

Shear wave propagation and estimation of material parameters in a nonlinear, fibrous material

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Abstract

This paper describes the propagation of shear waves in a Holzapfel-Gasser-Ogden (HGO) material and investigates the potential of magnetic resonance elastography (MRE) for estimating parameters of the HGO material model from experimental data. In most MRE studies the behavior of the material is assumed to be governed by linear, isotropic elasticity or viscoelasticity. In contrast, biological tissue is often nonlinear and anisotropic with a fibrous structure. In such materials, application of a quasi-static deformation (pre-deformation) can influence plays an important role in shear wave propagation. Closed form expressions for shear wave speeds in an HGO material with a single family of fibers were found in a reference (undeformed) configuration and after imposed pre-deformations. These analytical expressions show that shear wave speeds are affected by the parameters (μ_0, k_1, k_2, κ) of the HGO model and by the direction and amplitude of the pre-deformations. Simulations of corresponding finite element models confirm the predicted influence of HGO model parameters on speeds of shear waves with specific polarization and propagation directions. Importantly, the dependence of wave speeds on the parameters of the HGO model and imposed deformations could ultimately allow non-invasive estimation of material parameters *in vivo* from experimental shear wave image data.

1 Introduction

Elastographic techniques, including both ultrasound elastography and magnetic resonance elastography (MRE) have great potential for non-invasive evaluation of the mechanics of soft tissues. Harmonic MRE is based on MR imaging of shear waves induced by external vibration of the tissue, followed by inversion of the displacement fields to estimate material parameters. MRE has been used to quantify non-invasively the material properties of many biological tissues, such as skeletal muscle [1], liver [2, 3], and brain [4]. Most MRE studies use linear elastic or viscoelastic material models, and typically the material is assumed to be isotropic. Recently, MRE has been extended to use anisotropic material models, such as a three-parameter model [5-7] for nearly-incompressible, transversely isotropic, fibrous materials. However, these models still rely on the assumptions of linear elasticity, while many biological tissues exhibit nonlinear strain-stress relationships [8].

Nonlinear hyperelastic models have been successfully applied to describe the mechanics of soft biological materials [9, 10]. The Holzapfel-Gasser-Ogden (HGO) model in particular is straightforward and widely used to model fibrous soft tissues [11, 12]; it contains separate terms to describe the contributions of fiber deformation to the strain energy, and can model an isotropic nonlinear material (with $\kappa = 1/3$), or a strongly anisotropic material with single or multiple “families” of fibers. Estimating the parameters of hyperelastic models from experimental data remains an important challenge. Here we demonstrate that wave speed data, such as those available from MRE studies, can be used for this purpose.

Shear waves in MRE consist of infinitesimal dynamic deformations, which may be superimposed on larger, quasi-static “pre-deformations.” Shear wave speeds in a nonlinear material are determined by both its mechanical properties and its deformation state. In this study, closed-form expressions for shear wave speeds in the HGO model are obtained in terms of the model parameters and imposed pre-deformations. Analytical expressions for wave speeds were confirmed by performing finite element simulations of shear waves in a pre-deformed cube of HGO material with a single fiber family. Local frequency estimation (LFE) was used to estimate speeds of shear waves with various polarization and propagation directions from

simulated displacement fields. Finally, we demonstrate, using simulated data, the feasibility of estimating the material parameters of the HGO model from shear wave speeds.

2 Theoretical Methods

2.1 Wave speeds in transversely isotropic elastic materials

Shear wave speeds in an elastic material are calculated from the eigenvalues of the acoustic tensor [13, 14], as in Eq. (1):

$$\rho c^2 \mathbf{m} = \mathbf{Q}(\mathbf{n}) \cdot \mathbf{m} \quad (1)$$

where ρc^2 is the eigenvalue of the acoustic tensor $\mathbf{Q}$, ρ is the density of material, c is the shear wave speed, $\mathbf{n}$ is the propagation direction of wave, $\mathbf{m}$ is the polarization direction vector of the shear wave. The acoustic tensor $\mathbf{Q}$ for a specific propagation direction, $\mathbf{n}$, is obtained from Eq. (2) [13, 14] below,:

$$\mathbf{Q} = \mathbf{n} \cdot \mathbf{A} \cdot \mathbf{n} \quad (2)$$

where $\mathbf{A}$ is a fourth-order elasticity tensor which relates the incremental strain tensor, $\tilde{\boldsymbol{\epsilon}}$, and incremental stress tensor, $\tilde{\boldsymbol{\sigma}}$. In Cartesian coordinates this relationship can be expressed in indicial notation, $\tilde{\sigma}_{pi} = A_{piqj} \tilde{\epsilon}_{qj}$. For nonlinear models, such as the HGO model, the components of the elasticity tensor can be obtained from the relationship $A_{piqj} = F_{p\alpha} F_{q\beta} \frac{\partial^2 W}{\partial F_{i\alpha} \partial F_{j\beta}}$, where $\mathbf{F}$ is the deformation gradient tensor which accounts for the effects of pre-deformation [13, 14], and W is the strain energy function. Thus, in principle, shear wave speeds can be used to estimate material parameters.

Since the acoustic tensor, $\mathbf{Q}$, depends on the propagation direction, $\mathbf{n}$, in general wave speeds depend on $\mathbf{n}$. Also, $\mathbf{Q}$, may have up to three distinct eigenvalues (wave speeds) and three corresponding eigenvectors (polarization directions) so that there may be three plane waves that propagate in the same direction. However, material symmetries and constraints reduce the number of possible wave speeds. In an isotropic linear elastic material with shear modulus, μ and bulk modulus, K , the acoustic tensor is the same for all propagation directions, and only two wave speeds exist: one longitudinal and one transverse (shear). Longitudinal waves in isotropic materials have $c^2 = (K + \frac{4\mu}{3})/\rho$ [15], and polarization parallel to the

propagation direction ($\mathbf{m} = \mathbf{n}$); corresponding shear waves have $c^2 = \mu/\rho$ and polarization $\mathbf{m} \perp \mathbf{n}$. In an isotropic, incompressible linear elastic material, the longitudinal wave speed becomes infinite, and only one parameter, μ , remains to estimate.

A linear elastic, transversely isotropic (TI) material requires five parameters to describe its constitutive behavior. These can be, for example two tensile moduli (E_1, E_2), two shear moduli (μ_1, μ_2) and a bulk modulus (K). If the material is incompressible, three parameters are sufficient, for example baseline shear modulus (μ_2) and two ratios: shear anisotropy $\phi_{TI} = \left(\frac{\mu_1}{\mu_2} - 1\right)$ and tensile anisotropy $\zeta_{TI} = \left(\frac{E_1}{E_2} - 1\right)$. In a linear elastic, nearly-incompressible, transversely isotropic (TI) material, in which anisotropy is due to a single family of aligned fibers, the shear wave speeds depends in a relatively simple fashion on the material properties and the wave propagation direction relative to the fibers [15]. Shear waves can be separated into “slow” (or “pure transverse”) and “fast” (or “quasi-transverse”) shear waves with different polarization directions. The slow and fast shear wave polarization directions under no pre-deformation are defined by the following relationship (Eq. (3)):

$$\mathbf{m}_s = \frac{\mathbf{n} \times \mathbf{a}}{|\mathbf{n} \times \mathbf{a}|} \quad \mathbf{m}_f = \mathbf{n} \times \mathbf{m}_s \quad (3)$$

where $\mathbf{n}$ is the propagation direction of the shear wave, $\mathbf{a}$ is the fiber orientation after deformation, and $\mathbf{m}_s$ and $\mathbf{m}_f$ are the slow and fast polarization directions, respectively. The corresponding wave speeds in this linear elastic material, which depend on the angle, θ , between fiber and propagation directions are:

$$c_s^2 = \frac{\mu_2}{\rho} [1 + \phi_{TI} \cos^2(\theta)]; \quad c_f^2 = \frac{\mu_2}{\rho} [1 + \phi_{TI} \cos^2(2\theta) + \zeta_{TI} \sin^2(2\theta)] \quad (4)$$

