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Kinetic theory of the electron strahl in the solar wind

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ABSTRACT

We develop a kinetic theory for the electron strahl, a beam of energetic electrons which propagate from the sun along the Parker-spiral-shaped magnetic field lines. Assuming a Maxwellian electron distribution function in the near-sun region where the plasma is collisional, we derive the strahl distribution function at larger heliospheric distances. We consider the two most important mechanisms that broaden the strahl: Coulomb collisions and interactions with oblique ambient whistler turbulence (anomalous diffusion). We propose that the energy regimes where these mechanisms are important are separated by an approximate threshold, $\mathcal{E}_c$; for the electron kinetic energies $\mathcal{E} < \mathcal{E}_c$ the strahl width is mostly governed by Coulomb collisions, while for $\mathcal{E} > \mathcal{E}_c$ by interactions with the whistlers. The Coulomb broadening decreases as the electron energy increases; the whistler-dominated broadening, on the contrary, increases with energy and it can lead to efficient isotropization of energetic electrons and to the formation of the electron halo. The threshold energy $\mathcal{E}_c$ is relatively high in the regions closer to the sun, and it gradually decreases with the distance, implying that the anomalous diffusion becomes progressively more important at large heliospheric distances. At 1 au, we estimate the energy threshold to be about $\mathcal{E}_c \sim 200\,\mathrm{eV}$.

Key words: plasmas – Sun: heliosphere – solar wind.

1 INTRODUCTION

The solar wind consists of a magnetized plasma nearly radially propagating from the sun. Observations show that the temperature of the expanding plasma declines with the radial distance r, following an approximate power-law trend $T(r) \propto r^{-\gamma}$, where $\gamma \approx 0.5$ (e.g. Köhnlein 1996; Cranmer et al. 2009; Štverák, Trávníček & Hellinger 2015; Bale et al. 2016; Chen 2016; Verscharen, Klein & Maruca 2019). In particular, at the distance of 1 au the solar wind cools down to about 10 eV as compared to the solar corona where the plasma temperature is of the order of 100 eV. The solar wind plasma is, however, weakly collisional, so while the temperature of its Maxwellian core follows the mentioned trend rather well, the velocity distribution function also has features that deviate significantly from the thermal Maxwellian distribution. In particular, the electron velocity distribution function (eVDF) can be represented as consisting of three major components: the nearly Maxwellian thermal core, the suprathermal beam aligned with the direction of the magnetic field (the so-called strahl), and the nearly isotropic and broad (non-Maxwellian) halo, which overlaps in energy with the strahl (e.g. Feldman et al. 1975; Pilipp et al. 1987; Pierrard et al. 2016). The strahl and halo are relatively tenuous, for instance, at 1 au they comprise about 5 per cent of the total electron density. However, since their energies exceed that of the core by an order of magnitude, the heat flux associated with the strahl is non-negligible and it can heat the solar wind at relatively large heliospheric distances (e.g. Štverák et al. 2009, 2015). Moreover, non-Maxwellian anisotropic distribution function can be a source of kinetic instabilities and small-scale turbulence that lead to formation of structures, particle heating, and acceleration (e.g. Forslund 1970).

In this work we develop a kinetic theory of the electron strahl. In order to understand how the strahl is formed and how it changes with the radial distance, we trace the evolution of the eVDF all the way from the hot inner region (~ 5–$10\,\mathrm{R}_\odot$) where the electrons are assumed to have a given distribution (say, a Maxwellian) to larger heliospheric distances. We assume that the magnetic field has a Parker-spiralled structure, and solve the drift-kinetic equation that describes the evolution of the electron distribution function along the magnetic field lines. The energetic, nearly collisionless electrons stream away from the sun along the magnetic field, and are collimated into a narrow beam (strahl) as they attempt to conserve their magnetic moment. Weak Coulomb collisions with the background plasma, on the other hand, tend to broaden their collimation angle.

Comparison with some strahl measurements in the fast solar wind (e.g. Štverák et al. 2009, 2015; Horaites et al. 2018a,b) demonstrates that our Coulomb theory allows one to describe the physics of strahl formation rather well, on both qualitative and quantitative levels. In

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particular, the theory is able to predict the strahl angular broadening and the number of particles in the strahl. When the Coulomb theory provides a good agreement with the observational data, it means that other possible scattering mechanisms, such as interactions of the electrons with plasma fluctuations, are relatively unimportant (Ogilvie, Fitzenreiter & Desch 2000; Pierrard, Maksimovic & Lemaire 2001; Horaites et al. 2015, 2018a,b; Horaites, Boldyrev & Medvedev 2019).

In some measurements, however, the angular distribution of the strahl electrons is wider than the distribution predicted by the Coulomb model (e.g. Hammond et al. 1996; Anderson et al. 2012; Graham et al. 2017, 2018). In such cases, it is reasonable to assume that the enhanced broadening is provided by ambient plasma turbulence that scatter the energetic electrons (anomalous scattering); such scattering should be taken into account in addition to that provided by Coulomb collisions (classical scattering). One of the natural candidates for the anomalous scattering is whistler turbulence (e.g. Gary et al. 1975, 1994; Vocks & Mann 2003; Vocks et al. 2005; Pagel et al. 2007; Saito & Gary 2007; Pierrard, Lazar & Schlickeiser 2011; Wilson et al. 2013; Lacombe et al. 2014; Kajdič et al. 2016; Stansby et al. 2016; Tang, Zank & Kolobov 2018). In order to analyse the strahl broadening caused by whistler turbulence, we assume that the turbulence is oblique with respect to the background magnetic field, and incorporate the anomalous scattering in the drift-kinetic equation.[1] In this respect our consideration is complementary to previous studies that considered electron-strahl broadening caused by the whistlers propagating *along* the background magnetic field (Pierrard et al. 2011; Tang et al. 2018).

