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Signal estimation and system identification with nonlinear dynamic sensors

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Abstract— **We consider the problem of estimating the output of an unknown discrete-time linear time-invariant system and identifying a model of the system, where only measurements via a nonlinear dynamic sensor with known dynamics are available. The main result of this paper is a rank-constrained semidefinite program, which provides an equivalent characterization of this identification and estimation problem. This extends existing results from Wiener system identification to the more general case that the nonlinear block exhibits dynamic behavior, which is a commonly found scenario in practical applications. Notably, the result can be applied in the presence of nonlinear sensors with general non-invertible system dynamics. Two examples are used to illustrate the applicability of our approach.**

I. INTRODUCTION

The identification of dynamical systems from measured data is a central topic in both theory and practice of automatic control. Once a sufficiently good model of the system is obtained, there exist various methods for analyzing and controlling these systems and therefore, numerous approaches for identifying linear time-invariant (LTI) systems have been developed [1]. These methods, which can be roughly divided into subspace techniques [2], prediction error methods [3], and set membership approaches [4], [5], require a set of input-output tuples sampled from the system of interest. In practice, however, the output of the system is often not directly accessible, but can only be measured via a sensor, which might itself exhibit nonlinear dynamic behavior [6]–[8]. Thus, to infer the actual output of the dynamical system, the effect of the sensor needs to be compensated. This problem, which is often referred to as *dynamic sensor compensation*, has been considered, e.g., in [9], [10], where the authors use neural networks to approximate the inverse of the (unknown) sensor dynamics, or in [11], where a linear time-varying filter is designed to smoothen the oscillatory dynamical behavior of load cell sensors.

We consider the problem of identifying an LTI system and estimating its output, when the measurements are only available via a sensor with nonlinear dynamics, i.e., when a cascade structure such as the one illustrated in Figure 1 is considered. When the sensor possesses a static characteristic, then this problem amounts to solving a Wiener system identification problem, which has been studied extensively in the literature (see, e.g., [12]–[15]). Contrary to the work from [9], [10], we will in this paper assume that the sensor's nonlinear dynamics are known. Notably, under this assumption, the problem of identifying the LTI part in Figure 1 is in general np-hard, even for the special case of Wiener systems [16]. Therefore, various (convex) relaxations to solve this problem for Wiener systems have been proposed in the literature [17]–[19]. Our results are based on the approach from [18], which develops a rank-constrained semidefinite program (SDP) reformulation for the Wiener system identification problem.

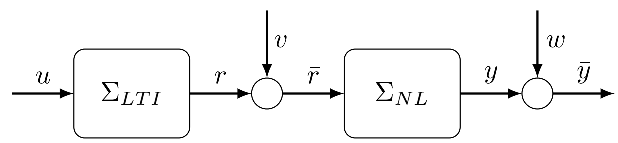

Fig. 1: Concatenation of an LTI system Σ_{LTI} and a nonlinear dynamical system Σ_{NL}, including process and measurement noise denoted by v and w, respectively.

In particular, we propose a rank-constrained feasibility problem, which characterizes all feasible output signals r and all feasible impulse responses g of Σ_{LTI}. This problem is then solved to obtain estimates of r and g from measured input-output data. While it is straightforward to obtain a reliable estimate of r, the impulse response g can only be estimated accurately if additional information on the system order is employed, which results in an increasing computational complexity. Contrary to the methods for dynamic sensor compensation from [9], [10], our approach can be applied to non-invertible sensor dynamics, as long as they are rational. Further, process as well as measurement noise is explicitly taken into account. As an additional assumption, we only require that the input to the excited system can be measured and that this system is linear. Knowledge of a model of this system is however not required.

The remainder of the paper is structured as follows: In Section II, we present the setting and recall a relevant result from LTI system identification. Thereafter, in Section III we state our main result, which allows for an equivalent reformulation of the original estimation and identification problem as a rank-constrained SDP. This result is then applied in Section IV to solve the signal estimation and system identification problem for two illustrative examples. Section V provides a conclusion as well as topics for future research.

