



Ram-pressure Stripping of a Kicked Hill Sphere: Prompt Electromagnetic Emission from the Merger of Stellar Mass Black Holes in an AGN Accretion Disk

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Abstract

Accretion disks around supermassive black holes (SMBHs) are promising sites for stellar mass black hole (BH) mergers due to mass segregation and merger acceleration by disk gas torques. Here we show that a gravitational-wave (GW) kick at BH merger causes ram-pressure stripping of gas within the BH Hill sphere. If $R_H \geq H$, the disk height, an off-center UV flare at $a_{BH} \sim 10^3 r_g$, emerges within $t_{UV} \sim O(2 \text{ days})(a_{BH}/10^3 r_g)(M_{SMBH}/10^8 M_\odot)(v_{kick}/10^2 \text{ km s}^{-1})$ postmerger and lasts $O(R_H/v_{kick}) \sim O(5 t_{UV})$. The flare emerges with luminosity $O(10^{42} \text{ erg s}^{-1})(t_{UV}/2 \text{ days})^{-1}(M_{Hill}/1 M_\odot)(v_{kick}/10^2 \text{ km s}^{-1})^2$. Active galactic nucleus optical/UV photometry is altered and asymmetric broad emission line profiles can develop after weeks. If $R_H < H$, detectability depends on disk optical depth. Follow-up by large optical sky surveys is optimized for small GW error volumes and for Laser Interferometer Gravitational-Wave Observatory/Virgo triggers $>50 M_\odot$.

Unified Astronomy Thesaurus concepts: Black holes (162); Active galactic nuclei (16); Gravitational waves (678); Gravitational wave astronomy (675)

1. Introduction

Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO; Aasi et al. 2015) and Advanced Virgo (Acernese et al. 2015) are revealing a surprisingly numerous population of merging stellar mass black holes (BHs; LIGO & Virgo Scientific Collaborations 2018), including a previously undetected population of BHs with masses $>20 M_\odot$. In the local universe, BH density seems greatest in our own Galactic nucleus (Hailey et al. 2018; Genozov et al. 2018), consistent with previous conjectures (Morris 1993; Miralda-Escudé & Gould 2000). Thus, a promising channel for the LIGO/Virgo BH mergers are active galactic nucleus (AGN) disks. This is because a fraction of the orbiting nuclear population (including BHs), are geometrically coincident with the AGN disk and another fraction is ground down into the disk (Syer et al. 1991; Artymowicz et al. 1993; Goodman & Tan 2004). Torques from AGN disk gas drive binary formation, migration, and mergers in these populations (McKernan et al. 2012, 2014, 2018, 2019; Bellovary et al. 2016; Bartos et al. 2017; Stone et al. 2017; Secunda et al. 2018). Here we show there can be a prompt, bright UV/optical counterpart from AGN disks after a kicked BH merger. Follow-up by wide-area optical photometric surveys (e.g., Zwicky Transient Facility (ZTF)) is optimized for LIGO/Virgo triggers $>50 M_\odot$ with small error volumes. Detection of such signatures would assign specific galactic counterparts to gravitational-wave (GW) sources and probe AGN disk interior conditions for the first time.

2. Hill Sphere Reaction Postmerger

Mass is lost in a BH merger during chirp and ringdown as GWs carry away energy and angular momentum. The final mass M_f is (Tichy & Marronetti 2008)

$$M_f = M_b[1 - 0.2\nu - 0.208\nu^2(a_1 + a_2)], \quad (1)$$

where $\nu \equiv \mu/M_b = q_b/(1 + q_b)^2$ is the symmetric mass ratio of binary $M_b \equiv M_1 + M_2$ where $q_b \equiv M_2/M_1$, and $a_{1,2}$ are the spin magnitudes of masses $M_{1,2}$. Typically, $\Delta M \equiv (M_b - M_f)/M_b \sim 0.05$ for $q_b \sim 1$ and small a_1, a_2 . The sphere of influence of a binary BH system of mass M_b in orbit around a supermassive black hole (SMBH) of mass M_{SMBH} is given by the Hill radius ($R_H \equiv r(q/3)^{1/3}$), where r is the semimajor axis of the binary center-of-mass around the SMBH and $q \equiv M_b/M_{SMBH}$. Postmerger R_H decreases at light speed by

$$\Delta R_H \approx r \left(\frac{M_b}{3M_{SMBH}} \right)^{1/3} \left(\frac{\Delta M}{3M_b} \right) \approx R_H \left(\frac{\Delta M}{3M_b} \right) \quad (2)$$

for small ΔM . Several effects result. First, gas within the Hill sphere that is orbiting too fast for new mass M_f moves outward, self-colliding. Second, gas formerly inside R_H now collides with gas orbiting the SMBH. Third, the postmerger BH accretes the low angular-momentum component of self-shocked Hill sphere gas in a burst. A jet or beamed outflow adds luminosity $\eta \dot{M}_f c^2$ to the emerging, observable hotspot outlined below, where η is the accretion efficiency onto M_f .

