Weak Target Detection in MIMO Radar via Beamspace Canonical Correlation

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***Abstract*—Reliable detection and accurate estimation of weak targets and their Doppler frequencies is a challenging problem in MIMO radar systems. Reflections from such targets are often overpowered by those from stronger nearby targets and clutter. Considering a 3-D data model where the coherent processing interval comprises multiple pulses, a novel weak target detection and estimation approach is proposed in this paper. The proposed method is based on creating partially overlapping spatial beams, and performing canonical correlation analysis (CCA) in the resulting beamspace. It is shown that if a target is present in the overlap sector, then its Doppler profile can be reliably estimated via beamspace CCA, even if hidden under much stronger interference from nearby targets and clutter. Numerical results are included to validate this theoretical claim, demonstrating that the proposed Beamspace Canonical Correlation (BCC) method yields considerable performance improvement over existing approaches.**

***Index Terms*—Weak target detection, Doppler estimation, multiple-input multiple-output (MIMO), radar, canonical correlation analysis (CCA)**

I. Introduction

An important problem in radar signal processing is to detect and accurately estimate the Doppler frequency of weak targets that evade detection and estimation using classical approaches [1], [2]. Towards this end, multiple-input multiple-output (MIMO) radar holds considerable promise, as it brings in new degrees of freedom to radar engineering and signal processing [3]–[8]. MIMO radar emits multiple orthogonal waveforms from the different transmit antennas, resulting in diverse target views after matched filtering, in addition to receive-diversity. MIMO radar can also leverage parametric spectral and spatial estimation tools, such as harmonic retrieval and direction-of-arrival (DOA) estimation [9]–[13] techniques and their multi-dimensional counterparts, as well as range-based target localization [14].

Existing parametric and non-parametric radar detection and estimation methods generally work well when the target of interest is received at relatively good signal to interference plus noise ratio after pulse compression (matched filtering), but often fail otherwise – especially if the target of interest in nearby other strong targets or clutter. The fundamental reason for this is that existing approaches directly or indirectly rely on a fitting criterion (often, least squares) that naturally pays attention to the stronger reflections. This renders reliable detection and estimation of weak targets extremely difficult using conventional approaches.

Breaking from the mold, we show in this paper that by creating two partially overlapping spatial beams and judiciously controlling the degree of their overlap, we can create two views of the signal space that share only the (potential) target of interest at a certain range-DOA cell. Using canonical correlation analysis (CCA) on these two views, it is possible to detect the sought target, even under strong nearby interference. CCA is a widely-used statistical learning tool that aims at reliably extracting highly correlated pairs of projections (linear combinations) of two random vectors [15], [16]. We have recently shown in [17] that CCA can be interpreted as *subspace intersection*: that is, it computes the shared subspace between a pair of matrices.

We consider the 3-D data model of [9] that assumes multiple pulse transmissions in a bi-static MIMO radar system where the transmit and receive arrays have co-located antennas. We design two narrow partly overlapping beamformers which, when applied to the 3-D array, yield two data matrices. We show that if there is a target located in the overlapped DOA region, then it can be reliably detected and its Doppler frequency can be accurately estimated via CCA. This is shown both theoretically and via simulations, which reveal the superiority of the proposed *Beamspace Canonical Correlation* (BCC) approach in detecting and accurately estimating the speed of the weak target over the state-of-the-art methods.

While CCA has found several applications in the signal processing literature, including DoA estimation [18], equalization [19], radar anti-jamming [20], and more recently cell-edge user detection [17], this paper is, to the best of our knowledge, the first attempt to utilize CCA for weak target detection and Doppler estimation in MIMO radar.

The rest of the paper is organized as follows. Section II describes the data model and defines the problem. The problem formulation is presented in Section III, while Section IV presents the proposed BCC method. Simulation results are provided in Section V, and conclusions are drawn in Section VI.

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II. Problem Statement

We consider a bistatic MIMO radar system with a transmitter comprising M co-located antennas and a receiver comprising N co-located antennas (the classical monostatic scenario is a special case). For the simulations, we assume for simplicity that both the transmitter and the receiver employ a uniform linear array (ULA), but such assumption is not needed for our method to work. We only need to know the manifold of the receive array, so that we can beamform it towards a direction (or sector) of interest. The rest of the setup is as follows:

- d_T and d_R are the transmit and receive array element spacing, respectively.
- K is the number of targets in a range bin of interest.
- v_k is the relative speed of the k-th target.
- Q is the number of transmitted pulses in the coherent processing interval.
- T_p is the radar pulse period.
- θ_k and ϕ_k are the direction of departure (DoD) and direction of arrival (DoA) associated with the k-th target.

