Published for SISSA by Springer

Received: December 3, 2019
Accepted: January 7, 2020
Published: January 20, 2020

Anomaly inflow, accidental symmetry, and spontaneous symmetry breaking

Ibrahima Bah and Federico Bonetti

Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, MD 21218, U.S.A.

E-mail: iboubah@jhu.edu, fbonett3@jhu.edu

Abstract: We consider the 6d (1,0) SCFT on a stack of N M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. In particular, we study its compactifications to four dimensions on a smooth genus-g Riemann surface with non-trivial flavor flux, yielding a family of 4d CFTs. By tracking the M-theory origin of the global symmetries of the 4d CFTs, we detect the emergence of an accidental symmetry and the spontaneous symmetry breaking of a U(1) generator. These effects are visible from geometric considerations and not apparent from the point of view of the compactification of the 6d field theory. These phenomena leave an imprint on the 't Hooft anomaly polynomial of the 4d CFTs, which is obtained from recently developed anomaly inflow methods in M-theory [1]. In the large-N limit, we identify the gravity dual of the 4d setups to be a class of smooth AdS_5 solutions first discussed by Gauntlett-Martelli-Sparks-Waldram. Using our anomaly polynomial, we compute the conformal central charge and a non-Abelian flavor central charge at large N, finding agreement with the holographic predictions.

Keywords: AdS-CFT Correspondence, Anomalies in Field and String Theories, M-Theory

ArXiv ePrint: 1910.07549

https://doi.org/10.1007/JHEP01(2020)117

JHEP01(2020)117

Contents

JHEP01(2020)117

1 Introduction and summary

Quantum field theory (QFT) provides a powerful framework to describe a variety of physical phenomena, ranging from particle physics, to condensed matter systems and cosmology. Symmetries and spontaneous symmetry breaking play a fundamental role in countless examples of applications of the QFT formalism. It is particularly interesting to investigate the symmetries and dynamics of QFTs in strongly coupled non-perturbative regimes. Geometric engineering is a remarkable tool in the construction and analysis of strongly coupled QFTs in various dimensions. Several non-trivial QFTs can be studied by examining the low-energy limit of brane configurations in string theory and M-theory. A prominent example is furnished by 6d (2,0) theories of type A_{N-1}, which emerge in the long-wavelength dynamics of a stack of N M5-branes extending along a flat worldvolume [2, 3]. By a similar token, an interesting class of 6d (1,0) theories is obtained by considering a stack of M5-branes probing an orbifold singularity [4–9]. A rich variety of 4d QFTs can be constructed by considering M-theory setups in which a stack of M5-branes is wrapped on a Riemann surface. These 4d QFTs are generically strongly coupled and fit into the larger Class $\mathcal{S}$ program, in which 6d superconformal field theories (SCFTs) are compactified to four dimensions on a Riemann surface, possibly with defects. The reduction of 6d (2,0) theories

to 4d $\mathcal{N}=2$ QFTs was first analyzed in [10, 11], and reduction to 4d $\mathcal{N}=1$ QFTs has been studied in [12–16]. The compactification of 6d (1,0) theories has been addressed in [17–32].

't Hooft anomalies are among the most important observables to compute in a geometrically engineered QFT, especially if a Lagrangian description of the theory is not available. It is worth emphasizing that anomalies are naturally geometric quantities. For the case of continuous 0-form symmetries — which is the case relevant for this work — the anomalies of a d-dimensional QFT (with d even) are encoded in the anomaly polynomial, which is a $(d+2)$-form constructed with the curvatures of the background fields associated to the symmetries [33–35]. The geometric nature of 't Hooft anomalies makes them particularly amenable to computation in the framework of geometric engineering. Building on seminal papers on anomaly inflow for M5-branes [36–39], a systematic toolkit for the computation of anomalies of QFTs from M5-branes has been developed in [1, 40–42].

Our strategy is to study the boundary conditions for the fields of 11d supergravity in the vicinity of the branes, instead of performing a direct field theory analysis of the worldvolume theory. The M-theory boundary conditions can be used as a proxy to track interesting QFT features, such as anomalies, accidental symmetries, and spontaneous symmetry breaking. In this paper we illustrate this point by considering a class of 4d $\mathcal{N}=1$ theories, obtained from compactification on a Riemann surface of the worldvolume theory of a stack of M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ singularity.

In order to describe in more detail the class of 4d theories we study in this work, let us recall some salient features of the 6d (1,0) SCFT on a stack of N M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Before modding out by $\mathbb{Z}_2$, the stack is surrounded in its five transvervse directions by a 4-sphere S^4. After quotienting by $\mathbb{Z}_2$, S^4 is replaced by $S^4/\mathbb{Z}_2$. The $\mathbb{Z}_2$ action has two fixed points, located at the north and south poles of S^4, which yield two orbifold singularities on $S^4/\mathbb{Z}_2$. The theory has global symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_\mathrm{N} \times \mathrm{SU}(2)_\mathrm{S}$. The factors $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ originate from isometries of $S^4/\mathbb{Z}_2$, while $\mathrm{SU}(2)_\mathrm{N} \times \mathrm{SU}(2)_\mathrm{S}$ originate from the two orbifold points (labeled N, S for "north", "south"). The factor $\mathrm{SU}(2)_R$ is the 6d R-symmetry, while the other factors are flavor symmetries.

The orbifold singularities at the north and south poles can be resolved by blowups, while preserving 8 supercharges. The orbifold $S^4/\mathbb{Z}_2$ is replaced by a smooth internal space M_4. After resolution, only the Cartan subgroup $\mathrm{U}(1)_\mathrm{N} \times \mathrm{U}(1)_\mathrm{S}$ of the flavor symmetry $\mathrm{SU}(2)_\mathrm{N} \times \mathrm{SU}(2)_\mathrm{S}$ is preserved. The 4d theories of interest in this work are obtained from the low-energy dynamics of these brane configurations wrapped on a smooth genus-g Riemann surface Σ_g (with $g \neq 1$). The compactification includes a twist for the $\mathrm{U}(1)_\mathrm{N} \times \mathrm{U}(1)_\mathrm{S}$ flavor symmetry, while $\mathrm{SU}(2)_L$ is untwisted. The resulting 4d theories are then labelled by the genus g and three flux quanta N, N_N, N_S, with N the number of M5-branes in the stack, and $N_{\mathrm{N,S}}$ the twist parameter for the flavor symmetry $\mathrm{U}(1)_{\mathrm{N,S}}$. The internal space M_4 of the resolved 6d theory is non-trivially fibered over the Riemann surface, yielding a 6d space M_6, with $M_4 \hookrightarrow M_6 \to \Sigma_g$. Our geometric construction is summarized schematically in figure 1.

We also expect that these 4d theories emerge in the IR of an RG flow across dimensions resulting from wrapping the parent 6d (1,0) SCFT on Σ_g with the appropriate R-symmetry and flavor twists. From this point of view, anomaly matching across dimensions implies that we can integrate the anomaly polynomial of the parent 6d theory on Σ_g to obtain 't

JHEP01(2020)117

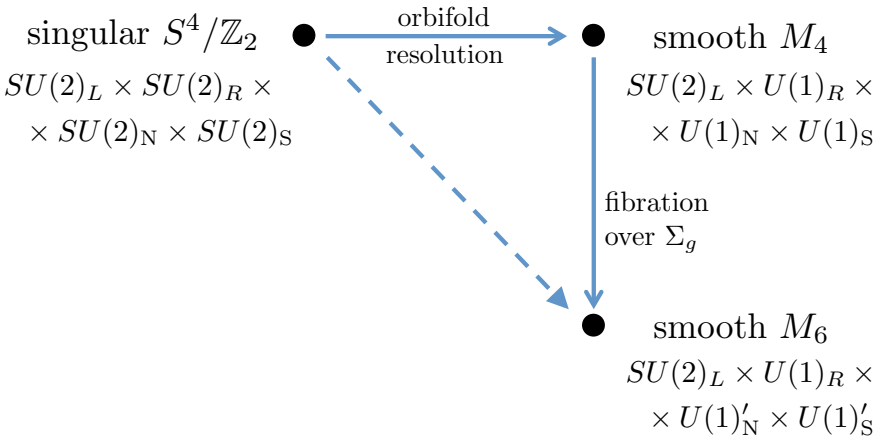

Figure 1. Schematic representation of the geometric construction considered in this work. The starting point is a configuration with M5-branes extending in six non-compact directions. The angular transverse directions form the singular space $S^4/\mathbb{Z}_2$. Along the horizontal arrow, the orbifold singularities of $S^4/\mathbb{Z}_2$ are blown up to yield a smooth manifold M_4. The latter is then fibered over a smooth Riemann surface Σ_g. For each stage of this process we give the corresponding global symmetries. The factors $\mathrm{U}(1)'_{\rm N} \times \mathrm{U}(1)'_{\rm S}$ are defined in (1.1). Finally, the diagonal dashed arrow corresponds to the expected RG flow across dimensions that results from wrapping the parent 6d (1,0) SCFT on Σ_g with the appropriate flavor twist.

Hooft anomalies in four dimensions. This method, however, does not capture accidental symmetries. The latter can generically contribute to the exact IR superconformal R-symmetry which governs the central charges of the 4d theory. In this work, we do not follow such approach and we do not construct explicitly the relevant RG flows from six to four dimensions. We can nonetheless track interesting physical phenomena through the 11d geometry.

The global symmetries of the 4d QFT are encoded in the geometry and topology of the space M_6. We notice that the global symmetries of the 4d QFT correspond to gauge symmetries of the 5d supergravity obtained by reduction of M-theory on the internal space M_6. In this 5d supergravity theory, we have a massless gauge field for each isometry generator of M_6. Furthermore, additional 5d gauge fields are obtained by expanding the M-theory 3-form C_3 onto harmonic 2-forms in M_6.

The space M_6 has isometry group $\mathrm{U}(1)_R \times \mathrm{SU}(2)_L$. It also admits three harmonic 2-forms, denoted $\omega_{\rm N}$, $\omega_{\rm S}$, $\omega_{\rm C}$. The 2-forms $\omega_{\rm N,S}$ are related to the 2-cycles associated with the resolution of the orbifold singularities of $S^4/\mathbb{Z}_2$. The 2-form $\omega_{\rm C}$, on the other hand, only emerges after reduction to four dimensions, by fibering M_4 non-trivially over Σ_g. From a field theory point of view, the existence of a third harmonic 2-form in the 4d setup is interpreted as the emergence of an accidental $\mathrm{U}(1)_C$ global symmetry, which is not visible in six dimensions.

A crucial feature of the 5d supergravity theory obtained from compactification on M_6 is the following. While all three harmonic 2-forms $\omega_{\rm N}$, $\omega_{\rm S}$, $\omega_{\rm C}$ yield a 5d vector, one linear combination of such vectors gets massive via Stückelberg mechanism, by coupling

JHEP01(2020)117

to a 5d axion. This phenomenon is described in greater detail in [1].[1] From a field theory perspective, the symmetry group $\mathrm{U}(1)_\mathrm{N}\times\mathrm{U}(1)_\mathrm{S}\times\mathrm{U}(1)_\mathrm{C}$ is spontaneouly broken to a $\mathrm{U}(1)'_\mathrm{N}\times\mathrm{U}(1)'_\mathrm{S}$ subgroup. The connection between Stückelberg mechanism in 5d supergravity, and spontaneous symmetry breaking of global symmetries in the 4d theory, is well-established in the holography literature, see e.g. [44, 45]. To summarize,

$$\mathrm{U}(1)_\mathrm{N}\times\mathrm{U}(1)_\mathrm{S}\xrightarrow[\text{symmetry}]{\text{accidental}}\mathrm{U}(1)_\mathrm{N}\times\mathrm{U}(1)_\mathrm{S}\times\mathrm{U}(1)_\mathrm{C}\xrightarrow[\text{symm. breaking}]{\text{spontaneous}}\mathrm{U}(1)'_\mathrm{N}\times\mathrm{U}(1)'_\mathrm{S}\,,$$

with the generators $T'_{\mathrm{N,S}}$ of $\mathrm{U}(1)'_{\mathrm{N,S}}$ given in terms of the generators $T_{\mathrm{N,S,C}}$ of $\mathrm{U}(1)_{\mathrm{N,S,C}}$ as

$$T'_\mathrm{N}=T_\mathrm{N}-\frac{N_\mathrm{N}}{N}T_\mathrm{C}\,,\qquad T'_\mathrm{S}=T_\mathrm{S}-\frac{N_\mathrm{S}}{N}T_\mathrm{C}\,. \tag{1.1}$$

The naïve symmetry $\mathrm{U}(1)_\mathrm{N}\times\mathrm{U}(1)_\mathrm{S}$ visible in 6d dimensions is replaced by $\mathrm{U}(1)'_\mathrm{N}\times\mathrm{U}(1)'_\mathrm{S}$. Even though the rank is unchanged, this process has deep implications for the 't Hooft anomalies of the theory, due to the non-trivial mixing of generators in (1.1).

We perform a careful analysis of the 't Hooft anomalies of the 4d QFT, using the techniques developed in [1], based on anomaly inflow from the M-theory ambient space. The main idea in [1] is to obtain the inflow anomaly polynomial I_6^{inflow} of the 4d QFT by integrating a 12-form characteristic class $\mathcal{I}_{12}$, which encodes the anomalous variation of the M-theory action in the presence of the M5-brane stack. Crucially, I_6^{inflow} counterbalances the anomalies of all degrees of freedom living on the stack, which in the IR can be organized into the interacting QFT of interest, plus possible decoupled sectors. We may then write

$$I_6^{\text{inflow}}=\int_{M_6}\mathcal{I}_{12}\,,\qquad I_6^{\text{inflow}}+I_6^{\text{QFT}}+I_6^{\text{decoupl}}=0\,. \tag{1.2}$$

While we do not have a complete understanding of decoupled sectors, we can assume that their contribution to the 't Hooft anomalies is subleading in the large-N limit, which is taken as $N, N_{\mathrm{N,S}}\to\infty$, keeping $N_{\mathrm{N,S}}/N$ finite. Under this assumption, the term I_6^{decoupl} in (1.2) can be neglected at leading order at large N, and we can infer the anomaly polynomial of the interacting QFT from I_6^{inflow}.

In order to test our large-N result, we investigate the gravity duals of our 4d field theory constructions. As it turns out, we identify the gravity duals to be a well-known class of AdS_5 solutions in M-theory, first discussed in Gauntlett-Martelli-Sparks-Waldram (GMSW) [46]. These solutions are warped products $\mathrm{AdS}_5\times_w M_6^{\text{hol}}$, where the internal space M_6^{hol} is smooth and has exactly the same topology and isometries as M_6 in the probe M5-brane picture. The existence of smooth dual geometries provides evidence that our construction yields a non-trivial interacting 4d SCFT in the IR, at least at large N.

