DISPLACEMENT-THICKNESS BASED RECYCLING INFLOW GENERATION METHOD FOR SPATIALLY DEVELOPING TURBULENT BOUNDARY LAYER SIMULATIONS

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ABSTRACT

A displacement thickness based inflow generation method, for simulation of a developing turbulent boundary layer, is proposed. Following existing rescaling/recycling methods, velocities from a plane sufficiently downstream of the inlet are recycled back and used as the inflow after re-scaling based on inner and outer length-scales. The inner length-scale is based on the viscous length-scale (for smooth walls) or surface specific scales (for rough walls). Prior recycling methods for smooth and rough boundary layers typically use δ_{99} as the outer length-scale. Since δ_{99} is a threshold based quantity, it is strongly dependent on the mean velocity profile and can have large undesired fluctuations, particularly if the profile shape is atypical or unsteady. Here, we propose the use of profile integrated quantities such as the displacement thickness (δ_1) to obtain a 'surrogate' for δ_{99} in order to mitigate the adverse effects of having to determine the outer scale from a point-wise measurement of the mean velocity profile. The outer length-scale at the downstream plane is determined based on the local displacement thickness and higher-order moments of the integrated velocity profile. The inlet displacement thickness is fixed at a desired value and the outer length-scale at the inlet is determined through an iterative method. The use of high-order moments of the velocity profile is tested a-priori on DNS data for a developing boundary layer. Also, an initial application to LES over a surface with roughness elements is presented.

INTRODUCTION

Simulations of developing turbulent boundary layers require specification of a realistic turbulent inflow velocity field. Recycling inflow generation methods, in which the velocity field from a plane sufficiently downstream of the inlet is recycled back to obtain the inflow (**?**), are a commonly used choice. The present study adopts such a method, based on previous studies (**?** and **?**). The velocity from a suitable plane is rescaled for use as the inflow, based on knowledge about inner and outer length-scales. The inner length-scale is based on the viscous length-scale (for smooth walls, **?**) or on surface geometry (for rough walls, **?**). Prior methods have selected δ_{99} as the outer length-scale. However, since this quantity is strongly mean profile dependent and also threshold based, it can suffer from large, undesirable fluctuations in simulations, especially during transients such as during initializations or in unsteady flow situations. These fluctuations can, in turn, generate large discontinuities in the location of threshold based velocities especially if the velocity profile is not monotonic. Instead, the use of profile-integrated quantities such as the displacement thickness (δ_1), less prone to such fluctuations, is therefore a better option for obtaining outer length-scales. We propose here that fixing the inlet δ_1 and determining the outer length-scales based on it through an iterative process gives an appropriate boundary inflow, without undesirable fluctuations and discontinuities.

BACKGROUND ON THE RESCALING-RECYCLING METHOD

The process of rescaling is based on the principle that the velocity field at the inlet and at a rescaling plane downstream, normalized by the local friction velocity, can be expressed as a combination of an inner (in near wall region) and an outer velocity (in the defect layer) profile. The inflow velocity field is determined based on the inner (l_d) and outer length-scales (l_δ) at the inlet(l_d^{inlet}, l_δ^{inlet}) and rescaling(l_d^{resc}, l_δ^{resc}) planes. Since velocities are normalized by the local friction velocity, the ratio (λ) of the friction velocities at the inlet and rescaling plane is also required for rescaling. This ratio can be calculated using empirical correlations such as the (1/n) velocity profile (**?**) or dynamically (**??**) using a "test-plane". Once the rescaling parameters have been obtained, the procedure for rescaling followed in the present study is the same as to the one fol-

lowed in **?** and similar to the pioneering approach in (**?**)
.