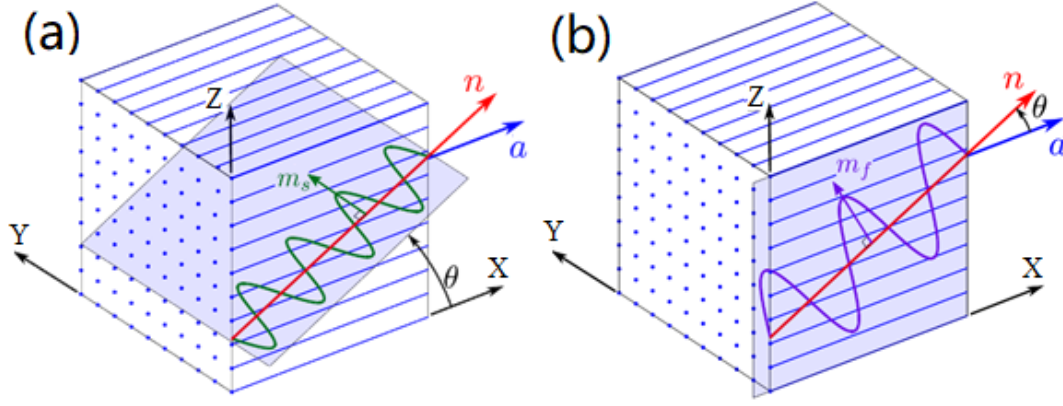


Fig. 1 Wave propagating with two polarization directions in a transversely isotropic material. (a) A “slow” shear wave is produced by harmonic shear displacement in the slow polarization direction, $\mathbf{m}_s$. (b) A “fast” shear wave arises from harmonic shear displacement in the fast polarization direction, $\mathbf{m}_f$. The wavelength of a fast shear wave is typically longer than that of a slow shear wave. Reproduced from reference [16].

2.2 The Holzapfel-Gasser-Ogden (HGO) model

The HGO model, which is an influential recent model for fibrous soft tissues, was proposed by Holzapfel et al. [17]. The strain energy density function is a sum of isotropic and anisotropic terms:

$$W = W_{iso} + W_{aniso} \quad (5)$$

The isotropic part of its strain energy density function contains both volumetric and isochoric terms:

$$W_{iso} = W_{vol} + W_{isochoric}$$

$$W_{vol} = \frac{K}{2} (J - 1)^2, \quad W_{isochoric} = \frac{\mu}{2} (\bar{I}_1 - 3), \quad (6)$$

where K and μ are the bulk modulus and the isotropic shear modulus respectively, $\bar{I}_1$ is the modified first invariant defined by $\bar{I}_1 = J^{-2/3} I_1$, ($J = \det \mathbf{F}$), where I_1 is the first variant of Cauchy-Green strain tensor $\mathbf{C}$. Many soft materials have shear moduli roughly between 10^2 - 10^5 Pa, spanning acellular collagen and fibrin gels [18-21], brain tissue [22, 23], and muscle [24]. Anisotropic terms in the strain energy density function can have different forms depending on fiber arrangement:

(1) Single fiber-family model (transversely isotropic, TI): Terms in the strain energy density function reflecting the effects of fibers with a distribution of orientations centered on the unit vector $\mathbf{a}_0$, which is the fiber direction before pre-deformation:

$$W_{aniso\ HGO1} = \frac{k_1}{2k_2} \{ \exp[k_2(\kappa\bar{I}_1 + (1 - 3\kappa)\bar{I}_4 - 1)^2] - 1 \}, \quad for\ \bar{I}_4 > 1. \quad (7)$$

where $\bar{I}_4$ is the modified pseudo-variant defined by $\bar{I}_4 = J^{-2/3}I_4$, $I_4 = \mathbf{a}_0 \cdot \mathbf{C} \cdot \mathbf{a}_0$ is the squared stretch in the fiber direction.

(2) Multiple fiber-family model (orthotropic): Additional fiber families (with a principal direction of $\mathbf{a}_{0i}$, and the same properties k_1 , k_2 , and κ) can be modeled by adding contributions from $I_{4i} = \mathbf{a}_{0i} \cdot \mathbf{C} \cdot \mathbf{a}_{0i}$ to the strain energy, as:

$$W_{aniso\ HGON} = \sum_i \frac{k_1}{2k_2} \left[\exp(k_2 \bar{E}_i^2) - 1 \right]; \quad \bar{E}_i = \kappa\bar{I}_1 + (1 - 3\kappa)\bar{I}_{4i} - 1, \quad for\ \bar{I}_{4i} > 1. \quad (8)$$

The effects of k_1 and k_2 on stress-strain behavior in simple shear are shown in Fig. 2. For example, for simple shear γ_{YZ} in a plane containing fibers at an angle of $\pi/4$ ($\mathbf{a} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$), k_1 describes the initial slope of the curve (Fig. 2a), k_2 describes the nonlinearity of the curve (Fig. 2b).

Fiber distributions corresponding to different values of κ are shown in Fig. 3 and κ captures the distribution of the fiber orientations, ranging from alignment in a single direction ($\kappa = 0$) to no preferred direction ($\kappa = 1/3$). When $\kappa = 0$, all fibers are assumed to be perfectly aligned, and when $\kappa = 1/3$, the material is isotropic. We note that, formally, fibers in the HGO model do not contribute to the stress or to the strain energy when they are in compression ($I_4 < 1$). We did not model the bi-linearity between fiber tension and compression for wave propagation in the HGO model in the undeformed case (assuming that the fibers can resist an infinitesimal compressive strain in wave propagation, or equivalently, an infinitesimal tensile pre-strain).

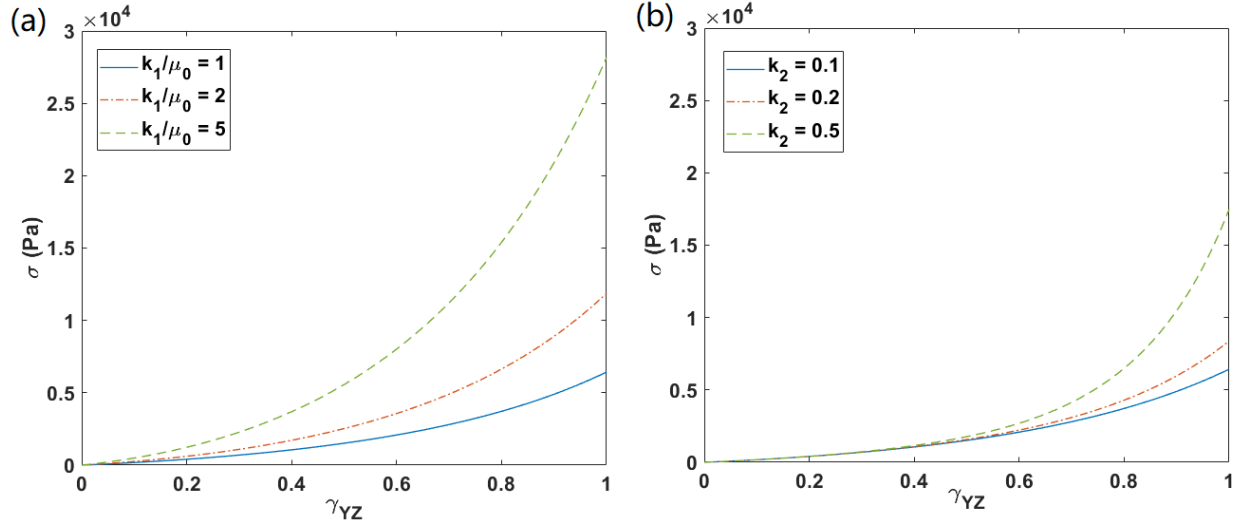


Fig. 2 Effects of parameters in the HGO model on the stress-strain relationship during simple shear in a plane containing fibers at an angle of $\pi/4$. (a) The parameter k_1 affects the initial slope of the stress-strain curves for deformations involving fiber stretch ($\mu_0 = 1000\text{Pa}, \kappa = 1/12, k_2 = 0.1$). (b) The parameter k_2 determines the nonlinearity of the relationships ($\mu_0 = 1000\text{Pa}, \kappa = 1/12, k_1/\mu_0 = 1$)