We find that the scattering by the whistlers rapidly increases with the intensity of the turbulence. As a result, the two scattering mechanisms (i.e. Coulomb scattering and scattering by the oblique whistlers) dominate in different regions of the phase space roughly separated by an energy threshold, $\mathcal{E}_c$. The strahl electrons with lower energies, $\mathcal{E} < \mathcal{E}_c$, are mostly scattered by classical Coulomb collisions, while the more energetic electrons, with energies exceeding $\mathcal{E}_c$, by whistler turbulence. As the anomalous scattering becomes far more significant than Coulomb collisions at high energies, it may lead to isotropization of the energetic strahl electrons and to formation of the electron halo. We estimate that at 1 au, the threshold energy may be of the order of 200 eV. The dominance of Coulomb collisions at lower energies and the predicted energy-dependent strahl broadening at higher energies is qualitatively consistent with the recent analytical and observational studies (e.g. Berčič et al. 2019; Horaites et al. 2019).

2 THE COULOMB THEORY OF THE STRAHL

In this section, we develop a kinetic theory for the strahl component of the eVDF, taking into account classical Coulomb collisions and neglecting anomalous scattering effects. The speed of the strahl electrons is significantly larger than that of the solar wind. The suprathermal electrons not only experience significantly weaker Coulomb collisions as compared to the core electrons, but they also stream from the sun to very large distances ($\sim$10 au) along nearly stationary magnetic field lines. Indeed, the magnetic field lines are advected with the speed of the solar wind, while the speed of the electrons is much higher.

When the collision frequency is much smaller than the gyrofrequency of the particles, the eVDF is gyrotropic; it can be averaged over the fast period or electron gyromotion. It can then be written using the variables of velocity v, the cosine of the angle between velocity and the (antisunward directed) background magnetic field $\mu \equiv \cos\theta = v_\parallel / v$, and the distance along a magnetic field line x. The distribution obeys the following drift-kinetic equation (e.g. Kulsrud 2005; Horaites et al. 2015):

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial x} - \frac{1}{2}\frac{\mathrm{d}\ln B}{\mathrm{d}x} v\left(1-\mu^2\right)\frac{\partial f}{\partial \mu} - \frac{eE_\parallel}{m_\mathrm{e}}\left[\frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu} + \mu\frac{\partial f}{\partial v}\right] = \hat{C}(f). \tag{1}$$

In this equation, $E_\parallel = -\nabla\phi(x)\cdot\hat{x}$ is the electric field along the magnetic field line, and $\hat{C}(f)$ denotes the collision integral.

Let us first consider a purely collisionless evolution, $\hat{C}(f) = 0$, and assume that we are interested in a steady-state distribution. Equation (1) then takes an especially simple form if one uses the following variables: the magnetic moment $M = m_\mathrm{e} v_\perp^2/(2B(x))$, the total energy $E = m_\mathrm{e} v^2/2 + e\phi(x)$, and the distance x. Note that e here refers to the (negative) electron charge. As can be directly verified, the eVDF $f(E, M, x)$ then obeys the equation

$$\mu v\, \partial f/\partial x = 0, \tag{2}$$

meaning that the distribution function is independent of the distance. The magnetic field lines that generally follow the Parker-spiral configuration are almost radial close to the sun. Using the observationally inferred trends for the temperature and density variations with the heliospheric distance (e.g. Köhnlein 1996; Štverák et al. 2015), one can expect that at a distance of approximately $x = r_0 \sim 5$–$10\,\mathrm{R}_\odot$, the plasma is dense and relatively more collisional than at larger radial distances. So one may assume that the electron distribution is Maxwellian with a temperature of about $T_0 \sim 100$ eV. (This simplifying assumption, although plausible, is not essential for our kinetic derivation. Our theory can be generalized for any gyrotropic distribution specified at distance r_0. Our goal is to find the electron distribution function at larger distances, $r > r_0$, once this inner-region distribution is known.)

In the new variables, the Maxwellian distribution looks like

$$f(r_0, E, M) = A_0 \exp\left\{-\frac{E}{T_0}\right\}\theta\left(E - MB_0\right), \tag{3}$$

where $A_0 = n_0(m_\mathrm{e}/2\pi T_0)^{3/2}$ is the normalization coefficient, $B_0 = B(r_0)$, and the theta function reflects the fact that our variables obey the restriction $E \geq MB_0$. According to equation (2), the distribution function in these variables is independent of the distance, $f(r, E, M) = f(r_0, E, M)$; we therefore obtain from equation (3), $r > r_0$:

$$f(r, E, M) = A_0 \exp\left\{-\frac{m_\mathrm{e} v^2}{2T_0} - \frac{e\phi(r)}{T_0}\right\}\theta \times \left(v^2 + \frac{2e}{m_\mathrm{e}}\phi(r) - \frac{B_0}{B(r)}v_\perp^2\right). \tag{4}$$

Away from the sun, $r > r_0$, the ambipolar potential energy $e\phi(r)$ fast approaches its maximal value, $e\phi_\infty$, which is a few times larger than T_0.[2] We can estimate the pitch-angle breadth (θ) of the beam

[1]Turbulent whistlers may, in fact, be expected to be oblique based on analytical and numerical studies, and also on observations (e.g. Cho & Lazarian 2009; Boldyrev & Perez 2012; Meyrand & Galtier 2013; Narita et al. 2016).