To give more supporting evidence for our claims, we carry out two quantitative checks, by computing the central charge c and the flavor central charge for the global symmetry $\mathrm{SU}(2)_L$ of the 4d theory. These quantities can be computed holographically at large N from the supergravity effective action. On the field theory side, they can be extracted

[1] This Stückelberg mechanism was also instrumental in [43] for the correct counting of symmetries of 4d $\mathcal{N}=2$ SCFTs from M5-branes from the perspective of their gravity duals.

JHEP01(2020)117

from the anomaly polynomial, because the superconformal algebra relates them to 't Hooft anomalies coefficients involving the $\mathrm{SU}(2)_L$ generators and the superconformal R-symmetry [47, 48]. The latter is the linear combination of $\mathrm{U}(1)_R \times \mathrm{U}(1)'_{\mathrm{N}} \times \mathrm{U}(1)'_{\mathrm{S}}$ determined by a-maximization [49]. We find a perfect agreement between the supergravity and field theory computations, at leading order in the large-N expansion.

The outcome of a-maximization depends crucially on the mixing (1.1) of the naïve symmetry generators $T_{\mathrm{N,S}}$ of $\mathrm{U}(1)_{\mathrm{N,S}}$ with the generator T_{C} of the emergent $\mathrm{U}(1)_{\mathrm{C}}$ symmetry. In particular, in order to match the supergravity results it is essential to take into account emergent symmetries and spontaneous symmetry breaking in the computation of the 't Hooft anomalies of the 4d theory. The methods of [1] provide a streamlined, geometric way of addressing these phenomena. Indeed, if we integrate the anomaly polynomial of the parent 6d SCFT of Σ_g, we do not reproduce the correct large-N central charge from holography.

The rest of this paper is organized as follows. In section 2 we describe the 6d M-theory setup with a stack of N M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Section 3 is devoted to the reduction of the 6d theory to four dimensions on Σ_g, and to the global symmetries of the 4d QFT. In section 4 we compute the anomalies of the 4d QFT using inflow. In section 5 we identify the gravity duals and we perform the aforementioned quantitative tests involving the central charges of the 4d theories. We conclude with a brief discussion in section 6. The computations in supergravity are collected in appendix A.

2 Six-dimensional setup

The main setup of interest is a stack of N M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ orbifold singularity in M-theory. First, we discuss some general aspects where the M5-branes are probing a $\mathbb{C}^2/\mathbb{Z}_k$ orbifold fixed point.

2.1 Aspects of M-theory on $\mathbb{C}^2/\mathbb{Z}_k$

First we consider the M-theory background with the orbifold $\mathbb{C}^2/\mathbb{Z}_k$. Let $(x^0, \cdots, x^5)$ be the coordinates along an $\mathbb{R}^6$ plane, and $(y^1, \cdots, y^5)$ be the coordinates along the transverse directions. The latter parametrize a five-dimensional space $\mathbb{C}^2 \times \mathbb{R}$ with complex coordinates $(z_1 = y^1 + iy^2, z_2 = y^3 + iy^4)$. The orbifold action is

$$(z_1, z_2, y^5) \sim (e^{\frac{2\pi i}{k}} z_1, e^{-\frac{2\pi i}{k}} z_2, y^5). \tag{2.1}$$

A local metric for the M-theory background is given as

$$ds_{11}^2 = ds^2(\mathbb{R}^6) + dr^2 + r^2\, ds^2(S^4/\mathbb{Z}_k) \tag{2.2}$$

$$ds^2(S^4/\mathbb{Z}_k) = \frac{d\mu^2}{1-\mu^2} + (1-\mu^2)\left[\frac{1}{k^2} D\varphi^2 + \frac{1}{4} ds^2(S^2_\psi)\right], \tag{2.3}$$

$$ds^2(S^2_\psi) = d\theta^2 + \sin^2\theta\, d\psi^2, \qquad D\varphi = d\varphi + \frac{k}{2}\cos\theta\, d\psi. \tag{2.4}$$

The radius r is constructed from y^5 and the radii of the two complex planes, in particular we have $y^5 = r\mu$. The circle coordinates have periodicity $(\Delta\psi = 2\pi, \Delta\varphi = 2\pi)$. For $k = 1$,

JHEP01(2020)117

the metric (2.3) is that of round four-sphere. When $k > 1$, the orbifold action admits two $\mathbb{R}^4/\mathbb{Z}_k$ fixed points at the poles of the sphere at $\mu = \pm 1$. At constant values of μ, the three dimensional sections of the four-sphere are S^1_ϕ bundles over S^2_ψ with degree k. The isometries of S^1_φ and S^2_ψ lead to a $\mathrm{U}(1)_\varphi \times \mathrm{SU}(2)_\psi$ gauge symmetry on the extended seven-dimensional directions of the M-theory background. This is the subgroup of the SO(5) isometry of the sphere preserved by the orbifold action.

The region near an orbifold fixed point of the sphere corresponds to a single center Taub-NUT space, the metric near each pole is

$$ds^2 \cong \frac{1}{V} D\varphi^2 + V\left[dR^2 + R^2\, ds^2(S^2_\psi)\right], \quad V = \frac{k}{2R}, \quad R = \frac{1}{k}(1 \mp \mu), \quad \text{for} \quad \mu = \pm 1. \tag{2.5}$$

The orbifold singularities can be resolved locally by replacing the single center Taub-NUT space to a Gibbons-Hawking space with k sources of unit charge. Such spaces are S^1_φ bundles over $\mathbb{R}^3$ with metric given as

$$ds^2 = V^{-1}(d\varphi + A)^2 + V\left(dX^2 + dY^2 + dZ^2\right), \qquad \text{with} \quad dV = \star_{\mathbb{R}^3} dA, \tag{2.6}$$

where V is a potential on the 3D base space and A is a connection one-form for the circle bundle. The potential satisfies Laplace's equation on the $\mathbb{R}^3$ base. A general solution of Gibbons-Hawking space is given by inserting k centers at positions $\vec{X}_I = (X_I, Y_I, Z_I)$ with charge n^I. The potential is

$$V = v_0 + \frac{1}{2}\sum_I \frac{n^I}{|\vec{X} - \vec{X}_I|}. \tag{2.7}$$

The parameter v_0 fixes the asymptotic size of the circle. The coordinate φ has period 2π. The space is Asymptotically Locally Flat (ALF) when $v_0 \neq 0$ with topology of $S^1 \times \mathbb{R}^3$, and asymptotically Locally Euclidean (ALE) when $v_0 = 0$ with topology of $\mathbb{R}^4$.

The region near each center is described by a $\mathbb{R}^4/\mathbb{Z}_{n^I}$ orbifold fixed point where S^1_φ shrinks. A two-cycle can be obtained by taking S^1_φ with a segment on $\mathbb{R}^3$ that connects two singularities. There are $k-1$ independent two-cycles with harmonic representatives, ω^i $(i \in \{1, \cdots, k-1\})$. When all $n^I = 1$, the space is smooth with $k-1$ two-cycles. This corresponds to a resolution of the orbifold singularity. The S^1_ψ circle of S^2_ψ in (2.5), is identified with the rotation on the (X, Y) plane. Resolving the singularity breaks the $\mathrm{SU}(2)_\psi$ isometry of S^2_ψ to a $\mathrm{U}(1)_\psi$ corresponding to the rotations of S^1_ψ.

M-theory, in the supergravity limit, can be studied on the space (2.2) where we replace the two orbifold singularities with their smooth resolutions. This local deformation is always possible since the asymptotic space of Gibbons-Hawking is fixed by the total charge. M-theory has a 2-form gauge symmetry with a 3-form potential, C_3. There are massless fluctuations of the C_3 potential on the extended seven-dimensional space coming from the resolutions of the singularities. In reducing M-theory on the compact space, we can add terms to C_3 as

$$C_3 = a^{\mathrm{N}}_i \wedge \omega^i_{\mathrm{N}} + a^{\mathrm{S}}_i \wedge \omega^i_{\mathrm{S}} + \cdots \tag{2.8}$$

where $(\omega^i_{\mathrm{N}}, \omega^i_{\mathrm{S}})$ are the resolution harmonic two-forms for the north and south singularities, respectively. The one-forms $(a^{\mathrm{N}}_i, a^{\mathrm{S}}_i)$ are massless gauge field on the seven-dimensional

JHEP01(2020)117

space. Each orbifold fixed point leads to a $\mathrm{U}(1)^{k-1}$ gauge symmetry in M-theory which enhances to an $\mathrm{SU}(k)$ gauge symmetry in the singular limit [50]. To summarize, the M-theory background admits an $\mathrm{SU}(2)_\psi \times \mathrm{SU}(k)_{\mathrm{N}} \times \mathrm{SU}(k)_{\mathrm{S}} \times \mathrm{U}(1)_\varphi$ bosonic gauge symmetry.

We are interested in the field theory that describe the low energy dynamics of a stack of N M5-branes probing the M-theory singularity at $r = 0$ in (2.2). The gauge symmetry of the M-theory background induces a global symmetry on the worldvolume directions of the M5-branes. The $\mathrm{SU}(2)_\psi$ symmetry from S^2_ψ corresponds to an $\mathrm{SU}(2)_{\mathcal{R}}$ R-symmetry, whiles the rest $\mathrm{SU}(k)_{\mathrm{N}} \times \mathrm{SU}(k)_{\mathrm{S}} \times \mathrm{U}(1)_\varphi$ imprints as a flavor symmetry. In particular, the 7d gauge fields $(a_i^{\mathrm{N}}, a_i^{\mathrm{S}})$ yield 6d background connections $(A^{\mathrm{N}}_{6d\,i}, A^{\mathrm{S}}_{6d\,i})$. This configuration preserves eight supercharges leading to a six-dimensional superconformal field theory with a $(1, 0)$ supersymmetry [4–9].

2.2 The case $k = 2$

The resolution space of $S^4/\mathbb{Z}_2$ has an enhanced symmetry. The $\mathrm{U}(1)_\varphi$ isometry of S^1_φ in $S^4/\mathbb{Z}_2$ enhances to an $\mathrm{SU}(2)_\varphi$ isometry group as can be seen by rewriting the metric as

$$ds^2(S^4/\mathbb{Z}_2) = \frac{d\mu^2}{1-\mu^2} + \frac{1}{4}(1-\mu^2)\left[D\psi^2 + d\theta^2 + \sin^2\theta\, d\varphi^2\right], \quad D\psi = d\psi + \cos\theta\, d\psi. \tag{2.9}$$

The resolution of the orbifold singularities at $\mu = \pm 1$ preserve the isometries of S^2_φ composed of (θ, ψ) in the metric of $S^4/\mathbb{Z}_2$ above. This follows from the fact that the resolution space of $\mathbb{R}^4/\mathbb{Z}_2$ is the Eguchi-Hanson space [51]. To see this more explicitly, write the potentials for the two center Gibbons-Hawking space with unit charge as

$$V = \frac{1}{2R_+} + \frac{1}{2R_-}, \qquad A = \frac{1}{2}\left(\frac{Z_+}{R_+} + \frac{Z_-}{R_-}\right) d\tan^{-1}(Y/X)\,, \tag{2.10}$$

$$R^2_\pm = X^2 + Y^2 + Z^2_\pm\,, \qquad Z_\pm = Z \pm Z_0\,. \tag{2.11}$$

The centers are sitting at $(0, 0, \pm Z_0)$. The metric of the resolved space can be written as

$$ds^2 = \frac{R^4}{R^4 - a^4}\, dR^2 + \frac{R^2}{4}\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + \frac{R^4 - a^4}{R^4}\,(d\psi + \cos\theta\, d\varphi)^2\right], \tag{2.12}$$

where we have made the coordinate transformation [52]

$$Z = \frac{R^2}{4}\cos\theta\,, \quad X = \frac{\sqrt{R^4 - a^4}}{4}\sin\theta\,\cos\psi\,, \quad Y = \frac{\sqrt{R^4 - a^4}}{4}\sin\theta\,\sin\psi\,. \tag{2.13}$$

The space smoothly caps off at $R = a$ where the S^1_ψ circle shrinks. The two-sphere, S^2_φ, composed of (θ, φ) in the resolved space has a finite size of $a/2$ at $R = a$. The two-sphere in this region also corresponds to the two-cycle of the resolution of the $\mathbb{R}^4/\mathbb{Z}_2$.

The singularities of $S^4/\mathbb{Z}_2$ are resolved by excising the singular region and gluing in the Eguchi-Hanson space describe in (2.12). The S^2_φ of (2.12) is identified with the S^2_φ of $S^4/\mathbb{Z}_2$ in (2.9). In this sense, the resolution of the singularities on the sphere preserves its isometries.

JHEP01(2020)117

The smooth geometry obtained from the resolution of $S^4/\mathbb{Z}_2$ is denoted as M_4. It has the topology of $S^2 \times S^2$ and corresponds to the Hirzebruch surface $\mathbb{F}_2$. It is useful to write a local metric for the space M_4,

$$ds^2(M_4) = h_1(\mu)\, ds^2(S^2_\varphi) + h_2(\mu)\, d\mu^2 + h_3(\mu)\, D\psi^2\,, \quad D\psi = d\psi + \cos\theta\, d\varphi\,. \tag{2.14}$$

The coordinate μ takes value in the interval $[\mu_{\rm S}, \mu_{\rm N}]$. The boundary conditions of the h's are fixed by regularity of the metric at the bounds, and in particular, h_3 must vanish on them. The two-sphere S^2_φ is not shrinking at the north and south poles of M_4 where the S^1_ψ is shrinking. The two-spheres at the tip of the μ-interval correspond to two-cycles denoted as $(\mathcal{S}^2_{\rm S}, \mathcal{S}^2_{\rm N})$ respectively. The volume of these cycles are then $(h_1(\mu_{\rm S}), h_1(\mu_{\rm N}))$ respectively.

The sizes of the two-cycles $(\mathcal{S}^2_{\rm S}, \mathcal{S}^2_{\rm N})$ are moduli parameters of M_4. In the singular limit where they vanish, $M_4 \to S^4/\mathbb{Z}_2$. The four-sphere admits a left SU(2) and a right SU(2) action; these are preserved by the $\mathbb{Z}_2$ orbifold action, and are related to the isometries of $\mathrm{SU}(2)_\psi \times \mathrm{SU}(2)_\varphi$ rotations. The $\mathrm{U}(1)_\psi \times \mathrm{SU}(2)_\varphi$ are manifest as isometries of $S^1_\psi \times S^2_\varphi$ of the metric (2.9). The total gauge symmetry in seven dimensions is $\mathrm{SU}(2)_\psi \times \mathrm{SU}(2)_\varphi \times \mathrm{SU}(2)_{\rm N} \times \mathrm{SU}(2)_{\rm S}$.