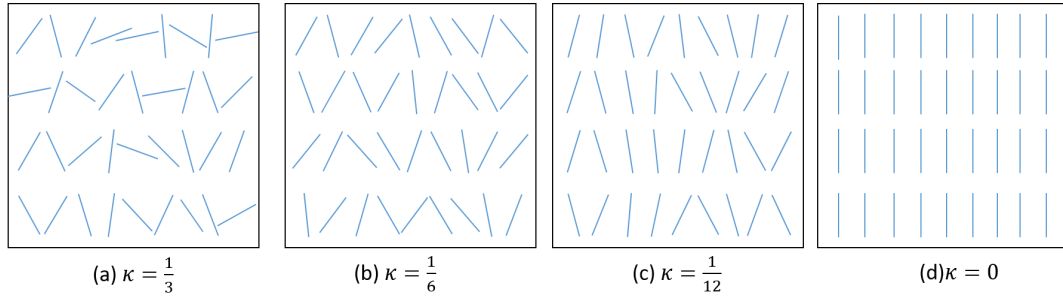


Fig. 3 The HGO dispersion parameter κ captures the distribution of fiber orientation in 2D domain, ranging from alignment in no preferred direction ($\kappa=1/3$) (a), $\kappa=1/6$ (b), $\kappa=1/12$ (c) and finally to a single direction ($\kappa=0$) (d).

2.3 Closed-form expressions for the relationships between model parameters and wave speeds

2.3.1 Closed-form expressions for speeds of waves superimposed on simple shear

Closed-form expressions that relate wave speeds to model parameters are highly desirable. Such expressions for the HGO model were determined from analytical solution of the eigenvalue problem (Eq. 1) for shear waves propagating in the negative Z direction ($\mathbf{n} = -\mathbf{k}$, Fig. 4) in the undeformed configuration

(Fig. 4a) and with imposed pre-deformations in simple shear corresponding to the configurations of Fig. 4b and Fig 4c. Symbolic solutions were obtained using Matlab Symbolic Toolbox (Mathworks, Natick, MA).

$$c_{s_{XZ}} = \sqrt{\frac{\mu}{\rho} + 2\frac{k_1}{\rho}\gamma_{XZ}^2 M^2 \exp(M^2 k_2 \gamma_{XZ}^4)} \quad (9)$$

$$c_{f_{XZ}} = \sqrt{\frac{\mu}{\rho} + \frac{k_1}{\rho} (N^2 + 6\gamma_{XZ}^2 M^2 + 2\gamma_{XZ}^4 k_2 M^2 (N^2 + 4\gamma_{XZ}^2 M^2)) \exp(M^2 k_2 \gamma_{XZ}^4)} . \quad (10)$$

$$c_{s_{YZ}} = \sqrt{\frac{\mu}{\rho} + 2\frac{k_1}{\rho}\gamma_{YZ} M (\gamma_{YZ} M + N) \exp((\gamma_{YZ} M + N)^2 k_2 \gamma_{YZ}^2)} \quad (11)$$

$$c_{f_{YZ}} = \sqrt{\frac{\mu}{\rho} + \frac{k_1}{\rho} (N^2 + 6\gamma_{YZ} M N + 6\gamma_{YZ}^2 N^2 + 2\gamma_{YZ}^2 (2\gamma_{YZ} M + N)^2 (\gamma_{YZ} M + N)^2) \exp((\gamma_{YZ} M + N)^2 k_2 \gamma_{YZ}^2)} \quad (12)$$

where $(c_{s_{XZ}}, c_{f_{XZ}})$ and $(c_{s_{YZ}}, c_{f_{YZ}})$ are the slow and fast shear wave speeds for pre-deformations γ_{XZ} and γ_{YZ} respectively. The terms M and N are combinations of the dispersion parameter κ and the angle ϕ between the fiber and the propagation direction:

$$M = 2((1 - 3\kappa) \cos^2 \phi + \kappa); \quad N = (1 - 3\kappa) \sin 2\phi. \quad (13)$$

With no imposed pre-deformation ($\gamma_{XZ} = \gamma_{YZ} = 0$), the speeds reduce to

$$c_s = c_0 = \sqrt{\frac{\mu}{\rho}}; \quad c_f = \sqrt{\frac{\mu}{\rho} + \frac{k_1 N^2}{\rho}} \quad (14)$$

2.3.2 Closed-form expressions for speeds of waves superimposed on stretching

Expressions for the speeds of slow and fast shear waves superimposed on isochoric, quasi-static lengthening deformations (Fig. 5) were also obtained. In this situation, the maximum stretch ratio, $\lambda_1 = \lambda$, is used to describe the imposed deformation; other stretches are $\lambda_2 = \lambda_3 = 1/\sqrt{\lambda}$. Wave speeds were obtained with the help of Matlab Symbolic Toolbox (Mathworks, Natick, MA).

$$C_{f_z} = \sqrt{\frac{\lambda^2 \mu}{\rho} + \frac{k_1}{\rho \lambda} (\lambda^2 N^2 + \lambda^2 M L + 2 N^2 L^2 k_2) \exp\left(\frac{k_2 L^2}{\lambda^2}\right)} \quad (15)$$

$$C_{s_z} = \sqrt{\frac{\lambda^2 \mu}{\rho} + \frac{\lambda L k_1}{\rho} \exp\left(\frac{k_2 L^2}{\lambda^2}\right)} \quad (16)$$

where (c_{sz}, c_{fz}) are the slow and fast shear wave speeds for stretch λ in Z direction. Definitions of M and N are from Eq. (13); the new variable L is defined in terms of M and the stretch ratio λ .

$$L = (\lambda - 1) \left(\frac{M}{2} (\lambda^2 + \lambda + 1) - 1 \right) \quad (17)$$

With no imposed pre-deformation ($\lambda = 1$), the wave speed expressions can be simplified, as above, to

$$c_s = c_0 = \sqrt{\frac{\mu}{\rho}}; \quad c_f = \sqrt{\frac{\mu}{\rho} + \frac{k_1 N^2}{\rho}} \quad (18)$$

which are identical to Eq. (14).

2.4 Computational modeling and simulations

To verify the analytical results, finite element (FE) simulations of shear wave propagation were performed using finite element software (COMSOL Multiphysics v5.3, Burlington, MA). A static pre-deformation step (either (i) simple shear or (ii) tension) and a frequency-domain perturbation step were performed in a cubic domain ($50 \times 50 \times 50 \text{ mm}^3$, Figs. 4 and 5). The HGO model was implemented in COMSOL to model elastic behavior; an isotropic loss factor of 0.1 was used to provide a small amount of viscoelastic damping. We set the frequency of excitation equal to 200 Hz, in order to provide multiple wavelengths in the model domain. The domain was discretized into 5000 hexahedral elements. To demonstrate convergence, the results were confirmed at higher resolution. In order to compare the undeformed case to the cases with finite pre-deformation, we assume the fibers can resist infinitesimal compressive loads during wave motion. A periodic boundary condition (PBC) was applied on the XZ plane and YZ plane, for fast and slow shear waves respectively. The PBCs eliminate boundary effects on the vertical sides of the cube, allowing for a closer comparison of the analytical and numerical results. The assigned default parameters are as follows: pre-deformation $\gamma_{XZ} = \gamma_{YZ} = 0.2$; initial isotropic shear modulus, $\mu_0 = 1 \text{ kPa}$; density $\rho = 1000 \text{ kg/m}^3$; initial anisotropy ratio, $k_1/\mu_0 = 2$; nonlinearity parameter, $k_2 = 5$; fiber dispersion parameter, $\kappa = 1/12$; and ratio of bulk modulus to initial shear modulus, $K/\mu_0 = 10^4$. To obtain either slow or fast shear waves, a harmonic displacement (simple shear case) or harmonic force (lengthening/shortening cases) was imposed to the top surface in the corresponding polarization

direction, defined by the $\mathbf{m}_s$ or $\mathbf{m}_f$ unit vector, respectively. The local frequency estimation (LFE) method [25] was applied to estimate wavelength (and thus wave speed) from simulated data. LFE provides an estimate of wave speed at each “voxel” in a discretized version of the 3D wave field. The mean values and standard deviations of wave speeds from all voxels in a central region of interest are used to generate the symbols and error bars in plots [25, 26]. The LFE parameters used in this study are $\rho_0 = 1$ for the center frequency and $L_0 = 11$ for the number of filters [26].