[2]We define the ambipolar potential in such a way that it is zero at $r = r_0$. Using standard methods (e.g. Boldyrev, Forest & Egedal 2019), one can

at such distances, by equating the argument of the theta function in equation (5) to zero:

$$\sin^2\theta = \frac{v_\perp^2}{v^2} = \frac{B(r)}{B_0}\left(1+\frac{2e\phi_\infty}{m_e v^2}\right). \tag{5}$$

For instance, at $r = 1$ au, we can estimate by order of magnitude that $B(r)/B_0 \sim 10^{-4}$, so for the electron kinetic energy of $m_e v^2/2 \sim 100$ eV, the strahl collimation angle would be rather narrow, $\theta \sim 10^{-2}$.

One can show, however, that such a narrow collimation angle cannot be established, since it will be broadened by weak Coulomb collisions. In order to describe the Coulomb collisions, we need to add the collision integral in equation (2). The energetic strahl electrons have relatively weak energy exchange with the plasma particles forming the Maxwellian core, however, they experience a significant pitch-angle scattering. In order to describe the strahl broadening, we therefore retain in the collision integral only the term describing the pitch-angle scattering (e.g. Helander & Sigmar 2002):

$$\hat{C}(f) = \left(\frac{4\pi n(x)e^4\Lambda\beta}{m_e^2 v^3}\right)\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial f}{\partial\mu}. \tag{6}$$

This collision integral describes the pitch-angle scattering of the suprathermal strahl electrons ($v^2 \gg v_{Te}^2$) by the Maxwellian core electrons and the core ions. In equation (6), $\beta = (1 + Z_{\rm eff})/2$, where $Z_{\rm eff}$ is the effective ion charge. For the solar wind plasma, β can be estimated as $\beta \approx 1.05$. The Coulomb logarithm can be estimated at 1 au as $\Lambda \approx 30$, it is a slowly varying function of the distance, and $n(r)$ is the density of the core electrons, which is approximately equal to the density of the ions, see e.g. Horaites et al. (2019).

The collision integral can be rewritten using the new variables E, M, and x, which gives for the steady-state drift-kinetic equation (Horaites et al. 2019):

$$\frac{\partial f(x,E,M)}{\partial x} = \frac{4\pi e^4\Lambda\beta n(x)}{\mathcal{E}(E,x)B(x)}\frac{\partial}{\partial M}M\sqrt{1-\frac{MB(x)}{\mathcal{E}(E,x)}}\frac{\partial f}{\partial M}, \tag{7}$$

where we have denoted $\mathcal{E}(E, x) \equiv E - e\phi(x) = m_e v^2/2$. The expression in the square root in equation (7) can be simplified since, as one can directly verify, $MB(x)/\mathcal{E}(E,x) = v_\perp^2/v^2 = \sin^2\theta \ll 1$, and this term can be neglected for the field-aligned strahl. Moreover, as we are interested in the runaway electrons, the total electron energy should exceed the ambipolar potential barrier, $E > e\phi_\infty$.

An equation similar to equation (7) was analysed in our previous treatment of the problem (Horaites et al. 2019), where we were interested in the evolution of the strahl at radial distances significantly exceeding the coronal region, $r \gg r_0$. As a result, we were able to obtain the angular distribution of the strahl electrons, but could not specify the electron distribution function uniquely – our solution contained an arbitrary function of the electron kinetic energy. In this work, we relate the electron distribution function to the boundary condition at $r \sim r_0$, and derive a complete solution for the suprathermal electron strahl.

It is convenient to represent the electron kinetic energy, $\mathcal{E} = E - e\phi(x)$, in the form $\mathcal{E} = \Delta E + \mathcal{T}(x)$, where the first term, $\Delta E = E - e\phi_\infty$, is independent of the distance, and all the radial dependence is included in the second term $\mathcal{T}(x) = e\phi_\infty - e\phi(x)$.[3] One can argue that the function $\mathcal{T}(x)$ is of the order of the local electron temperature, i.e. the temperature of the electron core $T(x) \approx \mathcal{T}(x)$.[4] In what follows, we therefore will approximate $\mathcal{E} \approx \Delta E + T(x)$.

The form of equation (7) suggests that we may introduce the following new variable (see also Horaites et al. 2019):

$$\mathrm{d}y = \left(\frac{4\pi e^4\Lambda\beta}{\mathcal{E}}\right)\left(\frac{n(x)}{B(x)}\right)\mathrm{d}x. \tag{8}$$

In this equation, dx is the length element along a magnetic field line. Since the magnetic field is frozen into the solar-wind flow, the combination d$xn(x)/B(x)$ is an invariant of the motion. We can therefore evaluate it at the distance r_0 close to the sun, where the magnetic field lines are nearly radial, $\mathrm{d}x n(x)/B(x) = \mathrm{d}r_0 n(r_0)/B(r_0)$. Next, we notice that the solar wind speed is nearly constant as a function of the radial distance at $r > r_0$. Therefore, any two points separated by the radial distance dr that corresponds to the separation dx along a field line do not change their radial separation during the motion, thus dr = dr_0. We can therefore write

$$\mathrm{d}y = \left(\frac{4\pi e^4\Lambda\beta}{\mathcal{E}}\right)\left(\frac{n_0}{B_0}\right)\mathrm{d}r = \left(\frac{4\pi e^4\Lambda\beta}{\Delta E+\mathcal{T}(r)}\right)\left(\frac{n_0}{B_0}\right)\mathrm{d}r. \tag{9}$$

This equation can, in principle, be integrated once the temperature profile is specified. For instance, if we assume the model power-law behaviour for the electron temperature $\mathcal{T}(r) \propto r^{-0.5}$, a straightforward calculation will give

$$y = \frac{4\pi e^4\Lambda\beta n_0}{B_0\Delta E}R(r), \tag{10}$$

where we have denoted

$$R(r) = r\left[1-2\left(\frac{\mathcal{T}(r)}{\Delta E}\right)+2\left(\frac{\mathcal{T}(r)}{\Delta E}\right)^2\log\left(\frac{\Delta E}{\mathcal{T}(r)}+1\right)\right]. \tag{11}$$

Of course, the function $R(r)$ may be evaluated numerically for an arbitrary $\mathcal{T}(r)$ profile, and our results in the following discussion do not assume any particular profile. For a given energy, $R(r)$ is a function of the heliospheric distance only.

