PRIMUS

Problems, Resources, and Issues in Mathematics Undergraduate Studies

ISSN: 1051-1970 (Print) 1935-4053 (Online) Journal homepage: https://www.tandfonline.com/loi/upri20

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To cite this article: Kristen N. Bieda, Aditya Visnawathan, Raven McCrory & Pavel Sikorskii (2020) The UTE Model: Enhancing Learning in Developmental Mathematics and Preparing Mathematics Teachers of the Future, PRIMUS, 30:7, 750-761, DOI: 10.1080/10511970.2019.1626958

To link to this article: https://doi.org/10.1080/10511970.2019.1626958

Accepted author version posted online: 04 Jun 2019.
Published online: 26 Jul 2019.

PRIMUS, 30(7): 750–761, 2020

ISSN: 1051-1970 print / 1935-4053 online
DOI: 10.1080/10511970.2019.1626958

The UTE Model: Enhancing Learning in Developmental Mathematics and Preparing Mathematics Teachers of the Future

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Abstract: Preparing students placing into developmental mathematics for success in undergraduate mathematics and preparing future teachers for the increasing demands of K–12 school settings are both persistent, yet seemingly divergent, problems facing higher education. In this paper, we shed light on a model that attempts to address aspects of each problem and share preliminary evidence of success in a multi-year application of this model. The paper highlights the benefits of implementing the model, particularly to support collaboration between faculty in mathematics and teacher education.

Keywords: Developmental mathematics, teacher preparation, mathematics education

1. INTRODUCTION

At first glance, the issues facing those working in post-secondary mathematics departments and teacher preparation programs may seem divergent. For the former, one persistent issue that seems to be growing is the large number of incoming freshman who place or enroll in mathematics classes, often non-credit-bearing, below college algebra. Mathematics department chairs and curriculum coordinators wrestle with the tensions of supporting students entering college underprepared for collegiate-level mathematics with non-credit-bearing courses that must efficiently address gaps in students' mathematical knowledge. Nationally, approximately 40% of incoming freshmen interested

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in STEM majors enroll either in developmental mathematics or introductory-level math courses, and approximately half of those students abandon STEM majors after their first year of post-secondary education [3].

Those in teacher preparation programs face a long-standing issue of addressing the need to provide apprenticeship experiences, prior to student teaching, that immerse prospective teachers (PTs) in learning from doing the work of teaching [2, 11]. Although increasing the amount of time prospective teachers spend in school-based placements is a predominant approach for improving the preparation of future teachers, there can be a disconnect between what PTs see and experience in K–12 classrooms and what they learn about effective teaching in on-campus methods courses [1,11]. A growing body of research shows that novice teachers may be better prepared for their first year of teaching when they have more field experience, but the quality of such field experience matters [8].

This paper will describe a model that bridges these two problems and provides a possible approach to their solution. The university-based teacher education (UTE) model is a field experience for secondary mathematics PTs where they take on responsibilities for instructing in an undergraduate developmental mathematics course designed in collaboration with mathematics education and mathematics faculty across mathematics and teacher education departments. In the UTE, secondary mathematics PTs receive ongoing mentoring and "on-the-spot" instructional support as they learn to implement inquiry-oriented lessons that focus on the "big" mathematical ideas of intermediate algebra. Our work to pilot, refine, and test this model within the context of a large university setting revealed promising results for the impact of the model on both PTs' learning and preparation to teach secondary mathematics, as well as developmental mathematics students' success in preparing for college-level mathematics.

2. DEVELOPMENT OF THE UTE MODEL

The UTE model was developed by the *Teacher Education and Mathematics* (TEAM) project through successive iterations in 2012, 2013, and 2014[1] in an undergraduate non-credit-bearing, remedial mathematics course (hereafter referred to as "developmental mathematics") at a large, midwestern university in the USA. One component of the model (*inquiry-oriented curriculum and task design*) involved a cross-departmental collaboration between mathematics teacher educators (MTEs) affiliated with the College of Education and Department of Mathematics faculty responsible for the developmental mathematics course curriculum. Faculty across both departments collaborated to design and select tasks (individual or series of problems) requiring a high level of

[1]Funded by an NSF TUES award (DUE-1245402).

cognitive demand [9] for the course, featuring core concepts related to understanding of functions, proportional reasoning, behavior of linear functions, behavior of quadratic functions, and behavior of rational functions.