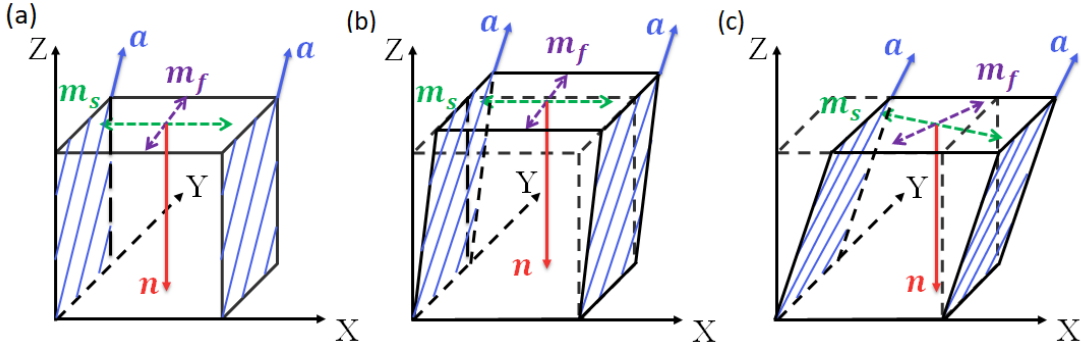


Fig. 4 Geometry used in numerical simulations ($50 \times 50 \times 50 \text{ mm}^3$) of waves superimposed on simple shear. Green dashed arrows represent harmonic perturbations in two perpendicular directions, which correspond to fast and slow shear wave polarizations for shear waves propagating along the negative Z-axis (red arrows). Blue solid lines are the mean fiber direction, where $\phi = 45^\circ$. One fiber family in (a) the undeformed configuration ($\mathbf{a} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$) and (b-c) pre-deformed configurations in simple shear (b) γ_{YZ} in the YZ plane ($\mathbf{a} = ((1 + \gamma_{YZ})\mathbf{j} + \mathbf{k})/\sqrt{(1 + \gamma_{YZ})^2 + 1}$) or (c) γ_{XZ} in the XZ plane ($\mathbf{a} = (\gamma_{XZ}\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{2 + \gamma_{XZ}^2}$).

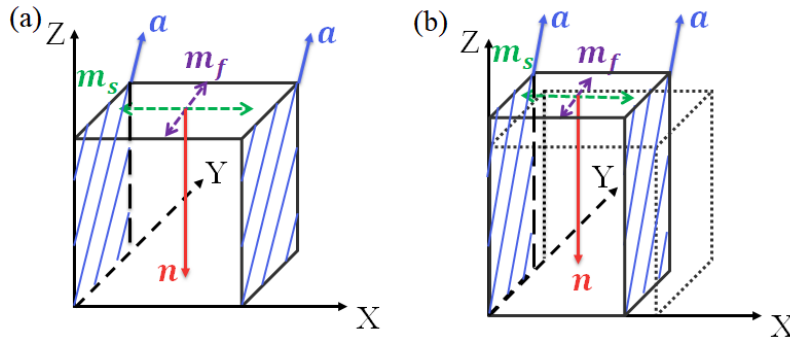


Fig. 5 Geometry used in simulations ($50 \times 50 \times 50 \text{ mm}^3$) of waves superimposed on lengthening. Green dashed arrows represent harmonic loading in two perpendicular directions, which correspond to fast and slow shear wave polarizations for shear waves propagating along the negative Z-axis (red arrows). Blue solid lines are the mean fiber direction, where $\phi = 45^\circ$. One fiber family in (a) the undeformed configuration ($\mathbf{a} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$) and (b) after imposed stretch λ in the Z direction ($\mathbf{a} = (\frac{1}{\sqrt{\lambda}}\mathbf{j} + \lambda\mathbf{k})/\sqrt{\lambda^2 + \frac{1}{\lambda}}$).

3 Results: shear wave speeds in undeformed and deformed configurations

Fig. 6 shows the simulation results for slow and fast shear waves propagating in the negative Z direction in the undeformed configuration and with shear pre-deformation in the YZ or XZ plane. Shear pre-deformation in either the YZ or XZ plane (Fig. 6e and Fig. 6f) increases the wavelength of the fast shear wave compared to the undeformed configuration (Fig. 6d), corresponding to an increase in the fast wave speed.

Shear wave speeds are compared in analytical predictions and simulations estimations for the three configurations of Figure 4 and 5, shown in Figures 7-10 below. In each configuration, one parameter was varied while holding the remaining parameters at the default parameter values given above. The vertical axes of the panels in the top row of each figure display c_f/c_0 , the normalized ratio between the fast shear wave speed and the initial wave speed $c_0 = \sqrt{\mu_0/\rho}$, where $\mu_0 = 1000Pa$ is the initial isotropic shear modulus, ρ is the density of material. Similarly, the vertical axes of the panels on the bottom row of each figure depict the ratio c_s/c_0 between the slow shear wave speed and the initial wave speed. Results are shown for ranges of the HGO parameters isotropic shear modulus μ , HGO model parameters k_1 and k_2 , dispersion parameter κ , and the imposed shear, γ . In all three figures, orange solid lines depict the analytical predictions, and blue solid lines with error bars display corresponding wave speeds estimated from FE simulations.

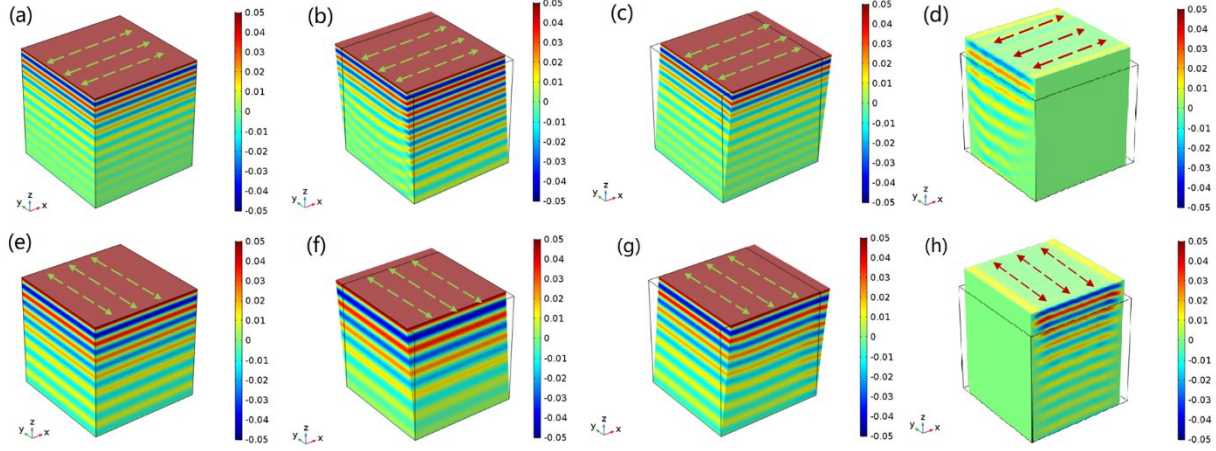


Fig. 6 Harmonic displacement fields due to slow and fast wave shear waves in configurations corresponding to Figs. 4 and 5. (a-d) x component of harmonic displacement showing slow shear waves propagating in the negative Z direction in the undeformed configuration (a) and after simple shear pre-deformation $\gamma_{YZ} = 0.2$ in the YZ plane (b) or $\gamma_{XZ} = 0.2$ in the XZ plane (c), or $\lambda = 1.2$ in the Z direction (d). (e-h) y component of harmonic displacement showing fast shear waves propagating in the negative Z direction in the undeformed configuration and after simple shear pre-deformation in the YZ, XZ plane, or tension deformation in the Z direction. Green arrows on the top surface indicate the direction of applied harmonic excitation (200 Hz) in each panel.