Anomalies of the 6d setup. Anomaly inflow for flat M5-branes probing orbifold singularities was studied in [53]. See also [1] for a review of the computation. The inflow computation yields an 8-form $I_8^{\rm inflow}$ which captures the variation of the M-theory action in the presence of the M5-brane stack. The inflow anomaly polynomial counterbalances the anomalies of the worldvolume degrees of freedom, which at low energies consists of an interacting SCFT and of modes that decouple in the IR,

$$I_8^{\rm inflow} + I_8^{\rm SCFT} + I_8^{\rm decoupl} = 0\,. \tag{2.15}$$

The inflow anomaly polynomial reads

$$\begin{aligned} -I_8^{\rm inflow} = {} & \frac{N^3}{6}\left[c_2^\varphi - c_2^\psi\right]^2 + \frac{N}{12}\left[4\,c_2^\varphi - 3\,c_2^\psi\right] c_2^\psi + \frac{N}{24}\left[2\,c_2^\varphi - c_2^\psi\right] p_1(TW_6) \\ & - \frac{1}{32\,N}\left[\frac{\mathrm{tr}\,(F^{\rm N}_{6d})^2}{(2\pi)^2} - \frac{\mathrm{tr}\,(F^{\rm S}_{6d})^2}{(2\pi)^2}\right]^2 + \frac{N}{4}\left[c_2^\varphi - c_2^\psi\right]\left[\frac{\mathrm{tr}\,(F^{\rm N}_{6d})^2}{(2\pi)^2} + \frac{\mathrm{tr}\,(F^{\rm S}_{6d})^2}{(2\pi)^2}\right] \\ & + \frac{N}{192}\left[p_1(TW_6)^2 - 4\,p_2(TW_6)\right]. \end{aligned} \tag{2.16}$$

We have introduced the compact notation

$$c_2^\varphi \equiv c_2(\mathrm{SU}(2)_\varphi)\,, \qquad c_2^\psi \equiv c_2(\mathrm{SU}(2)_\psi)\,. \tag{2.17}$$

The anomaly of the decoupling modes is given by

$$I_8^{\rm decoupl} = I_8^{\rm tensor} + \frac{1}{2}\, I_8^{\rm vec,N} + \frac{1}{2}\, I_8^{\rm vec,S}\,. \tag{2.18}$$

The 8-form $I_8^{\rm tensor}$ is the anomaly polynomial of a 6d (1,0) tensor multiplet,

$$I_8^{\rm tensor} = \frac{1}{24}\,(c_2^\psi)^2 + \frac{1}{48}\, c_2^\psi\, p_1(TW_6) + \frac{23}{5760}\, p_1(TW_6)^2 - \frac{29}{1440}\, p_2(TW_6)\,. \tag{2.19}$$

JHEP01(2020)117

The 8-form $I_8^{\text{vec,N}}$ is the anomaly polynomial of a 6d (1,0) vector multiplet of $\text{SU}(2)_{\text{N}}$,

$$I_8^{\text{vec,N}} = -\frac{1}{8}(c_2^\psi)^2 - \frac{1}{16}c_2^\psi\, p_1(TW_6) - \frac{1}{1920}\left[7\,p_1(TW_6)^2 - 4\,p_2(TW_6)\right] - \frac{1}{2}c_2^\psi\,\frac{\text{tr}\,(F_{6d}^N)^2}{(2\pi)^2} - \frac{1}{24}p_1(TW_6)\,\frac{\text{tr}\,(F_{6d}^N)^2}{(2\pi)^2} - \frac{1}{12}\left[\frac{\text{tr}\,(F_{6d}^N)^2}{(2\pi)^2}\right]^2. \tag{2.20}$$

The quantity $I_8^{\text{vec,S}}$ is completely analogous.[2]

3 Four-dimensional setup

In this section we describe aspects of the geometric setup when a stack of N M5-branes wrapping a genus, g, Riemann Surface $\Sigma_{g,n}$ with n punctures, probe a $\mathbb{C}^2/\mathbb{Z}_2$ singularity in M-theory. We will discuss various aspects of the geometric setup in M-theory and the symmetries they induce for the field theory that describe the low-energy dynamics of the branes. We will then use the geometric set-up and compute the anomaly polynomial of the field theory by using anomaly inflow techniques developed in [1].

3.1 Geometric setup for 4d systems

We consider an eleven-dimensional background given as

$$M_{11} = W_4 \times \Sigma_{g,n} \times \mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{R}. \tag{3.1}$$

The M-theory background preserves supersymmetry when the space $\Sigma_{g,n} \times \mathbb{C}^2/\mathbb{Z}_2$ satisfies the Calabi-Yau threefold condition, i.e. the first Chern class of the space must vanish. The worldvolume of the branes decompose as $W_6 = W_4 \times \Sigma_{g,n}$ where W_4 is the external spacetime. The low-energy dynamics of the branes is captured by a field theory that live on W_4. In the region near the branes, the spacetime decomposes as

$$M_{11} = \mathbb{R}^+ \times M_{10}, \quad \text{where} \quad M_6 \hookrightarrow M_{10} \to W_4, \quad M_4 \hookrightarrow M_6 \to \Sigma_{g,n}. \tag{3.2}$$

The line $\mathbb{R}^+$ is the overall radius of the transverse directions of the worldvolume spacetime W_4. The space M_{10} describes the tubular neighborhood of the branes, it also corresponds to an internal boundary of the M-theory spacetime near the branes. Finally, M_4 is the resolution space of $S^4/\mathbb{Z}_2$ describe in section 2.2.

The Calabi-Yau condition on the M-theory background is satisfied by twisting the R-symmetry circle S^1_ψ over the Riemann surface. This twist breaks the six-dimensional $\text{SU}(2)_{\mathcal{R}}$ R-symmetry to a $\text{U}(1)_{\mathcal{R}}$ symmetry for the worldvolume theory on W_4. At the level of the geometry, this is achieved by shifting the connection of S^1_ψ as

$$D\psi \to \widehat{D}\psi = d\psi + \cos\theta\, d\varphi - 2\pi\,\chi\, A_\Sigma, \quad \text{with} \quad \int_{\Sigma_{g,n}} dA_\Sigma = 1, \quad \chi = -2(g-1) - n. \tag{3.3}$$

The Euler characteristic of the Riemann surface is denoted by χ. The twisting, A_Σ, is a nontrivial component of the six-dimensional gauging of the R-symmetry of the 6d theory.

[2] The trace 'tr' is normalized in such a way that, if $n_{1,2}$ are the Chern roots of an SU(2) bundle with $n_1 + n_2 = 0$, then $\text{tr}F^2/(2\pi)^2 = -2\,(n_1^2 + n_2^2)$. Since $c_2(\text{SU}(2)) = n_1 n_2$, we have $\text{tr}F^2/(2\pi)^2 = 4\,c_2(\text{SU}(2))$.

JHEP01(2020)117

In addition to twisting the R-symmetry, a large family of four-dimensional SCFTs can be engineered by turn on background fields along the Riemann surface for the Cartan elements of the $\mathrm{SU}(2)_{\mathrm{N}} \times \mathrm{SU}(2)_{\mathrm{S}} \times \mathrm{SU}(2)_{\varphi}$ flavor symmetry. Such background fields break the flavor symmetry of the six-dimensional theory to $\mathrm{U}(1)_{\mathrm{N}} \times \mathrm{U}(1)_{\mathrm{S}} \times \mathrm{U}(1)_{\varphi}$ for the four-dimensional theory. At the level of the M-theory background, this is achieved by turning on the following flux parameters

$$d\varphi \to \widehat{D}\varphi = d\varphi + 2\pi\, z\, A_\Sigma \tag{3.4}$$

$$C_3 \to \ \widehat{C}_3 = C_3 + 2\pi\, (N_{\mathrm{N}}\, A_\Sigma \wedge \omega_{\mathrm{N}} + N_{\mathrm{S}}\, A_\Sigma \wedge \omega_{\mathrm{S}})\,. \tag{3.5}$$

The connection forms $(N_{\mathrm{N}}\, A_\Sigma, N_{\mathrm{S}}\, A_\Sigma)$ are background fields for the U(1) gauge fields discussed in (2.8) in the case of $k = 2$. The two-forms $(\omega_{\mathrm{N}}, \omega_{\mathrm{S}})$ are the closed representatives that measure the volumes of the resolution two-cycles $(\mathcal{S}^2_{\mathrm{N}}, \mathcal{S}^2_{\mathrm{S}})$ in M_4. For every choice of a Riemann surface, $\Sigma_{g,n}$, there is a family of four-dimensional systems labeled by the three flux parameters $(z, N_{\mathrm{N}}, N_{\mathrm{S}})$ and the number of branes, N.

In this paper, we restrict to four-dimensional theories that preserve the $\mathrm{SU}(2)_{\varphi}$ symmetry, this corresponds to fixing $z = 0$. We will also restricted to cases with no punctures, i.e. $n = 0$, and non-vanishing curvature, $g \neq 0$. For this family, a local metric for M_6 can be written as

$$ds^2(M_6) = h_0(\mu) ds^2(\Sigma_g) + h_1(\mu) ds^2(S^2_\varphi) + h_2(\mu) d\mu^2 + h_3(\mu) \widehat{D}\psi^2. \tag{3.6}$$

The interval μ takes value in $[\mu_{\mathrm{S}}, \mu_{\mathrm{N}}]$ where the endpoints are fixed by the loci where h_3 vanish. The functions (h_0, h_1) parametrize the radii of Σ_g and S^2_φ in M_6, they are non-vanishing on the interval of μ.

At a fixed point on the surface Σ_g, there is a fiber that is a copy of M_4 composed of S^2_φ, the circle S^1_ψ and the interval μ. Similarly, we can consider a fixed point on the sphere S^2_ψ, there are four-dimensional fibers which are copies of a space M_4^Σ which are composed of the Riemann surface Σ_g, the circle S^1_ψ and the interval μ. These fibers do not shrink in M_6.

3.2 Flux quantization and four-cycles of M_6

One of the main object of interest is the boundary condition for the four-flux G_4 when the branes wrap the Riemann surface. Following [38, 39], we can write the boundary term for G_4 by using a bump function $\rho(r)$ as

$$G_4 = 2\pi\rho(r)\, \overline{G}_4 + \ldots, \qquad \text{with} \qquad \int_{M_4} \overline{G}_4 = N\,. \tag{3.7}$$

The flux $\overline{G}_4$ supports the non-trivial geometry M_6.

The space M_4 is the fiber over the Riemann Surface, Σ_g. Since it is non-shrinking, we can thread N units of flux on it at a fixed point of the Riemann surface. Similarly, at a fixed point of the sphere, S^2_φ, we can thread flux on the fiber M_4^Σ given as

$$\int_{M_4} \overline{G}_4 = N_\Sigma. \tag{3.8}$$

JHEP01(2020)117

The space M_6 also admits four-cycles localized at the north and south poles of the μ interval where S^1_ψ shrinks. These correspond to the product of the resolution cycles of the original orbifold singularities of the $S^4/\mathcal{Z}_2$ with the Riemann surface, Σ_g. They are denoted as

$$\mathcal{C}^4_{\rm N} = \mathcal{S}^2_{\rm N} \times \Sigma_g \qquad \text{localized at} \qquad \mu = \mu_{\rm N}\,, \tag{3.9}$$

$$\mathcal{C}^4_{\rm S} = \mathcal{S}^2_{\rm S} \times \Sigma_g \qquad \text{localized at} \qquad \mu = \mu_{\rm S}\,. \tag{3.10}$$

We can thread flux on these cycles given as

$$\int_{\mathcal{C}^4_{\rm N}} \overline{G}_4 = N_{\rm N} \in \mathbb{Z}, \qquad \int_{\mathcal{C}^4_{\rm S}} \overline{G}_4 = N_{\rm S} \in \mathbb{Z}. \tag{3.11}$$

It seems then that there exist four flux quanta $(N, N_\Sigma, N_{\rm N}, N_{\rm S})$ that label the class of $\overline{G}_4$ in M_6. These flux parameters are not all independent. This reflects the fact that the space M_6 actually admits only three four-cycles denoted as $(\mathcal{C}^4_{\rm N}, \mathcal{C}^4_{\rm S}, \mathcal{C}^4_{\rm C})$. The last one $\mathcal{C}^4_{\rm C}$ is not localized on M_6. It consists of $S^2_\varphi \times S^2_{\mu\psi}$ at a generic point on Σ_g.

To see that there are only three independent flux parameters, we consider the most general local expression for $\overline{G}_4$ that is closed and consistent with the symmetries of M_6

$$\overline{G}_4 = \left[d\gamma_\varphi \wedge V_2^\varphi + d\gamma_\Sigma \wedge V_2^\Sigma \right] \wedge \frac{\widehat{D}\psi}{2\pi} - (\chi\,\gamma_\varphi + 2\,\gamma_\Sigma)\, V_2^\varphi \wedge V_2^\Sigma\,, \tag{3.12}$$

where γ_φ, γ_Σ are functions of μ only and V_2^φ, V_2^Σ are proportional to the volume forms of S^2_φ and the Riemann surface, respectively, normalized according to

$$\int_{S^2_\varphi} V_2^\varphi = 1\,, \qquad \int_{\Sigma_g} V_2^\Sigma = 1\,. \tag{3.13}$$

The flux parameters are given as

$$\gamma^{\rm N}_\varphi - \gamma^{\rm S}_\varphi = N\,, \quad \gamma^{\rm N}_\Sigma - \gamma^{\rm S}_\Sigma = N_\Sigma\,, \quad \chi\,\gamma^{\rm N}_\varphi + 2\,\gamma^{\rm N}_\Sigma = -N_{\rm N}\,, \quad \chi\,\gamma^{\rm S}_\varphi + 2\,\gamma^{\rm S}_\Sigma = -N_{\rm S}\,, \tag{3.14}$$

where the γ parameters are defined as $\gamma^{\rm N,S}_\bullet \equiv \gamma_\bullet(\mu_{\rm N,S})$. The γ relations imply[3]

$$2N_\Sigma + \chi N = -(N_{\rm N} - N_{\rm S})\,. \tag{3.16}$$

The class G_4 is labeled by three flux parameters that we denote as $(N, N_{\rm N}, N_{\rm S})$. The flux parameter N can be associated with the four-cycle $\mathcal{C}^4_{\rm C}$.

