Offline Contextual Bandits with High Probability Fairness Guarantees

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Abstract

We present RobinHood, an offline contextual bandit algorithm designed to satisfy a broad family of fairness constraints. Our algorithm accepts multiple fairness definitions and allows users to construct their own unique fairness definitions for the problem at hand. We provide a theoretical analysis of RobinHood, which includes a proof that it will not return an unfair solution with probability greater than a user-specified threshold. We validate our algorithm on three applications: a user study with an automated tutoring system, a loan approval setting using the Statlog German credit data set, and a criminal recidivism problem using data released by ProPublica. To demonstrate the versatility of our approach, we use multiple well-known and custom definitions of fairness. In each setting, our algorithm is able to produce fair policies that achieve performance competitive with other offline and online contextual bandit algorithms.

1 Introduction

Machine learning (ML) algorithms are increasingly being used in *high-risk* decision making settings, such as financial loan approval [7], hiring [37], medical diagnostics [12], and criminal sentencing [3]. These algorithms are capable of unfair behavior, and when used to guide policy and practice, can cause significant harm. This is not merely hypothetical: ML algorithms that influence criminal sentencing and credit risk assessment have already exhibited racially-biased behavior [3; 4]. Prevention of unfair behavior by these algorithms remains an open and challenging problem [14; 20; 24]. In this paper, we address issues of unfairness in the *offline contextual bandit* setting, providing a new algorithm, designed using the recently proposed Seldonian framework [47] and called RobinHood, which is capable of satisfying multiple fairness definitions with high probability.

Ensuring fairness in the bandit setting is an understudied problem. While extensive research has been devoted to studying fairness in classification, recent work has shown that the decisions made by fair ML algorithms can affect the well-being of a population over time [34]. For example, criminal sentencing practices affect criminal recidivism rates, and loan approval strategies can change the amount of wealth and debt in a population. This delayed impact indicates that the feedback used for training these algorithms or defining fairness is more *evaluative* in nature, i.e., training samples quantify the (delayed) outcome of taking a particular action given a particular context. Therefore, it is important that fairness can be ensured for methods that are designed to handle evaluative feedback, such as contextual bandits. For example, instead of predicting the likelihood of violent recidivism, these methods can consider what actions to take to minimize violent recidivism.

Within the bandit setting, prior work has mostly focused on ensuring fairness in the *online* setting [24; 25; 27; 35], in which an *agent* learns the quality of different actions by interacting with the system of interest. However, for many fairness applications, e.g., medical treatment suggestion [29], the online setting is not feasible, as direct interaction might be too costly, risky, or otherwise unrealistic. Instead, these problems can be framed in the *offline* bandit setting, where a finite amount of data is collected from the system over time and then used by the agent to construct a fair solution.

Issues of ensuring fairness in the offline contextual bandit setting are similar to those in other ML settings. For instance, contextual bandit algorithms need to manage the trade-off between performance optimization and fairness. Ideally, these algorithms should also be capable of handling a large set of user-defined fairness criteria, as no single definition of fairness is appropriate for all problems [16]. Importantly, when it is not possible to return a fair solution, i.e., when fairness criteria are in conflict, or when too little data is present, the algorithm should indicate this to the user. We allow our algorithm, `RobinHood`, to return *No Solution Found* (`NSF`) in cases like this, and show that if a fair solution *does* exist, the probability `RobinHood` returns `NSF` goes to zero as the amount of available data increases.

In summary, we present the first Seldonian algorithm for contextual bandits. Our contributions are: **1)** we provide an offline contextual bandit algorithm, called `RobinHood`, that allows users to mathematically specify their own notions of fairness, including combinations of fairness definitions already proposed by the ML community, and novel ones that may be unique to the application of interest; **2)** we prove that `RobinHood` is (quasi-)Seldonian: that it is guaranteed to satisfy the fairness constraints defined by the user with high probability, **3)** we prove that if a fair solution exists, as more data is provided to `RobinHood`, the probability it returns `NSF` goes to zero; and **4)** we evaluate `RobinHood` on three applications: a tutoring system in which we conduct a user study and consider multiple, unique fairness definitions, a loan approval setting in which well-known fairness definitions are applied, and criminal recidivism. Our work complements fairness literature for classification (e.g., [1; 8; 14; 50]), contextual bandits (e.g., [24; 25; 35; 14; 27; 22]), and reinforcement learning (e.g., [40; 49]), as described in Section 7.

2 Contextual Bandits and Offline Learning

This section defines a contextual bandit, or *agent*, which iteratively interacts with a system. At each iteration $\iota \in \{1, 2, ...\}$, the agent is given a context, represented as a random variable $X_\iota \in \mathbb{R}^\nu$ for some ν. We assume that the contexts during different iterations are *independent and identically distributed* (i.i.d.) samples from some distribution, d_X. Let $\mathcal{A}$ be the finite set of possible actions that the agent can select. The agent's *policy*, $\pi : \mathbb{R}^\nu \times \mathcal{A} \to [0, 1]$ characterizes how the agent selects actions given the current context: $\pi(x, a) = \Pr(A_\iota = a | X_\iota = x)$. Once the agent has chosen an action, A_ι, based on context X_ι, it receives a stochastic real-valued reward, R_ι. We assume that the conditional distribution of R_ι given X_ι and A_ι is given by d_R, i.e., $R_\iota \sim d_R(X_\iota, A_\iota, \cdot)$. The agent's goal is to choose actions so as to maximize the expected reward it receives.

For concreteness, we use a loan approval problem as our running example. For each loan applicant (iteration ι), a single action, i.e., whether or not the applicant should be given a loan, is chosen. The resulting reward is a binary value: 1 if the loan is repaid and -1 otherwise. The agent's goal using this reward function is to maximize the expected number of loan repayments.

This paper focuses on enforcing fairness in the offline setting, where the agent only has access to a finite amount of *logged* data, D, collected from one or more different policies. Policies used to collect logged data are known as *behavior* policies. For simplicity of notation, we consider a single behavior policy, π_b. D consists of the observed contexts, actions, and rewards: $D = \{(X_\iota, A_\iota, R_\iota)\}_{\iota=1}^m$, where m is the number of iterations for which D was collected, and where $A_\iota \sim \pi_b(X_\iota, \cdot)$.

The goal in the offline setting is to find a policy, π', which maximizes $r(\pi') := \mathbf{E}[R_\iota | A_\iota \sim \pi'(X_\iota, \cdot)]$ using samples from D only. Algorithms that solve this problem are called offline (or batch) contextual bandit algorithms. At the core of many offline contextual bandit algorithms is an *off-policy estimator*, $\hat{r}$, which takes as inputs D and a policy to be evaluated, π_e, in order to produce an estimate, $\hat{r}(\pi_e, D)$, of $r(\pi_e)$. We call $\hat{r}(\pi_e, D)$ the off-policy reward of π_e.

3 Problem Statement

Following the Seldonian framework for designing ML algorithms [47], our goal is to develop a fair offline contextual bandit algorithm that satisfies three conditions: **1)** the algorithm accepts multiple user-defined fairness definitions, **2)** the algorithm guarantees that the probability it returns a policy that violates each definition of fairness is bounded by a user-specified constant, and **3)** if a fair solution exists, the probability that it returns a fair solution (other than NSF) converges to one as the amount of training data goes to infinity. The first condition is crucial because no single definition of fairness is appropriate for all problems [16]. The second condition is equally important because it allows the user to specify the necessary confidence level(s) for the application at hand. However, if an algorithm only satisfies the first two conditions, this does not mean it is qualitatively helpful. For example, we can construct an algorithm that always returns NSF instead of an actual solution—this technically satisfies **1)** and **2)**, but is effectively useless. Ideally, if a fair solutions exists, a fair algorithm should be able to (given enough data) eventually find and return it. We call this property *consistency* and show in Section 5 that RobinHood is consistent.

Since condition **1)** allows users to specify their own notions of fairness, the set of fair policies is not known *a priori*. Therefore, the algorithm must reason about the fairness of policies using the available data. We consider a policy to be either fair or unfair with respect to condition **2)**. For example, we may say that a lending policy is fair if for all pairs of applicants who are identical in every way except race, the policy takes the same action, i.e., approving or denying the loan for both. This criterion is known as causal discrimination (with respect to race) [17; 30]. Then, a policy that adheres to this definition is fair, and a policy that violates this definition is unfair. The algorithm that produces a policy from data must ensure that, with high probability, it will produce a policy that is fair. Note that the setting in which each *action* is fair or unfair can be captured by this approach by defining a policy to be fair if and only if the probability it produces unfair actions is bounded by a small constant.

Formally, assume that policies are parameterized by a vector $\theta \in \mathbb{R}^l$, so that $\pi(X_\iota, \cdot, \theta)$ is the conditional distribution over action A_ι given X_ι for all $\theta \in \mathbb{R}^l$. Let $\mathcal{D}$ be the set of all possible logged data sets and D be the logged data (a random variable, and the only source of randomness in the subsequent equations). Let $a : \mathcal{D} \to \mathbb{R}^l$ be an offline contextual bandit algorithm, which takes as input logged data and produces as output the parameters for a new policy.

Let $g_i : \mathbb{R}^l \to \mathbb{R}$ be a user-supplied function, called a *constraint objective*, that measures the fairness of a policy. We adopt the convention that if $g_i(\theta) \leq 0$ then the policy with parameters θ is fair, and if $g_i(\theta) > 0$ then the policy with parameters θ is unfair. Our goal is to create a contextual bandit algorithm that enforces k behavioral constraints, where the i^{th} constraint has the form:

$$\Pr\Big(g_i(a(D)) \leq 0\Big) \geq 1 - \delta_i, \tag{1}$$

where $\delta_i \in [0, 1]$ is the required confidence level for the i^{th} constraint. Together, the constraint objective g_i and the confidence level δ_i constitute a *behavioral constraint*. Any algorithm that satisfies (1) is called *Seldonian*, or *quasi-Seldonian* if it relies on reasonable but false assumptions, such as appeals to the central limit theorem [47].

Some constraints might be impossible to enforce if, for example, the user provides conflicting constraints [28], or if δ_i cannot be established given the amount of data available. Algorithms that provide high-probability guarantees are often very conservative [26]; if only a small amount of data is available, it may be impossible for the algorithm to produce a policy that satisfies all behavioral constraints with sufficiently high confidence. In such cases, RobinHood returns NSF to indicate that it is unable to find a fair solution. When this occurs, the user has control over what to do next. For some domains, deploying a known fair policy may be appropriate; for others, it might be more appropriate to issue a warning and deploy no policy. We define $g_i(\text{NSF}) = 0$, so that NSF is always fair.

Notice that the creation of an algorithm that satisfies each of the three desired conditions is difficult. Condition **1)** is difficult to enforce because the user must be provided with an interface that allows them to specify their desired definition of fairness without requiring the user to know the underlying data distribution. Condition **2)** is difficult to achieve because of the problem of multiple comparisons—testing b solutions to see if they are fair equates to performing b hypothesis tests, which necessitates measures for avoiding the problems associated with running multiple hypothesis tests using a single data set. Condition **3)** is particularly difficult to achieve in conjunction with the second condition—the algorithm must carefully trade-off the maximizing expected reward with predictions about the outcomes of future hypothesis tests when picking the candidate solution that it considers returning.

4 RobinHood Algorithm

This section presents RobinHood, our fair bandit algorithm. At a high level, RobinHood allows users to specify their own notions of (un)fairness based on statistics related to the performance of a potential solution. It then uses concentration inequalities [36] to calculate high-probability bounds on these statistics. If these bounds satisfy the user's fairness criteria, then the solution is returned.