3.1 One family of fibers in the undeformed configuration

Fig. 7 shows the relationships between shear wave speeds and the parameters in the HGO model in the undeformed configuration. The horizontal axis represents three normalized parameters ($\mu/\mu_0, k_1/\mu_0, \kappa$) in HGO model. There is no effect of changing k_2 or γ because no pre-deformation is applied. The fast wave speed increases with increasing k_1 and μ and decreases with increasing κ . In contrast, the slow wave speed is affected only by μ .

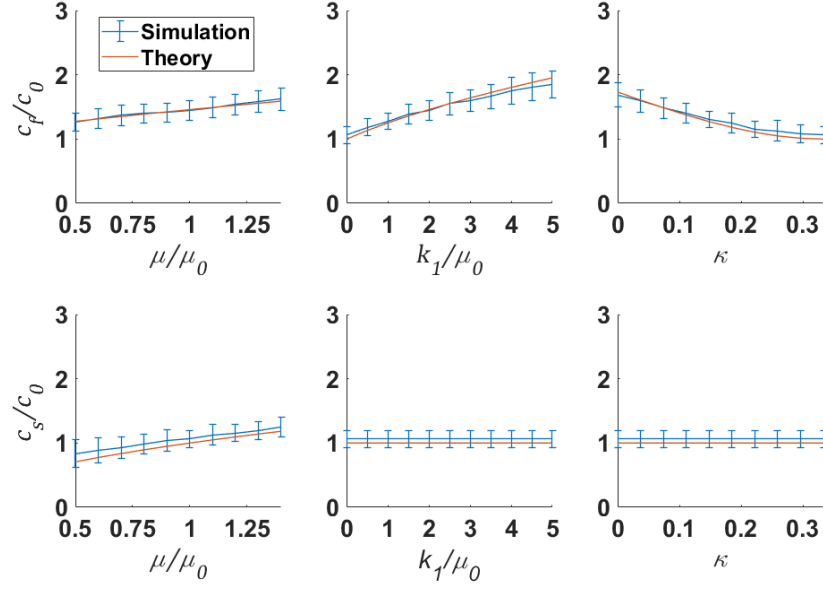


Fig. 7 Fast and slow wave speeds in relation to three parameters ($\mu/\mu_0, k_1/\mu_0, \kappa$) in the HGO model when no pre-deformation is applied. Default parameters are: $\mu_0 = 1000$ Pa, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$.

3.2 One fiber family with pre-deformation by simple shear in the YZ plane

Fig. 8 shows the dependence of wave speed on HGO parameters when simple shear pre-deformation is imposed in the YZ plane, i.e, in the direction that induces fiber stretch. The horizontal axis of each panel displays values of one of the four parameters ($\mu/\mu_0, k_1/\mu_0, \kappa, k_2$) in the HGO model or the magnitude of shear γ_{YZ} . The slow and fast wave speeds all increase with increasing k_1, k_2, μ , and γ_{YZ} , and decrease with fiber dispersion, κ . The fast wave speed is larger than the slow wave speed due to the stiffening effect of the fibers.

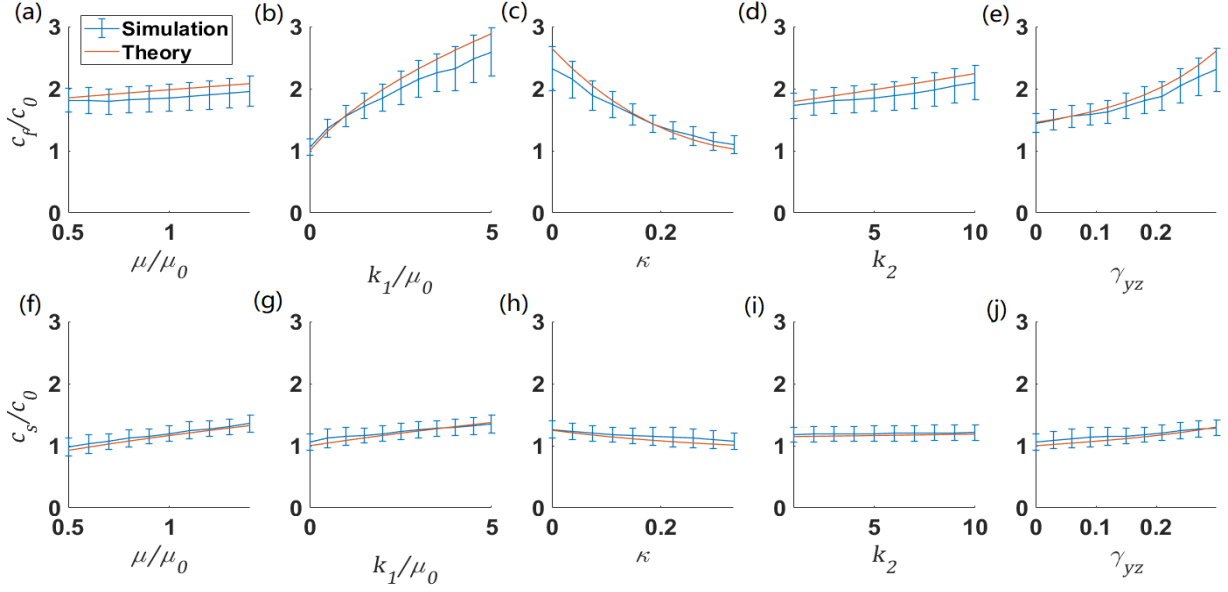


Fig. 8 The effects on fast and slow wave speeds of the parameters $(\mu/\mu_0, k_1/\mu_0, \kappa, k_2)$ of the HGO model. The parameter γ_{YZ} is the pre-deformation magnitude of simple shear in the YZ plane, inducing fiber stretch of order γ_{YZ} in fibers at $\phi = \pi/4$. Default parameters are: $\mu_0 = 1000 \text{ Pa}$, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $\gamma_{YZ} = 0.2$.

3.3 One fiber family with pre-deformation by simple shear in the XZ plane

The effects of the HGO parameters on shear wave speeds are illustrated in Fig. 9 for the configuration in which pre-deformation is applied perpendicular to the original fiber axis. The vertical axis of each panel on the top row shows the (normalized) fast wave speed, and on the bottom row depicts slow wave speed. The horizontal axis of each panel shows the value of the HGO model parameter or the magnitude of shear. Wave speed is influenced by all five parameters μ/μ_0 , k_1/μ_0 , κ , k_2 and γ_{YZ} . Similar to pre-deformation in the YZ plane, the fast wave speed increases with increasing μ/μ_0 , k_1/μ_0 , k_2 , γ_{XZ} and decreases with increasing κ . Slow wave speeds follow the same trend as the fast wave speeds, but to a lesser extent.