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II. SETTING

We say that a symmetric matrix H is a Hankel matrix if it is of the form

$$H := \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ a_2 & a_3 & \dots & a_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_n & a_{n+1} & \dots & a_{2n-1} \end{pmatrix}$$

for some $a_i \in \mathbb{R}$. Given a vector sequence $\{x_k\}_{k=1}^N$, we denote the corresponding Toeplitz matrix by

$$T_x^N := \begin{pmatrix} x_1 & 0 & \dots & 0 \\ x_2 & x_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ x_N & \dots & x_2 & x_1 \end{pmatrix}.$$

Further, for a symmetric real-valued matrix P, $P \geq 0$ means that P is positive semidefinite. Moreover, we write $\|x\|_\infty := \sup_i \|x_i\|_\infty$ for the ℓ_∞-norm of x, with $\|x_i\|_\infty$ being the standard ∞-norm in a Euclidean vector space. We define the following set of bounded, analytic transfer functions

$$\mathcal{H}_\infty(\rho, K) = \{G \text{ analytic in } \mathcal{D}_\rho \mid \sup_{z \in \mathcal{D}_\rho} |G(z)| < K\},$$

where $\mathcal{D}_\rho = \{z \in \mathbb{C} \mid |z| < \rho\}$ is an open disk.

In this paper, we consider the cascade system depicted in Figure 1. Σ_{LTI} is an unknown LTI system with transfer function G, i.e.,

$$r_k = (g \star u)_k, \tag{1}$$

where g is the corresponding impulse response, $\star$ denotes convolution, and $u_k \in \mathbb{R}^m, r_k \in \mathbb{R}^q, k \in \mathbb{N}$. Although (1) implies that the initial state of Σ_{LTI} is zero, the results of this paper can be directly extended to account for non-zero initial conditions. Σ_{NL} is a known nonlinear dynamical system of the form

$$\begin{aligned} x_{k+1} &= f(x_k, \bar{r}_k), \qquad x(0) = x_0 \\ y_k &= h(x_k, \bar{r}_k), \end{aligned} \tag{2}$$

for some $f : \mathbb{R}^{n+q} \to \mathbb{R}^n, h : \mathbb{R}^{n+q} \to \mathbb{R}^p, x_0 \in \mathbb{R}^n$. The initial condition x_0 of Σ_{NL} is not assumed to be known. Note that for $f = 0$, $h(x_k, \bar{r}_k) = \Psi(\bar{r}_k)$ with some static function Ψ, the cascade of the LTI system (1) and the nonlinearity (2) (cf. Figure 1) is a Wiener system. Further, we consider process and measurement noise, represented by $v_k \in \mathbb{V}, w_k \in \mathbb{W}$, respectively, where $\mathbb{V} \subset \mathbb{R}^q, \mathbb{W} \subset \mathbb{R}^p$ are described by some ℓ_∞ bound, i.e., $\mathbb{V} := \{v \in \mathbb{R}^q \mid \|v\|_\infty \leq \varepsilon_v\}$, $\mathbb{W} := \{w \in \mathbb{R}^p \mid \|w\|_\infty \leq \varepsilon_w\}$ for some $\varepsilon_v, \varepsilon_w > 0$.

Given some input-output data $\mathbb{A} = \{u_k, \bar{y}_k\}_{k=1}^N$, we want to find a feasible point in the consistency set

$$\begin{aligned} \mathcal{T}(\mathbb{A}) = \Big\{ &\{r_k\}_{k=1}^N, \{x_k\}_{k=1}^N \mid r_k = (g \star u)_k, \ x_{k+1} = f(x_k, r_k + v_k) \\ &\bar{y}_k = h(x_k) + w_k, \ v_k \in \mathbb{V}, w_k \in \mathbb{W} \Big\}. \end{aligned} \tag{3}$$

Many results on Wiener system identification are essentially extensions of LTI system identification concepts. Similarly, the following technical result combines an LTI identification result from [20, Theorem 2] with the specific dynamics in (2) in order to arrive at an equivalent characterization of the consistency set $\mathcal{T}(\mathbb{A})$.