Since we are interested in suprathermal electrons, we can often approximate $\mathcal{E} \approx \Delta E \gg \mathcal{T}(r)$, and therefore in this limit the function $R(r)$ can be replaced by r in equation (10). We, however, note that in cases when we need to evaluate exponential functions, we may need to keep the small $\mathcal{T}(r)$ term in the general expression for the energy $E = e\phi_\infty + \Delta E = e\phi_\infty + \mathcal{E} - \mathcal{T}(r)$.

demonstrate that it increases to the values comparable to its asymptotic value ϕ_∞ at a typical distance of order $r \gtrsim r_0(m_e/m_i)^{1/4}$, which for a hydrogen plasma gives $r \gtrsim 6\,r_0$.

[3]We remind the reader that we denote by x the distance along a magnetic field line, while keeping the variable r for the radial distance. One variable can be expressed through the other using the Parker-spiralled shape of the magnetic field.

[4]There are two ways how these results can be understood. First, at $r \gg r_0$, the electron temperature $T(r)$ can be related to the ambipolar electric potential if one assumes that the large-scale electron flow obeys the hydrodynamic momentum equation. The radial force-balance condition in this equation then gives $-(1/n)\nabla(nT) - e\nabla\phi = 0$ (e.g. Hollweg 1970; Jockers 1970), where the electron temperature is assumed to be isotropic. Substituting here $n(r) \propto r^{-2}$ and $T(r) \propto r^{-1/2}$, by integrating we obtain $e\phi_\infty - e\phi(r) = 5\,T(r)$. Therefore, $\mathcal{T}(r)$ is proportional to the electron temperature. The second, more general, consideration (e.g. Boldyrev et al. 2019) notices that the local thermal electron populations can consist only of the electrons that cannot run away to infinity, i.e. whose kinetic energy does not exceed the potential barrier $e\phi_\infty - e\phi(x)$. Therefore, $\mathcal{T}(x)$ is of the order of the local kinetic energy of the background electrons and proportional to their temperature.

Equation (7) now turns into a two-dimensional diffusion equation in M-space:

$$\frac{\partial}{\partial y} f(y, E, M) = \frac{\partial}{\partial M} M \frac{\partial}{\partial M} f. \tag{12}$$

The standard solution of equation (7) takes the form (equations 5 and 6, Horaites et al. 2019):

$$f(y, E, M) = \frac{C(E)}{y} \exp\left(-\frac{M}{y}\right), \tag{13}$$

where $C(E)$ is an arbitrary function. We would now like to match the solution (13) with our formula for the collisionless case (4). We will accomplish this by imposing that the two solutions have the same width (ΔM) and amplitude at some distance y_m. First let us consider that the width of the strahl (in terms of M) inferred from equation (13), ΔM, is of the order $\Delta M \sim y$. By comparison, the width of the strahl described by equation (4) is estimated as $\Delta M \sim E/B_0$. So we find that the solutions will have the same width, and can be approximately matched, at the (energy-dependent) distance $y_m \sim E/B_0$. Additionally equating the amplitudes of solutions (13), (4) leads to the expression, which we use to model the strahl[5] at distances $y \gtrsim y_m$:

$$f = A_0 \exp\left(-\frac{E}{T_0}\right) \frac{E}{B_0} \frac{1}{y} \exp\left(-\frac{M}{y}\right). \tag{14}$$

The obtained solution can be rewritten in a more compact form if we introduce the electron mean free path at $r = r_0$, defined as

$$\lambda_0 = \frac{T_0^2}{4\pi n_0 e^4 \Lambda \beta}. \tag{15}$$

We then get for the suprathermal part of the distribution function

$$f \approx A_0 F_0 \frac{\lambda_0}{R(r)} \left[\frac{\Delta E + e\phi_\infty}{e\phi_\infty}\right] \frac{\Delta E}{T_0} \exp\left(-\frac{\Delta E}{T_0}\right) \times \exp\left(-\frac{\mathcal{E}\Delta E \sin^2\theta}{T_0^2} \frac{\lambda_0}{R(r)} \frac{B_0}{B(r)}\right), \tag{16}$$

where

$$F_0 = \frac{e\phi_\infty}{T_0} \exp\left(-\frac{e\phi_\infty}{T_0}\right) \approx \left(\frac{T_{i,0}}{T_0}\right)^{1/2} \left(\frac{m_e}{m_i}\right)^{1/2}. \tag{17}$$

We remind the reader that $\mathcal{E} = m_e v^2/2$ is the electron kinetic energy, $\Delta E \approx \mathcal{E} - T(r)$ is the excess of the kinetic energy of the strahl electrons over the thermal energy of the background plasma, and the distance parameter $R(r)$ is given by formula (11).

In formula (17), $T_{i,0}$ is the temperature of the ions at the source location r_0. The estimate for F_0 comes from the fact that the ambipolar potential barrier ϕ_∞ is established as to ensure that the proton and electron currents from the sun balance each other. The electrons, as lighter particles, escape with higher velocities therefore ϕ_∞ is negative and $e\phi_\infty$ is positive. Observations and analytical modelling suggests that the ions are heated more efficiently in the corona, so that $T_{i,0}/T_0 \approx 10$ (e.g. Chandran et al. 2011). A kinetic calculation, assuming that at r_0 the distributions of both the ions and the electrons are Maxwellian and the outflows are radially symmetric, then leads to the estimate (17) and to $e\phi_\infty/T_0 \approx 4$.[6] Formula (16) is the main result of our Coulomb theory of the electron strahl.