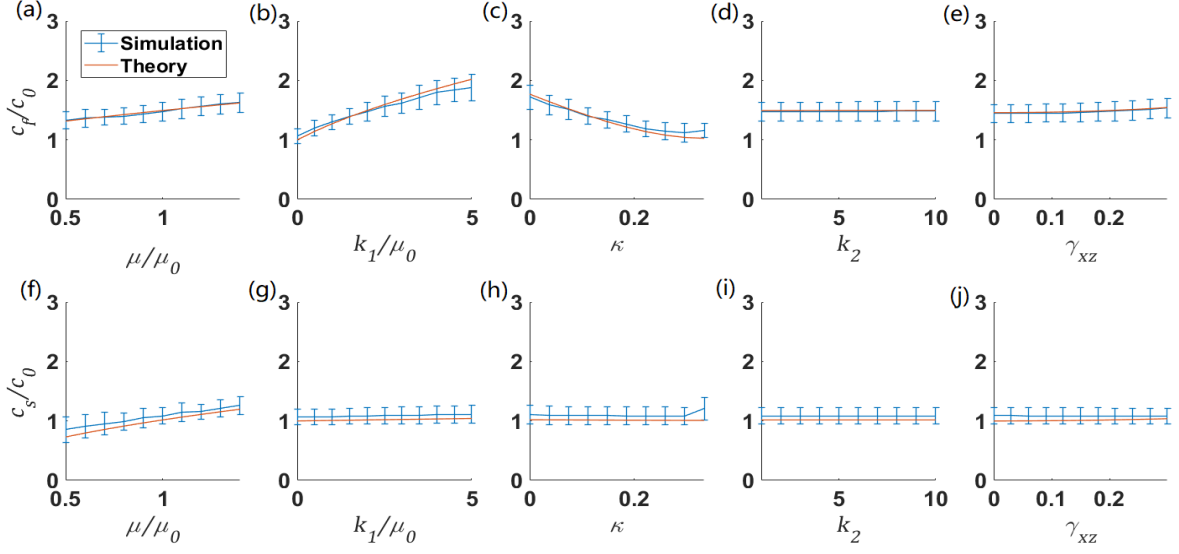


Fig. 9 The effects on fast and slow wave speeds of parameters $(\mu/\mu_0, k_1/\mu_0, \kappa, k_2)$ of the HGO model and pre-deformation magnitude, γ_{XZ} , when pre-deformation is simple shear in the XZ plane, inducing fiber stretch of order (γ_{XZ}^2) . Default parameters are: $\mu_0 = 1000$ Pa, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $k_2 = 5$, $\gamma_{XZ} = 0.2$.

3.4 One fiber family with pre-deformation consisting of imposed extension

The effect of stretch ratio on wave speed is shown in Fig. 10 for the case of imposed extension. Both fast and slow wave speeds increase with stretch ratio and the simulation results agree well with the analytical predictions.

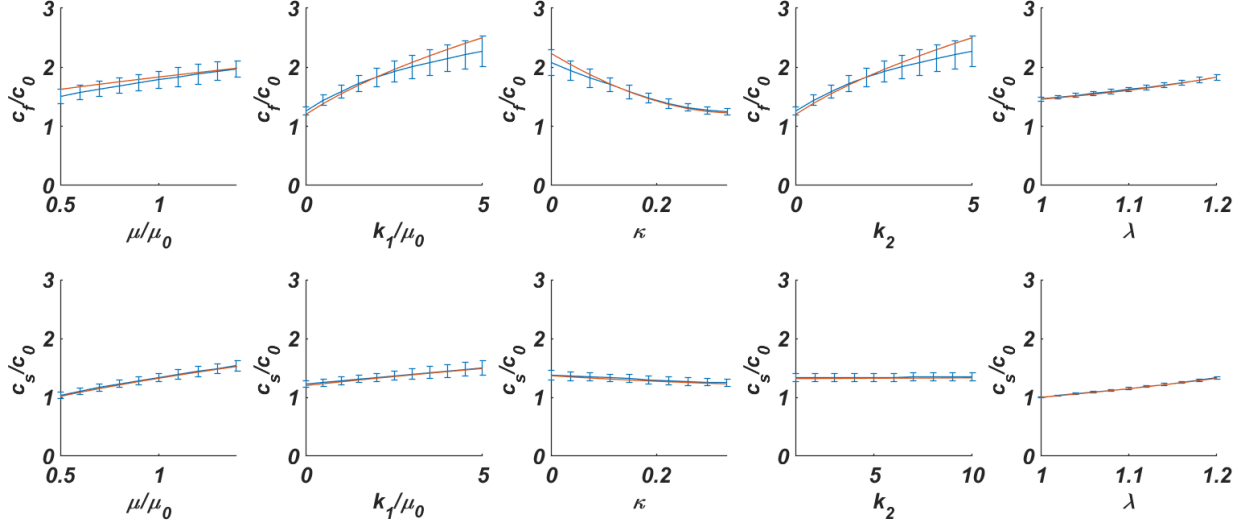


Fig. 10 The effects on fast and slow wave speeds of imposed extension with stretch ratio λ in the Z direction. Default parameters are: $\frac{\mu}{\mu_0} = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $k_2 = 5$, $\lambda = 1.2$.

4 Estimation of Parameters in the HGO model

In the previous section we demonstrated that shear wave speeds can be calculated analytically from parameter values of the HGO model, for specific propagation direction and polarization directions. We also confirmed that the analytical solutions agree with simulated wave speeds in a finite cube-shaped domain. Conversely, the parameters of the HGO model can, in principle, be estimated from measured shear wave speeds, for given propagation and polarization directions, in pre-deformed specimens. In the following section we demonstrate the feasibility of this approach to parameter estimation.

4.1 Estimation method

The example system is shown in Fig. 11. The angle ϕ of the fiber axis is chosen to be $\pi/4$ radians from the base of the specimen in the undeformed configuration, as in Figs. 4 and 5. The propagation direction $\mathbf{n}$ is along negative Z-axis, and fiber direction $\mathbf{a}$ is in the YZ plane. Experiments are separated into two steps. In the first step, the fast wave speed and slow wave speed are measured without pre-deformation, by applying horizontal, harmonic displacements to the top surface in the fast or slow polarization directions. The slow wave speed is a function of only the isotropic shear modulus, μ , and density ρ , but the fast wave

speed is a function of μ , k_1 , and κ (Eq. (19)). In the second step, the fast wave speed and slow wave speed are measured after applying a pre-deformation of simple shear in the YZ plane). In this configuration, both fast and slow wave speeds are functions of all five parameters (μ , k_1 , k_2 , κ , and γ_{YZ}) (Eq. (20)).

$$c_{s0} = \sqrt{\mu/\rho}; \quad c_{f0} = f(k_1, \mu, \kappa) \quad (19)$$

$$c_s = f(k_1, k_2, \mu, \kappa, \gamma_{YZ}); \quad c_f = f(k_1, k_2, \mu, \kappa, \gamma_{YZ}) \quad (20)$$

For the analogous situation using imposed extension (stretch ratio λ), Eq. (20) can be written as:

$$c_s = f(k_1, k_2, \mu, \kappa, \lambda); \quad c_f = f(k_1, k_2, \mu, \kappa, \lambda) \quad (21)$$

In the proposed experiment, the density ρ of the material is a known value, and the simple shear ratio γ_{YZ} (or stretch ratio λ) can be controlled and measured. For a single value of the pre-deformation, the four independent linear equations can be solved simultaneously to determine the four independent parameters. If more data are available, the over-determined system can be solved in the least-squares sense. The MATLAB optimization tool *lsqnonlin* for solving nonlinear least square problems was used to find parameters that minimized the difference between predicted and measured values of wave speed.