To set notation, the stream wise, wall normal and cross stream directions are referred to as x, y, and z respectively and the corresponding velocities are u, v and w or u_1, u_2 and u_3. To obtain the inlet velocity field, the velocity at the rescaling plane is first decomposed into a fluctuation and a (spanwise and temporal) mean.

$$u_i = \langle \bar{u}_i \rangle_z(y) + u_i'(y,z) \quad (1)$$

The temporal mean is calculated by the temporal one-sided filtering method used in **?** and **?**. The inner length-scales at the inlet (l_d^{inlet}) and at the rescaling (l_d^{resc}) planes are then used to calculate the inner mean velocity field ($(\langle \bar{u}_i \rangle_z)_{inner}^{inlet}(y)$) and the inner fluctuation field ($(u_i')_{inner}^{inlet}(y,z)$) field at the inlet. The mean ($(\langle \bar{u}_i \rangle_z)_{outer}^{inlet}(y)$) and fluctuation ($({u_i}')_{outer}^{inlet}(y,z)$) outer velocity profiles at the inlet are calculated using the respective outer length-scales (l_δ^{inlet}, l_δ^{resc}). A weight function($W(y, l_d^{inlet}, l_\delta^{inlet})$) (**?**) is used to superpose the outer and inner profiles to obtain the mean ($(\langle \bar{u}_i \rangle_z)^{inlet}(y)$) and fluctuation ($(u_i')^{inner}(y,z)$) inlet velocities. The weight function ensures that while combining the two profiles, more importance is given to the inner profile close to the wall and that far away from it, the outer profile dominates. The mean and fluctuation inflow velocities are then added together to obtain the total inlet velocity fields.

PROPOSED METHODOLOGY

The method proposed in the present study is for determining the length-scales required for rescaling. In prior studies using the rescaling-recycling method, δ_{99}^{inlet} is generally kept fixed at a desired value and is taken to be the outer length-scale at the inlet plane ($l_\delta^{inlet} = \delta_{99}^{inlet}$) also. At the rescaling plane, δ_{99}^{resc} measured from the mean flow profile, is taken to be the outer length-scale ($l_\delta^{resc} = \delta_{99}^{resc}$). However, since δ_{99}^{resc} is a threshold based quantity, it can be very sensitive to minor changes in the mean velocity profile shape. Large and rapid changes in the outer length-scale can in turn cause sudden changes in the inlet velocity profile. The use of δ_{99} as the outer length-scale, therefore, makes the inlet velocity susceptible to large undesirable fluctuations, particularly if the profile shape is non-monotonic. This challenge is illustrated in a sample simulation of turbulent boundary layer over roughness elements (details of the simulation to be provided below) that has not yet fully converged in time and hence displays some non-monotonicities in the partially converged mean velocity profile at the rescaling plane. In the profiles shown in Figs. **??**(a) and (b), where the presence of a small 'bulge' in the mean profile at the rescaling plane causes a large jump in δ_{99}^{resc}, which results in a sudden change in the mean inlet velocity profile in a very short time period between (a) and (b).

Motivated by this difficulty, we propose that instead of δ_{99}, a profile integrated quantity such as the displacement thickness (δ_1) be used. This should cushion the effects of an unusual mean velocity profile and reduce unwanted fluctuations in the inlet velocity profile. The benefit of using the displacement thickness is illustrated in Figs. **??** (c) and (d), where even the presence of a considerable 'bump' in the mean velocity profile at the rescaling plane does not result in significant changes in the mean inlet velocity in time. Instead of prescribing δ_{99}^{inlet}, the inlet displacement thickness δ_1^{inlet} is set to a desired value.

However, the existing expressions for the weight function $W(y, l_d^{inlet}, l_\delta^{inlet})$ must be expressed in terms of δ_{99}, and we prefer to maintain the well-tested expression rather than developing another merging function. Thus, we need to find a relation between the outer scale and δ_1, $l_\delta = \alpha\ \delta_1$ that for a canonical standard velocity profile reverts to the relation between the traditional δ_{99} scale and δ_1. At the inlet we write the relation as

$$l_\delta^{inlet} = \alpha\ \delta_1^{inlet}, \quad (2)$$

while at the outlet we write

$$l_\delta^{resc} = \beta\ \delta_1^{resc}, \quad (3)$$

since the ratio between both scales can be dependent on streamwise development distance of the boundary layer.

Rescaling Plane

The outer length-scale at the rescaling plane (l_δ^{resc}) is obtained based on the displacement thickness (δ_1^{resc}) and the coefficient β. The canonical $1/n$ mean velocity profile reads

$$\frac{(\langle \bar{u} \rangle_z)^{resc}(y)}{U} = \left(\frac{y}{\delta_{99}^{resc}} \right)^{1/n}. \quad (4)$$

For this profile it is known that the ratio among standard thicknesses is given by

$$\beta = \frac{\delta_{99}^{resc}}{\delta_1^{resc}} = n+1 \quad (5)$$

so that if we knew the precise value of n we could determine β and relate δ_{99} to δ_1. However, n is not necessarily known, and can depend on Reynolds number and other parameters.