3 ANALYSIS OF THE COULOMB STRAHL SOLUTION

In this section we discuss what predictions follow from the strahl solution mediated by Coulomb collisions (equation 16) and to what extent they agree with the available observations.

Our first result is the width of the strahl, which can be found from the exponential factor in equation (16):

$$\sin^2\theta \approx \frac{T_0^2}{\mathcal{E}\Delta E} \frac{R(r)}{\lambda_0} \frac{B(r)}{B_0}. \tag{18}$$

The formula is valid as long as our main assumption $\sin^2\theta \ll 1$ is satisfied. In the limit $\mathcal{E} \gg \mathcal{T}(r)$, this formula is consistent with the result derived previously in Horaites et al. (2019), where it was found to be in good agreement with the Wind measurements using the SWE instrument (Ogilvie et al. 2000). We remind that the Parker-spiral magnetic field strength has the form

$$B(r) = B_0 \frac{r_0^2}{r^2} \sqrt{1 + \frac{r^2}{r_{45}^2}}, \tag{19}$$

where r_{45} is the heliospheric distance where the magnetic field line makes an angle of 45° with the radial direction. From observations, one can estimate that $r_{45} \approx 1$ au. From equations (18) and (19) one can see that the strahl becomes progressively more collimated with the distance in the inner heliosphere, $r < r_{45}$. However, at $r \gg r_{45}$ the width of the strahl saturates, i.e. it becomes independent of the distance. This effect was discovered in Horaites et al. (2019). It can be explained in the following way. At large heliospheric distances, the Parker spiral becomes progressively better aligned with the azimuthal heliospheric direction, so that the travel distance of the electrons increases as they propagate away from the sun, which enhances the efficiency of the Coulomb collisions. Simultaneously, the strength of the magnetic field (19) declines with the distance more slowly in the outer heliosphere, which reduces the magnetic focusing effect. The solution presented above demonstrates that in this case the magnetic focusing and Coulomb pitch-angle broadening balance each other at $r \gg r_{45}$, which leads to a universal saturated width of the strahl.

Second, formula (16) also allows us to estimate the number of particles in the strahl. First, we note that due to the exponential cut-off, only the energies $\Delta E \approx \mathcal{E} \lesssim T_0$ will contribute significantly to the integral of the distribution function (16). Therefore, the expression in the square brackets in equation (16) is of order unity. Next, we assume that the strahl is narrow, so we can approximate $\sin\theta \sim \theta$. The strahl distribution function (16) can then be easily integrated over the velocity space, and we obtain

$$\frac{n_{st}(r)}{n(r)} \approx \frac{F_0}{2} \frac{B(r)}{B_0} \frac{n_0}{n(r)} \exp\left(\frac{\mathcal{T}(r)}{T_0}\right). \tag{20}$$

We remind that $\mathcal{T}(r)$ is of the order of the local electron temperature, and $\mathcal{T}(r)$ is smaller than T_0. At 1 au, we estimate from this formula that $n_{st}(r)/n(r) \approx 0.05$. This simple derivation provides a rather good

[5]From equation (10), one can estimate that the distance at which the two solutions match is several times larger than r_0, so that the collimation angle of the suprathermal electrons is smaller than one, and the diffusion equation (12) derived in the limit of small collimation angles is applicable.

[6]Strictly speaking, the kinetic calculation gives the following condition for the potential barrier $\left[1 + \frac{e\phi_\infty}{T_0}\right] \exp\left(-\frac{e\phi_\infty}{T_0}\right) = \left(\frac{T_{i,0}}{T_0}\right)^{1/2} \left(\frac{m_e}{m_i}\right)^{1/2}$ (e.g. Boldyrev et al. 2019). However, as $e\phi_\infty/T_0 \approx 4$, we may neglect unity in the square brackets and use equation (17) as an estimate.

agreement with the values inferred from observations (e.g. Štverák et al. 2009). Due to the slowly changing function $\exp(\mathcal{T}(r)/T_0)$, our formula (20) also predicts that in the inner heliosphere, the fraction of particles in the strahl slowly declines with the distance, which is in agreement with the observations by Štverák et al. (2009).

In the outer heliosphere $r \gg r_{45}$, however, our Coulomb formula predicts a relative increase of the strahl fraction, while the observations demonstrate the opposite trend. This may be not surprising, however, since our Coulomb model does not include possible strong angular scattering and isotropization of the strahl electrons due to non-Coulomb effects, and therefore it overestimates the strahl population. Non-Coulomb (anomalous) broadening may also explain the instances where the strahl width was observed to be broader than that predicted by the Coulomb model or where the width of the strahl was found to increase with the heliospheric distance rather than decrease or saturate (e.g. Anderson et al. 2012; Graham et al. 2017, 2018; Horaites et al. 2019). The non-Coulomb scattering effects are discussed in Section 4.

Third, as follows from equation (18), the strahl width is independent of the parameters of the source – the electron temperature T_0 and the magnetic field B_0. The information about the electron distribution function of the source is, however, imprinted in the strahl amplitude. Our formula (16) demonstrates that for the Maxwellian velocity distribution of the source electrons, the strahl amplitude is proportional to $(\mathcal{E}/T_0)\exp(-\mathcal{E}/T_0)$. This result agrees with the exponential fall-off of the strahl amplitude previously reported in the SWE measurements by Ogilvie et al. (2000), where the characteristic temperature scales of about $T_0 \sim 100$ eV were detected. We also note that the strahl amplitude, as given by our formula (16), is rather low. An estimate shows that at 1 au, the strahl component of the distribution function starts to exceed the Maxwellian core component at about $\mathcal{E} \gtrsim 4\,T(r)$, which also agrees with available observations (e.g. Štverák et al. 2009).

