Jayaswal, and G. Mistura, in *Frontiers in Optics* (Optical Society of America, 2014), pp. FW1C–6.
[6] O. S. Magaña-Loaiza, M. Mirhosseini, B. Rodenburg, and R. W. Boyd, Phys. Rev. Lett. **112**, 200401 (2014).
[7] F. Ferreyrol, M. Barbieri, R. Blandino, S. Fossier, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. **104**, 123603 (2010).
[8] M. A. Usuga, C. R. Müller, C. Wittmann, P. Marek, R. Filip, C. Marquardt, G. Leuchs, and U. L. Andersen, Nat. Phys. **6**, 767 (2010).
[9] A. Zavatta, J. Fiurášek, and M. Bellini, Nat. Photonics **5**, 52 (2011).
[10] C. I. Osorio, N. Bruno, N. Sangouard, H. Zbinden, N. Gisin, and R. T. Thew, Phys. Rev. A **86**, 023815 (2012).
[11] S. Kocsis, G.-Y. Xiang, T. C. Ralph, and G. J. Pryde, Nat. Phys. **9**, 23 (2013).
[12] J. Ho, A. Boston, M. Palsson, and G. Pryde, New J. Phys. **18**, 093026 (2016).
[13] R. McConnell, H. Zhang, J. Hu, S. Ćuk, and V. Vuletić, Nature (London) **519**, 439 (2015).
[14] W. Chen, J. Hu, Y. Duan, B. Braverman, H. Zhang, and V. Vuletić, Phys. Rev. Lett. **115**, 250502 (2015).
[15] F. Fröwis, P. C. Strassmann, A. Tiranov, C. Gut, J. Lavoie, N. Brunner, F. Bussières, M. Afzelius, and N. Gisin, Nat. Commun. **8**, 907 (2017).
[16] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A **64**, 052312 (2001).
[17] K. M. Beck, M. Hosseini, Y. Duan, and V. Vuletić, Proc. Natl. Acad. Sci. U.S.A. **113**, 9740 (2016).
[18] A. Feizpour, X. Xing, and A. M. Steinberg, Phys. Rev. Lett. **107**, 133603 (2011).
[19] M. Hallaji, A. Feizpour, G. Dmochowski, J. Sinclair, and A. M. Steinberg, Nat. Phys. **13**, 540 (2017).
[20] Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, Nature (London) **450**, 272 (2007).
[21] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Nature (London) **436**, 87 (2005).
[22] H. P. Specht, C. Nölleke, A. Reiserer, M. Uphoff, E. Figueroa, S. Ritter, and G. Rempe, Nature (London) **473**, 190 (2011).
[23] A. Stute, B. Casabone, P. Schindler, T. Monz, P. Schmidt, B. Brandstätter, T. Northup, and R. Blatt, Nature (London) **485**, 482 (2012).
[24] M. Hosseini, K. M. Beck, Y. Duan, W. Chen, and V. Vuletić, Phys. Rev. Lett. **116**, 033602 (2016).
[25] S. E. Harris, Phys. Today **50**, No. 7, 36 (1997).
[26] R. J. Thompson, G. Rempe, and H. J. Kimble, Phys. Rev. Lett. **68**, 1132 (1992).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.124.223602 for mathematical and experimental details.
[28] J. Martínez-Rincón, Quantum Inf. Process. **16**, 45 (2017).
[29] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal, and W. K. Wootters, Phys. Rev. Lett. **87**, 077902 (2001).
[30] N. A. Peters, J. T. Barreiro, M. E. Goggin, T.-C. Wei, and P. G. Kwiat, Phys. Rev. Lett. **94**, 150502 (2005).
[31] S. Pandey, Z. Jiang, J. Combes, and C. M. Caves, Phys. Rev. A **88**, 033852 (2013).
[32] N. A. McMahon, A. P. Lund, and T. C. Ralph, Phys. Rev. A **89**, 023846 (2014).
[33] V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, M. Dušek, N. Lütkenhaus, and M. Peev, Rev. Mod. Phys. **81**, 1301 (2009).
[34] A. Zavatta, J. Fiurášek, and M. Bellini, Nat. Photonics **5**, 52 (2011).
[35] P. G. Kwiat, S. Barraza-Lopez, A. Stefanov, and N. Gisin, Nature (London) **409**, 1014 (2001).
[36] H. Takahashi, J. S. Neergaard-Nielsen, M. Takeuchi, M. Takeoka, K. Hayasaka, A. Furusawa, and M. Sasaki, Nat. Photonics **4**, 178 (2010).
[37] D. V. Sychev, A. E. Ulanov, A. A. Pushkina, M. W. Richards, I. A. Fedorov, and A. I. Lvovsky, Nat. Photonics **11**, 379 (2017).
[38] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).
[39] G. J. Pryde, J. L. O'Brien, A. G. White, T. C. Ralph, and H. M. Wiseman, Phys. Rev. Lett. **94**, 220405 (2005).
[40] G. T. Foster, L. A. Orozco, H. M. Castro-Beltran, and H. J. Carmichael, Phys. Rev. Lett. **85**, 3149 (2000).
[41] J. S. Lundeen and A. M. Steinberg, Phys. Rev. Lett. **102**, 020404 (2009).
[42] P. B. Dixon, D. J. Starling, A. N. Jordan, and J. C. Howell, Phys. Rev. Lett. **102**, 173601 (2009).
[43] C. Ferrie and J. Combes, Phys. Rev. Lett. **112**, 040406 (2014).
[44] A. N. Jordan, J. Martínez-Rincón, and J. C. Howell, Phys. Rev. X **4**, 011031 (2014).
[45] J. Harris, R. W. Boyd, and J. S. Lundeen, Phys. Rev. Lett. **118**, 070802 (2017).
[46] J. Sinclair, M. Hallaji, A. M. Steinberg, J. Tollaksen, and A. N. Jordan, Phys. Rev. A **96**, 052128 (2017).
[47] J. P. Torres and L. J. Salazar-Serrano, Sci. Rep. **6**, 19702 (2016).
[48] F. Haas, J. Volz, R. Gehr, J. Reichel, and J. Estève, Science **344**, 180 (2014).
[49] P. Zarkeshian, C. Deshmukh, N. Sinclair, S. Goyal, G. Aguilar, P. Lefebvre, M. G. Puigibert, V. Verma, F. Marsili, M. Shaw *et al.*, Nat. Commun. **8**, 906 (2017).