Constructing Constraint Objectives. Users can specify their desired fairness definitions with constraint objectives, $\{g_i\}_{i=1}^k$, that accept a parameterized policy θ as input and produce a real-valued measurement of fairness. For simplicity of notation, we remove the subscript i and discuss the construction of an arbitrary constraint objective, g. In our loan approval example, we might define $g(\theta) = \text{CDR}(\theta) - \epsilon$, where $\text{CDR}(\theta)$ indicates the causal discrimination rate of θ. However, computing g for this example requires knowledge of the underlying data distribution, which is typically unknown. In practice, each g might depend on distributions that are unavailable, so it is unreasonable to assume that the user can compute $g(\theta)$ for an arbitrary g.

Instead of explicitly requiring $g(\theta)$, we could instead assume that the user is able to provide unbiased estimators for g. However, this is also limiting because it may be difficult to obtain unbiased estimators for certain constraint objectives, e.g., unbiased estimators of the standard deviation of a random variable can be challenging (or impossible) to construct. Importantly, our algorithm does *not* explicitly require an unbiased estimate of $g(\theta)$. Instead, it computes high-probability upper bounds for $g(\theta)$. Even if the random variable of interest does not permit unbiased estimators, if it is a function of random variables for which unbiased estimators exist, then valid upper bounds can be computed.

With this in mind, we propose a general interface in which the user can define g by combining d real-valued *base variables*, $\{z_j\}_{j=1}^d$, using addition, subtraction, division, multiplication, absolute value, maximum, inverse, and negation operators. Base variables may also be multiplied and added to constants. Instead of specifying the base variable $z_j(\theta)$ explicitly, we assume the user is able to provide an unbiased estimator, $\hat{z}_j$, of each base variable: $z_j(\theta) := \mathbf{E}[\text{Average}(\hat{z}_j(\theta, D))]$. That is, each function $\hat{z}_j$ takes a parameterized policy θ and a data set D and returns an arbitrary number of i.i.d. outputs, $\hat{z}_j(\theta, D)$. In the definition of z_j, the average of the outputs is taken so $z_j(\theta)$ is a scalar.

A base variable estimator $\hat{z}_j$ for our loan approval example could be defined as an integer indicating whether or not causal discrimination is satisfied for applicable data points in D. To do this, $\hat{z}(\theta, D)$ should produce 1 if for points h and f that differ only by race, θ chooses the same action, and 0 otherwise. Defining $\hat{z}$ in this way gives us an unbiased estimate of the CDR. We could then define $g(\theta) = z(\theta) - \epsilon$, requiring the CDR to be within some value ϵ, with probability at least $1 - \delta$.

There may be some base variables that the user wants to use when defining fairness that do not have unbiased estimators, e.g., standard deviation. To handle these cases, we also allow the user to use base variables for which they can provide high-probability upper and lower bounds on $z(\theta)$ given any θ and data D. As an example, in Appendix G we show how the user can define a base variable to be the largest possible expected reward for any policy with parameters θ in some set Θ, i.e., $\max_{\theta \in \Theta} r(\theta)$.

In summary, users can define constraint objectives that capture their desired definitions of fairness. Constraint objectives are mathematical expressions containing operators (including summation, division, and absolute value) and base variables (any variable, including constants, for which high-confidence upper and lower bounds can be computed). In Section 6, we construct constraint objectives for different fairness definitions and find solutions that are fair with respect to these definitions. In Appendix A, we provide more examples of how to construct constraint objectives for other common fairness definitions used in the ML community.

Pseudocode. `RobinHood` (Algorithm 1) has three steps. In the first step, it partitions the training data into the candidate selection set D_c and the safety set D_s. In the second step, called *candidate selection*, `RobinHood` uses D_c to construct a candidate policy, $\theta_c \in \Theta$, that is likely to satisfy the third step, called the *safety test*, which ensures that the behavioral constraints hold. Algorithm 1 presents the `RobinHood` pseudocode. Because the candidate selection step is constructed with the safety step in mind, we first discuss the safety test, followed by candidate selection. Finally, we summarize the helper methods used during the candidate selection and safety test steps.

The safety test (lines 3–4) applies concentration inequalities using D_s to produce a high-probability upper bound, $U_i := U_i(\theta_c, D_s)$, on $g_i(\theta_c)$, the value of the candidate solution found in the candidate selection step. More concretely, U_i satisfies $\Pr(g_i(\theta_c) \leq U_i) \geq 1 - \delta_i$. If $U_i \leq 0$, then θ_c passes the safety check and is returned. Otherwise, `RobinHood` returns `NSF`.

The goal of the candidate selection step (line 2) is to find a solution, θ_c, that maximizes expected reward and is likely to pass the safety test. Specifically, θ_c is found by maximizing the output of

Algorithm 1 `RobinHood` $(D, \Delta = \{\delta_i\}_{i=1}^k, \hat{Z}\{\hat{z}_j^i\}_{j=1}^d, \mathcal{E} = \{E_i\}_{i=1}^k)$

```
1: $D_c, D_s = \texttt{partition}(D)$
2: $\theta_c = \arg\max_{\theta \in \Theta} \texttt{CandidateUtility}(\theta, D_c, \Delta, \hat{Z}, \mathcal{E})$    ▷ Candidate Selection
3: $[U_1, ..., U_k] = \texttt{ComputeUpperBounds}(\theta_c, D_s, \Delta, \mathcal{E}, \texttt{inflateBounds=False})$    ▷ Safety Test
4: if $U_i \leq 0$ for all $i \in \{1, ..., k\}$ then return $\theta_c$ else return NSF
```

Algorithm 2 CandidateUtility$(\theta, D_c, \Delta, \hat{Z}, \mathcal{E})$

```
1: $[\hat{U}_1, ..., \hat{U}_k] = \texttt{ComputeUpperBounds}(\theta, D_c, \Delta, \hat{Z}, \mathcal{E}, \texttt{inflateBounds=True})$
2: $r_{\min} = \min_{\theta' \in \Theta} \hat{r}(\theta', D_c)$
3: if $\hat{U}_i \leq -\xi$ for all $i \in \{1, ..., k\}$ then return $\hat{r}(\theta, D_c)$ else return $r_{\min} - \sum_{i=1}^{k} \max\{0, \hat{U}_i\}$
```

Algorithm 2, which uses D_c to compute an estimate, $\hat{U}_i$, of U_i. The same concentration inequality used to compute U_i in the safety test is also used to compute $\hat{U}_i$, e.g., Hoeffding's inequality is used to compute U_i and $\hat{U}_i$. Multiple comparisons are performed on a single data set (D_c) during the search for a solution. This leads to the candidate selection step over-estimating its confidence that the solution it picks will pass the safety test. In order to remedy this issue, we inflate the width of the confidence intervals used to compute $\hat{U}_i$ (this is indicated by the Boolean `inflateBounds` in the pseudocode). Another distinction between the candidate selection step and the safety test is that, in Algorithm 2, we check whether $\hat{U}_i \leq -\xi$ for some small constant ξ instead of 0. This technical assumption is required to ensure the consistency of `RobinHood`, and is discussed in Appendix E.

We define the input $\mathcal{E} = \{E_i\}_{i=1}^k$ in the pseudocode to be analytic expressions representing the constraint objectives $\{g_i\}_{i=1}^k$. For example, if we had the constraint objective $g(\theta) = z_1(\theta) \times z_2(\theta) + z_3(\theta) - \epsilon$, then $E = E_1 \times E_2 + E_3 - \epsilon$, where $E_1 = z_1(\theta)$, $E_2 = z_2(\theta)$, and $E_3 = z_3(\theta)$. Each expression E_i is used in `ComputeUpperBounds` (Algorithm 3) and related helper functions, the pseudocode for which is located in Appendix B. At a high level, these helper functions recurse through sub-expressions of each E_i until a base variable is encountered. Once this occurs, real-valued upper and lower $(1 - \delta_i)$-confidence bounds on the base variable's estimators are computed and subsequently propagated through E_i. Using our example from earlier, bounds for $E_{1,2} = E_1 \times E_2$ would first be computed, followed by bounds for $E_{1,2} + E_3$.

5 Theoretical Analyses

This section proves that given reasonable assumptions about the constraint objectives, $\{g_i(\theta)_{i=1}^k\}$, and their sample estimates, $\{\text{Average}(\hat{g}_i(\theta, D))\}_{i=1}^k$, **1)** `RobinHood` is guaranteed to satisfy the behavioral constraints and **2)** `RobinHood` is consistent.

To prove that `RobinHood` is Seldonian [47]: it is guaranteed to satisfy the behavioral constraints, i.e., that `RobinHood` returns a fair solution with high probability, we show that the safety test only returns a solution if the behavioral constraints are guaranteed to be satisfied. This follows from the use of concentration inequalities and transformations to convert bounds on the base variables, $z_j(\theta_c)$, into a high-confidence upper bound, U_i, on $g_i(\theta_c)$. We therefore begin by showing (in Appendix C) that the upper bounds computed by the helper functions in Appendix B satisfy $\Pr(g_i(\theta) > U_i) \leq \delta_i$ for all constraint objectives. Next, we show that the behavioral constraints are satisfied by `RobinHood`. We denote `RobinHood` as $a(D)$, a batch contextual bandit algorithm dependent on data D.

Theorem 1 (Fairness Guarantee)**.** *Let* $\{g_i\}_{i=0}^n$ *be a sequence of behavioral constraints, where* $g_i : \Theta \to \mathbb{R}$*, and let* $\{\delta_i\}_{i=0}^n$ *be a corresponding sequence of confidence thresholds, where* $\delta_i \in [0, 1]$*. Then, for all* i*,* $\Pr(g_i(a(D)) > 0) \leq \delta_i$*.* **Proof**. See Appendix D.

We define an algorithm to be consistent if, when a fair solution exists, the probability that the algorithm returns a solution other than NSF converges to 1 as the amount of training data goes to infinity. We state this more formally in Theorem 2.

Theorem 2 (Consistency)**.** *If Assumptions 1 through 5 (specified in Appendix E) hold, then* $\lim_{n\to\infty} \Pr(a(D) \neq \texttt{NSF},\ g(a(D)) \leq 0) = 1$. **Proof.** See Appendix E.

In order to prove Theorem 2, we first define the set $\bar{\Theta}$, which contains all unfair solutions. At a high level, we show that the probability that θ_c satisfies $\theta_c \notin \bar{\Theta}$ converges to one as $n \to \infty$. To establish this, we **1)** establish the convergence of the confidence intervals for the base variables, **2)** establish the convergence of the candidate objective for all solutions, and **3)** establish the convergence of the probability that $\theta_c \notin \bar{\Theta}$. Once we have this property about θ_c, we establish that the probability of the safety test returning NSF converges to zero as $n \to \infty$.

In order to build up the properties discussed above, we make a few mild assumptions, which we summarize here. To establish **1)**, we assume that the confidence intervals on the base variables converge almost surely to the true base variable values for all solutions. Hoeffding's inequality and Student's t-test are examples of concentration inequalities that provide this property. We also assume that the user-provided analytic expressions, $\mathcal{E}$, are continuous functions of the base variables. With the exception of division, all operators discussed in Section 4 satisfy this assumption. In fact, this assumption is still satisfied for division when positive base variables are used in the denominator. To establish **2)** and **3)**, we assume that the sample off-policy estimator, $\hat{r}$, converges almost surely to r, the actual expected reward. This is satisfied by most off-policy estimators [46]. We also make particularly weak smoothness assumptions about the output of Algorithm 2, which only requires the output of Algorithm 2 to be smooth across a countable partition of Θ. Lastly, we assume that at least one fair solution exists and that this solution is not on the fair-unfair boundary.

Note that consistency does not provide bounds on the amount of data necessary for the return of a fair solution. Although a high-probability bound on how much data our algorithm requires to return a solution other than NSF would provide theoretical insights into the behavior of our algorithm, our focus is on ensuring that our algorithm can return solutions other than NSF using practical amounts of data on real problems. Hence, in the following section we conduct experiments with real data (including data that we collected from a user study).