[3]The constraint on the flux parameters can be understood as the condition that must be satisfied by the first Chern class of the S^1_ψ bundle over the rest of the space:

$$\int_{\mathcal{C}^4_{\rm N}} \overline{G}_4 - \int_{\mathcal{C}^4_{\rm S}} \overline{G}_4 = \int_{M_6} \overline{G}_4\, d\frac{\widehat{D}\psi}{2\pi}. \tag{3.15}$$

The minus sign is due the fact that the two cycles have opposite orientations in M_6. A similar phenomenon was observed in [54].

JHEP01(2020)117

The flux parameters can be equivalently regarded as the coefficients of the expansion of $\overline{G}_4$ onto coholomogy classes of M_6,

$$\overline{G}_4 = N\,\mathcal{V}_4^{\mathrm{C}} + N_{\mathrm{N}}\,\mathcal{V}_4^{\mathrm{N}} + N_{\mathrm{S}}\,\mathcal{V}_4^{\mathrm{S}} + (\text{exact terms})\,. \tag{3.17}$$

The 4-forms $\mathcal{V}_4^\alpha$, $\alpha = \mathrm{N}, \mathrm{S}, \mathrm{C}$ are closed but not exact, and define a basis of cohomology classes in M_6. We can parametrize them uniformly by writing

$$\mathcal{V}_4^\alpha = d\left[U_\varphi^\alpha\, V_2^\varphi \wedge \frac{\widehat{D}\psi}{2\pi} + U_\Sigma^\alpha\, V_2^\Sigma \wedge \frac{\widehat{D}\psi}{2\pi}\right] + C^\alpha\, V_2^\varphi \wedge V_2^\Sigma\,. \tag{3.18}$$

In the above expression, U_φ^α, U_Σ^α are functions of μ, while C^α is constant. The parametrization (3.18) is subject to a 2-parameter redundancy, related to shifts of U_φ^α, U_Σ^α by constants. A way to fix this redundancy is to demand

$$(U_\varphi^\alpha)^{\mathrm{N}} + (U_\varphi^\alpha)^{\mathrm{S}} = 0\,, \qquad (U_\Sigma^\alpha)^{\mathrm{N}} + (U_\Sigma^\alpha)^{\mathrm{S}} = 0\,. \tag{3.19}$$

We want the 4-forms $\mathcal{V}_4^\alpha$ to be dual to the 4-cycles $\mathcal{C}_\alpha^4$ defined above,

$$\int_{\mathcal{C}_\alpha^4} \mathcal{V}_4^\beta = \delta_\alpha^\beta\,. \tag{3.20}$$

This condition determines the constants C^α and the quantities $(U_\varphi^\alpha)^{\mathrm{N}}$, $(U_\Sigma^\alpha)^{\mathrm{N}}$, according to the following table,

	$(U_\varphi^\alpha)^{\mathrm{N}}$	$(U_\Sigma^\alpha)^{\mathrm{N}}$	C^α
$\alpha = \mathrm{C}$	$\frac{1}{2}$	$-\frac{\chi}{4}$	0
$\alpha = \mathrm{N}$	0	$-\frac{1}{4}$	$\frac{1}{2}$
$\alpha = \mathrm{S}$	0	$\frac{1}{4}$	$\frac{1}{2}$

(3.21)

Another way to interpret the fluxes $(N_{\mathrm{N}}, N_{\mathrm{S}})$ is to consider the fate of the fluctuations of the C_3 potential from the $\mathrm{SU}(2)_{\mathrm{N}} \times \mathrm{SU}(2)_{\mathrm{S}}$ symmetry before the compactification on the Riemann surface. The curvatures associated to $\mathrm{SU}(2)_{\mathrm{N}} \times \mathrm{SU}(2)_{\mathrm{S}}$ are $(F_{6d}^{\mathrm{N}}, F_{6d}^{\mathrm{S}})$. We use the notation $(n_{6d}^{\mathrm{N}}, n_{6d}^{\mathrm{S}})$ for the corresponding Chern roots. Since these forms have legs on the worldvolume directions of the M5-branes, W_6, when we compactify on the Riemann surface, they decompose as [22]

$$n_{6d}^{\mathrm{N}} = N_{\mathrm{N}}\, V_2^\Sigma + N\,\frac{F^{\mathrm{N}}}{2\pi}, \qquad n_{6d}^{\mathrm{S}} = N_{\mathrm{S}}\, V_2^\Sigma + N\,\frac{F^{\mathrm{S}}}{2\pi}\,. \tag{3.22}$$

The quantities $(F^{\mathrm{N}}, F^{\mathrm{S}})$ are 4d external connections. We have introduced a factor N in such a way that both terms on the r.h.s. of (3.22) scale linearly with the flux parameters N, N^{N}, N^{S}. In the reduction, this decomposition implies the flux terms in $\overline{G}_4$ as

$$\overline{G}_4 = N_{\mathrm{N}}\, V_2^\Sigma \wedge \omega_{\mathrm{N}} + N_{\mathrm{S}}\, V_2^\Sigma \wedge \omega_{\mathrm{S}} + \ldots \tag{3.23}$$

In field theory, the flux correspond to twisting the Cartan elements of the six-dimensional $\mathrm{SU}(2)_{\mathrm{N}} \times \mathrm{SU}(2)_{\mathrm{S}}$ symmetry over the Riemann surface.

JHEP01(2020)117

There are three harmonic two-forms associated to the four-cycles by Poincaré duality, we denote them as $(\omega_{\rm N}, \omega_{\rm S}, \omega_{\rm C})$. Indeed, the first two are just the resolution two cycles of M_4 which are preserved in M_6. The flux quantization conditions can be equivalently written as

$$\int_{M_6} \overline{G}_4 \wedge \omega_{\rm N} = N_{\rm N}, \qquad \int_{M_6} \overline{G}_4 \wedge \omega_{\rm S} = N_{\rm S}, \qquad \int_{M_6} \overline{G}_4 \wedge \omega_{\rm C} = N. \tag{3.24}$$

These formulas will be useful in the computation of the anomalies for the four-dimensional theories of interest.

In the reduction of M-theory on M_6, there are a class of fluctuations we can add for the C_3 potential. For each of the harmonic two-forms, we can add a gauge field in the external spacetime given as $(a^{\rm N}, a^{\rm S}, a^{\rm C})$ with field strength $(f^{\rm N}, f^{\rm S}, f^{\rm C})$. We are using lowercase letters to emphasize that these are gauge fields in 7d supergravity. Naively, each one of these fields should lead to a U(1) gauge symmetry on the external spacetime which then induces a U(1) flavor symmetry on the worldvolume theory on the branes. However, the C_3 potential can also have a three-form fluctuation, c_3, on the external spacetime with field strength, $g_4 = dc_3$. These terms can be collected as

$$C_3 = a^{\rm N} \wedge \omega_{\rm N} + a^{\rm S} \wedge \omega_{\rm S} + a^{\rm C} \wedge \omega_{\rm C} + c_3 + \dots \tag{3.25}$$

When we reduce the effective action of M-theory, the effective action of the seven-dimensional theory will have terms

$$S = \int b_1 g_4 \wedge \star g_4 + b_2 N_\alpha a^\alpha \wedge g_4 + \dots \tag{3.26}$$

where the b's are numbers. We observe that g_4 couples to a linear combination of the gauge field. This coupling implies that g_4 can be dualized to a Stückelberg field which is eaten by the gauge field $N_\alpha a^\alpha$. Here and in what follows, the index α enumerates harmonic 2-forms and takes the values $\alpha = {\rm N}, {\rm S}, {\rm C}$. We refer the reader to [1] for a more detailed discussion of this Stückelberg mechanism.

Symmetries of the system. The low-energy quantum field preserves a $\mathrm{U}(1)_{\mathcal{R}}$ R-symmetry and the $\mathrm{SU}(2)_\varphi$ flavor symmetries corresponding to the isometries of S^1_ψ and S^2_φ. Naively there is an $\mathrm{U}(1)_{\rm N} \times \mathrm{U}(1)_{\rm S} \times \mathrm{U}(1)_{\rm C}$ corresponding to the fluctuations of C_3 along the harmonic two-forms $(\omega_{\rm N}, \omega_{\rm S}, \omega_{\rm C})$. One linear combination is broken and only a $\mathrm{U}(1)^2$ is preserved, which we denote as $\mathrm{U}(1)_{\rm N} \times \mathrm{U}(1)_{\rm S}$. In the special cases when $g = 0$, the Riemann surface is a two-sphere, in the low-energy limit it can admit an SU(2) isometry group. The flavor symmetry will further enhance by an SU(2) flavor symmetry.

Now we are in a position to construct the gauge invariant and globally defined boundary condition for the G_4 flux in presences of curved branes. We have the action of the symmetries in $\overline{G}_4$ by adding suitable connection forms and curvature terms to make it closed:

$$\begin{aligned} \overline{G}_4 \;\to\; E_4 = \big(d\gamma_\varphi \wedge e_2^\varphi + d\gamma_\Sigma \wedge e_2^\Sigma\big) \wedge \frac{\widetilde{D}\psi}{2\pi} + \big(\gamma_\varphi\, e_2^\varphi + \gamma_\Sigma\, e_2^\Sigma\big) \wedge \left(-2\, e_2^\varphi - \chi\, e_2^\Sigma + 2\,\frac{F^\psi}{2\pi}\right) \\ + N\,\frac{F^{\rm N}}{2\pi} \wedge \widetilde{\omega}_{\rm N} + N\,\frac{F^{\rm S}}{2\pi} \wedge \widetilde{\omega}_{\rm S} + N\,\frac{F^{\rm C}}{2\pi} \wedge \widetilde{\omega}_{\rm C}\,, \qquad \text{with} \quad F^\alpha N_\alpha = 0\,. \end{aligned} \tag{3.27}$$

JHEP01(2020)117

The last condition imposes the fact that one of gauge fields in the fluctuations of the C_3 potential is massive. In the expression for E_4 we have gauged the isometry group as

$$\frac{\widetilde{D}\psi}{2\pi} = \frac{d\psi}{2\pi} - 2\,\mathcal{A}^{\varphi} - \chi\,\mathcal{A}^{\Sigma} + 2\,\frac{A^{\psi}}{2\pi}\,, \quad dA^{\psi} = F^{\psi}, \quad d\mathcal{A}^{\varphi} = e_2^{\varphi}, \quad d\mathcal{A}^{\Sigma} = e_2^{\Sigma}\,. \tag{3.28}$$

The quantity A_ψ is the 4d connection for $\mathrm{U}(1)_\psi$. The two-forms, e_2^{φ} and e_2^{Σ} are the closed and gauge invariant volume forms of S^2_ψ and Σ_g respectively. The expression for e_2^{φ} is

$$e_2^{\varphi} = \frac{1}{8\pi}\,\epsilon_{abc}\,(Dy^a \wedge Dy^b\,y^c - F^{ab}\,y^c)\,, \qquad Dy^a = dy^a - A^{ab}\,y_b\,. \tag{3.29}$$

The indices $a, b, c = 1, 2, 3$ are vector indices of $\mathrm{SO}(3)_\varphi$, raised and lowered with δ_{ab}. The three quantities y^a are constrained coordinates on S^2_φ, with $y^a\,y_a = 1$.[4] The 1-forms A^{ab} are the components of the external $\mathrm{SO}(3)_\varphi$ connection, and F^{ab} are the components of the field strength. When $g > 1$, e_2^{Σ} is simply the volume form on the Riemann surface, $e_2^{\Sigma} = V_2^{\Sigma}$. When $g = 0$ and the surface is a sphere, e_2^{Σ} is given similarly as (3.29). We need the integrals

$$\int_{\Sigma_g} e_2^{\Sigma} = 1\,, \quad \int_{S^2} e_2^{2s+2} = 0\,, \quad \int_{S^2} e_2^{2s+1} = 2^{-2s}\,[p_1(\mathrm{SO}(3))]^s\,, \quad s = 0, 1, 2, \ldots \tag{3.31}$$

where $p_1(\mathrm{SO}(3)_\varphi)$ is the first Pontryagin class of the $\mathrm{SO}(3)_\varphi$ bundle over W_4.

The tilde over the harmonic 2-forms $\omega_{\mathrm{N,S,C}}$ in (3.27) signals the fact that we have gauged the isometry group and we have restored closure, as explained below.

Harmonic 2-forms. Before gauging the isometry group, the harmonic 2-forms ω_α, $\alpha = \mathrm{N, S, C}$, can be uniformly parametrized as

$$\omega_\alpha = d\left[H_\alpha(\mu)\,\frac{\widehat{D}\psi}{2\pi}\right] + t_\alpha^{\varphi}\,V_2^{\varphi} + t_\alpha^{\Sigma}\,V_2^{\Sigma}\,, \tag{3.32}$$

where t_α^{φ}, t_α^{Σ} are suitable constants and H_α is a suitable function of μ. We promote ω_α to $\widetilde{\omega}_\alpha$ by writing

$$\widetilde{\omega}_\alpha = d\left[H_\alpha(\mu)\,\frac{\widetilde{D}\psi}{2\pi}\right] + t_\alpha^{\varphi}\,e_2^{\varphi} + t_\alpha^{\Sigma}\,e_2^{\Sigma}\,. \tag{3.33}$$

This object is indeed manifestly closed and gauge-invariant.

[4] More explicitly,

$$y^1 = \sin\theta\,\cos\varphi\,, \qquad y^2 = \sin\theta\,\sin\varphi\,, \qquad y^3 = \cos\theta\,. \tag{3.30}$$

JHEP01(2020)117

Our parametrization of ω_α is subject to a 1-parameter redundancy related to shifts of H_α by a constant. We fix this redundancy by demanding $H^{\rm N}_\alpha + H^{\rm S}_\alpha = 0$. Moreover, we want to basis ω_α to be dual to the basis $\mathcal{V}_4^\alpha$ of 4-forms defined in (3.18), in the sense that

$$\int_{M_6} \mathcal{V}_4^\alpha \wedge \omega_\beta = \delta^\alpha_\beta \qquad \Rightarrow \qquad \begin{array}{c|c|c|c} & H^{\rm N}_\alpha & t^\varphi_\alpha & t^\Sigma_\alpha \\ \hline \alpha = {\rm C} & 0 & 0 & 1 \\ \alpha = {\rm N} & \frac{1}{2} & -1 & -\frac{\chi}{2} \\ \alpha = {\rm S} & \frac{1}{2} & 1 & \frac{\chi}{2} \end{array} . \tag{3.34}$$

The table summarize all information about ω_α that is needed for the computation of anomaly inflow in the next section.