The second component (*plan and implement*) in the UTE model involves PTs planning and teaching lessons in the developmental mathematics class using the tasks designed and selected as described in component one above. While enrolled in a secondary mathematics teaching methods course (hereafter referred to as the "methods course"), the PTs plan and teach in pairs in the developmental mathematics class to provide an opportunity for them to experience an approximation of teaching practice [6] in an authentic learning setting. Other peer PTs enrolled in the methods course observe as pairs teach. Finally, a third component of the UTE is the *mentoring* that is provided by MTEs during the planning, implementation, and reflection stages of PTs' teaching in the developmental mathematics course. In our implementation, the MTEs include the secondary mathematics methods instructor (also the instructor of record for the developmental mathematics class) as well as a graduate teaching assistant (TA) with secondary mathematics teaching background. The MTEs model teaching practices and provide in-the-moment coaching, if needed, during PTs' teaching episodes. After each developmental math class session, MTEs orchestrate a debrief discussion with the lead PT(s) for each class and their peer PTs who observed the lesson to discuss the development of mathematics students' thinking and react to the decision making of the lead PT(s). In addition, the debrief discussion offers an opportunity for peer PTs preparing to teach subsequent lessons to rehearse the beginning, or task set-up, phase of the lesson prior to actual enactment in the developmental mathematics course. All of these components are summarized in Table 1.

The UTE model is novel not only in its approach to field experiences in teacher preparation, but also in its reform of developmental mathematics courses. Several peer tutoring or peer assistant models have been designed and tested that support developmental mathematics students' learning [4, 10]. Although an affordance of these approaches compared to the UTE model is that peer assistants or tutors can work in a one-to-many configuration, a key difference between these approaches and the UTE model is that the PT instructors in the UTE assume responsibility for teaching in the course. This necessitates that they understand the scope and sequence of the curriculum, and the added responsibility encourages PTs to take ownership of the developmental mathematics students' learning.

2.1. A Sample UTE Lesson

To illustrate one product of the first component of the UTE model, curriculum and task design, an actual task taught by PTs in the developmental math course during all implementations of the model is described below. The task

Table 1. Components of the UTE model

	Developmental Math Course (1 section)	Secondary Mathematics Teaching Methods Course	University Teaching Experience (joint class with Developmental Math section and Secondary Mathematics Teaching Laboratory Experience)
Instructor(s)	TA or mathematics dept. faculty	MTE faculty and mathematics education TA	MTEs, mathematics teacher education TA or mathematics dept. faculty ~20 PTs (10 PTs attend each session – two mathematics class sessions per week). Each PST co-teaches two lessons per semester, beginning in week 3
Students	~30–50 freshman and sophomores	~20 secondary PTs	~30–50 freshman and sophomores in math class
Course credits	Three credits (does not count toward degree)	Five credits (three credits for seminar and two credits for laboratory experiences)	Three credits (does not count toward degree)
Contact hours per week	3 hours 40 minutes	3 hours for seminar and 4 total hours for laboratory (two laboratory hours for each student)	3 hours 40 minutes for Developmental Math students Each PST attends one Developmental Math class for ~1 hr. 50 mins per week (~20 PTs attend one debrief session per week after developmental math class for 30 minutes)
Resources for learning	ALEKS online course	Seminar curriculum	MTE supports planning and provides in-the-moment coaching for PTs MTE and TAs develop high cognitive demand tasks for Developmental Math class to be implemented in group work setting PTs observing sit at tables with Developmental Math students and support group work

focuses on the goal of strengthening students' proportional reasoning and understanding of the behavior of rational functions, with a focus on the conceptual foundation of simplifying rational expressions or, put another way, writing equivalent rational expressions. These include tasks such as showing $(x^2 - 5x + 6)/(x - 3)$ is equivalent to $(x - 2)$ when $x \neq 3$.