In order to determine n and β, additional information about the velocity profile is required, which we propose to diagnose using higher order moments of the mean velocity profile. Specifically, we propose to define a new mth order length-scale according to

$$\delta_m^{resc} = \int_0^\infty \left[1 - \left(\frac{(\langle \bar{u} \rangle_z)^{resc}(y)}{U} \right)^m \right] dy. \quad (6)$$

For a $1/n$ profile, it can be deduced that

$$\frac{\delta_{99}^{resc}}{\delta_m^{resc}} = \frac{n+m}{m} \quad (7)$$

which is consistent with Eq. **??** for the classic displacement length with $m = 1$. Eliminating n from Eqs. **??** and **??** we obtain

$$\delta_{99}^{resc} = \frac{\delta_m^{resc}(m-1)}{m\delta_1^{resc} - \delta_m^{resc}} \times \delta_1^{resc}, \quad (8)$$

or

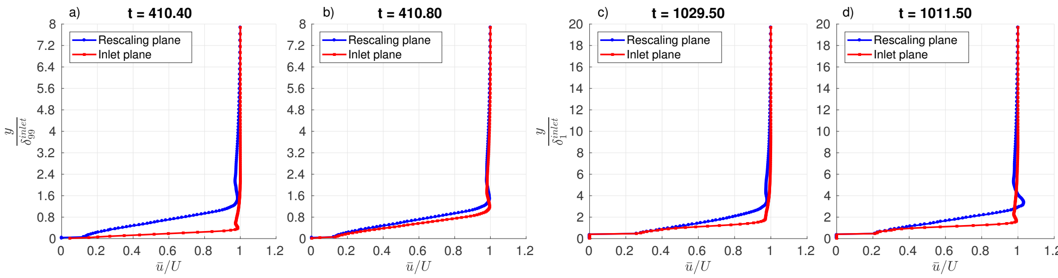

Figure 1. Illustration of recycle method in LES of boundary layer flow over rough wall with wall-mounted cubes. The figure shows partially converged mean stream wise velocity profiles at the inlet and rescaling planes. a) Rescaling based on δ_{99} : the mean velocity profile shape at the rescaling plane is only a slight deviation from a typical profile, hence the inlet profile is as expected. b) Rescaling based on δ_{99}: a short time after (a), although the bulge in the mean profile is very slight, it is large enough to cause a sudden change in the threshold-based measurement of δ_{99}^{resc}, giving a spurious inflow velocity field, very different from (a). (c) Rescaling based on δ_1 : for a typical mean profile, the inlet velocity field is as expected. (d) Rescaling based on δ_1 : the significant bulge in the mean profile at the rescaling plane also shows up in the inlet velocty field, but its overall effect on the inlet profile is small.

$$\beta = \frac{\delta_m^{resc}(m-1)}{m\delta_1^{resc} - \delta_m^{resc}}. \tag{9}$$

If the mean velocity profile satisfies the $1/n$ profile given in (**??**) exactly, the relation (**??**) yields the same β for all moment orders $m > 0$. However, for real profiles that condition may not be exactly true, but the relation (**??**) still provides a good approximation to the outer length-scale, as we have verified empirically that it varies very slowly with m.

Inlet Plane

The inlet displacement thickness (δ_1^{inlet}) is set to a desired value. Since it is a profile integrated quantity, the outer length-scale (l_δ^{resc}) must be determined via iterations so that a profile with the desired displacement thickness in obtained. This is done by determining α via iterations. Consider α_k, where k is the iteration index, which can be used to calculate the corresponding $l_\delta^{inlet,k}$. This length-scale is then used for rescaling, and since the inner lengths and the length-scales at the rescaling plane remain unchanged, a mean inlet velocity profile for that iteration step $((\langle\bar{u}\rangle_z^k)^{inlet}(y))$ is obtained. The inlet velocity is thus obtained and is used to evaluate the corresponding displacement thickness δ_1^k:

$$\delta_1^k = \int_0^\infty \left(1 - \frac{(\langle\bar{u}\rangle_z^k)^{inlet}(y)}{U}\right) dy \tag{10}$$

which may still differ from the desired inlet displacement thickness. Using a bisection iteration method we update the value of α_k until δ_1^k is sufficiently close to the prescribed value δ_1^{inlet}. We have checked that the l_δ^{inlet} determined in this manner is very close to the traditional δ_{99}^{inlet} if the velocity profile is of a shape well approximated by a $1/n$ profile (Eq. **??**). However the value may be significantly different for profiles not of that type.