The transmitted baseband pulse waveform from the m-th transmit antenna is denoted by $\mathbf{s}_m \in \mathbb{C}^{T\times 1}$, where T is the number of samples per pulse duration and $\mathbf{s}_{m_1}^H\mathbf{s}_{m_2} = \begin{cases} T, & m_1 = m_2, \\ 0, & \text{otherwise}, \end{cases}$ for $m_1, m_2 \in \{1, \cdots, M\}$. The received $N \times T$ complex space-time baseband signal after synchronization is given by [9]

$$\mathbf{Y}_q = \sum_{k=1}^{K} \alpha_{kq}\mathbf{a}_R(\phi_k)\mathbf{a}_T^T(\theta_k)\mathbf{S} + \mathbf{W}_q, \quad q = 1, \cdots, Q \quad (1)$$

where $\mathbf{Y}_q \in \mathbb{C}^{N\times T}$ holds in its columns the T samples received by the N antennas for the q-th pulse period, $\mathbf{S} \in \mathbb{C}^{M\times T}$ holds in its columns the transmitted orthogonal pulse waveforms, and $\mathbf{W}_q$ is the noise matrix that contains independent identically distributed (i.i.d.) entries with zero mean and variance σ^2. The term $\alpha_{kq} = r_k e^{j2\pi(q-1)f_k T_p}$ accounts for the Doppler frequency, f_k, and the radar cross section (RCS), r_k. Throughout this work, we assume that the RCS coefficient, r_k, of the k-th target is constant across all pulses. The vectors $\mathbf{a}_T(\theta_k) \in \mathbb{C}^M$ and $\mathbf{a}_R(\phi_k) \in \mathbb{C}^N$ represent the transmit and receive steering vectors associated with the k-th target, respectively, where

$$\mathbf{a}_T(\theta_k) = \frac{1}{\sqrt{M}}[1, e^{(j2\pi\frac{d_T}{\lambda}\sin\theta_k)}, \cdots, e^{(j2\pi\frac{d_T}{\lambda}(M-1)\sin\theta_k)}]^T$$

$$\mathbf{a}_R(\phi_k) = \frac{1}{\sqrt{N}}[1, e^{(j2\pi\frac{d_R}{\lambda}\sin\phi_k)}, \cdots, e^{(j2\pi\frac{d_R}{\lambda}(N-1)\sin\phi_k)}]^T$$

where $\lambda := \frac{c}{f_c}$ with c and f_c are the speed of light and the carrier frequency, respectively.

The received signals in (1) are matched by $\frac{1}{T}\mathbf{S}^H$ so the output of the matched filters is given by

$$\overline{\mathbf{Y}}_q = \mathbf{A}_R\mathbf{D}_q\mathbf{A}_T^T + \overline{\mathbf{W}}_q \quad (2)$$

where $\overline{\mathbf{Y}}_q = \mathbf{Y}_q\mathbf{S}^H/T \in \mathbb{C}^{N\times M}$, $\overline{\mathbf{W}}_q = \mathbf{W}_q\mathbf{S}^H/T \in \mathbb{C}^{N\times M}$, $\mathbf{A}_R = [\mathbf{a}_R(\phi_1), \cdots, \mathbf{a}_R(\phi_K)] \in \mathbb{C}^{N\times K}$, $\mathbf{D}_q = \text{Diag}([\alpha_{1q}, \cdots, \alpha_{Kq}]) \in \mathbb{C}^{K\in K}$, $\mathbf{A}_T = [\mathbf{a}_T(\theta_1), \cdots, \mathbf{a}_T(\theta_K)] \in \mathbb{C}^{M\times K}$. Assuming ULAs at both ends, this is a 3-D harmonic retrieval model which can be tackled using specialized algorithms, e.g., [8]–[11]. A lot is known about its identifiability properties, algorithms, and performance bounds. However, the ULA model is rather fragile (e.g., mutual coupling), and radars often employ other kinds of transmit and receive arrays, e.g., circular. Even without ULAs, the model above can be viewed as low-rank canonical polyadic tensor decomposition (CPD), and estimated as such – CPD is unique under mild conditions. However, the difficulty in detecting weak targets remains, because CPD works under a least-squares fitting criterion. Our method can work with all types of arrays, provided the manifold of the receive array is known, and it can provably detect weak targets, as we will see.