4 Anomalies for the low energy QFT

Now we are in a position to compute anomalies for M5-branes probing the $k = 2$ orbifold singularity. The construction of the boundary data above will allow for an explicit computation for the anomaly polynomial for the field theories that describe the low energy dynamics of the branes [1, 40, 42]. These are captured by a 12-form M-theory anomaly polynomial given as

$$\mathcal{I}_{12} = -\frac{1}{6} E_4^3 - E_4 \wedge X_8, \qquad X_8 = \frac{1}{192}\left[p_1(TM_{11})^2 - 4\, p_2(TM_{11})\right] , \tag{4.1}$$

where $p_1(TM_{11})$, and $p_2(TM_{11})$ are the first and second Pontryagin classes of the tangent bundle of the eleven-dimensional M-theory spacetime, TM_{11}. The four-form E_4 is precisely the gauge invariant boundary and globally defined boundary condition for the G_4 flux. The anomaly six-form for the four-dimensional theories discussed above is given as

$$I_6^{\rm inflow} = \int_{M_6} \mathcal{I}_{12} \, , \tag{4.2}$$

where we use the corresponding E_4 given in (3.27).

The task at hand is the computation of the 8-form X_8 for the geometry (3.6) and of the integrals $\int_{M_6} E_4\, X_8$, $\int_{M_6} E_4^3$ with E_4 as in (3.27). The full derivation is reported in detail in [1]. Here we point out some salient features of the analysis.

The 8-form X_8 is constructed with the first and second Pontryagin classes of the 11d tangent bundle TM_{11}, see (4.1). For the class of 4d theories under examination, these classes can be computed using the following splitting of TM_{11},

$$TM_{11} \to TW_4 \oplus T\Sigma_g \oplus TS^2_\varphi \oplus TS^2_{\mu\psi} \, . \tag{4.3}$$

The above expression is motivated recalling that the space M_6 is a fibration of M_4 (the resolved orbifold $S^4/\mathbb{Z}_2$) over Σ_g, and that M_4 is a fibration of the 2-sphere $S^2_{\mu\psi}$ spanned by μ, ψ over the 2-sphere S^2_φ. The gauging of the isometry $\mathrm{SO}(3)_\varphi$ shifts the Chern root of TS^2_φ with a contribution with legs along W_4. By a similar token, the Chern root of $TS^2_{\mu\psi}$ is

JHEP01(2020)117

shifted by the fact that S^1_ψ is non-trivially fibered over S^2_φ, Σ_g, and W_4, as can be inferred from the expression of $\widetilde{D}\psi$ in (3.28). The split (4.3) implies

$$\begin{aligned} p_1(TM_{11}) &= p_1(TW_4) + p_1(\mathrm{SO}(3)_\varphi) + \left[d\frac{\widetilde{D}\psi}{2\pi}\right]^2 , \\ p_2(TM_{11}) &= \left[p_1(TW_4) + p_1(\mathrm{SO}(3)_\varphi)\right]\left[d\frac{\widetilde{D}\psi}{2\pi}\right]^2 . \end{aligned} \tag{4.4}$$

We noticed that terms with more than six external legs, such as $p_1(TW_4)\, p_1(\mathrm{SO}(3)_\varphi)$, can be dropped, because they cannot contribute to the inflow anomaly polynomial. We notice that, if the Riemann surface is a sphere, we can keep track of its $\mathrm{SO}(3)_\Sigma$ isometry. To this end, we simply have to replace $p_1(\mathrm{SO}(3)_\varphi)$ with $p_1(\mathrm{SO}(3)_\varphi) + p_1(\mathrm{SO}(3)_\Sigma)$ in (4.4).

As pointed out earlier, the background curvatures for the U(1) symmetries associated to harmonic 2-forms are subject to the constraint $F^\alpha N_\alpha = 0$. This is due to the argument given around (3.26) for the emergence of a massive vector in 7d supergravity. This argument, however, is valid under the technical assumption

$$\int_{M_6} \overline{G}_4 \wedge \omega_\psi = 0\,. \tag{4.5}$$

In the previous expression $\overline{G}_4$ is as in (3.12) and the 2-form ω_ψ is the coefficient of the linear term in F^ψ inside E_4,

$$E_4 = F^\psi \wedge \omega_\psi + \dots\,, \qquad \omega_\psi = 2\,(2\pi)^{-1}\,(\gamma_\varphi\, e_2^\varphi + \gamma_\Sigma\, e_2^\Sigma)\,. \tag{4.6}$$

The requirement (4.5) ensures that the linear combination of 7d U(1) vectors that gets massive via Stückelberg mechanism is built exclusively with the vectors associated to harmonic 2-forms, without any mixing with the vector A^ψ associated to the isometry $\mathrm{U}(1)_\psi$. The interested reader can find a more detailed discussion of this point in [1]. If we combine (4.5) with the relations (3.14), we can express the four quantities $\gamma^{\mathrm{N}}_\varphi$, $\gamma^{\mathrm{S}}_\varphi$, $\gamma^{\mathrm{N}}_\Sigma$, $\gamma^{\mathrm{S}}_\Sigma$ in terms of the three flux quanta $(N, N_{\mathrm{N}}, N_{\mathrm{S}})$,

$$\gamma^{\mathrm{N,S}}_\varphi = -\frac{N\,(N_{\mathrm{N}} + N_{\mathrm{S}})}{2\,(2\chi N + N_{\mathrm{N}} - N_{\mathrm{S}})} \pm \frac{1}{2}\,N\,, \tag{4.7}$$

$$\gamma^{\mathrm{N,S}}_\Sigma = -\frac{(N_{\mathrm{N}} + N_{\mathrm{S}})(\chi N + N_{\mathrm{N}} - N_{\mathrm{S}})}{4\,(2\chi N + N_{\mathrm{N}} - N_{\mathrm{S}})} \mp \frac{1}{4}(\chi N + N_{\mathrm{N}} - N_{\mathrm{S}})\,. \tag{4.8}$$

It is worth noticing that the values of the integrals $\int_{M_6} E_4\, X_8$, $\int_{M_6} E_4^3$ are insensitive to the specific profile of the functions γ_φ, γ_Σ, H_α entering E_4, but only depend on the values that these functions attain at the endpoints of the μ interval. For γ_φ and γ_Σ these values are given in (4.7), while for H_α they are collected in (3.34). We also notice that integration over S^2_φ of powers of e_2^φ is conveniently preformed making use of (3.31), and similarly for integration along the Riemann surface.

JHEP01(2020)117

After these preliminary remarks, we can give the full expression for the inflow anomaly polynomial I_6^{inflow} computed via (4.2). We solve the constraint $F^\alpha N_\alpha = 0$ by expressing F^{C} in (3.27) in terms of $F^{\text{N,S}}$, $F^{\text{C}} = -\frac{1}{N}\,(N_{\text{N}}\,F^{\text{N}} + N_{\text{S}}\,F^{\text{S}})$. We introduce the notation

$$c_1^\psi \equiv c_1(\text{U}(1)_\psi) = \frac{F^\psi}{2\pi}\,, \qquad c_1^{\text{N,S}} \equiv c_1(\text{U}(1)_{\text{N,S}}) = \frac{F^{\text{N,S}}}{2\pi}\,, \tag{4.9}$$

and we write I_6^{inflow} in terms of the quantities

$$N_\Sigma = -\frac{1}{2}(N\,\chi + N_{\text{N}} - N_{\text{S}})\,, \qquad M = \frac{1}{2}(N_{\text{N}} + N_{\text{S}})\,. \tag{4.10}$$

We find

$$\begin{aligned}
-I_6^{\text{inflow}} = &\left[\frac{N^2 N_\Sigma(\chi N-2N_\Sigma+2M)(\chi N-2N_\Sigma-2M)}{(\chi N-2N_\Sigma)^2}+\frac{\chi N}{2}\right](c_1^\psi)^2\,(c_1^{\text{N}}+c_1^{\text{S}}) \\
&-\frac{N^2(\chi N-2N_\Sigma+2M)(\chi N+2N_\Sigma-2M)}{2(\chi N-2N_\Sigma)}\,c_1^\psi\,(c_1^{\text{N}})^2 \\
&-\frac{N^2(\chi N+2N_\Sigma+2M)(\chi N-2N_\Sigma-2M)}{2(\chi N-2N_\Sigma)}\,c_1^\psi\,(c_1^{\text{S}})^2 \\
&-\frac{N^2(\chi^2N^2-4N_\Sigma^2-4M^2)}{\chi N-2N_\Sigma}\,c_1^\psi\,c_1^{\text{N}}\,c_1^{\text{S}} \\
&+\frac{1}{6}N^2(\chi N-6N_\Sigma+6M)\,(c_1^{\text{N}})^3+\frac{1}{6}N^2(\chi N-6N_\Sigma-6M)\,(c_1^{\text{S}})^3 \\
&+\frac{1}{2}N^2(\chi N+2N_\Sigma+2M)\,(c_1^{\text{N}})^2\,c_1^{\text{S}}+\frac{1}{2}N^2(\chi N+2N_\Sigma-2M)\,(c_1^{\text{S}})^2\,c_1^{\text{N}} \\
&+\frac{\chi N+2N_\Sigma}{24}\,c_1^\psi\,p_1(TW_4)-\frac{\chi N}{24}(c_1^{\text{N}}+c_1^{\text{S}})\,p_1(TW_4)-\frac{\chi N+2N_\Sigma}{6}\,(c_1^\psi)^3 \\
&+\left[-\frac{N^2(\chi^2N^2-12N_\Sigma^2+4\chi NN_\Sigma+12M^2)}{24(\chi N-2N_\Sigma)}-\frac{\chi N}{12}\right]c_1^\psi\,p_1(\text{SO}(3)_\varphi) \\
&+\frac{1}{8}N^2(\chi N+2N_\Sigma-2M)\,c_1^{\text{N}}\,p_1(\text{SO}(3)_\varphi)+\frac{1}{8}N^2(\chi N+2N_\Sigma+2M)\,c_1^{\text{S}}\,p_1(\text{SO}(3)_\varphi) \\
&+\left[-\frac{N_\Sigma^2(3N^2-N_\Sigma^2-2NN_\Sigma-3M^2)}{12(N-N_\Sigma)}-\frac{N_\Sigma}{6}\right]c_1^\psi\,p_1(\text{SO}(3)_\Sigma) \\
&+\frac{1}{4}N_\Sigma(N+M)(N+N_\Sigma-M)\,c_1^{\text{N}}\,p_1(\text{SO}(3)_\Sigma) \\
&+\frac{1}{4}N_\Sigma(N-M)(N+N_\Sigma+M)\,c_1^{\text{S}}\,p_1(\text{SO}(3)_\Sigma)\,.
\end{aligned} \tag{4.11}$$

In the last three lines we have collected the terms related to the symmetry $\text{SO}(3)_\Sigma$, which is only present if the Riemann surface is a sphere.

5 Holographic solutions

In the previous sections we have adopted a UV point of view: we fixed the supersymmetric M-theory background (3.1) and we inserted a stack of M5-branes extended along $W_4 \times \Sigma_g$ and sitting at the origin of $\mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{R}$, specifying also the appropriate background fluxes along the Riemann surface. In this section we argue that, in the large-N limit, this class

JHEP01(2020)117

of UV setups corresponds in the IR to a well-known class of AdS_5 solutions in M-theory, first described in GMSW [46].

In the vicinity of the M5-branes, the UV picture of 11d spacetime is described in (3.2). Our expectation for the near-horizon IR picture, based on [55], is that the overall radial direction $\mathbb{R}^+$ combines with W_4 to yield an AdS_5 factor, leaving the geometry M_6 as internal space. Taking into account backreaction effects, the Ansatz for the 11d metric in the near-horizon limit has the form

$$ds^2(M_{11}) = e^{2\lambda}\left[ds^2(\mathrm{AdS}_5) + ds^2(M_6)\right], \tag{5.1}$$

with $ds^2(M_6)$ of the form (3.6), and λ a warp factor depending on M_6. Let us stress that all metric functions in the ansatz (3.6) for the metric on M_6 depend on the interval coordinate μ only. It is natural to also demand that the warp factor λ be a function of μ only.

In [46] a class of solutions is described, in which M_6 and λ have exactly the properties described in the previous paragraph. More precisely, the fully backreacted geometry of M_6 is

$$ds^2(M_6) = e^{-6\lambda}\Big[F_\varphi\, ds^2(S^2_\varphi) + F_\Sigma\, ds^2(\Sigma_g)\Big] + \frac{e^{-6\lambda}}{\cos^2\zeta}\, d\mu^2 + \frac{\cos^2\zeta}{9}\,\widehat{D}\psi^2\,, \tag{5.2}$$

where the warp factor λ and the metric functions F_φ, F_Σ, $\cos\zeta$ depend on μ only. Their expressions are recorded in appendix A, where we summarize some key features of the GMSW solutions. The G_4-flux configuration of the holographic solution is given in (A.4). As expected, it has exactly the same structure as $\overline{G}_4$ in (3.12).

The fact that the topology of the internal space M_6 in the GMSW solutions matches exactly with the topology of M_6 in our UV setup is a strong hint that the GMSW solution provides the gravity dual to the field theory setups we discussed in section 3. Furthermore, the GMSW solutions provide evidence for the fact that the 4d construction yields a non-trivial IR fixed point, at least at large N.

In the remainder of this section we perform two quantitative checks of our proposed field theory interpretation of the GMSW solutions. Before entering the details of the computation, let us briefly discuss our stategy.

On the field theory side, the inflow anomaly polynomial (4.11) is expected to be exact in N, but to contain both the anomalies of the interacting SCFT of interest and of decoupled sectors. It is natural to assume that the decoupled sectors do not contribute to the leading order N^3. As a result, from (4.11) we can safely extract the anomaly polynomial for the interacting SCFT at large N. We then perform a-maximization [49] at large N in order to identify which linear combination of $\mathrm{U}(1)_\psi$, $\mathrm{U}(1)_{\mathrm{N,S}}$ is the superconformal R-symmetry. Once the latter is determined, its 't Hooft anomaly coefficients give us the central charge c and the flavor central charge B for the $\mathrm{SO}(3)_\varphi$ symmetry originating from isometries of S^2_φ. The quantities c and B can also be computed holographically in the GMSW solutions. This supergravity computation is reported in appendix A. We find a perfect agreement with the field theory results.