Separately, a GW kick at merger (e.g., Baker et al. 2008; Zlochower et al. 2011) causes ram-pressure stripping of the original Hill sphere gas as it collides with a comparable mass of disk gas. Many of these effects have been studied in the context of SMBH mergers in gas disks (e.g., Bogdanović et al. 2008; Kocsis & Loeb 2008; Shields & Bonning 2008; Megevand et al. 2009; O’Neill 2009; Bode et al. 2010). However, the physics of the kicked Hill sphere colliding with surrounding gas has no direct analogy in circumbinary disks for SMBH mergers.

2.1. Collisions Involving Hill Sphere Gas

Post-mass loss, parcels of gas within the Hill sphere are at the pericenter of a new eccentric orbit. The Hill sphere gas will self-collide on timescales $> O(t_{\text{orb}})$ (O’Neill 2009). Supersonic collisions occur deepest in the Hill sphere ($r = R_1 < R_H$), where the collisional Mach number (\mathcal{M}) is

$$\mathcal{M} \approx \frac{1}{2} \frac{1}{(r/R_H)^{1/2}} \frac{q^{1/3}}{r(r_g)^{1/2}} \left(\frac{c}{c_s} \right) \left(\frac{\Delta M}{M_b} \right), \quad (3)$$

where c_s is the gas sound speed and $\Delta v/v_{K,\text{Hill}} \approx \Delta M/2M_b$ where $v_{K,\text{Hill}}(r) = c/\sqrt{r_{g,b}}$ is the gas Keplerian velocity with $r_{g,b} = GM_b/c^2$. For $q \geq 10^{-4}$, supersonic collisions can extend to $R_1 \sim R_H$. If $\mathcal{M} \gg 1$, ($r \ll R_1$) the postshock temperature is $T \sim (5/16)\mathcal{M}^2 T_{\text{disk}}$ from Rankine–Hugoniot jump conditions, where T_{disk} is the model disk temperature at the merger radius. If $\mathcal{M} \sim 1 + \epsilon$, ($r \sim R_1$), then $T \sim (1 + \epsilon)T_{\text{disk}}$. The collisions at $R_1 < R_H$ generate energy $E_1 \approx (3/2)M_{\text{Hill}}(R_1/R_H)^3(k_B/m_H)T_{\text{disk}}$ on orbital timescales around M_f .

In the subsonic collision zone ($R_1 < r < R_H - \Delta R_H$), $\mathcal{M} < 1$ and the average gas temperature is $T \sim (1/3)(m_H/k_B)(\Delta M \Delta v_2/M_b)^2$, where Δv_2 is the velocity difference between $[R_1, R_H]$. Subsonic collisions contribute $E_2 \approx (1/2)M_{\text{Hill}}(\Delta M \Delta v_2/M_b)^2$. Gas between $R_H - \Delta R_H$ and R_H , with total mass ΔM_{Hill} , collides with gas orbiting the SMBH at velocity differential $\Delta v(r) \approx (c/2r(r_g)^{1/2})(q/3)^{1/3}$, yielding an average uniform temperature of $T = (1/3)(m_H/k_B)(\Delta v(r))^2$ and energy $E_3 \approx (1/2)\Delta M_{\text{Hill}}\Delta v(r)^2$.

The luminosity of the self-collisions above are limited by the timescales on which the gas collides, which could be $\gg t_{\text{orb}}$ (O’Neill 2009). The most luminous electromagnetic (EM) contribution postmerger occurs as the kicked BH remnant attempts to carry its original Hill sphere gas with it. This is because (1) it involves most of M_{Hill} and (2) the timescale of the disk response is short (R_H/v_{kick}) compared to most self-collision timescales. Physically, once the Hill sphere gas collides with an equivalent mass of disk gas (in time R_H/v_{kick}), much of the original Hill sphere gas is lost via ram-pressure stripping. As a result, energy $E_{\text{kick}} = 1/2 M_H v_{\text{kick}}^2 \sim 10^{47} \text{erg} (M_{\text{Hill}}/10^9 M_\odot) (v_{\text{kick}}/10^2 \text{km s}^{-1})^2$ is dissipated via shocks at temperature $T \sim O(10^5) \text{K} (v_{\text{kick}}/10^2 \text{km s}^{-1})^2$ over a duration $O(R_H/v_{\text{kick}})$. Here we assume an adiabatic shock; however, the timescales for high-mass SMBHs, especially for merging binaries on large orbits, imply nonadiabatic processes will be important. For such circumstances, our estimates represent upper limits.

