Transmit Pattern Analysis for Active Incoherent Microwave Imaging

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***Abstract*—A microwave imaging technique that enables fast image reconstruction without beamforming using low cost receivers has recently been demonstrated by the authors. Using multiple noise transmitters separated by a large number of wavelengths, an incoherent radiation pattern illuminates the scene mimicking the properties of thermal radiation. The receive array takes advantage of the spatial incoherence of the reflections, producing two-dimensional imagery of the scene. In this paper a study to quantify the incoherence of the transmit radiation pattern is presented using the average mutual coherence metric.**

I. Introduction

Electronic imaging has recently attracted a lot of attention because of the large variety of applications that can benefit from low cost and fast imaging of moving and stationary objects. Microwave and millimeter-wave imaging has significant potential due to good propagation characteristics through tissue, clouds, fog and some building materials compared to optical and infrared imaging.

Following the trend of smaller, more compact and mobile electronics, using an array of small antenna elements to perform imaging instead of mechanically or electronically scanning a larger system is an attractive choice. Such motivations have led to the development of passive interferometric imaging systems in security sensing [1], using techniques first developed in radio astronomy [2]. These systems capture the thermally generated electromagnetic radiation from humans and other objects, terrestrial or galactic. Their biggest drawback is that their receivers are very expensive because such systems are passive and thermal radiation has very lower power. The receivers require very high sensitivity, high gain, wide bandwidth, long integration time and low loss components. For this reason, the authors in their previous work proposed illumination of the scene with noise transmitters separated at a large number of wavelengths, constructively and destructively interfering, leading to a spatially incoherent radiation pattern, and increasing the signal-to-noise ratio (SNR) and thus the sensitivity, thereby alleviating the need for wide bandwidth and long integration time [3].

In this paper, an approach is presented to lead toward the design and analysis of the effects of illuminating the scene using noise transmitters. Starting from the spatial incoherence requirement and borrowing the average mutual coherence metric from the compressed sensing field, this work presents a framework for determining the incoherence of the radiation pattern from three noise transmitters.

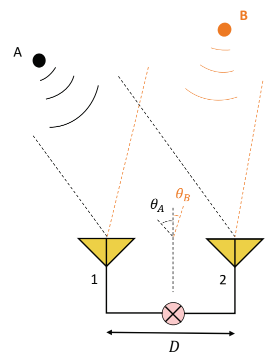

Fig. 1. Correlation interferometer observing two radiating point sources.

II. Active Incoherent Microwave Imaging

For an antenna pair in the interferometric array observing two point sources as shown in Fig. 1, the voltage outputs on the two receivers can be given by [4]:

$$V_1 = s_{1A} + s_{1B} + n_1 \quad (1)$$

$$V_2 = s_{2A} + s_{2B} + n_2 \quad (2)$$

where s_{iA}, s_{iB} are the responses of the i^{th} element due to the point sources A and B respectively, and n_i is the noise received by the i^{th} element. Assuming that the signals from the point sources are uncorrelated with each other, uncorrelated with the noise samples, and the noise samples are uncorrelated with each other, the output voltage of the correlation interferometer can be written as

$$V_{out} = \langle V_1 V_2 \rangle = \langle s_{1A} s_{2A} \rangle + \langle s_{1B} s_{2B} \rangle \quad (3)$$

where $\langle \cdot \rangle$ indicates time averaging.

The requirement that the terms in Eq. (1),(2) need to be uncorrelated with each other lead to the development of passive interferometric imaging systems, because thermal radiation is inherently noise-like. Using multiple noise transmitters with a larger separation from the array, as shown in Fig. 2, can make the point responses partially independent. However in the case of partial coherence between two points, additional terms may arise which may lead to unwanted information in the image and Eq. (3) can be written as

$$\langle V_1 V_2 \rangle = \langle s_{1A} s_{2A} \rangle + \langle s_{1B} s_{2B} \rangle + \langle s_{1B} s_{2A} \rangle + \langle s_{1A} s_{2B} \rangle . \quad (4)$$

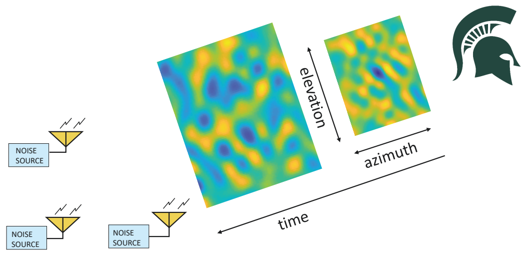

Fig. 2. Three noise transmitters separated at a large number of wavelengths illuminating an object. The sidelobe structure is a result of the constructive and destructive interference across the azimuth and elevation planes. The yellow points represent higher intensities, while the blue ones lower. The time vector shows that the frame furthest away was the one transmitted first.