Lemma 2.1: Given $K > 0$, $\rho > 1$ as well as experimental data $\mathbb{A} = \{u_k, \bar{y}_k\}_{k=1}^N$, there exists an LTI system Σ_{LTI} with transfer function $G \in \mathcal{H}_\infty(\rho, K)$ such that the consistency set $\mathcal{T}(\mathbb{A})$ is non-empty if and only if there exist $g, \bar{r}_k, r_k, x_k$ such that

$$\begin{bmatrix} KR^{-2} & \left(T_g^N\right)^\top \\ T_g^N & KR^2 \end{bmatrix} \geq 0, \tag{4}$$

$$r = T_g^N u, \ \ \bar{r}_k - r_k \in \mathbb{V}, \tag{5}$$

$$x_{k+1} = f(x_k, \bar{r}_k), \tag{6}$$

$$\bar{y}_k - h(x_k, \bar{r}_k) \in \mathbb{W}, \tag{7}$$

for $k = 1, \dots, N$, where $R = \text{diag}[1, \rho, \rho^2, \dots, \rho^{N-1}]$.

Proof: Follows directly from combining [20, Theorem 2] with the above cascade structure. ■

In Lemma 2.1, the decision variable g is the impulse response corresponding to the transfer function G, and the signals $r, \bar{r}$ and x are the same as in Figure 1. Hence, the above result provides a direct equivalent characterization of all feasible signals $\bar{r}, r, x$ and impulse responses g. In particular, determining the desired quantities is equivalent to finding a feasible point in (4)-(7). Further, Lemma 2.1 is a combination of an LTI identification result and the nonlinear dynamics from (2): The conditions (4) and (5) represent the LTI identification part, whereas (6) and (7) reflect the nonlinear dynamic parts in the considered cascade system. While the former conditions are a combination of a linear matrix inequality (LMI) with linear equality constraints, the latter constraints are in general non-convex and thus the above feasibility problem can usually not be solved efficiently. In the next section, we show that, when f and h are rational, then the feasibility problem is equivalent to a rank-constrained SDP, for which various solution methods exist.

III. A RANK-CONSTRAINED SDP RELAXATION

In this section, we provide a solution to the above-defined system identification and signal estimation problem, i.e., to the problem of finding a feasible point in (4)-(7). In [18], a rank-constrained convex relaxation for Wiener system identification is proposed. Transferred to our setting, the main idea is that, when the system dynamics (2) are rational, then the constraints (6) and (7) can be replaced by rank-constraints on suitably defined moment matrices.

Suppose for the following considerations that all signals in Figure 1 as well as the internal state x are scalar[1]. Further,

[1] This is only assumed for notational simplicity and does not pose a limitation on the presented approach. We will comment on the multidimensional case after our main result.

we assume that f and h are rational functions, i.e.,

$$f(x_k, \bar{r}_k) = \frac{P_f(x_k, \bar{r}_k)}{Q_f(x_k, \bar{r}_k)},$$
$$h(x_k, \bar{r}_k) = \frac{P_h(x_k, \bar{r}_k)}{Q_h(x_k, \bar{r}_k)},$$

where P_f, Q_f, P_h, Q_h are multivariate polynomials. In this case, the constraints (6) and (7) can be written equivalently as

$$x_{k+1} Q_f(x_k, \bar{r}_k) - P_f(x_k, \bar{r}_k) = 0, \tag{8}$$
$$y_k Q_h(x_k, \bar{r}_k) - P_h(x_k, \bar{r}_k) = 0, \tag{9}$$
$$\|\bar{y}_k - y_k\|_\infty \leq \varepsilon_w, \tag{10}$$

for all $k = 1, \ldots, N$. Note that (8) and (9) are polynomial constraints and thus, it follows directly from Lemma 2.1 that the set $\mathcal{T}(\mathbb{A})$ is semi-algebraic. Hence, convex relaxations for the original feasibility problem can be found using, e.g., moments [21] or tools from semi-algebraic geometry [22]. In [17], for instance, a moments-based approach to find feasible points in $\mathcal{T}(\mathbb{A})$ was presented for the special case of Wiener systems. However, the computational complexity of such approaches grows quickly with the number of decision variables and thus with the number of sampled data points. A slightly different approach was pursued in [18], where the polynomial constraints in $\mathcal{T}(\mathbb{A})$ were replaced by rank constraints. Similarly, we will in the following replace the nonlinear constraints induced by (8) and (9) by rank-constraints where the corresponding matrices are linear in the decision variables. To do this, we introduce the Hankel matrices