Let us discuss in greater detail the field theory derivation of the quantities c, B. The first step is simply to isolate the leading terms in (4.11) at large N. For simplicity, in this

JHEP01(2020)117

section we do not keep track of the $\mathrm{SO}(3)_\Sigma$ symmetry that is present in the case in which the Riemann surface is a sphere. We may then write

$$
\begin{aligned}
I_6^{\text{SCFT, large } N} = & \frac{N^2 N_\Sigma(\chi N - 2N_\Sigma + 2M)(\chi N - 2N_\Sigma - 2M)}{(\chi N - 2N_\Sigma)^2}\,(c_1^\psi)^2\,(c_1^{\mathrm{N}} + c_1^{\mathrm{S}}) \\
& - \frac{N^2(\chi N - 2N_\Sigma + 2M)(\chi N + 2N_\Sigma - 2M)}{2(\chi N - 2N_\Sigma)}\,c_1^\psi\,(c_1^{\mathrm{N}})^2 \\
& - \frac{N^2(\chi N + 2N_\Sigma + 2M)(\chi N - 2N_\Sigma - 2M)}{2(\chi N - 2N_\Sigma)}\,c_1^\psi\,(c_1^{\mathrm{S}})^2 \\
& - \frac{N^2(\chi^2 N^2 - 4N_\Sigma^2 - 4M^2)}{\chi N - 2N_\Sigma}\,c_1^\psi\,c_1^{\mathrm{N}}\,c_1^{\mathrm{S}} \\
& + \frac{1}{6}N^2(\chi N - 6N_\Sigma + 6M)\,(c_1^{\mathrm{N}})^3 + \frac{1}{6}N^2(\chi N - 6N_\Sigma - 6M)\,(c_1^{\mathrm{S}})^3 \\
& + \frac{1}{2}N^2(\chi N + 2N_\Sigma + 2M)\,(c_1^{\mathrm{N}})^2\,c_1^{\mathrm{S}} + \frac{1}{2}N^2(\chi N + 2N_\Sigma - 2M)\,(c_1^{\mathrm{S}})^2\,c_1^{\mathrm{N}} \\
& - \frac{N^2(\chi^2 N^2 - 12N_\Sigma^2 + 4\chi N N_\Sigma + 12M^2)}{24(\chi N - 2N_\Sigma)}\,c_1^\psi\,p_1(\mathrm{SO}(3)_\varphi) \\
& + \frac{1}{8}N^2(\chi N + 2N_\Sigma - 2M)\,c_1^{\mathrm{N}}\,p_1(\mathrm{SO}(3)_\varphi) \\
& + \frac{1}{8}N^2(\chi N + 2N_\Sigma + 2M)\,c_1^{\mathrm{S}}\,p_1(\mathrm{SO}(3)_\varphi)\,.
\end{aligned}
\tag{5.3}
$$

Next, we perform a-maximization. The trial superconformal R-symmetry is a linear combination of $\mathrm{U}(1)_\psi$ with $\mathrm{U}(1)_{\mathrm{N,S}}$, parametrized as

$$
R = T_\psi + s^{\mathrm{N}}\,T_{\mathrm{N}} + s^{\mathrm{S}}\,T_{\mathrm{S}}\,, \tag{5.4}
$$

where T_ψ, $T_{\mathrm{N,S}}$ denote the generators of $\mathrm{U}(1)_\psi$, $\mathrm{U}(1)_{\mathrm{N,S}}$, and $s^{\mathrm{N,S}}$ are parameters to be fixed. At leading order at large N,

$$
I_6^{\mathrm{SCFT}} = \frac{1}{6}\,\mathrm{tr}\,R^3\,(c_1^R)^3\,, \qquad a = c = \frac{9}{32}\,\mathrm{tr}\,R^3\,, \qquad\qquad (\text{large } N) \tag{5.5}
$$

where c_1^R is the first Chern class of the background curvature for the superconformal R-symmetry. At the level of the anomaly polynomial, (5.4) is equivalent to the replacements

$$
c_1^\psi \to c_1^R\,, \qquad c_1^{\mathrm{N,S}} \to s^{\mathrm{N,S}}\,c_1^R\,. \tag{5.6}
$$

It follows that a-maximization at large N can be carried out by taking (5.3), performing the replacements (5.6), and maximizing the coefficient of $(F^R)^3$ with respect to the parameters $s^{\mathrm{N,S}}$. The result of this computation is most conveniently written in terms of the quantities N_Σ, M defined in (4.10). The central charge reads

$$
\begin{aligned}
c = & \frac{9N^2N_\Sigma^2\left[\chi^2N^2 + 2\chi N N_\Sigma + 4\,N_\Sigma^2 - 3M^2\right]^{3/2}}{4\,(3M^2 + 2\chi N N_\Sigma)^{\,2}} \\
& - \frac{9N^2N_\Sigma^2\,(2N_\Sigma + \chi N)\left(2\chi^2N^2 + 2\chi N N_\Sigma + 8N_\Sigma^2 - 9M^2\right)}{8\,(3M^2 + 2\chi N N_\Sigma)^{\,2}}\,.
\end{aligned}
\tag{5.7}
$$

JHEP01(2020)117

For completeness, let us also record the values of the parameters $s^{\mathrm{N,S}}$,

$$s^{\mathrm{N,S}} = \frac{2\chi^2 N^2 N_\Sigma - 8N_\Sigma^3 \pm M\,(\chi^2 N^2 - 4\chi N N_\Sigma - 4N_\Sigma^2) \mp 6M^3}{2\,(\chi N - 2N_\Sigma)\,(3M^2 + 2\chi N N_\Sigma)} - \frac{2N_\Sigma \mp M}{2\,(3M^2 + 2\chi N N_\Sigma)}\sqrt{\chi^2 N^2 + 2\chi N N_\Sigma + 4\,N_\Sigma^2 - 3M^2}\,. \tag{5.8}$$

Let us now discuss the flavor central charge B for the $\mathrm{SO}(3)_\varphi$ symmetry. The quantity B appears in the 2-point function of two $\mathrm{SO}(3)_\varphi$ symmetry currents. For its normalization, we follow the conventions of [56]. The superconformal algebra relates B to the 't Hooft anomaly between the superconformal R-symmetry and $\mathrm{SO}(3)_\varphi$ [47, 48]. Let us define the 't Hooft anomaly coefficient $\mathcal{A}_{\mathrm{SO}(3)_\varphi}$ by

$$I_6^{\mathrm{SCFT}} = \mathcal{A}_{\mathrm{SO}(3)_\varphi}\, c_1^R\, p_1(\mathrm{SO}(3)_\varphi) + \dots \tag{5.9}$$

We then have

$$B = 4\,\mathcal{A}_{\mathrm{SO}(3)_\varphi}\,. \tag{5.10}$$

The quantity $\mathcal{A}_{\mathrm{SO}(3)_\varphi}$ is extracted from (5.3) by performing the replacements (5.6) and using (5.8). The result for B then reads

$$B = \frac{N^2\left(4\chi^2 N^2 N_\Sigma + 12\chi N N_\Sigma^2 + 24N_\Sigma^3 - 30M^2 N_\Sigma - 9\chi M^2 N\right)}{6\,(3M^2 + 2\chi N N_\Sigma)} - \frac{N^2\left(\chi N N_\Sigma + 2N_\Sigma^2 - M^2\right)}{3M^2 + 2\chi N N_\Sigma}\sqrt{\chi^2 N^2 + 2\chi N N_\Sigma + 4\,N_\Sigma^2 - 3M^2}\,. \tag{5.11}$$

For definiteness, the supergravity computation of appendix A is performed in the case in which the Riemann surface has genus $g \geq 2$ and the flux parameter M is set to zero. The results for c and B are given in (A.25), (A.26) in terms of the quantity $\tilde{r} = -2N_\Sigma/(N\chi)$. They agree perfectly with (5.7), (5.11), respectively.

5.1 Comments on the reduction of the 6d anomaly polynomial

In this section we contrast the approach of section 4 with the direct reduction on the Riemann surface of the anomaly polynomial of the parent 6d (1,0) theory. More precisely, let us consider the inflow anomaly polynomial I_8^{inflow} in (2.16), and let us integrate it on Σ_g. To this end, it is useful to express c_2^ψ and $\mathrm{tr}(F_{6d}^{\mathrm{N,S}})^2$ in terms of 6d Chern roots. Following [22], we have

$$c_2^\psi = -(n_{6d}^\psi)^2\,, \qquad \mathrm{tr}\,\frac{(F_{6d}^{\mathrm{N,S}})^2}{(2\pi)^2} = -4\,(n_{6d}^{\mathrm{N,S}})^2\,. \tag{5.12}$$

The 6d Chern roots split as

$$n_{6d}^\psi = c_1^\psi - \frac{\chi}{2}\,V_2^\Sigma\,, \qquad n_{6d}^{\mathrm{N,S}} = N\,c_1^{\mathrm{N,S}} + N_{\mathrm{N,S}}\,V_2^\Sigma\,, \tag{5.13}$$

JHEP01(2020)117

with V_2^Σ normalized as in (3.13). We are not twisting $\mathrm{SU}(2)_\varphi$, whose connection is thus purely external. Upon integration on Σ_g, we obtain

$$\begin{aligned}-\int_{\Sigma_g} I_8^{\text{inflow}} = &-\frac{1}{3}\chi\left(N^3-\frac{3}{2}N\right)(c_1^\psi)^3 - 2N^2\left(N_{\mathrm{N}}c_1^{\mathrm{N}}+N_{\mathrm{S}}c_1^{\mathrm{S}}\right)(c_1^\psi)^2\\ &+\chi N^3 c_1^\psi\left[(c_1^{\mathrm{N}})^2+(c_1^{\mathrm{S}})^2\right]-2N^2\left[N_{\mathrm{N}}(c_1^{\mathrm{N}})^3+N_{\mathrm{S}}(c_1^{\mathrm{S}})^3\right]\\ &+2N^2\left[N_{\mathrm{N}}c_1^{\mathrm{N}}(c_1^{\mathrm{S}})^2+N_{\mathrm{S}}c_1^{\mathrm{S}}(c_1^{\mathrm{N}})^2\right]-\frac{1}{24}\chi N c_1^\psi p_1(TW_4)\\ &-\frac{1}{3}\chi(N^3-N)c_1^\psi c_2^\varphi-2N^2\left(N_{\mathrm{N}}c_1^{\mathrm{N}}+N_{\mathrm{S}}c_1^{\mathrm{S}}\right)c_2^\varphi\,. \end{aligned} \tag{5.14}$$

By virtue of anomaly matching across dimensions, the above polynomial captures the anomalies of symmetries that are manifest in six dimensions. We stress that the quantity (5.14) generically receives contributions from modes that decouple along the RG flow from the 6d SCFT to the 4d SCFT. As a result, caution must exercised in extracting 4d central charges from (5.14).

For example, if we perform a-maximization at large N using (5.14) as an input, we get a central charge c that does not agree with (5.7). Working for simplicity in the case $M=0$, or equivalently $N^{\mathrm{N}}+N^{\mathrm{S}}=0$, we obtain the results

$$c=-\frac{9N\left(5\chi^2N^2+12\chi NN_\Sigma+12N_\Sigma^2\right)}{32\chi}\,,\qquad s^{\mathrm{N,S}}=\mp\frac{N_\Sigma}{\chi N}\mp\frac{1}{2}\,,\qquad (M=0) \tag{5.15}$$

which have a different structure compared to (5.7), (5.8) at $M=0$, due to the absence of radicals. We have verified numerically in a few examples that the discrepancy between the correct central charge (5.7) and the central charge obtained from (5.14) persists for $M\neq 0$. This test can be regarded as a basis-independent check that (5.14) and (4.11) are inequivalent anomaly polynomials. We interpret this discrepancy as due to decoupling modes in the RG flow from six to four dimensions. A more detailed understanding of this RG flow would be useful in studying the decoupling sector, but is beyond the scope of this work.[5]

If the Riemann surface has genus $g\geq 2$, we can consider the limit $M=0$, $N_\Sigma=-\frac{\chi}{2}N$. This is equivalent to setting $N^{\mathrm{N,S}}=0$. As a result, we are blowing down the resolution 2-cycles, and the geometry re-develops orbifold singularities. In this scenario, the reduction of the 6d anomaly polynomial gives a large-N central charge that agrees with our 4d inflow anomaly polynomial (4.11). We detect, however, a mismatch in the 't Hooft anomaly coefficients for $\mathrm{U}(1)_{\mathrm{N,S}}$. We interpret this discrepancy as being due to decoupled modes in the resolved phase, which have to be re-included in the limit $N^{\mathrm{N,S}}\to 0$.

6 Discussion

In this work we have mainly focused on the 6d (1,0) theory living on a stack of N M5-branes probing a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. We expect, however, that many features of this setup should persist for branes probing $\mathbb{C}^2/\mathbb{Z}_k$ for $k\geq 3$. By resolving the orbifold singularities

[5] We thank Shlomo Razamat for interesting correspondence on this matter.

JHEP01(2020)117

at the north and south poles of $S^4/\mathbb{Z}_k$, the flavor symmetry $\mathrm{SU}(k)_\mathrm{N} \times \mathrm{SU}(k)_\mathrm{S}$ is broken to $[\mathrm{U}(1)^{k-1}]_\mathrm{N} \times [\mathrm{U}(1)^{k-1}]_\mathrm{S}$. We can then compactify on a Riemann surface with a non-trivial twist for this symmetry. We expect the emergence of an accidental U(1) symmetry and the spontaneous breaking of a U(1) generator to occur in such setups.

In the case $k = 2$ the geometry of the resolution of $S^4/\mathbb{Z}_2$ is particularly simple. This facilitates the identification of the gravity duals. Nonetheless, it would be interesting to investigate the gravity duals also for $k \geq 3$. In this case, the internal geometry M_4, associated to the 6d QFT in its resolved phase, is expected to have a smaller $\mathrm{U}(1)_\psi \times \mathrm{U}(1)_\varphi$ isometry, and a more complicated topology. The identification of the dual AdS_5 solutions would be particularly useful, since it would allow us to perform large-N supergravity tests similar to the ones considered in this work for $k = 2$. These solutions should be obtained from BPS system described in [57].

The examples studied in this paper show the power of geometric methods in the study of strongly coupled dynamics of 4d QFTs. In particular, by constructing the 4-form E_4 that governs anomaly inflow from the M-theory ambient space, we are able to track directly the emergence of accidental symmetries and spontaneous symmetry breaking. Our analysis fits into a broader geometrization program, aimed at using geometric and string theoretic tools define and classify non-trivial QFTs, and to uncover their non-perturbative dynamics.

Acknowledgments

We would like to thank Chris Beem, Nikolay Bobev, Ken Intriligator, Ruben Minasian, Emily Nardoni, Shlomo Razamat, Alessandro Tomasiello, Peter Weck for interesting conversations and correspondence. The work of IB and FB is supported in part by NSF grant PHY-1820784. IB gratefully acknowledges the Aspen Center for Physics, supported by NSF grant PHY-1607611, for hospitality during part of this work.