6 Empirical Evaluation

We apply RobinHood to three real-world applications: tutoring systems, loan approval, and criminal recidivism. Our evaluation focuses on the three research questions. **1)** When do solutions returned by RobinHood obtain performance comparable to those returned by state-of-the-art methods? **2)** How often does RobinHood return an unfair solution, as compared to state-of-the-art methods? **3)** In practice, how often does RobinHood return a solution besides NSF?

6.1 Experimental Methodology and Application Domains

To the best of our knowledge, no other fair contextual bandit algorithms have been proposed in the offline setting. We therefore compare to two standard offline methods: POEM [44] and Offset Tree [6]. In Appendix F, we also compare to Rawlsian Fairness, a fair *online* contextual bandit framework [23]. Existing offline bandit algorithms are not designed to adhere to multiple, user-defined fairness constraints. One seemingly straightforward fix to this is to create an algorithm that uses all of the data, D, to search for a solution, θ, that maximizes the expected reward (using a standard approach), but with the additional constraint that an estimate, $\hat{g}(\theta, D)$, of how unfair θ is, is at most zero. That is, this method enforces the constraint $\hat{g}(\theta, D) \leq 0$ without concern for how well this constraint generalizes to future data. We construct this method, called NaïveFairBandit, as a baseline for comparison. In RobinHood and NaïveFairBandit, we use Student's t-test to calculate upper and lower confidence bounds on base variables for a particular θ.

Note that Algorithm 1 relies on the optimization algorithm (and implicitly, the feasible set) chosen by the user to find candidate solutions. If the user chooses an optimization algorithm incapable of finding fair solutions, e.g., they choose a gradient method when the fairness constraints defined make it difficult or impossible for it to find a solution, then RobinHood will return NSF. We chose CMA-ES [19] as our optimization method. Further implementation details, including pseudocode for NaïveFairBandit, can be found in Appendix F.

Tutoring Systems. Our first set of experiments is motivated by *intelligent tutoring systems* (ITSs), which aim to teach a specific topic by providing personalized and interactive instruction based on a student's skill level [38]. Such adaptations could have unwanted consequences, including inequity with respect to different student populations [13]. We conduct our experiments in the *multi-armed bandit* setting—a special case of the contextual bandit setting in which context is the same for every iteration. To support these experiments, we collected user data from the crowdsourcing marketplace Amazon Mechanical Turk. In our data collection, users were presented with one of two different versions of a tutorial followed by a ten-question assessment. Data including gender, assessment score, and tutorial type was collected during the study. Let R_ι be the assessment score achieved by user ι and $S_\iota \in \{\texttt{f},\texttt{m}\}$ represent the gender of user ι. (Due to lack of data for users identifying their gender as "other," we restricted our analysis to male- and female-identifying users.) D was collected using a uniform-random behavior policy. Further details are provided in Appendix F.

To demonstrate the ability of our algorithm to satisfy multiple and novel fairness criteria, we develop two behavioral constraints for these experiments: $g_f(\theta) \coloneqq |F|^{-1}\sum_{\iota=0}^{|D|} R_\iota \mathbb{I}(\texttt{f}) - \mathbf{E}[R|\texttt{f}] - \epsilon_f$ and $g_m(\theta) \coloneqq |M|^{-1}\sum_{\iota=0}^{|D|} R_\iota \mathbb{I}(\texttt{m}) - \mathbf{E}[R|\texttt{m}] - \epsilon_m$, where $|F|$ and $|M|$ denote the respective number of female- and male-identifying users, $\mathbb{I}(x)$ is an indicator function for the event $X_\iota = x$, where $x \in \{\texttt{f},\texttt{m}\}$, and $\mathbf{E}[R|x]$ is the expected reward conditioned on the event $X_\iota = x$ for x previously defined. In words, for the constraint g_f to be less than 0, the expected reward for females may only be smaller than the empirical average for females in the collected data by at most ϵ_f. The second constraint g_m is similarly defined with respect to males. Note that different values of ϵ_f and ϵ_m can allow for improvement to female performance and decreased male performance and vice versa. Defining the constraints in this way may be beneficial if the user is aware that bias towards a certain group exists, and then hypothesizes that performance towards this group may need to decrease to improve performance of a minority group, as is the case in this data set. We also highlight that a fair policy for this experiment is one such that $g_f(\theta) \leq 0$ and $g_m(\theta) \leq 0$.

In our first experiment, which we call the *similar proportions* experiment, males and females are roughly equally represented in D: $|F| \approx |M|$. In our second ITS experiment, which we call the *skewed proportions* experiment, we simulate the scenario in which females are under-represented in the data set (and elaborate on the process for doing this in Appendix F). We perform this experiment because **1)** biased data collection is a common problem [9] and **2)** methods designed to maximize reward may do so at the expense of under-represented groups.

In the skewed proportions experiment we introduce a purposefully biased tutorial that responds differently to users based on their gender identification in order to experiment with a setting where unfairness is likely to occur. This tutorial provides information to male-identifying users in an intuitive, straightforward way but gives female-identifying users incorrect information, resulting in high assessment scores for males and near-zero assessment scores for females. The average total score for the biased tutorial in D is higher than that of the other tutorials—because of this, methods that optimize for performance without regard for g_m and g_f will often choose the biased tutorial for deployment. In practice, a similar situation could occur in an adaptive learning system, where tutorials are uploaded by different sources. The introduction of a bug might compel a fairness-unaware algorithm to choose an unfair tutorial.

Loan Approval. Our next experiments are inspired by decision support systems for loan approval. In this setting, a policy uses a set of features, which describe an applicant, to determine whether the applicant should be approved for a loan. We use the Statlog German Credit data set, which includes a collection of loan applicants, each one described by 20 features, and labels corresponding to whether or not each applicant was assessed to have good financial credit [32]. A policy earns reward 1 if it approves an applicant with good credit or denies an applicant with bad credit (the credit label of each applicant is unobserved by the policy); otherwise the agent receives a reward of -1. We conduct two experiments that focus on ensuring fairness with respect to sex (using the `Personal Status and Sex` feature of each applicant in the data set to determine sex). Specifically, we enforce *disparate impact* [51] in one and *statistical parity* [14] in the other. In Appendix F we define statistical parity for this domain and discuss experimental results.

Disparate impact is defined in terms of the *relative magnitudes* of positive outcome rates. Let `f` and `m` correspond to females and males and let $A = 1$ if the corresponding applicant was granted a loan and $A = 0$ otherwise. Disparate impact can then be written as: $g(\theta) \coloneqq$

$\max\{\mathbf{E}[A|\mathtt{m}]/\mathbf{E}[A|\mathtt{f}], \mathbf{E}[A|\mathtt{f}]/\mathbf{E}[A|\mathtt{m}]\} - (1+\epsilon)$. To satisfy this, neither males nor females may enjoy a positive outcome rate that is more than $100\epsilon\%$ larger than that of the other sex.

Criminal Recidivism. This experiment uses recidivism data released by ProPublica as part of their investigation into the racial bias of deployed classification algorithms [3]. Each record in the data set includes a label indicating if the person would later re-offend (decile score), and six predictive features, including juvenile felony count, age, and sex. In this problem, the agent is tasked with producing the decile score given a feature vector of information about a person (the decile score label of each applicant is unobserved by the policy). The reward provided to the bandit is 1 if recidivism occurs and 0 otherwise. We apply *approximate* statistical parity here, where features of interest are race (Caucasian and African American). A policy exhibits statistical parity if the probability with which it assigns a beneficial outcome to individuals belonging to protected and unprotected classes is equal: $g(\theta) \coloneqq |\Pr(A=1|\text{Caucasian}) - \Pr(A=1|\text{African American})| - \epsilon$.

6.2 Results and Discussion

Figure 1 shows our experimental results over varying training set sizes. The leftmost plots in each row show the off-policy reward of solutions returned by each algorithm, and the middle plots show how often solutions are returned by each algorithm. The solution rate for each baseline is 100% because `RobinHood` is the only algorithm able to return `NSF`. The purpose of plotting the solution rate is to determine how much data is required for solutions other than `NSF` to be returned by our algorithm. The rightmost plots show the probability that an algorithm violated the fairness constraints. The dashed line in these plots denotes the maximum failure rate allowed by the behavioral constraints ($\delta = 5\%$ in our experiments).

In all of our experiments, unless a certain amount of data is provided to `NaïveFairBandit`, it returns unfair solutions at an unacceptable rate. This seems workable at first glance—one could argue that so long as enough data is given to `NaïveFairBandit`, it will not violate the behavioral constraints. In practice, however, it is not known in advance how much data is needed to obtain a fair solution. `NaïveFairBandit`'s failure rate varies considerably in each experiment, and it is unclear how to determine the amount of data necessary for `NaïveFairBandit`'s failure rate to remain under δ. In essence, `RobinHood` is a variant of `NaïveFairBandit` that includes a mechanism for determining when there is sufficient data to trust the conclusions drawn from the available data.

In some of our experiments, the failure rates of the fairness-unaware baselines (Offset Tree and POEM) approach δ as more data is provided. To explain this behavior, note that when reward maximization and fairness are *non*conflicting, there may exist fair high-performing solutions. In the case that *only* high-performing solutions meet the fairness criteria, the failure rate of these algorithms should decrease as more data is provided. Importantly, while these baselines might be fair in some cases, unlike `RobinHood`, these approaches do not come with fairness guarantees.

In the similar proportions experiment, fairness and performance optimization are nonconflicting. In this case, `RobinHood` performs similarly to the state-of-the-art—it is able to find and return solutions whose off-policy reward is comparable to the baselines. The same pattern can be seen in the loan approval and criminal recidivism experiments. In these applications, when high-performing fair solutions exist, `RobinHood` is able to find and return them. In the skewed proportions experiment, the biased tutorial maximizes overall performance but violates the constraint objectives. As expected, POEM and Offset Tree frequently choose to deploy this tutorial regardless of the increase in training data, while `RobinHood` frequently chooses to deploy a tutorial whose off-policy reward is high (with respect to the behavioral constraints). In summary, in each of our experiments, `RobinHood` is able to return fair solutions with high probability given a reasonable amount of data.

7 Related Work

Significant research effort has focused on fairness-aware ML, particularly classification algorithms [1; 8; 14; 50], measuring fairness in systems [17], and defining varied notions of fairness [33; 17; 14]. Our work complements these efforts but focuses on the contextual bandit setting. This section describes work related to our setting, beginning with online bandits.