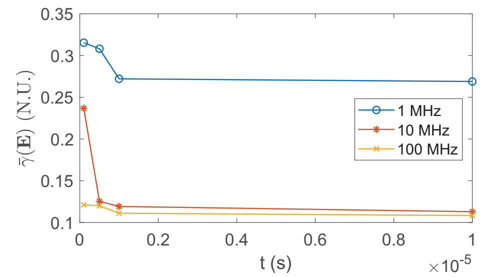

Fig. 3. The values of the average mutual coherence for different values of receiver bandwidth and integration time. As both bandwidth and integration time increase the values of average mutual coherence decrease.

The use of multiple elements which capture the common source responses will also act coherently and eventually the total unwanted information will be very small compared to the actual information from the image. The following presents an approach to determine the impact of the cross terms, which represent unwanted artifacts.

III. Incoherence of the Radiation Pattern

To analyze a system using the incoherent radiation from three noise transmitters, separated at a large number of wavelengths, using the average mutual coherence of the spatio-temporal transmit pattern, the transmitted electric field is first defined by

$$E(\alpha,\beta,t) = \sum_{i=1}^{3} X_i(t) \int_{f_c-\frac{1}{2}\Delta f}^{f_c+\frac{1}{2}\Delta f} e^{-j2\pi\frac{f}{c}(d_{xi}\alpha+d_{yi}\beta)} df \quad (5)$$

where $X_i(t) = n_a(i,t)e^{j\omega t}e^{jn_{ph}(i,t)})$ is the noise signal coming from the i-th transmitter, $n_a(i,t)$ is random amplitude noise and $n_{ph}(i,t)$ is random phase noise, f_c is the carrier frequency, Δf the passband around f_c, the d_{xi}, d_{yi} terms represent the location of the i-th transmitter in the x and y directions accordingly, and α, β are the directional cosines relative to azimuth and elevation planes, respectively.

The spatio-temporal transmit pattern can be modeled as a 3-D matrix $\mathbf{E}$ where the first two dimensions are selected as the two angular dimensions that the antenna array observes the scene from, and the third dimension is time. Reshaping it into a 2-D matrix can be easily accomplished by keeping the columns as the individual point responses and the rows to be the time. To satisfy the Van Cittert-Zernike theorem [2], the columns in the matrix must be statistically independent.

The maximum degree of coherence γ of a matrix $\mathbf{E}$ with K columns and T rows is defined as the maximum absolute value of the cross-correlation between the columns of the matrix by

$$\gamma(\mathbf{E}) = \max_{1\leq k\neq j\leq K} \frac{|\epsilon_k^H \epsilon_j|}{||\epsilon_k|| \, ||\epsilon_j||} \quad (6)$$

where ϵ_k is the k-th column of $\mathbf{E}$ [5], [6]. In our case K is the number of spatial points (azimuth and elevation) and T is the time duration of the calculated electric field. Low values of maximum degree of coherence for a matrix correspond to low dependency its vectors, but it is generally a worst case scenario and therefore a more general metric can be adopted, which is the average mutual coherence [7], given by

$$\bar{\gamma}(\mathbf{E}) = \frac{1}{K(K-1)} \sum_{k\neq j} \frac{|\epsilon_k^H \epsilon_j|^2}{||\epsilon_k||^2 ||\epsilon_j||^2} \quad (7)$$

The average mutual coherence, as defined in (7) will give a measure of the normalized power of the cross terms compared to the power of the first two terms from the sources in (4).

The locations of three 4 GHz noise transmitters were randomized in a square space of 3 m by 3 m and their electric field was calculated in MATLAB with (5) for 100 Monte-Carlo simulations. The integration took place over 0.1, 0.5, 1 and 10 μs captures. The average mutual coherence was calculated with (7) for receiver bandwidths equal to 1, 10 and 100 MHz, as shown in Fig. 3. The bandwidth and integration time represent realistic examples. Both the 10 and 100 MHz plot are close to 0.1 for 1 and 10 μs captures, which means that the unwanted cross terms will have on average approximately 10 times less power than the actual spatial information from the scene. Narrowing the bandwidth to 1 MHz increased the average mutual coherence considerably, which is expected. Changing the spatial and time sampling (i.e. the parameters K, T) did not affect the calculations significantly.

References

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