A Supergravity computations

A.1 Review of the GMSW solutions

In this appendix we review a class of M-theory solutions with 4d $\mathcal{N} = 1$ superconformal symmetry, first described in GMSW [46]. The 11d metric reads

$$
\begin{aligned}
ds_{11}^2 &= e^{2\lambda}\left[ds^2(\mathrm{AdS}_5) + ds^2(M_6)\right], \\
ds^2(M_6) &= e^{-6\lambda}\left[F_\varphi\, ds^2(S^2_\varphi) + F_\Sigma\, ds^2(\Sigma_g)\right] + \frac{e^{-6\lambda}}{\cos^2\zeta} d\mu^2 + \frac{\cos^2\zeta}{9}\widehat{D}\psi^2, \\
d\widehat{D}\psi &= (2\pi)\left(-2\,V_2^\varphi - \chi\, V_2^\Sigma\right), \qquad \int_{S^2_\varphi} V_2^\varphi = 1, \qquad \int_{\Sigma_g} V_2^\Sigma = 1.
\end{aligned}
\tag{A.1}
$$

We have set the AdS_5 radius to 1, so that the Ricci scalar of AdS_5 is $R = -20$. The metric on the Riemann surface Σ_g has curvature $k = \pm 1$, with Ricci scalar $R = 2\,k$. Compared with [46], we have flipped the sign of ψ and we have renamed y into μ. All metric functions

JHEP01(2020)117

depend on μ only. They are given by

$$e^{6\lambda} = \frac{2\,(a_\varphi - \mu^2)\,(a_\Sigma - k\,\mu^2)}{a_\Sigma + k\,a_\varphi + 2\,k\,\mu\,(\mu - 3\,\gamma_0)}\,, \quad F_\varphi = \frac{1}{3}(a_\varphi - \mu^2)\,, \quad F_\Sigma = \frac{1}{3}(a_\Sigma - k\,\mu^2)\,, \tag{A.2}$$

where a_φ, a_Σ, γ_0 are constant parameters. The quantity $0 \leq \cos\zeta \leq 1$ is determined by

$$e^{3\lambda}\,\sin\zeta = 2\,\mu\,. \tag{A.3}$$

The G_4-flux configuration is given by

$$\begin{aligned} G_4 &= 2\pi\, d\Big[\big(\widetilde{\gamma}_\varphi\, V_2^\varphi + \widetilde{\gamma}_\Sigma\, V_2^\Sigma\big) \wedge \widehat{D}\psi\Big] \\ &= 2\pi\,\Big[d\widetilde{\gamma}_\varphi \wedge V_2^\varphi + d\widetilde{\gamma}_\Sigma \wedge V_2^\Sigma\Big] \wedge \widehat{D}\psi - (2\pi)^2\,(2\,\widetilde{\gamma}_\Sigma + \chi\,\widetilde{\gamma}_\varphi + 2\,\chi\,\gamma_0)\,V_2^\varphi \wedge V_2^\Sigma\,, \end{aligned} \tag{A.4}$$

with the functions $\widetilde{\gamma}_\varphi$, $\widetilde{\gamma}_\Sigma$ given as

$$\begin{aligned} \widetilde{\gamma}_\varphi &= 2\,\frac{2\,k\,a_\Sigma\,\mu - 6\,k\,a_\Sigma\,\gamma_0 + a_\varphi\,\mu + \mu^3}{9\,k\,\,(a_\Sigma - k\,\mu^2)}\,, \\ \widetilde{\gamma}_\Sigma &= \chi\,\frac{2\,k\,a_\varphi\,\mu - 6\,k\,a_\varphi\,\gamma_0 + a_2\,y + k\,\mu^3}{9\,k\,\,(a_\varphi - \mu^2)}\,. \end{aligned} \tag{A.5}$$

We have put a tilde on $\widetilde{\gamma}_\varphi$, $\widetilde{\gamma}_\Sigma$ to distinguish these functions, coming from the holographic solution, from the functions γ_φ, γ_Σ that enter the parametrization (3.27) of E_4 in the main text. In this appendix, we are adopting conventions in which the quantization of G_4 flux reads

$$\int_{\mathcal{C}_4} \frac{G_4}{(2\pi\ell_p)^3} \in \mathbb{Z}\,, \tag{A.6}$$

where $\mathcal{C}_4$ is a 4-cycle and ℓ_p is the 11d Planck length.

Let us now focus on solutions with $\gamma_0 = 0$. We verify that in this class of solutions the fluxes $N^{\rm N,S}$ defined in (3.11) satisfy

$$N^{\rm N} + N^{\rm S} = 0\,. \tag{A.7}$$

The range of the coordinate μ is determined from the zeros of $\cos\zeta$ and has the form $[-\mu_{\rm N}, \mu_{\rm N}]$, with $\mu_{\rm N} > 0$. We find it useful to distinguish the cases in which the genus g of the Riemann surface is $g \geq 2$ contrasted to $g = 0$. We find

$$\begin{aligned} g = 0\ :\quad & \mu_{\rm N}^2 = -\frac{1}{2}\,(a_\varphi + a_\Sigma) + \frac{1}{2}\,\sqrt{(a_\varphi + a_\Sigma)^2 + \frac{4}{3}\,a_\varphi\,a_\Sigma}\,, \quad a_\varphi, a_\Sigma > 0\,, \\ g \geq 2\ :\quad & \mu_{\rm N}^2 = \frac{1}{2}\,(a_\Sigma - a_\varphi) - \frac{1}{2}\,\sqrt{(a_\Sigma - a_\varphi)^2 - \frac{4}{3}\,a_\varphi\,a_\Sigma}\,, \qquad a_\Sigma > 3\,a_\varphi > 0\,. \end{aligned} \tag{A.8}$$

Recall that the fluxes N and N_Σ are defined as

$$N = \int_{S^2_\varphi \times S^2_{\mu\psi}} \frac{G_4}{(2\pi\ell_p)^3}\,, \qquad N_\Sigma = \int_{\Sigma_g \times S^2_{\mu\psi}} \frac{G_4}{(2\pi\ell_p)^3}\,. \tag{A.9}$$

JHEP01(2020)117

We can express the ratio a_Σ/a_φ in terms of the ratio N_Σ/N,

$$
\begin{aligned}
g=0\ :\quad & \frac{a_\Sigma}{a_\varphi}=\frac{2\,r^2-r+2+2\,(r-1)\,\sqrt{r^2+r+1}}{3\,r}\,, \qquad r:=\frac{N_\Sigma}{N}\,,\\
g\geq 2\ :\quad & \frac{a_\Sigma}{a_\varphi}=\frac{2\,\tilde{r}^2+\tilde{r}+2+2\,\sqrt{\tilde{r}^4+\tilde{r}^3+\tilde{r}+1}}{3\,\tilde{r}}\,, \qquad \tilde{r}:=-\frac{2\,N_\Sigma}{\chi\,N}>1\,.
\end{aligned}
\tag{A.10}
$$

Moreover, we can express the ratio between the Planck length and the AdS_5 scale (which was set to 1 in the line element) in terms of a_Σ, a_φ, $\mu_{\rm N}$, N,

$$
\begin{aligned}
g=0\ :\quad & \frac{\ell_p^3}{L_{\rm AdS}^3}=\frac{2\,\mu_{\rm N}\,(2\,a_\Sigma+a_\varphi+\mu_{\rm N}^2)}{9\,\pi\,N\,(a_\Sigma-\mu_{\rm N}^2)}\,,\\
g\geq 2\ :\quad & \frac{\ell_p^3}{L_{\rm AdS}^3}=\frac{2\,\mu_{\rm N}\,(2\,a_\Sigma-a_\varphi-\mu_{\rm N}^2)}{9\,\pi\,N\,(a_\Sigma+\mu_{\rm N}^2)}\,.
\end{aligned}
\tag{A.11}
$$

A.2 Effective action in five dimensions

In order to compute holographically the central charge c and the flavor central charge for the $SO(3)_\varphi$ isometry of S^2_φ, we need to extract the coefficients of the Einstein-Hilbert term in the 5d effective action, as well as the coefficient of the kinetic terms for the $SO(3)_\varphi$ vectors. To this end, we only need two terms in the 11d M-theory action,

$$
S_{11}=\frac{1}{2\kappa_{11}^2}\int_{M_{11}}\left[R_{(11)}\ *_{11}1-\frac{1}{2}\,G_4\wedge *_{11}G_4+\dots\right]\,, \qquad 2\kappa_{11}^2=(2\pi)^8\,\ell_p^9\,.
\tag{A.12}
$$

The dimensional reduction from 11d to 5d is performed activating the external 5d metric and the gauge fields for $SO(3)_\varphi$. The 11d line element then reads

$$
\begin{aligned}
ds_{11}^2 &= e^{2\lambda}\left[ds^2(M_5)+ds^2(M_6)\right]\,,\\
ds^2(M_6) &= e^{-6\lambda}\left[F_\varphi\,ds^2(S^2_\varphi)^{\rm g}+F_\Sigma\,ds^2(\Sigma_g)\right]+\frac{e^{-6\lambda}}{\cos^2\zeta}d\mu^2+\frac{\cos^2\zeta}{9}\widetilde{D}\psi^2\,,\\
d\widetilde{D}\psi &= (2\pi)\,(-2\,e_2^\varphi-\chi\,V_2^\Sigma)\,, \qquad \int_{S^2_\varphi}e_2^\varphi=1\,, \qquad \int_{\Sigma_g}V_2^\Sigma=1\,,\\
ds^2(S^2_\varphi)^{\rm g} &= Dy^a\,Dy_a\,, \quad y^a\,y_a=1\,, \quad Dy^a=dy^a-A^{ab}\,y_b\,, \quad A^{ab}\equiv\epsilon^{abc}\,A_c\,.
\end{aligned}
\tag{A.13}
$$

The 2-form e_2^φ is defined in (3.29). The gauge fields A^a for $SO(3)_\varphi$ also enter G_4, as described around (3.27). We replace V_2^φ with e_2^φ, and $\widehat{D}\psi$ with $\widetilde{D}\psi$. Therefore, the form of G_4 we use for the reduction is

$$
G_4=2\pi\,d\Big[\big(\widetilde{\gamma}_\varphi\,e_2^\varphi+\widetilde{\gamma}_\Sigma\,V_2^\Sigma\big)\wedge\widetilde{D}\psi\Big]\,.
\tag{A.14}
$$

The dimensional reduction of the 11d Ricci scalar yields

$$
R_{(11)}=e^{-2\lambda}\,R_{(5)}-\frac{1}{4}\,e^{-4\lambda}\left[\frac{1}{9}\,e^{2\lambda}\,\cos^2\zeta\,y^a\,y^b+e^{-4\lambda}\,F_\varphi\,(\delta^{ab}-y^a\,y^b)\right]F^a_{mn}\,F^{b\,mn}+\dots
\tag{A.15}
$$

JHEP01(2020)117

where $m, n = 0, \ldots, 4$ are indices in external 5d spacetime, $R_{(5)}$ is the Ricci scalar of the external 5d metric, and we have only written down the terms that are relevant for our discussion. We also have

$$*_{11}\, 1 = \frac{1}{3}\, e^{-4\lambda}\, F_\varphi\, F_\Sigma\, (4\pi\, V_2^\varphi) \wedge (-2\pi\chi\, V_2^\Sigma) \wedge d\mu \wedge d\psi \wedge (*_5 1)\,, \tag{A.16}$$

with V_2^φ, V_2^Σ normalized as in (A.1), and $*_5$ denoting the Hodge star with respect to the external 5d metric. Finally, one computes

$$\begin{aligned} G_4 \wedge *_{11} G_4 = \frac{e^{-6\lambda}\, F_\varphi}{12\, \chi^2\, F_\Sigma} \left[9\, \chi^2\, F_\Sigma^2 \left(\frac{d\widetilde{\gamma}_\varphi}{d\mu} \right)^2 + e^{6\lambda}\, (2\, \widetilde{\gamma}_\Sigma + \chi\, \widetilde{\gamma}_\varphi)^2 \right] z^a\, z^b\, F_a \wedge *_5 F_b \wedge \\ \wedge\, (4\pi\, V_2^\varphi) \wedge (-2\pi\chi\, V_2^\Sigma) \wedge d\mu \wedge d\psi + \ldots \end{aligned} \tag{A.17}$$

where we have only written down the terms that can saturate the integration along the internal directions. Notice that our conventions for the Hodge star is such that $F^a \wedge *_5 F_a = \frac{1}{2}\, F^a_{mn}\, F_a^{mn}\; *_5\, 1$.