Recall (from Section 2) that in the standard online bandit setting, an agent's goal is to maximize expected reward, $\rho(a) = \mathbf{E}[R_\iota | A_\iota = a]$, as it interacts with a system. Over time, estimates of ρ are

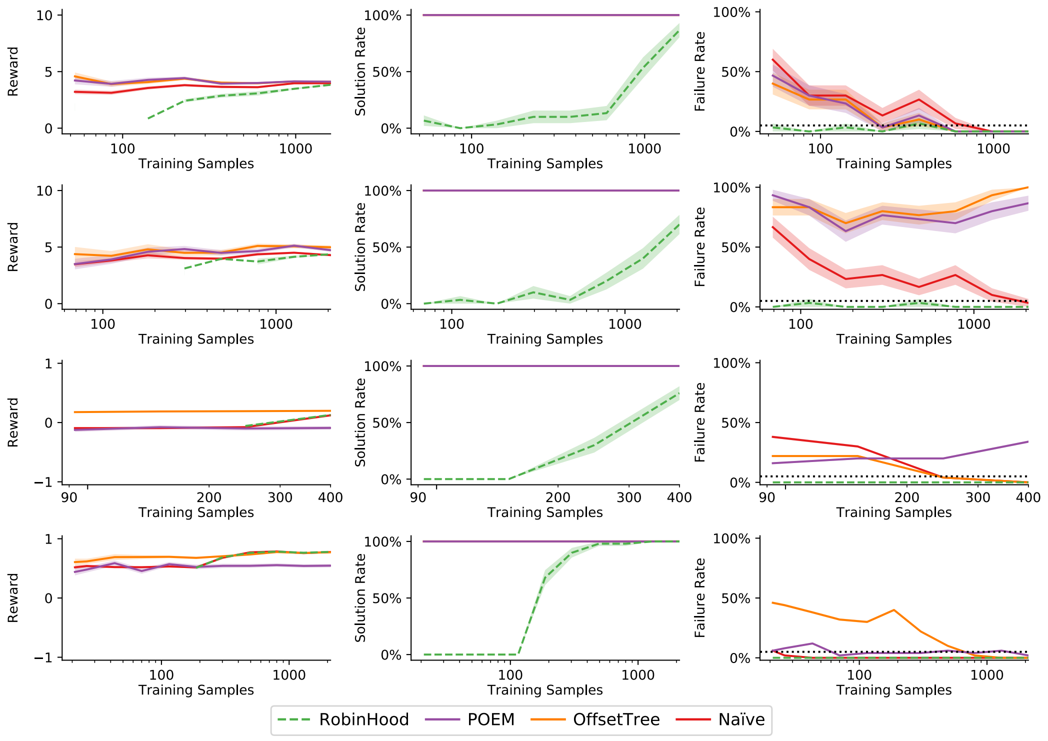

Figure 1: Each row presents results for different experiments generated over 30, 30, 50, and 50 trials respectively. Top row: tutoring sytsem, similar proportions with $\epsilon_m = 0.5$, $\epsilon_f = 0.0$. Second row: tutoring system, skewed proportions with $\epsilon_m = 0.5$, $\epsilon_f = 0.0$. Third row: enforcing disparate impact in the loan approval application with $\epsilon = -0.8$. Fourth row: enforcing statistical parity in the criminal recidivism application with $\epsilon = 0.1$. The dashed line denotes the maximum failure rate allowed by the behavioral constraints ($\delta = 5\%$ in our experiments).

computed, and the agent must trade-off between *exploiting*, i.e., taking actions that will maximize ρ, and *exploring*, i.e., taking actions it believes are suboptimal to build a better estimate of ρ for taking that action. Most fairness-unaware algorithms eventually learn a policy that acceptably maximizes ρ, but there are no performance guarantees for policies between initial deployment and the acceptable policy, i.e., while the agent is exploring. These intermediate polices may choose suboptimal actions too often—this can be problematic in a real-world system, where choosing suboptimal actions could result in unintended inequity. In effect, fairness research for online methods has mostly focused on conservative exploration [24; 25; 35; 14; 27; 22]. The notion of action exploration does not apply in the offline setting because the agent does not interact iteratively with the environment. Instead, the agent has access to data collected using previous policies not chosen by the agent. Because of this, fairness definitions involving action exploration are not applicable to the offline setting.

Related work also exists in online metric-fairness learning [18; 41], multi-objective contextual bandits (MOCB) [45; 48], multi-objective reinforcement learning (MORL) [40; 49], and data-dependent constraint satisfaction [11; 47]. `RobinHood` can address metric-based definitions of fairness that can be quantitatively expressed using the set of operations defined in Section 4. MOCB and MORL largely focus on approximating the Pareto frontier to handle multiple and possibly conflicting objectives, though a recent batch MORL algorithm proposed by Le et al. [31] is an exception to this trend—this work focuses on problems of interest (with respect to fair policy learning) that can be framed as chance-constraints, and assumes the convexity of the feasible set Θ. `RobinHood` represents a different approach to the batch MOCB setting with high probability constraint guarantees. The interface for specifying fairness definitions (presented in Section 4) makes `RobinHood` conceptually related to algorithms that satisfy data-dependent constraints [11]. In fact, `RobinHood` can more generally be thought of as an algorithm that ensures user-defined properties or behaviors with high probability, e.g., properties related to fairness. Finally, `RobinHood` belongs to a family of methods called *Seldonian* algorithms [47]. This class of methods satisfies user-defined safety constraints with high probability.

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A Constructing Constraint Objectives for Common Fairness Definitions

This appendix provides examples of how to construct constraint objectives for several common fairness definitions. Our example domain will be the loan approval problem described in Section 2. In this problem, a bank is interested in maximizing the expected number of loan repayments it receives. The bank chooses to formulate this as an offline bandit problem such that, for each loan applicant, a single action, i.e., whether or not the applicant should be given a loan, is chosen.

We consider an action to belong to the *positive class* if the action corresponds to approving a loan, and we say it belongs to the *negative class* otherwise. We define an outcome to be whether or not an applicant repays (or would have repaid) a loan. We consider an outcome to belong to the positive class if the applicant repays the loan, and we say it belongs to the negative class otherwise. Many of the statistical measures of fairness we consider in this section rely on metrics we introduce below, framed in the context of the loan approval example.

- *True positive* (TP): the event in which the action chosen by the policy and the actual outcome both belong to the positive class. A true positive in the loan approval setting occurs when an applicant who would repay a loan is given a loan.
- *False positive* (FP): the event in which the action chosen by the algorithm is in the positive class when the actual outcome belongs to the negative class. A false positive in the loan approval setting is when an applicant who would not repay a loan is given a loan.
- *False negative* (FN): the event in which the action chosen by the policy is in the negative class but the actual outcome is in the positive class. A false negative in the loan approval setting is when an applicant who would have repaid a loan is denied a loan.
- *True negative* (TN): the event in which the action chosen by the policy and the actual outcome both belong to the negative class. A true negative in the loan approval setting is when an applicant who would not have repaid a loan is denied a loan.

Let TPR=TP/(TP+FN), FPR=FP/(TP+FN), FNR=FN/(TP+FN), and TNR=TN/(FP+TN) be the true positive, false positive, false negative, and true negative rates, respectively. Assume that the bank has unbiased estimators of these terms.

We now provide examples of how to construct objective constraints for some common definitions of fairness. In each definition, the bank is interested in guaranteeing fairness with respect to gender (the protected group), which we will assume to be binary in order to simplify notation.

Predictive Equality [10]. A policy exhibits predictive equality if FP rates are equal between groups. In our loan approval problem, this implies that the probability that an applicant who would not have repaid a loan be incorrectly approved for a loan should be the same for male- and female-identifying applicants. Predictive equality can be defined as $\mathbf{E}[\text{FPR}|\text{f}] = \mathbf{E}[\text{FPR}|\text{m}]$. To construct a constraint objective that satisfies $g(\theta) \leq 0$ if θ is fair, we can set $g = |\mathbf{E}[\text{FPR}|\text{f}] - \mathbf{E}[\text{FPR}|\text{m}]| - \epsilon$. This gives us a constraint objective for (approximate) predictive equality.

Equal Opportunity [20]. A policy exhibits equal opportunity if FN rates are equal between groups. In our loan approval problem, this implies that the probability that an applicant is denied a loan when they would have repaid it is equal between male- and female-identifying applicants. Equal opportunity can be defined as $\mathbf{E}[\text{FNR}|\text{f}] = \mathbf{E}[\text{FNR}|\text{m}]$. To construct a constraint objective that satisfies $g(\theta) \leq 0$ if θ is fair, we can set $g = |\mathbf{E}[\text{FNR}|\text{f}] - \mathbf{E}[\text{FNR}|\text{m}]| - \epsilon$. This gives us a constraint objective for (approximate) equal opportunity.

Equalized Odds [20] (Conditional Procedure Accuracy Equality [5]). This definition combines predictive equality and equal opportunity: a policy exhibits equalized odds if FPR and TPR are equal between protected and unprotected groups. Assume the bank has unbiased estimators of FPR and TPR. Then equalized odds can be defined as $(\mathbf{E}[\text{FPR}|\text{m}] = \mathbf{E}[\text{FPR}|\text{f}]) \wedge (\mathbf{E}[\text{TPR}|\text{m}] = \mathbf{E}[\text{TPR}|\text{f}])$. To satisfy $g(\theta) \leq 0$ if θ is fair, we can set $g = |\mathbf{E}[\text{FPR}|\text{f}] - \mathbf{E}[\text{TPR}|\text{m}]| + |\mathbf{E}[\text{FPR}|\text{m}] - \mathbf{E}[\text{TPR}|\text{f}]| - \epsilon$.

Treatment Equality [5]. This definition focuses on the ratio of FN and FP errors for each group. A policy exhibits treatment equality if the ratio of FNs and FPs is equal for both female-

Algorithm 3 ComputeUpperBounds($\theta, D, \Delta, \hat{Z}, \mathcal{E}$, `inflateBounds`)

```
1: out = [ ]
2: for $i = 1, ..., k$ do
3:     $\hat{Z}_i = \{\hat{z}^i_j\}_{j=1}^{d_i} \subseteq \hat{Z}$
4:     $L_i, U_i =$ Bound($E_i, \theta, D, \delta_i/d_i, \hat{Z}_i$, inflateBounds)
5:     out.append($U_i$)
6: return out
```

and male-identifying applicants. For the loan approval problem, this definition can be written as $(\mathbf{E}[\text{FN}|\text{f}]/\mathbf{E}[\text{FP}|\text{f}]) = (\mathbf{E}[\text{FN}|\text{m}]/\mathbf{E}[\text{FP}|\text{m}])$. For intuition, if the left-hand side of this expression were greater than the right-hand side, then either fewer female-identifying applicants were incorrectly assigned to the negative class than male-identifying applicants or more females-identifying applicants were incorrectly assigned to the positive class than male-identifying applicants. To construct a constraint objective that satisfies $g(\theta) \leq 0$ if θ is fair, we can set $g = |(\mathbf{E}[\text{FN}|\text{f}]/\mathbf{E}[\text{FP}|\text{f}]) - \mathbf{E}[\text{FN}|\text{m}]/\mathbf{E}[\text{FP}|\text{m}])| - \epsilon$.

B RobinHood Algorithm

This section elaborates on the helper methods (Algorithms 3 through 6) our approach uses. Recall that during candidate selection, $\hat{U}_i$ is calculated as the estimate of U_i, the upper bound of a behavioral constraint function g_i. This is done by substituting all quantities that depend on D_s with approximations computed from D_c, and inflating the confidence intervals used to reflect this substitution, which is indicated by the Boolean variable `inflateBounds`. When set to `True`, the number of $g(\theta_c)$ estimates obtained using D_s is approximated as $m = m|D_c|/|D_s|$ (Algorithm 5).

Algorithm 6 uses concentration inequalities [36] to produce high probability upper and lower bounds for a given behavioral constraint. It does this by recursively looking at sub-expressions of E until a base variable is encountered, upon which, Algorithm 4 is called to calculate a real-valued high-confidence bound on the base variable's estimators. Correctness of Algorithm 6 is shown in Section C, and mostly follows from correct use of interval arithmetic and probability theory.

Different concentration inequalities can be substituted into Algorithm 5 to calculate upper and lower $(1-\delta_i)$-confidence bounds on each g_i. The pseudocode in Algorithm 5 presents both Hoeffding's inequality and Student's t-test as examples, where σ is the sample standard deviation with Bessel's correction and $t_{1-\delta_i,m}$ is the $100(1-\delta_i)$ percentile of the Student's t distribution with m degrees of freedom. Note that Hoeffding's inequality often requires algorithms to be very conservative, i.e., high-probability guarantees may require an impractical amount of data. We could instead use Student's t-test, which is more data-efficient but assumes that the amount of many random variables is normally distributed. Due to the central limit theorem this is a reasonable assumption, but does not hold in general.