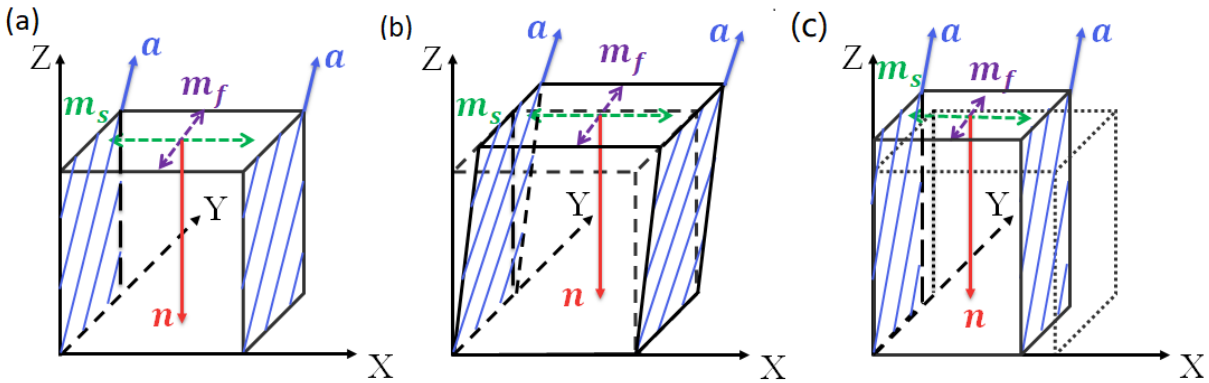


Fig. 11 Idealized experimental setup for estimating parameters of the HGO model. (a) No pre-deformation. (b) Simple shear $\gamma_{YZ} = 0.2$ imposed in the YZ plane. (c) Imposed extension with stretch ratio $\lambda = 1.2$ in Z direction.

4.2 Sensitivity to noise

Because experimental data inevitably contain noise or measurement errors, it is necessary to quantify the robustness of parameter estimates. For each wave speed estimate, random noise was applied from a normal distribution, as shown in Eq. (22) and Eq. (23).

$$c_f^{(noise)} = c_f(1 + \psi\tau) \quad (22)$$

$$c_s^{(noise)} = c_s(1 + \psi\tau) \quad (23)$$

Here, τ is a random value in the standard normal distribution (mean=0, std. dev. = 1), and ψ is defined as a noise factor to control the range of noise variance. In this paper, the noise is defined on three levels. Values of $\psi = 0.01, 0.02$ and 0.03 indicate wave speed variance ranges of $\pm 3.3\%, \pm 6.6\%$ and $\pm 10\%$ from the expected values, respectively.

Table 1 Comparison of HGO model parameter estimates for different noise levels (imposed shear).

Noise Level	μ [Pa]	k_1 [Pa]	κ	k_2
Expected	1000	2000	0.083	5
($\psi=0.01$)	1000 ± 11	2034 ± 367	0.083 ± 0.023	5 ± 0.0
($\psi=0.02$)	1000 ± 23	2050 ± 739	0.077 ± 0.044	4.8 ± 0.1
($\psi=0.03$)	1001 ± 34	2161 ± 1107	0.079 ± 0.057	4.7 ± 0.2

Table 2 Comparison of HGO model parameter estimates for different noise levels (imposed extension).

Noise Level	μ [Pa]	k_1 [Pa]	κ	k_2
Expected	1000	2000	0.083	5
($\psi=0.01$)	1000 ± 11	2007 ± 390	0.081 ± 0.023	4.9 ± 0.1
($\psi=0.02$)	1000 ± 20	2044 ± 752	0.077 ± 0.043	4.7 ± 0.3
($\psi=0.03$)	1002 ± 32	2068 ± 1125	0.075 ± 0.050	4.5 ± 0.5

For wave speed data without noise, the material parameters can be determined by four equations corresponding to two configurations, the undeformed configuration and one value of pre-deformation.

However, if wave speed data are noisy, more data are needed. In a Monte Carlo approach, ten additional simulated experiments with different pre-deformations were added to the original two simulated experiments, and these simulated experiments were repeated 1000 times with different random values. For various noise levels, the mean values ($\pm$ std. dev.) of all four parameters were calculated (Table 1 and Table 2). To improve accuracy, outliers (greater than three standard deviations from the mean) were excluded from wave speed data.

As expected, the standard deviation of each parameter estimate increases with noise level. The mean value of some parameters also deviates from the expected value as noise increases. The parameters k_2 and μ_0 are relatively insensitive to the noise level; estimates of k_1 and κ deviate more when noise increases.

5 Discussion

In materials that can be modeled as nonlinear, anisotropic, and nearly incompressible, slow and fast wave speeds can be measured from MRE and used to estimate parameters of the material model. In the examples above, theoretical predictions of shear wave speed values in different configurations (undeformed configuration, simple shear in the XZ plane or YZ plane) agreed well with simulation results.

Fast and slow shear wave speeds provide complementary information. The fast shear wave speed is affected by the stiffness of the fibers, while the slow shear wave speed is not. In transversely isotropic materials, displacements in the direction of slow shear wave polarization do not induce fiber stretch. In addition, in the example of this paper, simple shear in the YZ plane (which contains the principal fiber axis, $\mathbf{a}$) directly stretches the fibers and significantly affects fast shear wave speeds. In contrast, simple shear in the XZ plane involves displacements perpendicular to the fibers, and does not stretch the fibers appreciably. The measured wave speed in this condition deviates little from the wave speed in the no pre-deformation condition. Extension in the Z direction also stretches the fibers, which increases shear wave speeds.

Using the closed-form expressions for wave speed, and data from either simulations or experiments, we can estimate the parameters of a nonlinear anisotropic material model. Unlike linear elastic materials, pre-deformation plays an important role in determining wave speeds. Without pre-deformation, the slow shear

wave speed varies only with the isotropic shear modulus μ of material and the fast shear wave depends on μ , k_1 and κ . If pre-deformations that stretch the fibers are imposed, the fast shear wave speed depends on all the HGO parameters, μ, k_1, k_2, κ , as well as the magnitude of the pre-deformation.

The accuracy of material parameter estimates is affected by the levels of noise in the measured wave speed data. The isotropic shear modulus μ is the least sensitive to noise because it is directly derived from the slow wave speed with no pre-deformation. For the other three parameters, the nonlinearity k_2 is less sensitive to noise than k_1 and κ , because k_2 has fewer interactions with other factors in the experiment.

Among the limitations of this paper, in computing wave speeds we do not impose the bi-linearity that excludes fibers from resisting infinitesimal levels of dynamic compressive strain. In practice, this assumption could be avoided by imposing a minimal pre-deformation greater than the wave amplitude. We have considered the original version of the HGO model, which is still widely used. A new version of HGO model has recently been proposed [27], which might also be analyzed by this approach. Parameter estimates improve, in terms of both increased accuracy and reduced variance, with more MRE experiments. Balancing the desired precision of the result and the cost of experiments must be considered carefully, as in all experimental studies.

Only one fiber family is considered in this paper, but it is plausible to generalize this approach to estimate wave speeds and material parameters in a material with multiple fiber families. Some special cases can be considered qualitatively. For simplicity, consider a second fiber family with $\phi = -45^\circ$. For the situation in which simple shear is imposed in the YZ plane, one fiber family will be stretched and the other will be compressed. In the original HGO model, fibers in compression ($I_4 < 1$) do not contribute to the stress or to the strain energy. Therefore, wave speeds in a material with two fiber families would be the same as with one fiber family. For the same reason, if the material is compressed in the Z direction ($\lambda < 1$), wave speeds are equal to those in an isotropic material because all fibers are under compression. For the idealized example of imposed extension, adding a second fiber family would simply double the effects of a single fiber family. For other configurations the addition of a second fiber family creates orthotropic material

symmetry, instead of the transverse isotropy considered in this paper, and would (in general) require further analysis.

Experimental studies that exploit this approach to characterize fibrous soft tissues are planned for future work; these studies would involve superimposing small amplitude shear waves on large finite deformations. Instead of the idealized simple shear deformations of the current paper, dense measurements of actual pre-deformations would need to be combined with regional measurements of shear wave speed. While challenging, this approach promises the possibility of comprehensive, non-invasive tissue characterization *in vivo*. Tissue may already be in a pre-deformed state (like white matter in the brain) [28, 29], or quasi-static loading might be imposed by respiration (liver [30]), ocular pressure (eye [31]), or external force (intervertebral disc, muscle or breast [32, 33]).

6 Conclusion

MR elastography can be used, in principle, to estimate parameters of the HGO material model in soft fibrous materials from the speeds of slow and fast shear waves. To demonstrate the ability to obtain accurate results, closed-form expressions for the wave speeds, as functions of pre-deformation and material parameters, were derived and confirmed by numerical simulations. These results illustrate the feasibility of a new approach to parameter estimation for nonlinear material models of fibrous soft matter.