2.2. Disk Opacity

Photons liberated by gas shock heating can be scattered inside the disk before escape. If the merger occurs where gas pressure dominates, then the disk atmospheric density is $\rho(z) = \rho_0 \exp(-z/H)$ where ρ_0 is the midplane density, z is the vertical distance, and H is the disk height. The mean free path length of photons in the disk is $\ell = 1/(\kappa\rho)$, where κ is the disk opacity. Total path length L traveled by photons to the disk surface is $L = N\ell$ where $N = H^2/\ell^2$ is the average number of steps. Therefore, $L = H^2/\ell$ and the mean photon travel time is $t_{\text{Hill}} = L/c$. Assuming constant dissipation per unit optical depth, the disk surface temperature (T_{eff}) is $T_{\text{eff}} \approx ((3/8)\tau + 1/(4\tau))^{-1/4} T_0$ where $\tau = \kappa\Sigma/2$ and T_0 is the midplane temperature (Sirko & Goodman 2003). Now we consider two illustrative AGN disk models where $R_H \geq H$ and $R_H < H$. This allows us to quantify EM signatures at the order-of-magnitude level.

2.3. $R_H \geq H$: Prompt UV Flare

Consider a kicked $M_b = 65 M_\odot$ binary located at $r = 10^3 r_g$ from an $M_{\text{SMBH}} = 10^9 M_\odot$ SMBH, so $R_H \sim 3r_g$. For a Thompson et al. (2005) disk model at radius $r = 10^3 r_g$, $H/r \sim 10^{-3}$, so $H \sim r_g$ and $R_H > H$ in this example. The volume of gas in the Hill sphere is

$$V_{\text{gas}} = \frac{4}{3}\pi R_H^3 - \frac{2}{3}\pi(R_H - H)^2[3R_H - (R_H - H)]. \quad (4)$$

Here, $V_{\text{gas}}/V_{\text{Hill}} \sim 0.5$, so $M_{\text{Hill}} = V_{\text{gas}}\rho \sim 0.8M_\odot$.

The ram-pressure stripping of the kicked Hill sphere gas releases shock energy $E_{\text{kick}} = 1/2 M_{\text{Hill}} v_{\text{kick}}^2 \sim 10^{47} \text{erg} (M_{\text{Hill}}/10^9 M_\odot) (v_{\text{kick}}/10^2 \text{km s}^{-1})^2$ at $T \sim (m_H/k_B) v_{\text{kick}}^2 \sim O(10^5) (v_{\text{kick}}/10^2 \text{km s}^{-1})^2 \text{K}$ over the timescale $O(R_H/v_{\text{kick}}) \sim 6 \text{mo}$, yielding a UV luminosity $\sim 10^{41} \text{erg s}^{-1}$. If disks similar to the Thompson et al. (2005) model are found around lower-mass SMBHs, the timescale R_H/v_{kick} drops considerably and the luminosity can reach $\sim 10^{42} \text{erg/s}$ around $M_{\text{SMBH}} \sim 10^6 M_\odot$ (see below).

2.4. $R_H < H$: Delayed, Weak Flare

An $M_b = 65 M_\odot$ binary located at $r = 10^3 r_g$ from an $M_{\text{SMBH}} = 10^8 M_\odot$ SMBH has a Hill sphere of radius $R_H \sim 6r_g$. In a Sirko & Goodman (2003) disk model at $r = 10^3 r_g$, $H/r \sim 10^{-2}$, so $H \sim 10 r_g$ and $R_H < H$. Therefore, $V_{\text{gas}} = V_{\text{Hill}}$ and $M_{\text{Hill}} = V_{\text{Hill}}\rho \sim 1.5M_\odot$. The photon diffusion length in the disk is $\langle L \rangle = \langle H \rangle^2/\ell$ where $\langle H \rangle$ spans $[H - R_H, H]$, or $[2.1, 10]r_g$ in this example. For an exponential atmosphere with $z \sim H$, $\langle L \rangle = \langle H \rangle^2/\ell \sim [0.4, 22] \times 10^{17} \text{cm}$ and the photon travel time spans $t_{\text{Hill}} = \langle L \rangle/c$ or $t_{\text{Hill}} \sim [15 \text{days}, 2.3 \text{yr}]$. For $\tau_0 \sim 10^4$, $T_{\text{eff}} \sim 0.1T_0$. So, while UV/optical photons emerge from the kick-shock on a timescale $O(\text{month})$ postmerger, the reprocessed optical signature is smeared out over several months with luminosity $\leq 10^{41} \text{erg s}^{-1}$. Upper limits on follow-up of LIGO/Virgo search volumes therefore constrains (H, ρ) in AGN disks.