Finally, different policy evaluation methods, concentration inequalities, and optimization methods can be used for this algorithm. Here, we describe the specific modules we use in our experiments. Our experiments use *importance sampling* [39] to compute the expected return of a potential solution π_c. Next, we use several concentration inequalities to bound the base variables used in our algorithm. As mentioned above, Hoeffding's inequality or the Student's t-test were used for most experiments. In addition, we use confidence intervals based on Bootstrap methods [15] due to their desirable variance-reduction properties in the tutoring experiment. Lastly, `RobinHood` uses a black box search method to solve the optimization problem on line 2 of Algorithm 1. In our experiments, we use the CMA-ES [19] implementation provided in the Python package cma (Python package).

Our algorithms were implemented in Python 3.6 (https://www.python.org/) using the numerical processing package, Numpy 1.15 (https://www.numpy.org/).

For the criminal recidivism and loan approval, a partition of 60% of the total data was used for testing and 40% for training. For the tutoring system experiments, we instead used 80% of the data for training. In all experiments, when training RobinHood, 40% of the training data was used for candidate selection and 60% for the safety test.

Algorithm 4 CIbound($\theta, D, \delta, \hat{Z}$, inflateBounds)

```
1: $z = \hat{z}_j(\theta, D)$
2: margin $=$ CIFunction($z, D, \delta$, inflateBounds)
3: return $\Big($mean$(z) -$ margin, mean$(z) +$ margin$\Big)$
```

Algorithm 5 CIfunction(z, D, δ, inflateBounds)

```
1: scale $= 1$
2: $m =$ length($z$)
3: if inflateBounds then
4:     scale $= 2$
5:     $m = m\frac{|D_s|}{|D_c|}$
6: case Hoeffding: CI $= (b-a)\sqrt{\frac{\ln 1/\delta}{2m}}$     case Student's $t$: CI $= \frac{\sigma(z)}{\sqrt{m}} t_{1-\delta, m-1}$
7: return scale $\times$ CI
```

C Proof of Correctness for Recursive Bounds

In this section, we show that the upper bounds computed by the helper functions in Appendix B (Algorithms 3 through 6) satisfy $\Pr(g_i(\theta) > U_i) \leq \delta_i$ for all constraint objectives. This follows from the use of concentration inequalities and transformations to convert bounds on the base variables $z_j(\theta_c)$, into a high-confidence upper bound, U_i on $g_i(\theta_c)$. More formally, we want to prove Lemma 1 below, whose assumptions are with respect to Hoeffding's inequality [21]. We show the proof for this lemma as well as a similar proof using Student's t-test [43] here.

Lemma 1 (Recursive Bounds: Hoeffding's Inequality)**.** *The upper bounds, $\{U_i\}_{i=1}^{k}$, returned by `ComputeUpperBounds` (Algorithm 3) satisfy the following inequality when `inflateBounds=False` and $\hat{z}_j(\theta_c, D)$ is bounded in some interval $[a, b]$ for θ_c and all $j \in \{1, ..., d\}$: $\forall i \in \{1, ..., k\}$ $\Pr(g_i(\theta) > U_i) \leq \delta_i$.*

For Student's t-test, assume that $\{U_i\}_{i=1}^{n}$ satisfies Lemma 1 when `inflateBounds=False` and Average$(\hat{z}_j(\theta_c, D))$ is normally distributed.

Proof. Assume l_1, u_1, and l_2, u_2 are $1-\delta/2$ confidence lower and upper bounds on base variables z_1 and z_2, respectively. Then $\Pr(z_1 \in [l_1, u_1]) \geq 1-\delta/2$ and $\Pr(z_2 \in [l_2, u_2]) \geq 1-\delta/2$. Below, we show that operations on base variables z_1 and z_2 result in high probability bounds. Note that throughout this proof, we make use of interval arithmetic and the following fact, which holds by the Union Bound:

$$\Pr(z_1 \in [l_1, u_1], z_2 \in [l_2, u_2]) \geq 1-\delta.$$

Addition: The event $(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$ implies that $z_1 + z_2 \in [l_1 + l_2, u_1 + u_2]$. Then, $\Pr((z_1 + z_2) \in [l_1 + l_2, u_1 + u_2]) \geq \Pr(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$, which is at least $1-\delta$. So, $\Pr((u_1 + u_2) \in [l_1 + l_2, u_1 + u_2]) \geq 1-\delta$.

Maximum: The event $(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$ implies that $\max\{z_1, z_2\} \in [\max\{l_1, l_2\} \wedge \max\{u_1, u_2\}]$. Then, $\Pr(\max\{u_1, u_2\} \in [\max\{l_1, l_2\} \wedge \max\{z_1, z_2\}]) \geq \Pr(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$, which is at least $1-\delta$. So, $\Pr(\max\{z_1, z_2\} \in [\max\{l_1, l_2\}, \max\{u_1, u_2\}]) \geq 1-\delta$.

Product: Let $A := \min\{l_1 l_2, l_1 u_2, l_2 u_1, l_2 u_2\}$ and $B := \max\{u_1 l_1, u_1 l_2, u_2 l_1, u_2 l_2\}$. The event $(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$ implies that $(z_1 \times z_2) \in [A, B]$. Then, $\Pr((z_1 \times z_2) \in [A, B]) \geq \Pr(z_1 \in [l_1, u_1] \wedge z_2 \in [l_2, u_2])$ which is at least $1-\delta$. So, $\Pr((z_1 \times z_2) \in [A, B]) \geq 1-\delta$.

This can extend to d base variables instead of two, in which case the operations on the probability inequalities described are less than or equal to $1-\delta/d$. Now, assume l and u are endpoints of an interval that satisfies $\Pr(z \in [l, u]) \geq 1-\delta$, for base variable z.

Negation: The event $z \in [l, u]$ implies that $-z \in [-u, -l]$. Then, $\Pr(-z \in [-u, -l]) \geq 1-\delta$.

Algorithm 6 Bound($E, \theta, D, \delta, \hat{Z}$, `inflate`)

```
1: $X = \{\theta, D, \delta, \hat{Z}, \texttt{inflate}\}$
2: **switch** $E$ **do**
3:     **case** $u(\theta)$
4:         **return** CIBound($X$)
5:     **case** $-E$
6:         **return** $-$Bound($E, X$)
7:     **case** $E_l + E_r$
8:         $(L_l, U_l)$ = Bound($E_l, X$)
9:         $(L_r, U_r)$ = Bound($E_r, X$)
10:        **return** $(L_l + L_r, U_l + U_r)$
11:    **case** $E_l \times E_r$
12:        $(L_l, U_l)$ = Bound($E_l, X$)
13:        $(L_r, U_r)$ = Bound($E_r, X$)
14:        **return** $(\min\{L_lL_r, U_lL_r, L_lU_r, U_lU_r\},\ \max\{L_lL_r, U_lL_r, L_lU_r, U_lU_r\})$
15:    **case** $E^{-1}$
16:        $(L, U)$ = Bound($E', X$)
17:        **if** $0 \in [l, u]$ **then return** NaN **else return** $(1/U, 1/L)$
18:    **case** $|E|$
19:        $(L, U)$ = Bound($E, X$)
20:        **if** $0 \in [l, u]$ **then**
21:            **return** $(\min\{0, |L|, |U|\}, \max\{|L|, |U|\})$
22:        **else**
23:            **return** $(\min\{|L|, |U|\}, \max\{|L|, |U|\})$
24:    **case** $\max\{E_l, E_r\}$
25:        $(L_l, U_l)$ = Bound($E_l, X$)
26:        $(L_r, U_r)$ = Bound($E_r, X$)
27:        **return** $(\max\{L_l, L_r\}, \max\{U_l, U_r\})$
```

Inverse $(1/z)$: Let E be the event $z \in [l, u]$. If $l = 0$, then E implies that $1/z \in [1/u, \infty]$, and if $u = 0$, then E implies that $1/z \in [-\infty, 1/l]$. If $0 \in [l, u]$, then E implies that $1/z \in ([-\infty, 1/l] \cup [1/u, \infty]) = [-\infty, \infty]$. In these cases, $\Pr(1/z \in [1/u, \infty]) \geq 1-\delta$ and $\Pr(1/z \in [-\infty, 1/l]) \geq 1-\delta$, respectively. If $0 \notin [l, u]$, E implies that $1/z \in [1/u, 1/l]$, and $\Pr(1/z \in [1/u, 1/l]) \geq 1-\delta$.

Absolute value: If $0 \in [l, u]$, then the event $z \in [l, u]$ implies $|z| \in [0, \max\{|l|, |u|\}]$. In this case, $\Pr(|z| \in [0, \max\{|L|, U\}]) \geq 1-\delta$. Otherwise, $z \in [l, u]$ implies $|z| \in [\min\{|l|, |u|\}, \max\{|l|, |u|\}]$, and $\Pr(|z| \in [\min\{|l|, |u|\}, \max\{|l|, |u|\}]) \geq 1-\delta$. $\square$

While our set of operations does not include much more than simple arithmetic, interval methods can also be applied to functions with certain behaviors, e.g., functions with properties of monotonicity. This can be used to extend the set of operations allowed on base variables.

D Proof of Theorem 1: High Probability Fairness Guarantee

Consider the contrapositive formulation of behavioral constraint i, $\Pr(g_i(a(D)) > 0) \leq \delta_i$. With respect to this expression, $g_i(a(D)) > 0)$ implies that $a(D)$ is not NSF, which further implies that $U_i \leq 0$ for all i, and thus $\Pr(g_i(a(D)) > 0) = \Pr(g_i(a(D)) > 0,\ U_i \leq 0)$. Next, we use the fact that the joint event, $(g_i(a(D)) > 0, U_i \leq 0)$ implies the event, $(g_i(a(D)) > U_i)$:

$$\Pr(g_i(a(D)) > 0) \leq \Pr\Big(g_i(a(D)) > U_i(\theta_c, D_s)\Big).$$

Lastly, we note that $g(a(D)) > 0$ implies that a solution was returned—that is, $a(D) = \theta_c$:

$$\Pr(g_i(a(D)) > 0) \leq \Pr(g_i(\theta_c) > U_i).$$

Assumption 1 shows that for any fixed parameter vector, $\theta \in \Theta$, the upper bounds estimated by Algorithm 3 using `inflateBounds=False` satisfy $\Pr(g_i(\theta) > U_i) \leq \delta_i$. Because $\theta_c \in \Theta$ for any D_c, it follows from the substitution, $\theta = \theta_c$, that $\Pr(g_i(a(D)) > U_i) \leq \delta_i$. This implies that $\Pr(g_i(a(D)) > 0) \leq \delta_i)$, which further implies $\Pr(g_i(a(D)) \leq 0) \geq 1 - \delta_i$, completing the proof.

E Proof of Theorem 2: Consistency Guarantee

Recall that the logged data, D, is a random variable. To further formalize this notion, let (Ω, Σ, p) be a probability space on which all relevant random variables are defined, and let $D_n : \Omega \to D$ be a random variable, where $D_n = D_c \cup D_s$. We will discuss convergence as $n \to \infty$. $D_n(\omega)$ is a particular sample of the entire set of logged data with n trajectories, where $\omega \in \Omega$. Below, we present formal definitions and assumptions necessary for presenting our main result. To simplify notation, we assume that there exists only a single behavioral constraint and note that the extension of our proof to multiple behavioral constraints is straightforward.

Definition 1. *We say that a function $f : M \to \mathbb{R}$ on a metric space (M, d) is piecewise Lipschitz continuous with Lipschitz constant K and with respect to a countable partition, $\{M_1, M_2, ...\}$, of M if f is Lipschitz continuous with Lipschitz constant K on all metric spaces in $\{(M_i, d)\}_{i=1}^{\infty}$.*

Definition 2. *If (M, d) is a metric space, a set $X \subseteq M$ is a δ-covering of (M, d) if and only if $\max_{y \in M} \min_{x \in X} d(x, y) \leq \delta$.*

Let $\hat{c}(\theta, D_c)$ denote the output of a call to `CandidateUtility`$(\Theta, D_c, \Delta, \hat{Z}, \mathcal{E})$ and let $c(\theta) := r_{\min} - g(\theta)$. Because we assume that there exists only a single behavioral constraint, the candidate utility function can be rewritten as `CandidateUtility`$(\Theta, D_c, \delta, \hat{Z}, E)$. That is, there is only a single threshold δ and a single analytic expression E. The next assumption ensures that c and $\hat{c}$ are piecewise Lipschitz continuous. Notice that the δ-covering requirement is straightforwardly satisfied if Θ is countable for $\Theta \subseteq \mathbb{R}^m$ for any positive natural number m.