$$M_k^r := \left(m_{ij}^{r,k}\right) \in \mathbb{R}^{n_{m,r} \times n_{m,r}}, \ m_{ij}^{r,k} = \bar{r}_k^{i+j-2},$$
$$M_k^x := \left(m_{ij}^{x,k}\right) \in \mathbb{R}^{n_{m,x} \times n_{m,x}}, \ m_{ij}^{x,k} = x_k^{i+j-2},$$

where $n_{m,r}$ and $n_{m,x}$ are chosen large enough, such that the above matrices contain every exponent appearing in (8) and (9). Note that the conditions $m_{11}^{r,k} = 1$, $m_{12}^{r,k} = \bar{r}_k$, $\operatorname{rank}\left(M_k^r\right) \leq 1$, and the fact that M_k^r is a Hankel matrix imply that it is of the above form, i.e., that $m_{ij}^{r,k} = \bar{r}_k^{i+j-2}$, and analogously for M_k^x. The polynomial equations (8) and (9) contain only monomials of the form $x_k^i \bar{r}_k^j$, $x_{k+1} x_k^i \bar{r}_k^j$ or $y_k x_k^i \bar{r}_k^j$. In the next step, we replace each of these monomials by a scalar decision variable. Define

$$R_{ij}^k := \begin{pmatrix} 1 & m_{1(i+1)}^{x,k} \\ m_{1(j+1)}^{r,k} & \alpha_{ij}^k \end{pmatrix}$$

for any monomial of the form $x_k^i \bar{r}_k^j$ appearing in (8) or (9) and note that $\operatorname{rank}\left(R_{ij}^k\right) \leq 1$ implies $\alpha_{ij}^k = x_k^i \bar{r}_k^j$. Furthermore, set

$$S_{ij}^k := \begin{pmatrix} 1 & \alpha_{ij}^k \\ m_{12}^{x,k+1} & \beta_{ij}^k \end{pmatrix}$$

for any occurrence of $x_{k+1}\alpha_{ij}^k$ in (8) and

$$T_{ij}^k := \begin{pmatrix} 1 & \alpha_{ij}^k \\ y_k & \gamma_{ij}^k \end{pmatrix}$$

for terms of the form $y_k \alpha_{ij}^k$ appearing in (9). The introduction of the above variables allows us to rewrite the equations (8) and (9), which are nonlinear in $x_k, \bar{r}_k, y_k$, as linear equations in $\alpha_{ij}^k, \beta_{ij}^k, \gamma_{ij}^k$, i.e., as

$$\begin{aligned} \sum_{i,j} a_{ij}^k \alpha_{ij}^k + b_{ij}^k \beta_{ij}^k = 0, \\ \sum_{i,j} c_{ij}^k \alpha_{ij}^k + d_{ij}^k \gamma_{ij}^k = 0, \end{aligned} \tag{11}$$

for $k = 1, \ldots, N$ and suitably chosen coefficients a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k. This allows us to state the following theorem, which provides an equivalent rank-constrained SDP reformulation of the original feasibility problem.

Theorem 3.1: Given $K > 0$, $\rho > 1$ as well as experimental data $\mathbb{A} = \{u_k, \bar{y}_k\}_{k=1}^N$, there exists an LTI system Σ_{LTI} with transfer function $G \in \mathcal{H}_\infty(\rho, K)$ such that the consistency set $\mathcal{T}(\mathbb{A})$ is non-empty if and only if there exist $g, r_k, m_{ij}^{r,k}, m_{ij}^{x,k}, \alpha_{ij}^k, \beta_{ij}^k, \gamma_{ij}^k, y_k$ such that