2.5. New Thermal Emission from the Kicked Hotspot

The temperature of an unperturbed thin AGN disk is $T(r) = T_{\text{max}} r^{-3/4}$, where $T_{\text{max}} \sim 6 \times 10^5 (M_{\text{SMBH}}/10^8 M_\odot)^{1/4} (\dot{m}/\dot{M}_{\text{Edd}})^{1/4} \text{K}$ and \dot{M}_{Edd} is the Eddington accretion rate. For $R_H > H$, T_{eff} can be $> T_{\text{max}}$

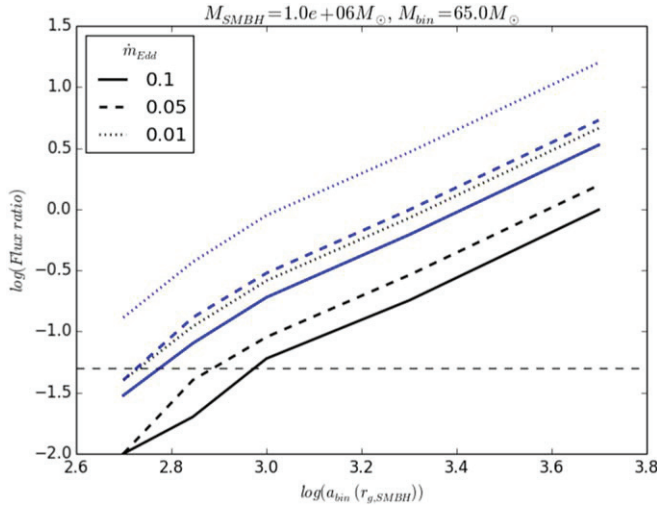


Figure 1. Ratio of optical (ZTF g -band) flux (black curves) and NUV (GALEX band) flux (blue curves) from kicked Hill sphere hotspot to (ZTF g -band, GALEX NUV) flux from an unperturbed AGN disk around an $M_{\text{SMBH}} = 10^6 M_{\odot}$ SMBH, as a function of binary distance (a_{bin}) from the SMBH. We assume $M_b = 65 M_{\odot}$, $\Delta M \sim 0.05$, and $v_{\text{kick}} = 10^2 \text{ km s}^{-1}$. Horizontal dashed line corresponds to a flux increase of $\sim 5\%$. Curves correspond to M_{SMBH} accreting at fractions 0.1 (solid), 0.05 (dashed), 0.01 (dotted) of the Eddington rate. At $a_b \geq 10^3 r_g$, the change in optical/NUV flux is detectable for $R_H > H$.

if τ_0 is small. The emitting area of the kicked Hill sphere is $(R_H/R_{\text{disk}})^2 \sim 5 \times 10^{-7}$ smaller than the disk (if $R_{\text{disk}} \sim 10^4 r_g$). The kicked hotspot has a temperature dependent only on v_{kick}^2 . Thus, a kicked merger in an AGN disk around a low-mass SMBH can generate a short-lived luminous hotspot that could dominate continuum emission in the UV or optical bands.

Figure 1 shows (black curves) the ratio of rest-frame g -band optical flux from the hotspot to that emitted by the AGN disk and (blue curves) the corresponding *Galaxy Evolution Explorer* NUV flux ratio. The curves assume a kicked $M_b = 65 M_{\odot}$ BH merger ($v_{\text{kick}} \sim 10^2 \text{ km s}^{-1}$) in a disk around an $M_{\text{SMBH}} = 10^6 M_{\odot}$ SMBH accreting at $[0.01, 0.1] \dot{M}_{\text{Edd}}$, with $R_H > H$ so flux escapes. Figure 2 is as Figure 1, but for $M_{\text{SMBH}} = 10^9 M_{\odot}$. Horizontal dashed lines show the detectability threshold of a 5% flux change. Figures 1 and 2 show the postmerger Hill sphere kick can be a substantial fraction of (or even dominate) g -band/NUV emission from the AGN disk if $a_b \geq 10^3 r_g$. ZTF and Large Synoptic Survey Telescope (LSST) can easily detect such changes ($>5\%$) in the g band for well-detected sources. If $R_H < H$, the hotspot is obscured. The curves in Figures 1 and 2 drop by a factor $O(T_0/T_{\text{eff}})$ or -1.2 in log flux ratio for $T_0/T_{\text{eff}} \sim 2$. Mergers in lower accretion rate AGN disks with $R_H < H$ may be detectable for small T_0/T_{eff} if $\dot{M}_{\text{Edd}} = 0.01$.