The lesson enacted by PTs in our UTE focused on reinforcing the application of the multiplicative identity and inverse to the set of rational expressions, namely that multiplying any rational expression by one does not change its value and that any expression divided by itself equals one.[2] So, as a first step, PTs spent considerable time having students in the developmental math class discuss the following questions:

1. What do you know about the number one?
2. Does anything divided by itself equal one? For example, does $16x/16x = 1$ when x is not zero?

Although some might feel that the first question is not mathematically important for the lesson, it is necessary for achieving the goals of the lesson. First, it is a "low floor, high ceiling" task. The open-ended nature of the question allows all students, whether they see themselves as mathematically capable or not, to participate and engage. Lack of motivation in developmental mathematics has been documented in research at the post-secondary level [7], and promoting engagement is critical for students to be successful in any math class. Second, the first question invites students to take a stance of curiosity about mathematics. Instead of passively waiting for the instructor to tell them what to think, students are invited to wonder about the properties of the number one. Beyond the affective dimensions of this task, it allows at least two important ideas to surface which are necessary to prepare students for discussion of the second question. One idea is that one has "special" properties that make it different from any other number, and another idea is that is has a "special" property that *anything multiplied by one remains the same*.

The second question, where students consider whether anything divided by its self is equal to one, follows from the multiplicative identity property. The example given to illustrate the question was carefully chosen to bring students to consider this claim for a member of the set of rational expressions. As the lesson unfolds, this property becomes the mechanism by which to find equivalent rational expressions, often describing as "simplifying rational expressions." In class, this action is called "finding the one." The PTs build from students' arguments about why dividing something by itself equals one in problem 2 to cases where they see the same algebraic expression in the numerator and denominator of a rational expression and, instead of marking

[2]We used Lesson 10.1.1 and 10.1.2 from *College Preparatory Mathematics* [5] as inspiration for our lesson design.

through the expressions in an act of “cancelling,” write a “1” to show the product of the division is one in those cases.

This introductory work sets students up to “find the one” rather than thinking of generating equivalent rational expressions as cancelling like terms. The next problem in the lesson works to surface a common issue students face if they hold the conception of cancelling like terms, shown below:

$$\frac{4x}{x} = ? \quad \frac{4+x}{x} = ?$$

Students who hold a conception that simplifying rational expressions involves cancelling like terms will conclude that both of these expressions equal four. We have found that showing these problems side-by-side is helpful in challenging this conception. It can present an opportunity for students to consider whether it could be true that both would equal four in all cases, if we know $4x = 4 + x$ only if $x = 4/3$. What seems to have even more of an impact is to ask student to “find the one.” The selection of these problems, where the only difference is if x is a factor or another term, facilitates students to think about how to re-write the expressions with a multiplicative identity element instead of looking to eliminate values that are “like.”

Students are then given some problems to practice the technique of “finding the one,” such as $(2x^2 - x - 10)/(3x^2 + 7x + 2)$. The problems are given with these instructions:

> In the previous problems, you may have noticed that the numerator and denominator of a rational expression must both be written as a product before any terms create a 1. Examine the expressions below. Factor the numerator and denominator of each fraction, if necessary. That is, rewrite each one as a product and look for “ones” and simplify. For each, assume the denominator is not zero.

3. IMPACT OF UTE MODEL ON DEVELOPMENTAL MATHEMATICS ACHIEVEMENT

Having secondary mathematics PTs teach lessons like the “find the one” lesson described above to students enrolled in a non-credit-bearing developmental math class undoubtedly provides a rich learning opportunity for PTs, but a core concern of our project team was how novice instructors might impact the learning of the developmental math students. Part-time instructors with the lowest levels of education and teaching experience are more likely to teach developmental math across 2-year and 4-year colleges (http://www.ccsse.org/docs/PTF_Special_Report.pdf). In contrast, students involved with the developmental math course taught in our UTE model are taught by PTs who receive mentoring from MTEs who have secondary math teaching experience and are either pursuing or hold a Ph.D. in mathematics education before, during *and* after instruction in the developmental math class.