$$\begin{aligned} &\begin{bmatrix} KR^{-2} & \left(T_g^N\right)^\top \\ T_g^N & KR^2 \end{bmatrix} \geq 0, \ \ r_k = [T_g^N u]_k \\ &\left\|m_{12}^{r,k} - r_k\right\|_\infty \leq \varepsilon_v, \ \ \|\bar{y}_k - y_k\|_\infty \leq \varepsilon_w, \\ &\operatorname{rank}\left(M_k^r\right) \leq 1, \ \operatorname{rank}\left(M_k^x\right) \leq 1, \\ &(11) \text{ hold}, \ \ m_{11}^{r,k} = m_{11}^{x,k} = 1, \\ &\operatorname{rank}\left(R_{ij}^k\right) \leq 1, \ \operatorname{rank}\left(S_{ij}^k\right) \leq 1, \ \operatorname{rank}\left(T_{ij}^k\right) \leq 1, \end{aligned} \tag{12}$$

for $k = 1, \ldots, N$, where $R = \operatorname{diag}[1, \rho, \rho^2, \ldots, \rho^{N-1}]$.

Proof: Follows directly from Lemma 2.1 and the above discussion. ■

Note that (12) is indeed a rank-constrained SDP and can thus be solved either using a convex relaxation of rank [23] or directly with LMIRank [24]. Theorem 3.1 is essentially an extension of [18, Theorem 1], which was stated for Wiener systems, to more general cascade systems with dynamic nonlinearities. It is not only a feasibility characterization for the original problem, but, similarly to Lemma 2.1, it also yields the desired unknown signals as well as the impulse response g. Although, for the above derivation, we considered scalar nonlinear dynamical systems Σ_{NL} as well as scalar signals u, r, and y, the result holds true in the multidimensional case as well, with obvious modifications: Any additional monomial term occurring in (6) and (7) simply needs to be replaced by a scalar variable with a corresponding rank-constraint, as in the scalar case. Further, we could also consider more general vector fields f and h which are linear combinations of rational basis functions with unknown coefficients.

Remark 3.2: For the special case of Wiener systems, [18] provides additional analysis results as well as extensions of the feasibility problem (12), which apply also in the present setting with dynamic nonlinearities. For instance, knowledge of an upper bound n_g on the order of Σ_{LTI} can be exploited by enforcing an upper bound on the rank of the Hankel matrix of the impulse response, in addition to the conditions of Theorem 3.1. This restricts the class of admissible impulse responses for the feasibility problem to those belonging to a system with McMillan degree less than or equal to n_g. As will be discussed in the next section, this might be beneficial if the main objective is the identification of the LTI system Σ_{LTI}. In practice, however, this extension increases the computational complexity of the resulting feasibility problem and should thus only be used if knowledge of an accurate model of Σ_{LTI} is indeed desired. Moreover, there might exist multiple solutions of the feasibility problem (12), due to the noise or the system Σ_{NL} being not invertible. In this case, an upper bound on the identification error is readily computed by formulating a related rank-constrained SDP [18, Section V.]

Remark 3.3: In practice, one is often not interested in identifying the system Σ_{LTI}, but rather in estimating the signal r online, i.e., in the dynamic compensation of the sensor Σ_{NL}. In particular in this case, a moving horizon implementation of the feasibility problem (12) might be a good alternative. To be more precise, Theorem 3.1 provides a characterization of the estimation problem over the full measurement length N. When $\tilde{N}$ additional data is obtained, the feasibility problem must be solved again, now with a total of $N + \tilde{N}$ measurements. Alternatively, one could only take the past T measurements into account, where $T < N + \tilde{N}$, thereby reducing the computational complexity of the problem. When new measurements are obtained, the horizon is shifted and the estimation process is carried out again for the past T measurements. For this, an additional constraint that the decision variables align with previous estimates in the overlapping interval might be beneficial. By employing this idea, the above result could also be applied to more general system classes for the unknown system such as (slowly) time-varying or (mildly) linear parameter-varying systems.

IV. ILLUSTRATIVE EXAMPLES

In this section, we apply Theorem 3.1 to two practical examples. First, we consider an unknown LTI system, whose output is measured via a force sensor with nonlinear elastic behavior. The nonlinear dynamics of the sensor are adopted from [10].