Assumption 1. *The feasible set of policies, Θ, is equipped with a metric, d_Θ, such that for all $D_c(\omega)$ there exist countable partitions of Θ, $\Theta^c = \{\Theta_1^c, \Theta_2^c, ...\}$ and $\Theta^{\hat{c}} = \{\Theta_1^{\hat{c}}, \Theta_2^{\hat{c}}, ...\}$, where $c(\cdot)$ and $\hat{c}(\cdot, D_c(\omega))$ are piecewise Lipschitz continuous with respect to Θ^c and $\Theta^{\hat{c}}$ respectively with Lipschitz constants K and $\hat{K}$. Furthermore, for all $i \in \mathbb{N}_{>0}$ and all $\delta > 0$ there exist countable δ-covers of Θ_i^c and $\Theta_i^{\hat{c}}$.*

Next we assume that for all $\theta \in \Theta$, the user-provided analytic expression E is a continuous function of the base variables. With the exception of division, all operators discussed in Section 4 satisfy this assumption. In fact, this assumption is still satisfied for division when positive base variables are used in the denominator.

Assumption 2. *For all $\theta \in \Theta$, $g(\theta)$ is a continuous function of the base variables, $z_1(\theta), z_2(\theta), ..., z_d(\theta)$.*

Next we assume that a fair solution, $\theta^\star$, exists such that $g(\theta^\star)$ is not precisely on the boundary of fair and unfair. This can be satisfied by solutions that are arbitrarily close to the fair-unfair boundary.

Assumption 3. *There exists an $\epsilon > \xi$ and a $\theta^\star \in \Theta$ such that $g(\theta^\star) \leq -\epsilon$.*

Next we assume that the sample off-policy estimator, $\hat{r}$, converges almost surely to r, the actual expected reward. This is satisfied by most off-policy estimators [46].

Assumption 4. $\forall \theta \in \Theta$, $\hat{r}(\theta, D_c) \xrightarrow{a.s.} r(\theta)$.

Lastly, we assume that the method used in `RobinHood` for constructing high-probability upper and lower bounds of a sample mean constructs confidence intervals that converge almost surely to the true mean, i.e., we assume that the confidence intervals on the base variables converge almost surely to the true base variable values for all solutions.. Hoeffding and Student's t-test are two example concentration inequalities that have this property: Hoeffding's inequality converges to the mean deterministically as $n \to \infty$, while the confidence interval used by Student's t-test converges almost surely to the mean assuming the standard deviation is finite (to see this, notice that the t statistic is bounded and $1/\sqrt{n} \to 0$).

Assumption 5. *The confidence intervals on the base variables, $z_1(\theta), z_2(\theta), ..., z_d(\theta)$, converge almost surely to the true base variable values for all $\theta \in \Theta$.*

We prove Theorem 2 by building up properties that culminate with the desired result, starting with a variant of the strong law of large numbers:

Property 1 (Khintchine Strong Law of Large Numbers)**.** *Let $\{X_\iota\}_{i=1}^{\infty}$ be independent and identically distributed random variables. Then $(\frac{1}{n}\sum_{i=1}^{n} X_\iota)_{n=1}^{\infty}$ is a sequence of random variables that converges almost surely to $\mathbf{E}[X_1]$, if $\mathbf{E}[X_1]$ exists, i.e., $\frac{1}{n}\sum_{i=1}^{n} X_\iota \xrightarrow{a.s.} \mathbf{E}[X_1]$.*

Proof. See Theorem 2.3.13 of Sen and Singer [42]. □

In this proof, we consider the set $\bar{\Theta} \subseteq \Theta$, which contains all solutions that are not safe, and some that are safe but fall beneath a certain threshold: $\bar{\Theta} := \{\theta \in \Theta : g(\theta) > -\xi/2\}$. At a high level, we will show that the probability that the candidate solution, θ_c, viewed as a random variable that depends on the candidate data set D_c, satisfies $\theta_c \notin \bar{\Theta}$ converges to one as $n \to \infty$, and then that the probability that θ_c is returned also converges to one as $n \to \infty$.

First, consider the confidence intervals produced for each base variable, z_j. Let $l_j(\theta, D_c)$ and $u_j(\theta, D_c)$ be the upper and lower confidence intervals on $z_j(\theta)$, respectively.

Property 2. *For all $\theta \in \Theta$, $l_j(\theta, D_c) \xrightarrow{a.s.} z_j(\theta)$ and $u_j(\theta, D_c) \xrightarrow{a.s.} z_j(\theta)$.*

Proof. Concentration inequalities construct confidence intervals around the mean by starting with the sample mean of the unbiased estimates (in our case, $\hat{z}(\theta, D_c)$) and then adding or subtracting a constant. While some concentration inequalities are naturally in this form, such as Hoeffding's inequality and Student's t-test, others can be restructured to produce this form. Thus, given Assumption 5, $l_j(\theta, D_c)$ and $u_j(\theta, D_c)$ can be be written as $\text{Average}(\hat{z}(\theta, D_c)) + Z_n$, where Z_n is a sequence of random variables that converges (almost surely) to zero. Since $\text{Average}(\hat{z}(\theta, D_c)) \xrightarrow{\text{a.s.}} z_j(\theta)$ by Property 1, we therefore have that both $l_j(\theta, D_c) \xrightarrow{\text{a.s.}} z_j(\theta)$ and $u_j(\theta, D_c) \xrightarrow{\text{a.s.}} z_j(\theta)$. □

Next, we are interested in showing that the upper bound, $\hat{U}$, returned by Algorithm 3 converges to $g(\theta)$. To clarify notation, here we write $\hat{U}(\theta, D_c)$ to emphasize that $\hat{U}$ depends on the solution, θ, and the data, D_c, passed to the off-policy estimator.

Property 3. *For all $\theta \in \Theta$, $\hat{U}(\theta, D_c) \xrightarrow{a.s.} g(\theta)$.*

Proof. We have from Property 2 that the confidence intervals on the base variables converge almost surely to $z_1(\theta), z_2(\theta), \ldots, z_d(\theta)$. Furthermore, by Assumption 2 we have that $g(\theta)$ is a continuous function of these base variables. Recall that $\hat{U}(\theta, D_c)$ is the upper bound produced by pushing the confidence intervals on the base variables though the analytic expression for $g(\theta)$. Since $g(\theta)$ is a continuous function of these base variables, $\hat{U}(\theta, D_c)$ is a continuous function of the confidence intervals on the base variables. So, by the continuous mapping theorem [2], $\hat{U}(\theta, D_c)$ converges almost surely to the value that it takes when applied to the converged values for the base variables, i.e., $g(\theta)$. □

Recall that we define $\hat{c}(\theta, D_c)$ as the output of Algorithm 2, i.e., `CandidateUtility`$(\Theta, D_c, \delta, \hat{Z}, E)$. Below, we show that given a fair solution $\theta^\star$ and data D_c, $\hat{c}(\theta^\star, D_c)$ converges almost surely to $r(\theta^\star)$, the expected reward of $\theta^\star$.

Property 4. $\hat{c}(\theta^\star, D_c) \xrightarrow{a.s.} r(\theta^\star)$.

Proof. By Property 3, we have that given Assumption 2, $\hat{U}(\theta^\star) \xrightarrow{\text{a.s.}} g(\theta^\star)$. By Assumption 3, we have that $g(\theta^\star) \leq -\epsilon$. Now, let

$$A = \{\omega \in \Omega : \lim_{n\to\infty} U(\theta^\star, D_c(\omega)) = g(\theta^\star)\}. \tag{2}$$

Recall that $\hat{U}(\theta^\star, D_c) \xrightarrow{\text{a.s.}} g(\theta^\star)$ means that $\Pr(\lim_{n\to\infty} \hat{U}(\theta^\star, D_c) = g(\theta^\star)) = 1$. So ω is in A almost surely, i.e., $\Pr(\omega \in A) = 1$. Consider any $\omega \in A$. From the definition of a limit and the previously established property that $g(\theta^\star) \leq -\epsilon$, we have that there exists an n_0 such that for all $n \geq n_0$, the candidate utility function, Algorithm 2, will return $\hat{r}(\theta^\star, D_c)$ (this avoids the discontinuity of the `if` statement in Algorithm 2 for values smaller than n_0).

Furthermore, we have from Assumption 4 that $\hat{r}(\theta^\star, D_c) \xrightarrow{\text{a.s.}} r(\theta^\star)$. Let

$$B = \{\omega \in \Omega : \lim_{n\to\infty} \hat{r}(\theta^\star, D_c(\omega)) = r(\theta^\star)\}. \tag{3}$$

From Assumption 4 we have that ω is in B almost surely, i.e., $\Pr(\omega \in B) = 1$, and thus by the countable additivity of probability measures, $\Pr(\omega \in (A \cap B)) = 1$.

Consider now any $\omega \in (A \cap B)$. We have that for sufficiently large n, Algorithm 2 will return $\hat{r}(\theta^\star, D_c)$ (since $\omega \in A$), and further that $\hat{r}(\theta^\star, D_c) \to r(\theta^\star)$ (since $\omega \in B$). Thus, for all $\omega \in (A \cap B)$, the output of the candidate utility function converges to $r(\theta^\star)$, i.e., $\hat{c}(\theta^\star, D_c(\omega)) \to r(\theta^\star)$. Since $\Pr(\omega \in (A \cap B)) = 1$, we conclude that $\hat{c}(\theta^\star, D_c(\omega)) \xrightarrow{\text{a.s.}} r(\theta^\star)$. □

We have now established that the candidate utility function converges almost surely to $r(\theta^\star)$ for the $\theta^\star$ assumed to exist in Assumption 3. We now establish a similar result for all $\theta \in \bar{\Theta}$—that the output of the candidate utility function converges almost surely to $c(\theta)$ (recall that $c(\theta)$ is defined as $r_{\min} - g(\theta)$).

Property 5. *For all $\theta \in \bar{\Theta}$, $\hat{c}(\theta, D_c) \xrightarrow{a.s.} c(\theta)$.*

Proof. By Property 3, we have that $\hat{U}(\theta, D_c) \xrightarrow{\text{a.s.}} g(\theta)$. If $\theta \in \bar{\Theta}$, then we have that $g(\theta) > -\xi/2$. We now change the definition of the set A from its definition in the previous property to a similar definition suited to this property. That is, let:

$$A = \{\omega \in \Omega : \lim_{n\to\infty} U(\theta, D_c(\omega)) = g(\theta)\}. \tag{4}$$

Recall that $\hat{U}(\theta, D_c) \xrightarrow{\text{a.s.}} g(\theta)$ means that $\Pr(\lim_{n\to\infty} \hat{U}(\theta, D_c) = g(\theta)) = 1$. So, ω is in A almost surely, i.e., $\Pr(\omega \in A) = 1$. Consider any $\omega \in A$. From the definition of a limit and the previously established property that $g(\theta) > -\xi/2$, we have that there exists an n_0 such that for all $n \geq n_0$ the candidate utility function will return $r_{\min} - \sum_{i=1}^k \max\{0, \hat{U}_i\}$. By the same argument, $\hat{U}(\theta, D_c(\omega)) \to g(\theta)$. So, for all $\omega \in A$, the output of the candidate utility function, $\hat{c}(\theta, D_c(\omega)) \to r_{\text{min}} - g(\theta) - \xi$, and since $\Pr(\omega \in A) = 1$ we therefore conclude that $\hat{c}(\theta, D_c(\omega)) \xrightarrow{\text{a.s.}} c(\theta)$. □

By Property 5 and one of the common definitions of almost sure convergence,

$$\forall \theta \in \bar{\Theta}, \forall \epsilon > 0, \Pr\left(\lim_{n\to\infty} \inf\{\omega \in \Omega : |\hat{c}(\theta, D_n(\omega)) - c(\theta)| < \epsilon\}\right) = 1.$$

Because Θ is not countable, it is not immediately clear that all $\theta \in \bar{\Theta}$ converge simultaneously to their respective $c(\theta)$. We show next that this is the case due to our smoothness assumptions.