Finally, it is interesting to point out that a non-monotone velocity profile of the strahl, as given by the Coulomb theory (16), may, in principle, lead to an instability, and if so, it would hardly persist at large heliospheric distances. If the core + strahl distribution function becomes unstable, it will quickly relax to a stable monotone profile. The relaxation process will smooth out the velocity profile at energies $\Delta E \lesssim T_0$, but will not change the number of particles in the strahl as estimated in equation (20), and the exponential decline of the strahl amplitude at higher energies, $\Delta E \gtrsim T_0$.

4 ANOMALOUS BROADENING OF THE STRAHL

Observations demonstrate that the electron strahl overlaps in energies with another suprathermal component of the electron distribution function, the so-called halo. The halo population is nearly isotropic in the velocity space, and its distribution is well approximated by a power-law function at large energies (e.g. Pierrard et al. 2016). The origin of the halo is currently not well understood. It is possible that several distinct mechanisms are at play in the halo formation. One mechanism is related to the possibility that the fast electrons can be trapped by the magnetic field lines at large heliospheric distances and directed back towards the sun by reflection by plasma inhomogeneities or by following looped magnetic field lines (e.g. Scudder & Olbert 1979; Gosling et al. 1993; Gosling, Skoug & Feldman 2001; Horaites, Boldyrev & Medvedev 2019). Indeed, the observed isotropy of the halo demonstrates the presence of sunward moving energetic electrons. Since these electron are rather energetic, they are virtually unaffected by Coulomb collisions and therefore they can come from very large radial distances ($\sim$10 au). As these electrons propagate closer to the sun in the regions of increasing magnetic field strength, magnetic de-focusing can efficiently isotropize their velocity distribution function. The halo electrons can thus be the population of suprathermal electrons that escaped the sun as a strahl but later trapped by magnetic field lines at global heliospheric scales, and isotropized by the combination of Coulomb collisions and magnetic de-focusing. This is consistent with the fact that the halo is nearly isotropic but the strahls are predominantly observed in the antisunward directions.

An alternative possibility, which will be discussed in more detail below, is that the halo is generated locally from the strahl electrons that experience very strong angular scattering by some mechanism (e.g. Štverák et al. 2009, 2015). The nature of such a mechanism can be debated, but a possible candidate for scattering is interaction with ambient plasma turbulence, in particular, the whistler modes. The wave–particle resonance condition, $\omega - k_\| v - n\Omega_e = 0$, can be easily satisfied for $n = \pm 1$. The quantitative description of this process depends on the model assumed for the whistler turbulence. For instance, one can assume that turbulence consists mostly of the whistlers propagating along the direction of the magnetic field lines, $k_\| \gg k_\perp$. Such models were developed in Pierrard et al. (2011) and Tang et al. (2018), as possible candidates for explaining the evolution of suprathermal electrons. The advection–diffusion kinetic equations describing the electron strahl were derived that could be analysed analytically and numerically.

In our consideration, we concentrate on the complementary case, when the whistler turbulence is oblique, i.e. $k_\perp > k_\|$. This assumption may be consistent with some phenomenological and numerical models (e.g. Cho & Lazarian 2009; Boldyrev & Perez 2012; Meyrand & Galtier 2013) and observations (Alexandrova et al. 2009; Kiyani et al. 2009; Chen et al. 2010, 2012; Sahraoui et al. 2013; Narita et al. 2016), and similarly to the case of quasi-parallel turbulence, it also allows for analytical treatment. In the case of oblique propagation, the whistler-mode frequency has the form

$$\omega = k_\| k_\perp v_A d_i, \tag{21}$$

where v_A is the Alfv´en speed and d_i is the ion inertial scale. Whistlers exist in the region of the phase space $\omega \gg k v_{Ti}$, where v_{Ti} is the thermal velocity of the ions. In the case when the ion plasma beta is of order one, $\beta_i = v_{Ti}^2/v_A^2 \approx 1$, this condition implies that $k_\| d_i \gg 1$. For the electron velocities satisfying $v \gg v_{Te}$, we see from equation (21) that $\omega \ll k_\| v$ therefore the resonance condition reads $k_\| v = \pm\Omega_e$.[7]

The simplest analytical description of the wave–particle interaction is in the form of quasi-linear diffusion, which demonstrates how the distribution function evolves under the action of a large number of particle interactions with an ensemble of linear waves (e.g. Stix 1992, Chapter 17). This is certainly an approximation as the whistler modes are not necessarily linear waves. However, it is known from analytic modelling and observation that even in the case of strongly non-linear turbulence, the linear and non-linear terms in the governing plasma equations are of the same order (the so-called critical balance condition) (Goldreich & Sridhar 1995; Cho & Lazarian 2009; Boldyrev & Perez 2012; TenBarge & Howes 2012).

[7] Indeed, in a plasma with the electron beta satisfying $\beta_e \approx 1$, the whistlers exist at $k_\perp d_e \lesssim 1$. This means that $\omega \lesssim k_\| v_{th,e} \ll k_\| v$ in case of suprathermal electron velocities v.