Example 4.1: Consider the unknown LTI system with transfer function

$$G(z) = \frac{0.16}{z^2 - 0.4z + 0.5},$$

cascaded with the known two-dimensional nonlinear dynamical system Σ_{NL}

$$\begin{aligned} x_1(k+1) &= x_1(k) + hx_2(k), \\ x_2(k+1) &= x_2(k) + \frac{h}{m}\left(\bar{r}(k) - cx_2(k) - k_1 x_1(k) - k_2 x_1(k)^3\right), \\ y(k) &= x_1(k), \end{aligned} \tag{13}$$

with unknown initial condition $x(0) = \begin{pmatrix} 0.005 & -0.1 \end{pmatrix}^\top$. The dynamics (13) are obtained from the continuous-time dynamics in [10, Example 1] via a Euler discretization with sampling rate $h = 0.005$. As in [10], the other parameters are chosen as $m = 1/6084, c = 49.6/6084, k_1 = 1, k_2 = 304.2/6084$. The LTI system is excited with a uniformly distributed input with values in $[-1, 1]$. We assume process and measurement noise levels of $\varepsilon_v = 0.02$ and $\varepsilon_w = 3 \cdot 10^{-4}$, corresponding to roughly 8% and 1.5% of the maximal simulated values of r and y, respectively.

Solving the rank-constrained SDP feasibility problem (3.1) with LMIRank [24] yields a trajectory r as well as an impulse response g, which are displayed in Figures 2 and 3, respectively. It can be seen that, for the signal r, the estimation error lies roughly within the noise tolerance. On the contrary, the estimate of the impulse response is inaccurate since we have not used information on the order of the system and therefore the order of the approximated impulse response is too large (cf. Remark 3.2).

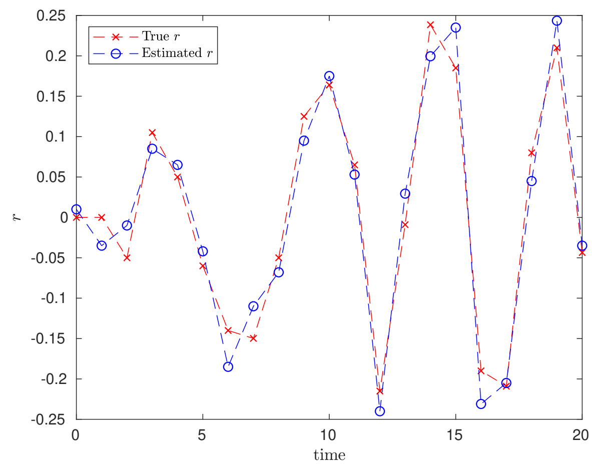

Fig. 2: Estimated (via Theorem 3.1) and true signal r from Example 4.1.

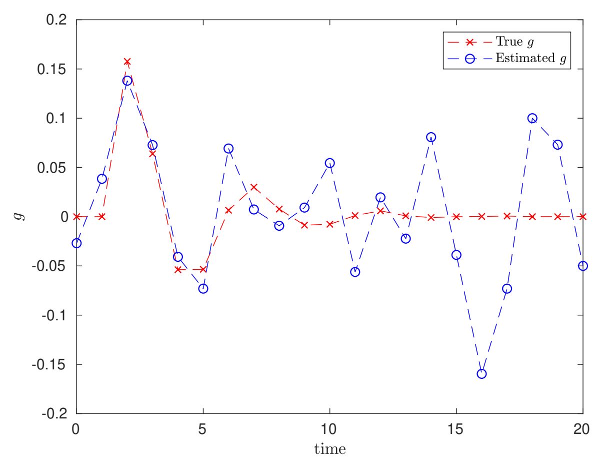

Fig. 3: Estimated (via Theorem 3.1) and true impulse response from Example 4.1.

As a second example, we consider the problem of estimating the output of an LTI system, where the sensor measures only the distance of this output to a certain reference point and is additionally perturbed by a PT1-behavior. This problem can be modelled using a sensor with non-invertible nonlinear dynamics.