Property 6. $\forall \epsilon' > 0, \Pr\left(\lim_{n\to\infty} \inf\{\omega \in \Omega : \forall \theta \in \bar{\Theta}, |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon'\}\right) = 1.$

Proof. Let $\mathrm{C}(\delta)$ denote the union of all the points in the δ-covers of the countable partitions of Θ assumed to exist by Assumption 1. Since the partitions are countable and the δ-covers for each region are assumed to be countable, we have that $\mathrm{C}(\delta)$ is countable for all δ. Then for all δ, we have convergence for all $\theta \in C(\delta)$ simultaneously:

$$\forall \delta > 0, \forall \epsilon > 0, \Pr\left(\lim_{n\to\infty} \inf\{\omega \in \Omega : \forall \theta \in C(\delta), |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon\}\right) = 1. \tag{5}$$

Now, consider a $\theta \notin C(\delta)$. By Definition 2 and Assumption 1, $\exists\, \theta' \in \bar{\Theta}_i^c, d(\theta, \theta') \leq \delta$. Moreover, because c and $\hat{c}$ are Lipschitz continuous on $\bar{\Theta}_i^c$ and $\bar{\Theta}_i^{\hat{c}}$ (by Assumption 1) respectively, we have that $|c(\theta) - c(\theta')| \leq K\delta$ and $|\hat{c}(\theta, D_c(\omega)) - \hat{c}(\theta', D_c(\omega))| \leq \hat{K}\delta$. So, $|\hat{c}(\theta, D_c(\omega)) - c(\theta)| \leq |\hat{c}(\theta, D_c(\omega)) - c(\theta')| + K\delta \leq |\hat{c}(\theta', D_c(\omega)) - c(\theta')| + \delta(K + \hat{K})$. This means that for all $\delta > 0$:

$$\left(\forall \theta \in \mathrm{C}(\delta), |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon\right) \implies \left(\forall \theta \in \bar{\Theta}, |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon + \delta(K + \hat{K})\right).$$

Substituting this into (5), we get:

$$\forall \delta > 0, \forall \epsilon > 0, \Pr\left(\lim_{n\to\infty} \inf\{\omega \in \Omega : \forall \theta \in \bar{\Theta},\ |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon + \delta(K + \hat{K})\}\right) = 1.$$

Now, let $\delta := \epsilon/(K + \hat{K})$ and $\epsilon' = 2\epsilon$. Thus, we have the following:

$$\forall \epsilon' > 0, \Pr\left(\lim_{n\to\infty} \inf\{\omega \in \Omega : \forall \theta \in \bar{\Theta}, |\hat{c}(\theta, D_c(\omega)) - c(\theta)| < \epsilon'\}\right) = 1.$$

□

So, given the appropriate assumptions, for all $\theta \in \bar{\Theta}$, we have that $\hat{c}(\theta, D_c(\omega)) \xrightarrow{\text{a.s.}} c(\theta)$ and that $\hat{c}(\theta^\star, D_c(\omega)) \xrightarrow{\text{a.s.}} r(\theta^\star)$. Due to the countable additivity property of probability measures and Property 6, we have the following:

$$\Pr\left(\left[\forall \theta \in \bar{\Theta}, \lim_{n\to\infty} \hat{c}(\theta, D_c(\omega)) = c(\theta)\right], \left[\lim_{n\to\infty} \hat{c}(\theta^\star, D_c(\omega)) = r(\theta^\star)\right]\right) = 1, \qquad (6)$$

where $\Pr(A, B)$ denotes the joint probability of A and B.

Let H denote the set of $\omega \in \Omega$ such that (6) is satisfied. Note that $r_{\min}$ is defined as the value always less than $r(\theta)$ for all $\theta \in \Theta$, and $-g(\theta) \leq 0$ for all $\theta \in \bar{\Theta}$. So, for all $\omega \in H$, for sufficiently large n, the candidate utility function will not define θ_c to be in $\bar{\Theta}$. Since ω is in H almost surely ($\Pr(\omega \in H) = 1$), we therefore have that $\lim_{n\to\infty} \Pr(\theta_c \notin \bar{\Theta}) = 1$.

The remaining challenge is to establish that, given $\theta_c \notin \bar{\Theta}$, the probability that the safety test returns `NSF` converges to zero as $n \to \infty$. By Property 3 (but using D_s in place of D_c), we have that $\hat{U}(\theta_c, D_s) \xrightarrow{\text{a.s.}} g(\theta_c)$. Furthermore, we have that for all $\omega \in H$, there exists an n_0 such that for all $n \geq n_0$, $\theta_c \notin \Theta$. From the definition of Θ, this means that there exists an n such that for all $n \geq n_0$, $g(\theta_c) < -\xi/2$, at which point the safety test would return θ_c. So, we have our desired result—the limit as $n \to \infty$ of the probability that `RobinHood` returns a solution, θ_c, is one, meaning the limit as $n \to \infty$ of the probability that `RobinHood` returns `NSF` is zero.

F Additional Experimental Details

This section provides pseudocode for the helper methods in `RobinHood`, details for the baselines, more details on our application domains, and a discussion of the results in our loan approval experiments.

F.1 Baseline Methods

Except for `NaïveFairBandit`, the baselines we compare to exist as repositories online and are linked below.

- Offset Tree [6]: https://github.com/david-cortes/contextualbandits
- POEM [44]: http://www.cs.cornell.edu/ adith/POEM/
- Rawlsian Fair Machine Learning for Contextual Bandits [23]: An online contextual bandit algorithm which enforces *weakly meritocratic fairness* at every step of the learning process. We used the repository located at https://github.com/jtcho/FairMachineLearning for this experiment, but made changes to the original code. This was done to accurately reflect the original work in Joseph et al. [23]. Specifically, the repository code showed the bandit the reward for each action at every round, and this was instead changed to returning the reward for only the action chosen by the algorithm.
- `NaïveFairBandit` does not employ a safety test, and therefore does not search for candidate solutions with inflated confidence intervals or return `NSF`. Pseudocode for `NaïveFairBandit` is located in Algorithm 7.

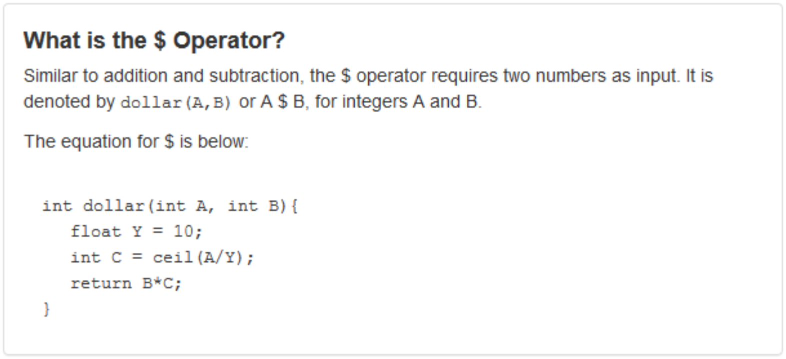
F.2 Tutoring Systems

User Study. We define a new mathematical operator called the \$ operator such that $A\$B = B \times \lceil A/10 \rceil$. In total, three tutorial versions are used in our experiments. The \$ operator is described in different ways in each of the three tutorials, as depicted in Figures F.1 through F.3. Tutorial 1 (Figure F.1) describes the \$ operator using code, and includes example problems. Tutorial 2 (Figure F.2 describes the \$ operator in a non-intuitive way, and uses fewer example problems.

Algorithm 7 `NaïveFairBandit`

```
1: function NAIVEFAIRBANDIT($D, \Delta, \hat{Z}, \mathcal{E}$)
2:     $\theta_c = \arg\max_{\theta \in \Theta}$ NaiveCandidateValue($\theta, D, \Delta, \hat{Z}, \mathcal{E}$)
3:
4: function NAIVECANDIDATEVALUE($\theta, D_c, \Delta, \hat{Z}, \mathcal{E}$)
5:     $[\hat{U}_1, ..., \hat{U}_k]$ = ComputeUpperBounds($\theta, D_c, \Delta, \hat{Z}, \mathcal{E}$, False)
6:     $r_{\min} = \min_{\theta' \in \Theta} \hat{r}(\theta', D_c)$
7:     if $\hat{U}_i \leq 0$ for all $i \in \{1, ..., k\}$ then return $\hat{r}(\theta, D_c)$ else return $r_{\min} - \sum_{i=1}^{k} \max\{0, \hat{U}_i\}$
```

Tutorial 1


What is the $ Operator?

Similar to addition and subtraction, the $ operator requires two numbers as input. It is denoted by `dollar(A,B)` or A $ B, for integers A and B.

The equation for $ is below:

```
int dollar(int A, int B){
   float Y = 10;
   int C = ceil(A/Y);
   return B*C;
}
```

Figure F.1: Tutorial that defines the $ operator using code.

Tutorial 3 (Figure F.3) responds differently to users based on their gender identification. Male-identifying users were given the straightforward definition of the $ operator, while female-identifying users were given an incorrect definition. The number of female-identified users in this data set is 1,403 and the number of male-identified users is 1,178. In the equal proportions experiment, two tutorials were used for a total of 1988 samples. In the skewed proportions experiment, three tutorials were used for a total of 2581 samples were used. The assessment shown to each user once they completed the provided tutorial consists of ten questions using the $ operator, e.g., 16$4. When computing importance weights, we re-scale female and male samples to simulate a similar data set containing 80% males and 20% females. Response format was fill-in-the-blank.

F.3 Loan Approval

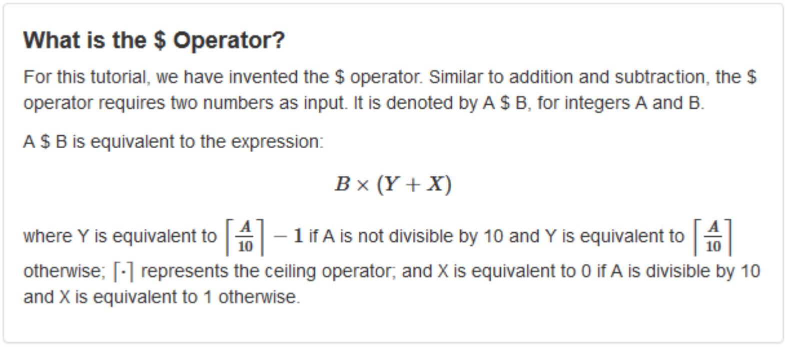
Statistical Parity. In this experiment, we enforce statistical parity. A policy is considered fair in this case if its probability of approving male applicants is equal to its probability of approving female applicants. We use the `Personal Status and Sex` feature of each applicant to determine sex. Statistical parity can be encoded as the following constraint objective:

$$g(\theta) := \Big| \mathbf{E}[A|\mathtt{m}] - \mathbf{E}[A|\mathtt{f}] \Big| - \epsilon, \tag{7}$$

where $A = 1$ if the corresponding applicant was granted a loan and $A = 0$ otherwise. To satisfy this constraint objective, the absolute difference between the conditional expectations must be less than ϵ with high probability.