We compared the scores of students in our UTE section of the developmental math class on common course exams written and administered in the Department of Mathematics and their final course grades with a matched sample of students in each of the 3 years of implementation (2012, 2013, and 2014) who were taking a different section of the developmental math course that met for an equivalent number of sessions and times per week. The matched sample was selected to correspond to the median household income, ethnicity, ACT math sub-score and mathematics placement exam score of the UTE section students. Other sections used a different curriculum than that used in the UTE section, one that had the same sequence of topics, but more emphasis on understanding and practice of procedures. The instructors of these other sections were predominately upper-level undergraduates hired by the Department of Mathematics and supervised by a full-time instructional specialist within the department. For each lesson, the specialist provided these undergraduate instructors with a set of worked examples to present to the class before offering one-on-one help to students as they completed worksheets provided in class. In summary, the major differences between the UTE section and the other sections of the course were the inquiry-oriented curriculum and the mentoring and coaching the undergraduate PTs received as they planned and taught each lesson.

There are a few important findings to note in Table 2. First, in 2012 and 2013, students in the UTE section had higher means on the course exams and their final grade than their counterparts in other sections of the course. In general, these were not statistically significant, but the trends suggest that students in the UTE section perform as well as students in other sections of the developmental mathematics course. However, results for 2014 appear to provide a rebuttal for this claim, as students in the UTE section in 2014 did not perform as well as their counterparts. We investigated the data for the 2014 cohort and noted a few differences between the 2014 experience and the 2012 and 2013 experiences. First, the total number of students who completed the 2014 UTE section of the developmental mathematics course was significantly lower than in previous years. Only 14 students completed the course compared with 34 and 24 students in 2012 and 2013, respectively. Second, four students performed much worse on all of the exams than the other 10 students in the UTE section. If we remove the exam scores of those four students, the mean final grade of the 2014 UTE section increases to 77.

3.1. Learning Experience for the PTs

Although the data from exams illustrates that students taking the section of developmental mathematics taught with the UTE model fared at least as well as, and in some cases better than, peers in other sections, the positive effects were pronounced for the PTs who gained experience teaching in the

Table 2. Matched sample comparisons with effect sizes (Cohen's *d*)

	UTE section 2012	Matched sample 2012 mean (effect size)	UTE section 2013	Matched sample 2013 mean (effect size)	UTE section 2014	Matched sample 2014 mean (effect size)
Exam 1	72.6	72.8(−0.10)	71.3	63.2(0.45)	55.2(−0.01)	64.1(−0.39)
Exam 2	78.2	76.4(0.09)	81.3	75.5(0.32)	76.3(−0.09)	84.9(−0.60*)
Exam 3	58.0	42.3(0.68)	57.9	54.6(0.15)	47.9(0.07)	66.9(−0.97**)
Exam 4	64.0	62.9(0.05)	74.0	70.8(0.15)	43.9(0.55*)	74.1(−0.81**)
Final Grade	64.6	59.7(0.25)	73.9	71.6(0.19)	63.2(0.41)	77.2(−0.58*)

* Indicates significance at the $p < 0.10$ level

** Indicates significance at the $p < 0.05$ level

UTE model. During discussions about the UTE experience in their teaching methods courses, PTs would remark:

> This experience is motivating me to learn how to be the best teacher I can be, so that my students don't end up in developmental mathematics when they go to college.

Such remarks show that the PTs recognize the challenges undergraduates face in completing their degree when they must take a non-credit-bearing course, and the awareness that their time in teacher preparation experiences is critical for helping them be the most effective novice teacher they can be.

In one-on-one interviews, PTs also shared that the UTE experience was in some ways more beneficial for their learning than their required weekly visits in a secondary mathematics class as a part of their teaching methods course. They frequently mentioned how the in-the-moment coaching during their UTE teaching, and the mentoring they received from the teacher educators, was invaluable in helping them understand the complex work of mathematics teaching. As one PST shared:

> in my placement class, um, I don't think the teacher would have stepped in unless it was a real like, um, train wreck, I guess? Um, meanwhile [Kristen] or the TAs in the UTE would be willing to step in for smaller things, just like, hey, think about this, or whisper in our ears, hey, think about this.