We are now in a position to perform the integral over the internal directions. The result reads

$$\int_{M_{11}} \left[R_{(11)}\, *_{11}\, 1 - \frac{1}{2}\, G_4 \wedge *_{11} G_4 \right] = \int_{M_5} \left[\alpha_1\, R_{(5)}\, *_5\, 1 + \alpha_2\, F^a \wedge *_5 F_a + \ldots \right], \tag{A.18}$$

with the coefficients α_1, α_2 given by

$$\begin{aligned} \alpha_1 &= \left[-\frac{8}{81}\, \pi^3\, \mu\, \chi \left(3\, a_\Sigma + 3\, k\, a_\varphi + 2\, k\, \mu^2 \right) \right]_{-\mu_{\rm N}}^{+\mu_{\rm N}}, \\ \alpha_2 &= \left[\frac{4\pi^3 \mu\chi}{2187\, (a_\Sigma - k\mu^2)^3} \left(6k\mu^6 a_\Sigma - 15 k\mu^2 a_\Sigma a_\varphi^2 + k\mu^2 a_\Sigma^3 + 3 k a_\Sigma a_\varphi^3 + 45 k a_\Sigma^3 a_\varphi \right. \right. \\ &\qquad \left. \left. + 18\mu^6 a_\varphi + 15\mu^4 a_\Sigma^2 + 9\mu^4 a_\varphi^2 - 9\mu^2 a_\Sigma^2 a_\varphi - \mu^2 a_\varphi^3 + 24 a_\Sigma^2 a_\varphi^2 + 24 a_\Sigma^4 + 8\mu^8 \right) \right]_{-\mu_{\rm N}}^{+\mu_{\rm N}}. \end{aligned} \tag{A.19}$$

We adopt the following parametrization of the 5d effective action,

$$S_5 = \int_{M_5} \left[\frac{1}{16\pi G_N^{(5)}}\, R_{(5)}\, *_5\, 1 + \frac{1}{g_{SG}^2}\, {\rm Tr}_{\rm f}(F \wedge *_5 F) + \ldots \right], \tag{A.20}$$

where the trace is in the fundamental representation of $\mathrm{SU}(2)_\varphi$, with conventions

$${\rm Tr}_{\rm f}(T^a\, T^b) = \frac{1}{2}\, \delta^{ab}\,, \qquad {\rm Tr}_{\rm f}(F \wedge *_5 F) = \frac{1}{2}\, F^a \wedge *_5 F_a\,, \tag{A.21}$$

where T^a are the generators of $\mathrm{SU}(2)_\varphi$. Our definition of g_{SG}^2 agrees with the conventions of [56]. Keeping into account the prefactor $1/(2\kappa_{11}^2)$ in the M-theory action (A.12), the reduction result (A.18) translates into

$$G_N^{(5)} = 2^4\, \pi^7\, \ell_p^9\, \alpha_1^{-1}\,, \qquad g_{SG}^2 = 2^7\, \pi^8\, \ell_p^9\, \alpha_2^{-1}\,. \tag{A.22}$$

JHEP01(2020)117

The holographic central charge c and $\mathrm{SO}(3)_\varphi$ flavor central charge B, in the notation of [56], are given in terms of $G_N^{(5)}$, g_{SG} as

$$c = \frac{\pi\, L_{\mathrm{AdS}}^3}{8\, G_N^{(5)}}\,, \qquad B = \frac{8\pi^2}{g_{SG}^2}\,, \tag{A.23}$$

so that we have the identifications

$$c = 2^{-7}\,\pi^{-6}\left[\frac{\ell_p}{L_{\mathrm{AdS}}}\right]^{-9}\alpha_1\,, \qquad B = 2^{-4}\,\pi^{-6}\left[\frac{\ell_p}{L_{\mathrm{AdS}}}\right]^{-9}\alpha_2\,. \tag{A.24}$$

For definiteness, we proceed in the case in which the Riemann surface has genus $g \geq 2$ and the parameter γ_0 is set to zero. We may then use the relations (A.8), (A.10), (A.11) and express c and B in terms of $\tilde{r}$,

$$c = \frac{9}{32}\,\chi\, N^3\left[(\tilde{r}-1)\,(2\,\tilde{r}^2-\tilde{r}+2) - 2\,(\tilde{r}^2-\tilde{r}+1)^{3/2}\right], \tag{A.25}$$

$$B = -\frac{1}{16}\,\chi\, N^3\left[-\frac{8}{3}\,(3\,\tilde{r}^2-3\,\tilde{r}+2) + 8\,(\tilde{r}-1)\,\sqrt{\tilde{r}^2-\tilde{r}+1}\right]. \tag{A.26}$$

To get the above expressions we had to de-nest some nested radicals. The supergravity results for c, B match perfectly with the large-N field theory analysis performed in the main text.

References

[1] I. Bah, F. Bonetti, R. Minasian and E. Nardoni, *Anomalies of QFTs from M-theory and Holography*, arXiv:1910.04166 [INSPIRE].

[2] E. Witten, *Some comments on string dynamics*, in *Future perspectives in string theory. Proceedings, Conference, Strings'95*, Los Angeles, U.S.A., 13–18 March 1995, pp. 501–523 (1995) [hep-th/9507121] [INSPIRE].

[3] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059] [INSPIRE].

[4] I. Brunner and A. Karch, *Branes and six-dimensional fixed points*, *Phys. Lett.* **B 409** (1997) 109 [hep-th/9705022] [INSPIRE].

[5] J.D. Blum and K.A. Intriligator, *Consistency conditions for branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 223 [hep-th/9705030] [INSPIRE].

[6] J.D. Blum and K.A. Intriligator, *New phases of string theory and 6-D RG fixed points via branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 199 [hep-th/9705044] [INSPIRE].

[7] K.A. Intriligator, *New string theories in six-dimensions via branes at orbifold singularities*, *Adv. Theor. Math. Phys.* **1** (1998) 271 [hep-th/9708117] [INSPIRE].

[8] I. Brunner and A. Karch, *Branes at orbifolds versus Hanany Witten in six-dimensions*, *JHEP* **03** (1998) 003 [hep-th/9712143] [INSPIRE].

JHEP01(2020)117

[9] A. Hanany and A. Zaffaroni, *Branes and six-dimensional supersymmetric theories*, *Nucl. Phys.* **B 529** (1998) 180 [hep-th/9712145] [INSPIRE].

[10] D. Gaiotto, $N = 2$ *dualities*, *JHEP* **08** (2012) 034 [arXiv:0904.2715] [INSPIRE].

[11] D. Gaiotto, G.W. Moore and A. Neitzke, *Wall-crossing, Hitchin Systems and the WKB Approximation*, arXiv:0907.3987 [INSPIRE].

[12] K. Maruyoshi, M. Taki, S. Terashima and F. Yagi, *New Seiberg Dualities from $N = 2$ Dualities*, *JHEP* **09** (2009) 086 [arXiv:0907.2625] [INSPIRE].

[13] F. Benini, Y. Tachikawa and B. Wecht, *Sicilian gauge theories and $N = 1$ dualities*, *JHEP* **01** (2010) 088 [arXiv:0909.1327] [INSPIRE].

[14] I. Bah and B. Wecht, *New $N = 1$ Superconformal Field Theories In Four Dimensions*, *JHEP* **07** (2013) 107 [arXiv:1111.3402] [INSPIRE].

[15] I. Bah, C. Beem, N. Bobev and B. Wecht, *AdS/CFT Dual Pairs from M5-Branes on Riemann Surfaces*, *Phys. Rev.* **D 85** (2012) 121901 [arXiv:1112.5487] [INSPIRE].

[16] I. Bah, C. Beem, N. Bobev and B. Wecht, *Four-Dimensional SCFTs from M5-Branes*, *JHEP* **06** (2012) 005 [arXiv:1203.0303] [INSPIRE].

[17] D. Gaiotto and S.S. Razamat, $\mathcal{N} = 1$ *theories of class* $\mathcal{S}_k$, *JHEP* **07** (2015) 073 [arXiv:1503.05159] [INSPIRE].

[18] K. Ohmori, H. Shimizu, Y. Tachikawa and K. Yonekura, *6d $\mathcal{N} = (1, 0)$ theories on T^2 and class S theories: Part I*, *JHEP* **07** (2015) 014 [arXiv:1503.06217] [INSPIRE].

[19] M. Del Zotto, C. Vafa and D. Xie, *Geometric engineering, mirror symmetry and* $6\mathrm{d}_{(1,0)} \to 4\mathrm{d}_{(\mathcal{N}=2)}$, *JHEP* **11** (2015) 123 [arXiv:1504.08348] [INSPIRE].

[20] K. Ohmori, H. Shimizu, Y. Tachikawa and K. Yonekura, *6d $\mathcal{N} = (1, 0)$ theories on S^1/T^2 and class S theories: part II*, *JHEP* **12** (2015) 131 [arXiv:1508.00915] [INSPIRE].

[21] S.S. Razamat, C. Vafa and G. Zafrir, *4d $\mathcal{N} = 1$ from 6d $(1, 0)$*, *JHEP* **04** (2017) 064 [arXiv:1610.09178] [INSPIRE].

[22] I. Bah, A. Hanany, K. Maruyoshi, S.S. Razamat, Y. Tachikawa and G. Zafrir, *4d $\mathcal{N} = 1$ from 6d $\mathcal{N} = (1, 0)$ on a torus with fluxes*, *JHEP* **06** (2017) 022 [arXiv:1702.04740] [INSPIRE].

[23] H.-C. Kim, S.S. Razamat, C. Vafa and G. Zafrir, *E-String Theory on Riemann Surfaces*, *Fortsch. Phys.* **66** (2018) 1700074 [arXiv:1709.02496] [INSPIRE].

[24] H.-C. Kim, S.S. Razamat, C. Vafa and G. Zafrir, *D-type Conformal Matter and SU/USp Quivers*, *JHEP* **06** (2018) 058 [arXiv:1802.00620] [INSPIRE].

[25] H.-C. Kim, S.S. Razamat, C. Vafa and G. Zafrir, *Compactifications of ADE conformal matter on a torus*, *JHEP* **09** (2018) 110 [arXiv:1806.07620] [INSPIRE].

[26] S.S. Razamat and G. Zafrir, *Compactification of 6d minimal SCFTs on Riemann surfaces*, *Phys. Rev.* **D 98** (2018) 066006 [arXiv:1806.09196] [INSPIRE].

[27] G. Zafrir, *On the torus compactifications of Z_2 orbifolds of E-string theories*, *JHEP* **10** (2019) 040 [arXiv:1809.04260] [INSPIRE].

[28] K. Ohmori, Y. Tachikawa and G. Zafrir, *Compactifications of 6d $N = (1, 0)$ SCFTs with non-trivial Stiefel-Whitney classes*, *JHEP* **04** (2019) 006 [arXiv:1812.04637] [INSPIRE].

[29] J. Chen, B. Haghighat, S. Liu and M. Sperling, *4d N=1 from 6d D-type $N = (1, 0)$*, arXiv:1907.00536 [INSPIRE].

JHEP01(2020)117

[30] S.S. Razamat, E. Sabag and G. Zafrir, *From 6d flows to 4d flows*, *JHEP* **12** (2019) 108 [arXiv:1907.04870] [INSPIRE].

[31] S. Pasquetti, S.S. Razamat, M. Sacchi and G. Zafrir, *Rank Q E-string on a torus with flux*, arXiv:1908.03278 [INSPIRE].

[32] S.S. Razamat and E. Sabag, *Sequences of 6d SCFTs on generic Riemann surfaces*, arXiv:1910.03603 [INSPIRE].

[33] L. Álvarez-Gaumé and E. Witten, *Gravitational Anomalies*, *Nucl. Phys.* **B 234** (1984) 269 [INSPIRE].

[34] L. Álvarez-Gaumé and P.H. Ginsparg, *The Structure of Gauge and Gravitational Anomalies*, *Annals Phys.* **161** (1985) 423 [*Erratum ibid.* **171** (1986) 233] [INSPIRE].

[35] W.A. Bardeen and B. Zumino, *Consistent and Covariant Anomalies in Gauge and Gravitational Theories*, *Nucl. Phys.* **B 244** (1984) 421 [INSPIRE].

[36] M.J. Duff, J.T. Liu and R. Minasian, *Eleven-dimensional origin of string-string duality: A One loop test*, *Nucl. Phys.* **B 452** (1995) 261 [hep-th/9506126] [INSPIRE].

[37] E. Witten, *Five-brane effective action in M-theory*, *J. Geom. Phys.* **22** (1997) 103 [hep-th/9610234] [INSPIRE].

[38] D. Freed, J.A. Harvey, R. Minasian and G.W. Moore, *Gravitational anomaly cancellation for M-theory five-branes*, *Adv. Theor. Math. Phys.* **2** (1998) 601 [hep-th/9803205] [INSPIRE].

[39] J.A. Harvey, R. Minasian and G.W. Moore, *NonAbelian tensor multiplet anomalies*, *JHEP* **09** (1998) 004 [hep-th/9808060] [INSPIRE].

[40] I. Bah and E. Nardoni, *Structure of Anomalies of 4d SCFTs from M5-branes and Anomaly Inflow*, *JHEP* **03** (2019) 024 [arXiv:1803.00136] [INSPIRE].

[41] I. Bah, F. Bonetti, R. Minasian and E. Nardoni, *Class $\mathcal{S}$ Anomalies from M-theory Inflow*, *Phys. Rev.* **D 99** (2019) 086020 [arXiv:1812.04016] [INSPIRE].

[42] I. Bah, F. Bonetti, R. Minasian and E. Nardoni, *Anomaly Inflow for M5-branes on Punctured Riemann Surfaces*, *JHEP* **06** (2019) 123 [arXiv:1904.07250] [INSPIRE].

[43] D. Gaiotto and J. Maldacena, *The Gravity duals of $N = 2$ superconformal field theories*, *JHEP* **10** (2012) 189 [arXiv:0904.4466] [INSPIRE].

[44] S.S. Gubser, *Breaking an Abelian gauge symmetry near a black hole horizon*, *Phys. Rev.* **D 78** (2008) 065034 [arXiv:0801.2977] [INSPIRE].

[45] S.A. Hartnoll, C.P. Herzog and G.T. Horowitz, *Building a Holographic Superconductor*, *Phys. Rev. Lett.* **101** (2008) 031601 [arXiv:0803.3295] [INSPIRE].

[46] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of M-theory*, *Class. Quant. Grav.* **21** (2004) 4335 [hep-th/0402153] [INSPIRE].

[47] D. Anselmi, D.Z. Freedman, M.T. Grisaru and A.A. Johansen, *Universality of the operator product expansions of SCFT in four-dimensions*, *Phys. Lett.* **B 394** (1997) 329 [hep-th/9608125] [INSPIRE].

[48] D. Anselmi, *Central functions and their physical implications*, *JHEP* **05** (1998) 005 [hep-th/9702056] [INSPIRE].

[49] K.A. Intriligator and B. Wecht, *The Exact superconformal R symmetry maximizes a*, *Nucl. Phys.* **B 667** (2003) 183 [hep-th/0304128] [INSPIRE].

JHEP01(2020)117

[50] A. Sen, *A Note on enhanced gauge symmetries in M and string theory*, *JHEP* **09** (1997) 001 [hep-th/9707123] [INSPIRE].

[51] T. Eguchi and A.J. Hanson, *Asymptotically Flat Selfdual Solutions to Euclidean Gravity*, *Phys. Lett.* **74B** (1978) 249 [INSPIRE].

[52] M.K. Prasad, *Equivalence of Eguchi-Hanson metric to two-center Gibbons-Hawking metric*, *Phys. Lett.* **83B** (1979) 310 [INSPIRE].

[53] K. Ohmori, H. Shimizu, Y. Tachikawa and K. Yonekura, *Anomaly polynomial of general 6d SCFTs*, *PTEP* **2014** (2014) 103B07 [arXiv:1408.5572] [INSPIRE].

[54] F. Benini, N. Bobev and P.M. Crichigno, *Two-dimensional SCFTs from D3-branes*, *JHEP* **07** (2016) 020 [arXiv:1511.09462] [INSPIRE].

[55] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *AdS spacetimes from wrapped M5 branes*, *JHEP* **11** (2006) 053 [hep-th/0605146] [INSPIRE].

[56] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the CFT_d/AdS_{d+1} correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058] [INSPIRE].

[57] I. Bah, *AdS_5 solutions from M5-branes on Riemann surface and D6-branes sources*, *JHEP* **09** (2015) 163 [arXiv:1501.06072] [INSPIRE].

JHEP01(2020)117