Example 4.2: Consider the unknown LTI system with transfer function

$$G(z) = \frac{-0.2z - 0.6}{z^2 + 0.4z - 0.2},$$

cascaded with the known nonlinear non-invertible dynamical system Σ_{NL}

$$\begin{aligned} x(k+1) &= 0.2 \cdot x(k) + 0.8 \cdot (\bar{r}(k) - r_{ref})^2, \\ y(k) &= x(k), \end{aligned} \tag{14}$$

with unknown initial condition $x(0) = 0.5$. The dynamics (14) can be interpreted as measuring the (squared) distance of $\bar{r}$ to the reference point r_{ref}, which is chosen to be $r_{ref} = 1.2$, where, e.g. due to ageing of the sensor, past measurements contribute to new measurements with a factor of 0.2. The noise levels are chosen as $\varepsilon_v = 0.05$ and $\varepsilon_w = 0.02$, corresponding to roughly 5% and 0.5% of the maximal simulated values of r and y, respectively. Further, the LTI system is excited with a uniformly distributed input with values in $[-2, 2]$. Figures 4 and 5 show the estimates of r and g, respectively, obtained by solving the rank-constrained SDP (12) with LMIRank [24]. Again, the estimated impulse response differs from the original one after the first time steps (cf. Remark 3.2). Nevertheless, a reliable estimate of the signal r could be obtained, despite Σ_{NL} being non-invertible.

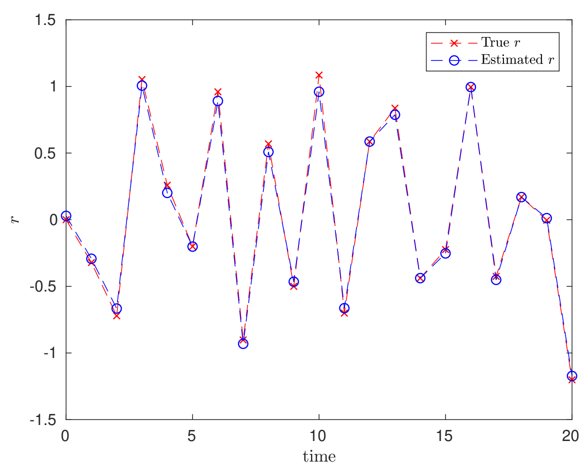

Fig. 4: Estimated (via Theorem 3.1) and true signal r from Example 4.2.

Remark 4.3: As the considered estimation and identification problem is in general np-hard [16], it is not surprising that the feasibility problem (12) does in practice not scale well for a large number of data tuples. Nevertheless, the present examples could be handled for more than 100 data points. Furthermore, we note that the approach could successfully be applied to a realistic example (Example 4.1) as well as to an example with a non-invertible nonlinearity (Example 4.2), where in particular the latter problem could not be solved using existing tools for the problem of dynamic sensor compensation.

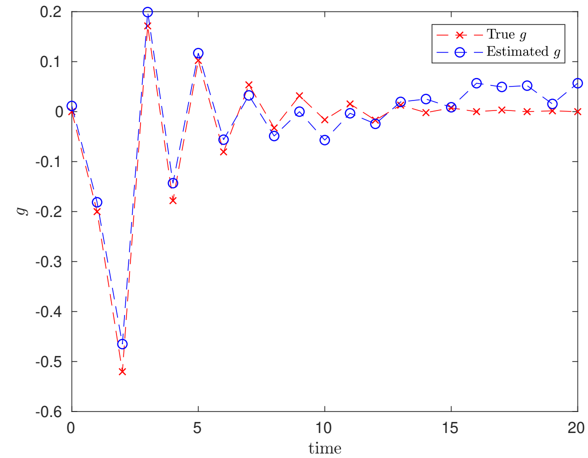

Fig. 5: Estimated (via Theorem 3.1) and true impulse response from Example 4.2.

V. CONCLUSION

In this paper, we presented an approach to the problem of estimating the output of an LTI system and identifying the system, when the measurements are obtained from a sensor with known nonlinear dynamics. The main result is an equivalent reformulation of the original feasibility problem as a rank-constrained SDP, which can be solved using either a direct solver such as LMIRank or convex relaxations of the rank constraints. Contrary to existing approaches, our approach can handle non-invertible nonlinear sensor dynamics. The applicability of the result was illustrated with two practical examples.

Future research will address the development of more efficient numerical reformulations of the original feasibility problem. Further, the present approach can only be applied in the presence of rational nonlinear systems. An extension to more general nonlinear system dynamics such as e.g. neural networks seems both promising and interesting. Finally, the presented examples were only simulation studies and thus the validation of the method in a practical experiment remains an open issue.

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