III. Problem Formulation

We start by designing two narrow partially-overlapped beamformers (assuming ULAs and Vandermonde structure only for simplicity of exposition):

$$\mathbf{a}_\ell(\phi^{(\ell)}) = \frac{1}{\sqrt{N}}[1, e^{(j2\pi\frac{d_R}{\lambda}\sin\phi^{(\ell)})}, \cdots, e^{(j2\pi\frac{d_R}{\lambda}(N-1)\sin\phi^{(\ell)})}]^T \quad (3)$$

where $\ell = 1, 2$. We will explain later in Section IV how we choose the directions $\phi^{(1)}$ and $\phi^{(2)}$. Upon applying the two beamformers at the receiver to the matrices $\{\overline{\mathbf{Y}}_q\}_{q=1}^Q$, we obtain the following vectors

$$\begin{aligned} \mathbf{x}_q^{(\ell)} &= \overline{\mathbf{Y}}_q^H\mathbf{a}_\ell(\phi^{(\ell)}) \\ &= (\mathbf{A}_T^T)^H\mathbf{D}_q^H\mathbf{A}_R^H\mathbf{a}_\ell(\phi^{(\ell)}) + \overline{\mathbf{W}}_q^H\mathbf{a}_\ell(\phi^{(\ell)}) \\ &= (\mathbf{A}_T^T)^H\mathbf{D}_q^H\mathbf{v}_\ell + \mathbf{n}_q^{(\ell)} \end{aligned} \quad (4)$$

where $\mathbf{x}_q^{(\ell)} \in \mathbb{C}^{M\times 1}$, $\mathbf{v}_\ell := \mathbf{A}_R^H\mathbf{a}_\ell(\phi^{(\ell)}) \in \mathbb{C}^{K\times 1}$ and $\mathbf{n}_q^{(\ell)} := \overline{\mathbf{W}}_q^H\mathbf{a}_\ell(\phi^{(\ell)}) \in \mathbb{C}^{M\times 1}$, for $\ell = 1, 2$. Let us define the matrices $\mathbf{X}_\ell := [\mathbf{x}_1^{(\ell)}, \cdots, \mathbf{x}_Q^{(\ell)}] \in \mathbb{C}^{M\times Q}$, $\mathbf{N}_\ell := [\mathbf{n}_1^{(\ell)}, \cdots, \mathbf{n}_Q^{(\ell)}] \in \mathbb{C}^{M\times Q}$, $\mathbf{\Lambda}_\ell = \text{Diag}(\mathbf{v}_\ell) \in \mathbb{C}^{K\times K}$, $\mathbf{B} = [\text{diag}(\mathbf{D}_1), \cdots, \text{diag}(\mathbf{D}_Q)]^T \in \mathbb{C}^{Q\times K}$ and $\mathbf{H} = (\mathbf{A}_T^T)^H$. Then, it follows that

$$\mathbf{X}_1 = \mathbf{H}\mathbf{\Lambda}_1\mathbf{B}^T + \mathbf{N}_1 \quad (5)$$

$$\mathbf{X}_2 = \mathbf{H}\mathbf{\Lambda}_2\mathbf{B}^T + \mathbf{N}_2 \quad (6)$$

Note that the above is a two-slab tensor model, which can again be estimated via CPD, under certain conditions. Furthermore, assume that there exist K_ℓ targets with DOAs close to the direction ϕ_ℓ of the ℓ-th beamformer, and one target with DOA in the overlap sector between the two beamformers (we refer to this target as the common target), where $K = K_1 + K_2 + 1$. Then (5) and (6) can be equivalently expressed as

$$\mathbf{X}_1 = \mathbf{H}_1\mathbf{\Lambda}_{11}\mathbf{B}_1^T + \sqrt{p_{1c}}\ \mathbf{h}_c\mathbf{b}_c^T + \mathbf{H}_2\mathbf{\Lambda}_{12}\mathbf{B}_2^T + \mathbf{N}_1 \quad (7)$$

$$\mathbf{X}_2 = \mathbf{H}_2\mathbf{\Lambda}_{22}\mathbf{B}_2^T + \sqrt{p_{2c}}\ \mathbf{h}_c\mathbf{b}_c^T + \mathbf{H}_1\mathbf{\Lambda}_{21}\mathbf{B}_1^T + \mathbf{N}_2 \quad (8)$$

where $\mathbf{h}_c \in \mathbb{C}^{M\times 1}$ is the transmit steering vector of the common target, $p_{\ell c}$ denotes the received power of the common target after beamforming with $\mathbf{a}_\ell(\phi^{(\ell)})$, and $\mathbf{b}_c$ is the Vandermonde vector that contains the Doppler information of the common target. Similarly, the columns of $\mathbf{H}_\ell \in \mathbb{C}^{M\times K_\ell}$ are the transmit steering vectors of the targets within the ℓ-th beamformer's look direction(s), the diagonal matrix $\mathbf{\Lambda}_{\ell j}$ accounts for the received power of these K_ℓ targets after applying the j-th beamformer, and $\mathbf{B}_\ell \in \mathbb{C}^{M\times K_\ell}$ holds in its columns the Vandermonde vectors which contain the Doppler information of the K_ℓ targets. Upon defining $\widetilde{\mathbf{N}}_\ell := \mathbf{H}_j\mathbf{\Lambda}_{\ell j}\mathbf{B}_j^T + \mathbf{N}_\ell$, (7) and (8) can be equivalently written as