Acknowledgements

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Nomenclature

$\mathbf{A}$ = Elasticity tensor

$\mathbf{a}$ = initial fiber direction vector

$\mathbf{C}$ = Cauchy-Green strain tensor

c = shear wave speed

c_0 = initial shear wave speed

c_f = fast wave speed under pre-deformation

c_{f0} = fast wave speed under no pre-deformation

$c_f^{(noise)}$ = slow wave speed with noise

c_s = slow wave speed under pre-deformation

c_{s0} = slow wave speed under no pre-deformation

$c_s^{(noise)}$ = slow wave speed with noise

E_1, E_2 = two tensile moduli in TI material

$\mathbf{F}$ = deformation gradient tensor

I_1 = first invariant of Cauchy-Green strain tensor

$\bar{I}_1$ = modified first invariant

I_4 = squared stretch in the fiber direction

$\bar{I}_4$ = modified pseudo-invariant

K = bulk modulus of the material

k_1 = initial slope of strain-stress curve in the HGO model

k_2 = nonlinearity of strain-stress curve in the HGO model

L_0 = number of filters

L = abbreviation in closed-form expression of HGO model

M = abbreviation in closed-form expression of HGO model

$\mathbf{m}$ = polarization direction vector

$\mathbf{m}_f$ = fast polarization direction

$\mathbf{m}_s$ = slow polarization direction

N = abbreviation in closed-form expression of HGO model

$\mathbf{n}$ = wave propagation direction vector

$\mathbf{Q}$ = acoustic tensor

W = strain energy function

γ = shear pre-deformation

λ = stretch ratio in Z direction

ψ = noise factor in sensitivity analysis

τ = random value from normal distribution

γ_{XZ} = simple shear ratio in XZ plane

γ_{YZ} = simple shear ratio in YZ plane

ϕ = deviation angle between fiber direction and wave propagation direction

ϕ_{TI} = parameter in TI material model

θ = angle between fiber and propagation directions

ρ = density of the material

ρ_0 = LFE parameter

κ = dispersion parameter of fibers in the HGO model

μ = isotropic shear modulus in the HGO model

μ_0 = initial value of isotropic shear modulus in the HGO model

μ_1, μ_2 = two shear moduli in TI material

ζ_{TI} = parameter in TI material model

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Table 1 Comparison of HGO model parameter estimates for different noise levels (imposed shear).

Table 2 Comparison of HGO model parameter estimates for different noise levels (imposed extension).

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Fig. 1 Wave propagating with two polarization directions in a transversely isotropic material. (a) A “slow” shear wave is produced by harmonic shear displacement in the slow polarization direction, $\mathbf{m}_s$. (b) A “fast” shear wave arises from harmonic shear displacement in the fast polarization direction, $\mathbf{m}_f$. The wavelength of a fast shear wave is typically longer than that of a slow shear wave. Reproduced from reference [16].

Fig. 2 Effects of parameters in the HGO model on the stress-strain relationship during simple shear in a plane containing fibers at an angle of $\pi/4$. (a) The parameter k_1 affects the initial slope of the stress-strain curves for deformations involving fiber stretch ($\mu_0 = 1000\text{Pa}$, $\kappa = 1/12$, $k_2 = 0.1$). (b) The parameter k_2 determines the nonlinearity of the relationships ($\mu_0 = 1000\text{Pa}$, $\kappa = 1/12$, $k_1/\mu_0 = 1$)

Fig. 3 The HGO dispersion parameter κ captures the distribution of fiber orientation in 2D domain, ranging from alignment in no preferred direction ($\kappa=1/3$) (a), $\kappa=1/6$ (b), $\kappa=1/12$ (c) and finally to a single direction ($\kappa=0$) (d).

Fig. 4 Geometry used in numerical simulations ($50 \times 50 \times 50 \text{ mm}^3$) of waves superimposed on simple shear. Green dashed arrows represent harmonic perturbations in two perpendicular directions, which correspond to fast and slow shear wave polarizations for shear waves propagating along the negative Z-axis (red arrows). Blue solid lines are the mean fiber direction, where $\phi = 45^\circ$. One fiber family in (a) the undeformed configuration ($\mathbf{a} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$) and (b-c) pre-deformed configurations in simple shear (b) γ_{YZ} in the YZ plane ($\mathbf{a} = ((1 + \gamma_{YZ})\mathbf{j} + \mathbf{k})/\sqrt{(1 + \gamma_{YZ})^2 + 1}$) or (c) γ_{XZ} in the XZ plane ($\mathbf{a} = (\gamma_{XZ}\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{2 + \gamma_{XZ}^2}$).

Fig. 5 Geometry used in simulations ($50 \times 50 \times 50 \text{ mm}^3$) of waves superimposed on lengthening. Green dashed arrows represent harmonic loading in two perpendicular directions, which correspond to fast and slow shear wave polarizations for shear waves propagating along the negative Z -axis (red arrows). Blue solid lines are the mean fiber direction, where $\phi = 45^\circ$. One fiber family in (a) the undeformed configuration ($\mathbf{a} = (\mathbf{j} + \mathbf{k})/\sqrt{2}$) and (b) after imposed stretch λ in the Z direction ($\mathbf{a} = (\frac{1}{\sqrt{\lambda}}\mathbf{j} + \lambda\mathbf{k})/\sqrt{\lambda^2 + \frac{1}{\lambda}}$)

Fig. 6 Harmonic displacement fields due to slow and fast wave shear waves in configurations corresponding to Figs. 4 and 5. (a-d) x component of harmonic displacement showing slow shear waves propagating in the negative Z direction in the undeformed configuration (a) and after simple shear pre-deformation $\gamma_{YZ} = 0.2$ in the YZ plane (b) or $\gamma_{XZ} = 0.2$ in the XZ plane (c), or $\lambda = 1.2$ in the Z direction (d). (e-h) y component of harmonic displacement showing fast shear waves propagating in the negative Z direction in the undeformed configuration and after simple shear pre-deformation in the YZ, XZ plane, or tension deformation in the Z direction. Green arrows on the top surface indicate the direction of applied harmonic excitation (200 Hz) in each panel.

Fig. 7 Fast and slow wave speeds in relation to three parameters ($\mu/\mu_0, k_1/\mu_0, \kappa$) in the HGO model when no pre-deformation is applied. Default parameters are: $\mu_0 = 1000 \text{ Pa}$, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$.

Fig. 8 The effects on fast and slow wave speeds of the parameters ($\mu/\mu_0, k_1/\mu_0, \kappa, k_2$) of the HGO model. The parameter γ_{YZ} is the pre-deformation magnitude of simple shear in the YZ plane, inducing fiber stretch of order γ_{YZ} in fibers at $\phi = \pi/4$. Default parameters are: $\mu_0 = 1000 \text{ Pa}$, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $k_2 = 5$, $\gamma_{YZ} = 0.2$.

Fig. 9 The effects on fast and slow wave speeds of parameters ($\mu/\mu_0, k_1/\mu_0, \kappa, k_2$) of the HGO model and pre-deformation magnitude, γ_{XZ} , when pre-deformation is simple shear in the XZ

plane, inducing fiber stretch of order (γ_{XZ}^2) . Default parameters are: $\mu_0 = 1000 \text{ Pa}$, $\mu/\mu_0 = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $k_2 = 5$, $\gamma_{XZ} = 0.2$.

Fig. 10 The effects on fast and slow wave speeds of imposed extension with stretch ratio λ in the Z direction.

Default parameters are: $\frac{\mu}{\mu_0} = 1$, $k_1/\mu_0 = 2$, $\kappa = 1/12$, $k_2 = 5$, $\lambda = 1.2$.

Fig. 11 Idealized experimental setup for estimating parameters of the HGO model. (a) No pre-deformation. (b) Simple shear $\gamma_{YZ} = 0.2$ imposed in the YZ plane. (c) Imposed extension with stretch ratio $\lambda = 1.2$ in Z direction.