Therefore, a consideration based on a linear dispersion relation, in addition to being analytically tractable, provides a good order-of-magnitude estimate. One can ask what contribution the quasi-linear interaction provides to the pitch-angle scattering. For that we write the collision operator as

$$\hat{C} = S\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial f}{\partial\mu}, \tag{22}$$

where $S = S_{\rm C} + S_{\rm QL}$ is the sum of the Coulomb collision term and the quasi-linear diffusion term. The Coulomb collision part is given by equation (6). The quasi-linear diffusion coefficient $S_{\rm QL}$ is proportional to the integral of the intensity of the electric-field fluctuations associated with the whistler waves (Stix 1992, page 498). In our case of $\omega \ll k_\| v = \Omega_{\rm e}$, this coefficient takes the form

$$S_{\rm QL} = \frac{\pi e^2\Omega_{\rm e}^2}{m_{\rm e}^2 v^3}\int {\rm d}^2k_\perp \frac{1}{\omega^2}\left|E(k_\|, k_\perp)\right|^2_{k_\|=\Omega_{\rm e}/v}. \tag{23}$$

The electric field of oblique whistler modes has a strong potential component, which is related to their magnetic component as $(\beta_{\rm e}/2)e\phi_k/T_{\rm e} \sim \delta B_k/B$, where B is a constant background magnetic field (e.g. Chen & Boldyrev 2017). This allows us to express the electric spectrum through the magnetic spectrum,

$$\left|E(k_\|, k_\perp)\right|^2 = k_\perp^2\frac{4T_{\rm e}^2}{e^2\beta_{\rm e}^2}\left|\frac{\delta B_k}{B}\right|^2. \tag{24}$$

Substituting this result and expression (21) for the whistler frequency, in the integral (23), we obtain

$$S_{\rm QL} = \frac{4\pi\Omega_{\rm e}^2 T_{\rm e}^2}{m_{\rm e}^2 v^3\beta_{\rm e}^2 v_{\rm A}^2 d_i^2 k_\|^2}\int {\rm d}^2k_\perp\left|\frac{\delta B_k}{B}\right|^2_{k_\|=\Omega_{\rm e}/v}. \tag{25}$$

Conveniently, the scattering coefficient provided by oblique whistler modes depends only on the field-parallel spectrum of the magnetic fluctuations, for which we will assume a power-law behaviour

$$\int {\rm d}^2k_\perp\left|\frac{\delta B_k}{B}\right|^2 = \left|\frac{\delta B_{k_\|}}{B}\right|^2 = Dk_\|^{-\alpha}. \tag{26}$$

Here D is the normalization coefficient. It is convenient to express this coefficient through the intensity of magnetic fluctuations in whistler turbulence. Since whistlers exist only at scales $k_\| d_i \gg 1$ (and they are strongly Landau damped at $k_\| d_i \approx 1$; Chen et al. 2013), we estimate the total magnetic energy in the whistler fluctuations as

$$\left(\frac{\delta B}{B}\right)^2 = \int_{1/d_i} {\rm d}k_\|\left|\frac{\delta B_{k_\|}}{B}\right|^2 = \frac{d_i^{\alpha-1}}{\alpha-1}D, \tag{27}$$

which gives

$$D = \frac{\alpha-1}{d_i^{\alpha-1}}\left(\frac{\delta B}{B}\right)^2. \tag{28}$$

The intensity of the whistler magnetic fluctuations is a parameter of the theory; it can be inferred, for example, from observations, or obtained from numerical simulations or analytical modelling. Substituting expression (28) into equation (26) and into the quasi-linear diffusion integral (25) we finally arrive at the expression for the scattering coefficient

$$S = \frac{4\pi n(r)e^4\Lambda}{m_{\rm e}^2 v^3}\left[1 + \frac{4\pi(\alpha-1)}{\beta_{\rm e}^2}\left(\frac{\lambda_{\rm e}}{d_i}\right)\left(\frac{m_{\rm e}}{m_i}\right)^\alpha \times\left(\frac{\delta B}{B}\right)^2\left(\frac{v}{v_{\rm A}}\right)^{\alpha+2}\right]. \tag{29}$$

The first term in the brackets corresponds to the classical Coulomb scattering, while the second term described the anomalous scattering by the whistlers.

For further consideration, one needs to specify the parameters of the whistler turbulence: its spectral scaling α, and the intensity of the turbulent fluctuations. As an example, we may perform a simple estimate assuming that the field-perpendicular spectrum of the turbulence scales as $k_\perp^{-8/3}$ and its anisotropy is $k_\| \propto (k_\perp)^{1/3}$. This is consistent with observations and numerical simulations (Howes et al. 2006; Alexandrova et al. 2009; Cho & Lazarian 2009; Kiyani et al. 2009; Chen et al. 2010, 2012; Meyrand & Galtier 2013; Sahraoui et al. 2013; Grošelj et al. 2018; Roytershteyn et al. 2019). We then derive that the field-parallel spectrum scales as $\sim k_\|^{-6}$, and therefore $\alpha = 6$. In addition, at the distance of 1 au, we may estimate $d_i = 10^7$ cm, $\lambda_{\rm e} = 10^{13}$ cm, $v_{\rm A} = 7\times 10^6\,{\rm cm\,s^{-1}}$, $v_{T{\rm e}} = 2\times 10^8\,{\rm cm\,s^{-1}} \approx 10\,{\rm eV}$, and $\beta_{\rm e} = 1$. For the intensity of the whistler fluctuations, we may follow observational results (e.g. Chen et al. 2013) and assume that the intensity of magnetic fluctuations is of the order of $(\delta B/B_0)^2 \sim 10^{-2}$. In fact, it is believed that the magnetic fluctuations in the observations are dominated by the kinetic-Alfv´en modes, with the whistlers contributing only a fraction of the fluctuation energy (e.g. Chen et al. 2013), so this expression can serve as a rather conservative upper boundary. We then obtain

$$S = \frac{4\pi n(r)e^4\Lambda}{m_{\rm e}^2 v^3}\left[1 + \left(\frac{\mathcal{E}}{\mathcal{E}_{\rm c}}\right)^4\right], \tag{30}$$

where $\mathcal{E}_{\rm c} \approx 200$ eV at 1 au.[8]

We see that the anomalous diffusion strongly depends on the energy. For energies below the characteristic energy $\mathcal{E}_{\rm c}$, the electron scattering is provided mostly by Coulomb collisions. As the energy increases above $\mathcal{E}_{\rm c}$, scattering by whistlers rapidly becomes dominant. This result is consistent, for instance, with the fast solar wind observations in (e.g. Horaites et al. 2019) that found that the electron strahl is rather well described by the Coulomb theory, i.e. it is not affected by anomalously strong scattering at relatively low energies, below 100–200 eV. Our results are also broadly consistent with the recent studies by (Berčič et al. 2019) who noticed that the strahl angular broadening is a function of the electron energy and it starts to increase at energies exceeding several hundred eV.