2.5.1. Asymmetry in BLR Line Profiles

Broad-line regions (BLRs) in AGNs are modeled as a distribution of clumpy clouds at median radius R_{BLR} (Krolik et al. 1981; Netzer & Marziani 2010). The hotspot on the disk photosphere emerges in days ($R_H > H$), whereupon the BLR is asymmetrically illuminated in time R_{BLR}/c (a few weeks). Off-center illumination generates a uniform asymmetry in all broad-line components, unlike central AGN variability. If the hotspot persists, the asymmetric illumination washes over the BLR in several months.

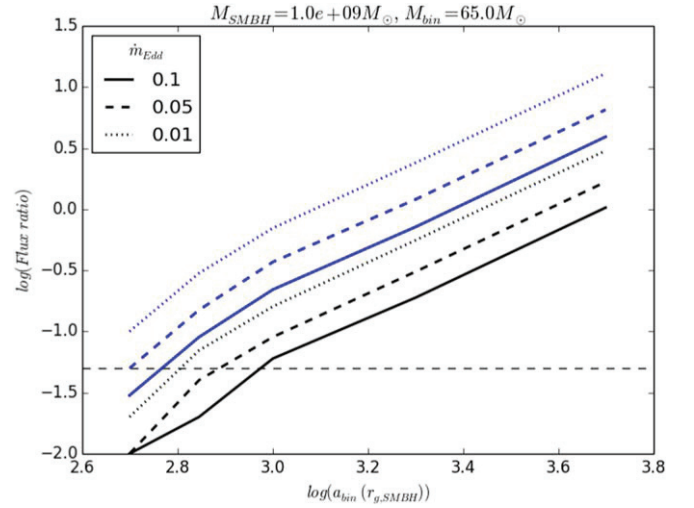


Figure 2. As Figure 1, except for $M_{\text{SMBH}} = 10^9 M_{\odot}$.

3. Strategy for Follow-up of a LIGO/Virgo GW Detection Volume

LIGO/Virgo are detecting BH mergers at a rate 1/wk in the third observing run (O3).¹⁰ If BH mergers are preferentially associated with an AGN, we must optimize searches for the EM signatures above. If LIGO/Virgo releases binary mass estimates ($>50 M_{\odot}$) with GW triggers, we increase the likelihood that $R_H > H$ so an EM counterpart is detectable.

Optimal searches involve rapid wide-field UV/optical follow-up of small LIGO/Virgo error boxes when $M_b > 50 M_{\odot}$. UV is preferred since the fractional signature is more pronounced (Figures 1 and 2), though such capabilities are currently lacking. Candidates displaying photometric jumps may have $R_H > H$, and optical spectroscopic follow-up can search for broad-line asymmetries, indicating an off-center hotspot. If $R_H < H$ then the disk hotspot shows up weeks/months post-GW trigger. Optical surveys can detect photometric changes if $R_H > H$ and $a_b > 2000(4000)r_g$ in AGN disks accreting at 0.01 (0.1) \dot{M}_{Edd} .

A vetted AGN catalog is a good starting point for optical/UV follow-up. Assef et al. (2018) provide a catalog consisting of 4.5 million AGN candidates across the full extragalactic sky (≈ 150 candidates deg^{-2}) with 90% reliability (their “R90” catalog), as well as a lower reliability catalog of 21 million AGN candidates across the full extragalactic sky (≈ 700 candidates deg^{-2}) with 75% completeness (their “C75” catalog). Both catalogs derive from the AllWISE Data Release (Wright et al. 2010), and are currently the widest-area published lists of AGN candidates across the full sky.

4. Conclusions



A merging BH binary in an AGN disk generates a prompt set of EM signatures if the Hill sphere radius is greater than the AGN disk height ($R_H > H$). The GW kick causes ram-pressure stripping of the Hill sphere. The resulting shock dominates the EM response and has no analog in SMBH mergers. Searches, including nondetections, constrain (H, ρ) in AGN disks. If $R_H < H$, detectability depends heavily on disk opacity. UV searches are optimal, but small LIGO/Virgo error boxes can be efficiently searched by large optical surveys with photo-z

¹⁰ <https://gracedb.ligo.org/latest/>

selected AGN catalogs. LIGO/Virgo triggers should include “high-mass” ($M_b > 50M_\odot$) estimates to optimize EM follow-up.

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