TESTS USING DNS DATA

The proposed method to determine β based on the mth order moment measurement at the rescaling plane and α from iterations is tested using data from DNS for a transitional boundary layer. The data are available at the Johns Hopkins Turbulence Databases (**??**). The flow is laminar close to the leading edge of the transition to turbulence around $Re_x \approx 4 \times 10^5$. The methodology to determine the length-scales and mean rescaled profiles is applied to this dataset and the mean velocity profiles obtained are compared to the real DNS profiles. The results are summarized in Fig **??**.

The plane corresponding to $Re_x = 7.8 \times 10^5$ is chosen to be the rescaling plane. The outer length-scale at this rescaling plane (l_δ^{resc}), is determined by **??**. In Fig. **??**(a) we show that the resulting scale ratio (β) is relatively insensitive to the value of m chosen. The value obtained is very close to the real ratio $\delta_{99}^{resc}/\delta_1^{resc}$, since the mean velocity profile satisfies (**??**) almost exactly, and is consistent with $n \approx 4.5$. For further applications, we thus choose $m = 5$ as the order of moment to diagnose the mean profile.

We then choose an inlet displacement thickness (δ_1^{inlet}) and apply the rescaling-recycling method to get an 'inlet' velocity profile. This profile is then compared to the profile with the same displacement thickness from the DNS data-set, which corresponds to a particular value of Re_x (Fig **??**(c)). This is done for several values of δ_1^{inlet} and several 'inlet' velocity profiles are obtained (a few are shown in Fig **??**(e)-(h)).

Comparison of the ratio l_δ/δ_1 with the ratio δ_{99}/δ_1 from the DNS data (Fig **??**(b)), and comparison of the computed outer length-scales (l_δ) with the δ_{99} from the database (Fig **??**(d)) yields satisfactory results. For an 'inlet' plane reasonably close to the rescaling plane, the velocity profiles are of the $1/n$ type described in (**??**) and $l_\delta \approx \delta_{99}$, as is seen for values of $Re_x \gtrsim 6 \times 10^5$. The obtained profiles for these cases match the profiles from the DNS dataset quite well, illustrated in Fig **??**(g) and Fig **??**(h). As the inlet plane is placed further upstream of the rescaling plane, the departure from a $1/n$ profile grows bigger. This is accompanied by a departure of the ratio l_δ/δ_1 from the ratio δ_{99}/δ_1 and of the length-scale l_δ from δ_{99}, as is observed for $Re_x \lesssim 6 \times 10^5$. The obtained profile shapes are still quite close to the profiles from the DNS dataset (Fig **??**(e), (f)), despite differences in shape from the rescaling plane.

Thus, the proposed method to diagnose the ratio l_δ/δ_1 at the rescaling and inlet planes appears to work well when