From equation (6) one can see that the threshold energy scales as

$$\mathcal{E}_{\rm c} \propto \left[(\delta B)^{-2}B(r)^6 n(r)^{-3/2}\right]^{1/4}. \tag{31}$$

According to observational estimates (e.g. Horbury et al. 1996; Horbury & Balogh 2001; Bruno & Carbone 2013), the intensity of magnetic fluctuations measured at a given frequency in the Alfv´enic inertial interval in a spacecraft frame (that, according to the Taylor hypothesis, corresponds to a given field-perpendicular scale), declines as $1/r^4$ with the heliospheric distance. Simultaneously, since the plasma density declines as $n(r) \propto 1/r^2$, the ion inertial scale increases with the distance as $d_i \propto r$. In order to estimate how the whistler component of the turbulence evolves with the distance, one needs to know the mechanism of turbulence generation, which is currently not well understood. One may, however, assume that the intensity of the whistler turbulence is

[8] This value, based on somewhat overestimated magnitude of the magnetic fluctuations $\delta B/B_0$, provides a lower boundary for the characteristic energy $\mathcal{E}_{\rm c}$. As follows from formula (31) given below, weaker magnetic fluctuations would lead to a larger characteristic energy.

proportional to the intensity of kinetic plasma turbulence at the scale d_i (in fact, whistler turbulence may be generated at this scale; Horaites et al. 2018b). Since at scales smaller than d_i, the spectrum of observed magnetic fluctuation is $\sim k_\perp^{-8/3}$, their intensity at d_i is $\int_{1/d_i} k_\perp^{-8/3}\mathrm{d}k_\perp \sim d_i^{5/3}$. When the intensity of the Alfv´enic fluctuations at a given scale decreases as $1/r^4$ while the transition scale to the kinetic regime increases as $d_i \sim r$, the intensity of magnetic fluctuations at the d_i scale varies as $(\delta B/B)^2 \propto r^{-4\,+\,5/3} = r^{-7/3}$. We therefore assume this behaviour as the upper boundary for the whistler fluctuations.

In the inner heliosphere ($r \ll 1$ au) the magnetic field strength scales approximately as $B(r) \propto 1/r^2$ therefore the threshold $\mathcal{E}_\mathrm{c}$ should vary approximately as $\mathcal{E}_\mathrm{c} \propto r^{-5/3}$. In the outer heliosphere ($r \gg 1$ au), the magnetic strength varies approximately as $B(r) \propto 1/r$ therefore the energy threshold scales as $\mathcal{E}_\mathrm{c} \propto r^{-1/6}$. The threshold is quite high in the inner heliosphere so it does not significantly affect the number of particles in the strahl. In the outer heliosphere, however, the threshold may be comparable to T_0, so its variations may significantly affect the number of strahl particles. This may be broadly consistent with the observational results that the fraction of the electrons forming the strahl decreases in the outer heliosphere.

5 CONCLUSIONS

In this work, we have developed a kinetic theory of the electron strahl, which describes the global evolution of the strahl electrons and relates their velocity distribution function at the hot coronal region to that at larger heliospheric distances. We have solved the drift-kinetic equation that traces the distribution function along the Parker-spiralled magnetic field lines. We have considered two pitch-angle scattering mechanisms that are believed to be relevant for the strahl broadening – Coulomb collisions (classical scattering) and scattering by plasma turbulence (anomalous scattering). The main prediction of our Coulomb theory is the strahl distribution function given by equation (16). We have found that this theory captures some essential physics of the strahl formation. In particular, the number of electrons forming the strahl, given by equation (20), and the angular width of the strahl, given by equation (18), are in good qualitative and sometimes quantitative agreement with the available observations where there is reason to believe that anomalous scattering is not significant (e.g. Horaites et al. 2018a,b, 2019).

When anomalous scattering mechanisms, e.g. pitch-angle diffusion caused by plasma turbulence, become important, the Coulomb theory is not applicable. To address such a situation, in addition to Coulomb collisions we have considered a quasi-linear diffusion provided by oblique whistler turbulence, as described by equations (22) and (29). We have found that in this case, the angular broadening of the strahl becomes energy dependent. In particular, it alters the Coulomb theory at high energies. Whistler turbulence may therefore efficiently scatter and isotropize very energetic electrons possibly leading to formation of the electron halo.

According to our results, the anomalous scattering becomes significant when the electron kinetic energy exceeds certain characteristic energy $\mathcal{E}_\mathrm{c}$. This energy threshold becomes lower at larger heliospheric distances, implying that the anomalous scattering mechanism becomes progressively more important with the distance. For a model spectral distribution of whistler turbulence we estimate that at 1 au, the anomalous scattering is not expected to be significant as compared to Coulomb collisions at the energies below 200 eV, but it becomes progressively more important at higher energies. These results are broadly consistent with the recent analytical and observational findings by Horaites et al. (2019) and Berčič et al. (2019).

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