The UTE model allows teacher educators to teach about teaching while PTs are *doing* the teaching, and, unlike school-based mentor teachers, while only responsible for a single mathematics course. In the UTE model, the PTs' mentors are teacher educators, compared with a school-based placement where their mentors' primary roles are as full-time mathematics teachers and have teacher education as a secondary role. For an initial teaching experience, the targeted attention from teacher educators is crucial.

4. DISCUSSION AND CONCLUSION

So, what do the results of this initiative tell us? The UTE model does not solve a problem of providing high-quality instruction to large numbers of students in non-credit-bearing developmental mathematics courses. Nor does the UTE model entirely solve a problem of providing optimal field placements for teacher candidates that are as instrumental for learning as being paired, one-on-one, with a secondary mathematics mentor teacher who models high-quality teaching practice. It is increasingly more the case that a solution does not exist to either problem. The real solution to the developmental mathematics course dilemma is for *students to not need developmental mathematics.* And that is why the UTE model is so important – the UTE model affords prospective secondary mathematics teachers with an opportunity to learn to teach mathematics with students who have, in some ways, been failed by the system of K–12 education. PTs who complete the UTE often say that one of the biggest take-aways they have from the experience is that they want to become the best teacher they can be so their future students do not end up taking a developmental mathematics course.

Although more work is needed to explore how the UTE model could be applied in different kinds of contexts – such as teacher preparation institutions partnering with a developmental math course in a 2-year college or having PTs work in a different course at a 4-year institution – our existing work provides an existence proof that the UTE model supports collaboration among faculty across Departments of Mathematics and Colleges of Education for the mutual benefit of both academic units. The collaboration is an integral part of the model; the first component (curriculum development) necessitates that faculty across departments discuss expectations and possibilities for the course curriculum. The curriculum reform conducted as part of our UTE implementation was an early precursor to what is now a campus-wide initiative to ensure that greater numbers of students, particularly students of color, are taking credit-bearing mathematics courses as freshman. This effort reflects that there is now joint attention of faculty in the Department of Mathematics and College of Education on the same problem, to provide greater access to all students for success in college mathematics, whether by ensuring students arrive well-prepared for college mathematics or ensuring that their college mathematics experiences prepare them for success in their major.

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BIOGRAPHICAL SKETCHES

Kristen N. Bieda is an associate professor of mathematics education at Michigan State University. Her research focuses on classroom practices related to reasoning and proof in middle grades and secondary mathematics, with the goal of informing teacher education, curriculum, and professional development programs. Other interests include the use of lesson study in teacher preparation and the development of pre-service teachers' mathematical knowledge for teaching through the use of curriculum as well as video-based representations of teaching.

Aditya Visnawathan is an assistant professor of mathematics at the University of Michigan – Dearborn. His research interests lie at the intersection of applied mathematics, signal processing and scientific computing. He received a Ph.D. and M.S. degrees in electrical engineering from Arizona State University under the supervision of Anne Gelb and Douglas Cochran and a B.E. in electronics and communication engineering from R.V. College of Engineering, Bangalore, India.

Raven McCrory is Professor Emeritus at Michigan State University, where she served for 3 years as the Associate Director for Mathematics for the CREATE for STEM Institute and as an associate professor for the Department of Teacher Education. Her research involved studying the mathematical education of teachers and exploring the knowledge needed for teaching K–12 mathematics. She also investigated the impact of textbooks on opportunities to learn; how teachers use resources including textbooks and digital technologies in their teaching; and how people teach and learn online.

Pavel Sikorskii is a senior teaching specialist for the Department of Mathematics at Michigan State University. He serves a co-director of undergraduate studies for the Department of Mathematics and has won numerous teaching and service awards for his contributions to undergraduate and K–12 students' mathematical preparation in Michigan. His work has been funded by the National Science Foundation, the DOW Foundation, the National Institutes of Health, and the Bosch Community Fund.