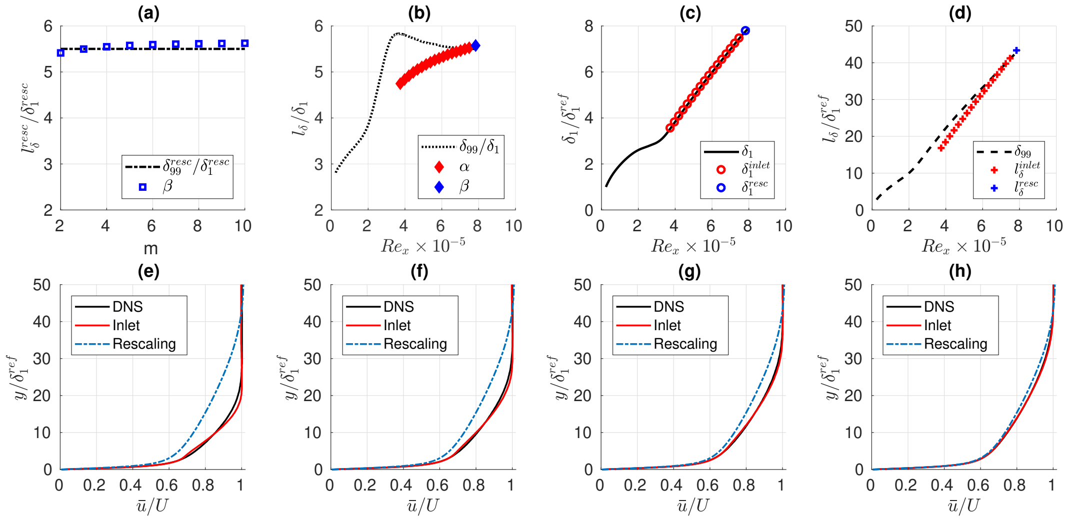

Figure 2. A-priori testing of the rescaling scheme using data from direct numerical simulation of a transitional boundary layer, taken from the Johns Hopkins Turbulence Databases (see **???**). (a) Variation of β with m compared to $\delta_{99}^{resc}/\delta_1^{resc}$ from the database at the rescaling plane (fixed at $Re_x = 7.8 \times 10^5$). (b) Ratio of the outer length-scale to the displacement thickness (l_δ/δ_1) at the inlet(α, calculated via iterations) and at the rescaling plane(β, for m=5) plotted against Re_x, corresponding to different δ_1^{inlet} is shown. The variation of δ_{99}/δ_1 with Re_x from the database is shown for comparison. (c) Displacement thickness plotted against Re_x. Reference length (δ_1^{ref}) corresponds to the displacement thickness at $Re_x = 2.4 \times 10^4$(from the database). (d) Outer length-scales at the inlet plane (l_δ^{inlet}) and at the rescaling plane (l_δ^{resc}) compared with δ_{99} from the database plotted against Re_x. (e)-(h) The mean streamwise velocity profile obtained from the scheme (Inlet) is compared to the profile from the database with equal displacement thickness (DNS). The mean profile at the rescaling plane (rescaling) is also shown for comparison. The values for Re_x corresponding to the different DNS profiles are: a) $Re_x = 4.0 \times 10^5$, b) $Re_x = 4.9 \times 10^5$, c) $Re_x = 5.9 \times 10^5$, d) $Re_x = 6.8 \times 10^5$.

tested on mean profiles in a direct numerical simulation of a developing turbulent boundary layer.

APPLICATION TO FLOW OVER ROUGHNESS ELEMENTS

The rescaling scheme is used to generate the inflow for large eddy simulation of a developing turbulent boundary layer over resolved roughness elements. For carrying out the simulations, a sharp-interface immersed boundary finite difference method (**?**) is used. The filtered Navier Stokes equations for large eddy simulations (LES) along with the Dynamic Vreman subgrid-scale model (**?**) are solved. For modelling wall effects, the integral wall model is used (**?**). A turbulent boundary layer over a rough wall is simulated with $Re_{\delta_1^{inlet}} = 30000$ over roughness elements. In units of δ_1^{inlet}, the computational domain size is (12 x 20 x 6) with roughness size (2 x 0.5 x 2). A 192 x 128 x 96 grid is used with 4% stretching in the wall normal direction.

The inlet displacement thickness δ_1^{inlet} is held fixed and is used as the reference length. In Fig **??**, instantaneous u, v, and w values are shown on various planes, scaled by the stream wise free stream velocity. The effect of the growing boundary layer is seen in the stream wise variation of u, v, and w in the $y-z$ plane. The effect of rescaling is apparent in the similarity between velocity fields close to the inlet and close to the rescaling plane. The height of the blocks is used as the inner length-scale l_d for the inlet and the rescaling planes. Time signals of the displacement thickness and the outer length-scales at the inlet and rescaling plane are shown in Figs. **??**(a) and (b). The displacement thickness at the inlet remains fixed at the desired value, while δ_1 at the rescaling plane 'settles' to a value after the initial transients pass. The mean velocity profiles after the flow has settled are as expected (Fig. **??**(c)). We conclude that the use of the inflow condition for this simple case appears to give satisfactory results.