Statistical Parity Experimental Results. We ran 50 trials for this experiment. We find that for larger training set sizes the estimated expected reward of solutions returned by `RobinHood` grows steadily more comparable to those returned by Offset Tree and `NaïveFairBandit`. Very quickly, we see that baselines begin to return solutions that are fair with respect to each constraint objective, indicating that solutions that optimize performance also are in line with the behavioral constraints. As can be seen in Figure F.4, `RobinHood` is able to find and return these solutions as well.

Tutorial 2


What is the \$ Operator?

For this tutorial, we have invented the \$ operator. Similar to addition and subtraction, the \$ operator requires two numbers as input. It is denoted by A \$ B, for integers A and B.

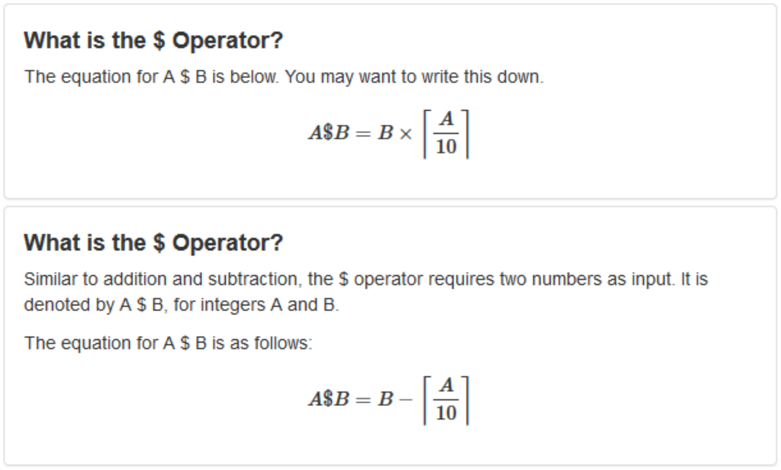
A \$ B is equivalent to the expression:

$$B \times (Y + X)$$

where Y is equivalent to $\left\lceil \frac{A}{10} \right\rceil - 1$ if A is not divisible by 10 and Y is equivalent to $\left\lceil \frac{A}{10} \right\rceil$ otherwise; $\lceil \cdot \rceil$ represents the ceiling operator; and X is equivalent to 0 if A is divisible by 10 and X is equivalent to 1 otherwise.

Figure F.2: Tutorial that explains the \$ operator in a convoluted manner.

Tutorial 3


What is the \$ Operator?

The equation for A \$ B is below. You may want to write this down.

$$A\$B = B \times \left\lceil \frac{A}{10} \right\rceil$$

What is the \$ Operator?

Similar to addition and subtraction, the \$ operator requires two numbers as input. It is denoted by A \$ B, for integers A and B.

The equation for A \$ B is as follows:

$$A\$B = B - \left\lceil \frac{A}{10} \right\rceil$$

Figure F.3: Biased tutorial that provides incorrect information to female-identifying users.

Comparison to Online Fairness Methods. Because most existing work on fairness for bandits considers the online setting, we performed an additional experiment to evaluate how `RobinHood` performs in comparison to these baselines. In particular, we compare `RobinHood` to the framework discussed by Joseph et al. [23], called Rawlsian Fair Machine Learning. This framework considers the contextual bandit setting in a slightly different way, where contexts are actions from which to choose. Importantly, we note that this experiment is not entirely fair, as `RobinHood` is formulated for the offline bandit setting. Nonetheless, the results of this experiment provide a rudimentary analysis of how our method might perform if used in this setting.

To compare our method, which learns using a batch of data, to baselines that make decisions online, we use the following procedure. First, we assume that the problem definition provides a reward function, $r : (\mathcal{X} \times \mathcal{A}) \to \mathbb{R}$, which determines the value of taking each action given a particular context. In our experiment, we obtain r by training a Gaussian process regression model to predict R given (X, A), using the data from the loan approval experiment. Importantly, because this experiment uses a simulator, which only roughly approximates the processes that generated the loan approval dataset, we expect the results to differ from those that were obtained in the offline loan approval experiments. Given r, we train `RobinHood` as in offline experiments, with the exception that reward is computed dynamically using r instead of being estimated using importance sampling. To train the online baseline algorithms, we allow them to iteratively learn over each context in the training data set in random order. As a result, despite the fact that `RobinHood` learns in batch while the baselines learn iteratively, our training procedure ensures that all models are trained on the same amount of data, and using the same set of contexts and reward function, r. Once the algorithms are trained, their parameters are fixed and they are evaluated on the remaining testing data. As in the offline

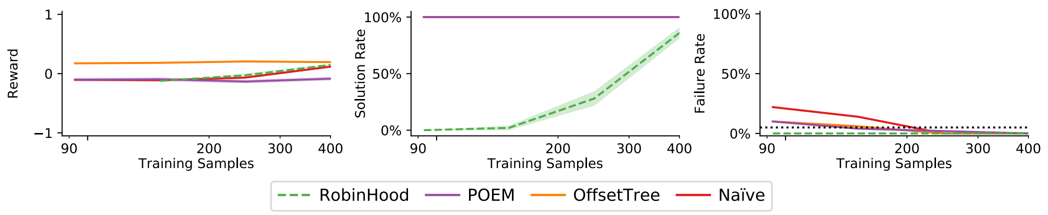


Figure F.4: Enforcing statistical parity in the Loan Approval domain ($\epsilon = 0.23$).

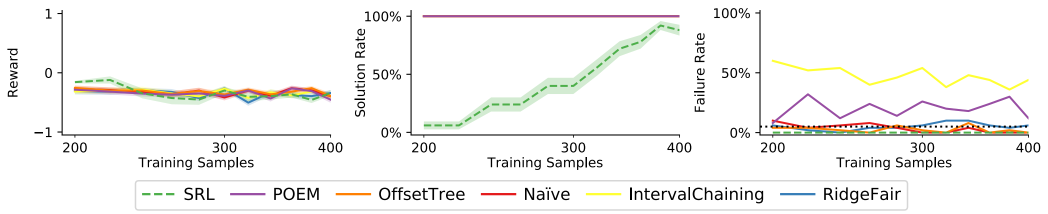


Figure F.5: Comparison to online methods for loan approval using disparate impact ($\epsilon = -0.8$).

experiments, we report evaluation statistics averaged over several randomized trials, where each trial uses 40% of the data for training and 60% for testing.

Figure F.5 shows the results of this experiment, which we averaged over 50 trials. Similar to our other results, `RobinHood` maintains an acceptable failure rate (5% in this experiment), and returns solutions other than `NSF` given a reasonable amount of data.

G Example Base Variable Without Unbiased Estimator

In Section 4 we mentioned that there may be some base variables that the user wants to use when defining fairness that do not have unbiased estimators (e.g., standard deviation). For example, say the user wanted to define a base variable to be the largest possible expected reward for any policy with parameters θ in some set Θ, i.e., $\max_{\theta\in\Theta} r(\theta)$. That is, this base variable is: $z(\theta) := \max_{\theta'\in\Theta} r(\theta')$. Note that because this quantity does not depend on the solution being evaluated, θ, we denote it as z instead of $z(\theta)$. This base variable is important in the context of this paper because it can be a useful component in definitions of fairness.

Consider an example application where each context corresponds to a person, actions correspond to deciding which tutorial on consumer economics to give to a person, and the reward is the person's fiscal savings during the following year. In this case, we might desire a system that is fair with respect to people from different locations, say Maryland and Mississippi. Since the mean income in Maryland in 2015 was $\$75{,}847$, while the mean income in Mississippi in 2015 was only $\$40{,}593$, it would not be reasonable to require the bandit algorithm to ensure that it selects actions in a way that ensures the expected return for people in both states is similar (the expected yearly income). That is, a tutoring system on resource economics could not be expected to remedy the income disparity between these two states.

Rather than compare the expected rewards for people in each state, it would be more reasonable to compare how far the expected reward is from the best possible expected reward for people in each state. This allows for behavioral constraints that require the expected yearly income (expected return) for people in each state to be within $\$500$ of the maximum possible expected income when considering the impact of different policies (tutoring systems). Alternatively, one might require the expected yearly income to be within 10% of the best possible (our of all of the tutoring systems considered) for people in each state. To use these definitions of fairness, we might desire a base variable that is equal to the maximum possible expected reward for people of a particular type. The base variable that we present here shows how this can be achieved (although we present the variable without conditioning on a person's type, this extension is straightforward).

The challenge when upper-bounding z is that we do not know which policy is optimal. It is straightforward to construct high-confidence upper bounds on the performance of a particular policy using importance sampling and a concentration inequality like Hoeffding's inequality. In this section only, let $U(\theta, D)$ denote a $(1-\delta)$-confidence upper bound on z, constructed from data, D. One approach to upper bound z would be to first produce a high-confidence upper bound, $U(\theta', D)$, on the performance of each solution $\theta' \in \Theta$, and then compute the bound on z using the set of upper bounds, $\{U(\theta', D)\}_{\theta' \in \Theta}$. Because $\theta^\star$ is the optimal policy with respect to r, i.e.,

$$\theta^\star \in \arg\max_{\theta' \in \Theta} r(\theta'), \tag{8}$$

one candidate for a $(1-\delta)$-confidence upper bound on z is the supremum of the upper bounds computed for each θ': $\sup_{\theta' \in \Theta} U(\theta', D)$.

However, it is not clear that this is a valid upper bound on z. Intuitively, if the upper bound computed for *any* of the $\theta' \in \Theta$ is too small, then this proposed upper bound on $r(\theta^\star)$ may be too small. So, it may seem that the probability that this upper bound fails is the probability that there is one or more $\theta' \in \Theta$ for which $U(\theta', D)$ does not upper bound z. If Θ has cardinality n, then this probability can be as large as $n\delta$. Here we show that this reasoning is incorrect: for this particular base variable, the probability of failure remains δ, not $n\delta$. That is:

Property 7. *Let Θ be a (possibly uncountable) set of policy parameters (policies). If, there exists at least one $\theta^\star$ satisfying* (8) *and for all $\theta' \in \Theta$, $U(\theta', D)$ is a $(1-\delta)$-confidence upper bound on $r(\theta')$, constructed from data D, then*

$$\Pr\left(\sup_{\theta' \in \Theta} U(\theta', D) \geq \max_{\theta' \in \Theta} r(\theta')\right) \geq 1-\delta. \tag{9}$$

Proof. Note that in (9), $U(\theta', D)$ is the only random variable—it is the source of all randomness. Furthermore, by the assumption that there exists at least one $\theta^\star$ satisfying (8), $\max_{\theta' \in \Theta} r(\theta')$ exists. However, $\max_{\theta' \in \Theta} U(\theta', D)$ may not exist, and so we use $\sup_{\theta' \in \Theta} U(\theta', D)$.

Let $\theta^\star$ be any policy satisfying (8). By assumption, $U(\theta^\star)$ is a $(1-\delta)$-confidence upper bound on $r(\theta^\star)$. That is,

$$\Pr\Big(U(\theta^\star, D) \geq r(\theta^\star)\Big) \geq 1-\delta. \tag{10}$$

Because $\theta^\star$ satisfies (8), we have that $r(\theta^\star) = \max_{\theta' \in \Theta} r(\theta')$, and so

$$\Pr\left(U(\theta^\star, D) \geq \max_{\theta' \in \Theta} r(\theta')\right) \geq 1-\delta. \tag{11}$$

Since $\theta^\star \in \Theta$, we have that $U(\theta^\star, D) \leq \sup_{\theta' \in \Theta} U(\theta', D)$, and so

$$\Pr\left(\sup_{\theta' \in \Theta} U(\theta', D) \geq \max_{\theta' \in \Theta} r(\theta')\right) \geq 1-\delta. \tag{12}$$

□