CONCLUSIONS

A rescaling-recycling inflow generation method for developing turbulent boundary layer simulations is proposed. The method is based on displacement thickness and avoids the use of threshold based quantities like δ_{99} which are strongly dependent on the shape of the mean velocity profile, which may lead to undesirable fluctuations. Additionally to determine the ratio of the outer scale to δ_1 a higher-order moment of the velocity profile at the rescaling plane is measured. Tests of the proposed method to data from direct numerical simulation of developing turbulent boundary layer, provides a good proof of concept and results in mean inflow profiles that are in good agreement with data. Application of the method to generate the inflow for large eddy simulations of turbulent boundary layer over surface with roughness elements show trends that are as expected.

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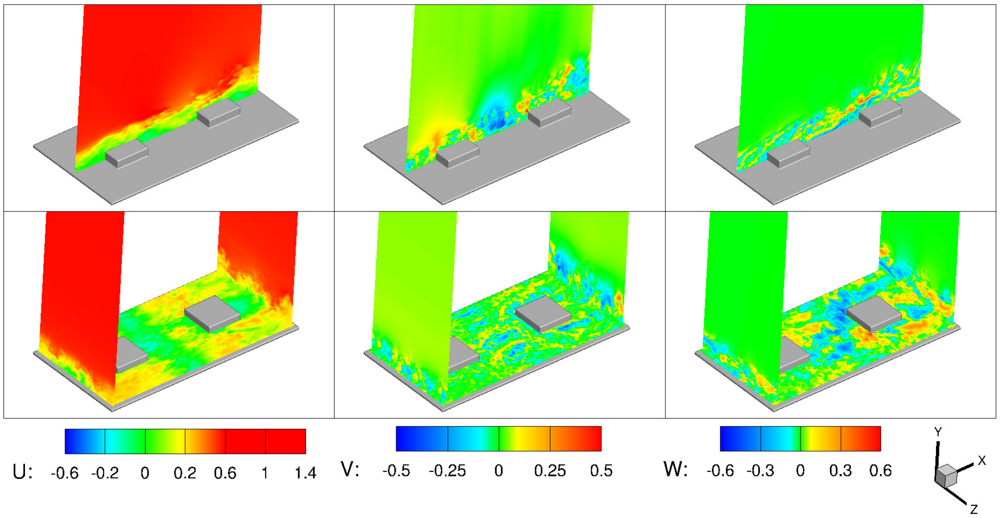

Figure 3. Instantaneous stream-wise (u), wall-normal (v) and span-wise (w) components of velocity on various planes. The upper row of figures shows the stream wise development of the velocity components on the central $(y-z)$-plane. The stream wise development of u, v and w indicates a developing boundary layer. The lower row shows u, v and w close to the inlet and the rescaling planes. The effect of the rescaling is clear in the velocity fields, since signatures of structures at the rescaling plane can be seen at the inlet plane. u, v and w are also shown on an $(x-z)$-plane in the region dominated by the inner layer.

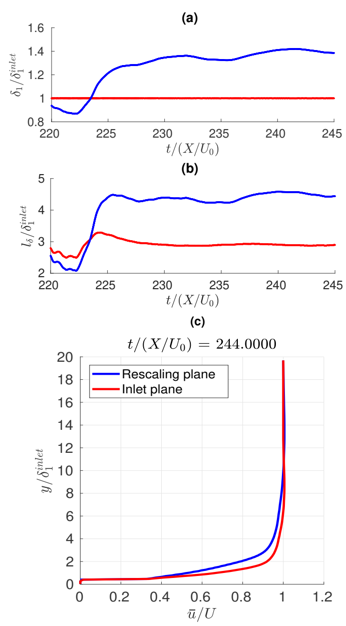

Figure 4. Time evolution of boundary layer scales in large eddy simulation of flow over wall-mounted roughness elements at $Re_{\delta_1^{inlet}} = 30000$. (a) Time variation of displacement thickness at the inlet and at the rescaling plane. (b) Temporal variation of the outer length-scale at the inlet and at the rescaling plane. After the initial transients pass, the flow settles and only small variations are seen in the length-scales. c) Mean stream-wise velocity profile at the inlet and rescaling planes when the flow has settled.