$$\mathbf{X}_\ell = \mathbf{H}_\ell\mathbf{\Lambda}_\ell\mathbf{B}_\ell^T + \sqrt{p_{\ell c}}\ \mathbf{h}_c\mathbf{b}_c^T + \widetilde{\mathbf{N}}_\ell \tag{9}$$

We will next present a CCA method in this beamspace, which can accurately estimate the Doppler frequency of the common target even if this target is present at low SNR and seemingly hidden under the interference of strong targets in the same range gate and adjacent look directions within the "purview" of one, but not both of the above beamformers. In practice, all signals will be present at the output of both beamformers, however CCA will first recover the ones that are present at approximately the same power at the output of the two beamformers, as we will see. The intuitive idea then is to tune the two beamformers to have the same gain only in the look direction of interest.

IV. Proposed Method

Our approach exploits CCA, a powerful statistical learning tool that seeks to uncover common (strongly correlated) information presented in two different data views – two views of the received signals in beamspace, in our context. In an optimization framework, the CCA problem can be posed as

$$\min_{\mathbf{q}_1,\mathbf{q}_2}\ \|\mathbf{X}_1^H\mathbf{q}_1 - \mathbf{X}_2^H\mathbf{q}_2\|_F^2 \tag{10a}$$

$$\text{s.t.}\quad \mathbf{q}_\ell^H\mathbf{X}_\ell\mathbf{X}_\ell^H\mathbf{q}_\ell = 1,\quad \ell = 1,2 \tag{10b}$$

which is referred to as the distance minimization formulation of the two view CCA [15]. It aims at finding two canonical vectors $\mathbf{q}_1 \in \mathbb{C}^M$ and $\mathbf{q}_2 \in \mathbb{C}^M$, such that the correlation between the projections of $\mathbf{X}_1$ and $\mathbf{X}_2$ onto these directions is maximized. The correlation coefficient associated with the canonical pair $\mathbf{q}_1$ and $\mathbf{q}_2$ is defined as

$$\rho = \mathbf{q}_1^H\mathbf{X}_1\mathbf{X}_2^H\mathbf{q}_2 \tag{11}$$

Note that (10) admits a simple algebraic solution via eigendecomposition [16].

In very recent work [21], we have provided an agebraic interpretation of CCA together with an insightful performance analysis. Our analysis shows how CCA can recover the common signal subspace between the two views in the presence of noise, even when the common signals are received at very low SNR. While the analysis in [21] was performed for a very different wireless communication setting assuming rich scattering, the obtained result can be translated to our present radar context. We have showed that under certain assumptions, the correlation coefficient associated with the optimal canonical pair $(\mathbf{q}_1^*, \mathbf{q}_2^*)$ can be expressed as a function of the relative received SNR of each target at different views. In the ideal case where the noise is absent, if there exist common components (the columns of the matrix $\mathbf{B}$ in (5) and (6)), then we can find corresponding pairs of canonical vectors with correlation coefficients equal to unity for all of them, and hence, we can perfectly recover the column space of the $\mathbf{B}$ matrix [17].

On the other hand, when noise is present, the correlation coefficient will be affected due to the addition of different noise terms to the different views. By translating our result in [21] to our present context, the maximum correlation coefficient between $\mathbf{X}_1^H\mathbf{q}_1^*$ and $\mathbf{X}_2^H\mathbf{q}_2^*$ is given by

$$\rho_k = \frac{\gamma_{k1}\gamma_{k2}}{(\gamma_{k1}+1)(\gamma_{k2}+1)} \tag{12}$$

where $\gamma_{k\ell}$ is the received SNR of the k-th target after applying the ℓ-th beamformer in (3) to obtain $\mathbf{X}_\ell$. Note that the higher the k-th correlation coefficient, the more reliably we can recover the signal of the k-th target. Equation (12) shows that as long as the received SNR of the k-th target is few dBs above the noise in both views, one can get a reasonable value for the correlation coefficient, and hence, recovering the k-th target signal is possible via CCA. In other words, our analysis shows that what matters is the "power (im)balance"; targets received at relatively equal SNR are *common* and can be recovered via CCA, whereas targets received at high SNR in one view and low SNR in the other cannot be recovered via CCA. Now one can see from (7) and (8) that the received power, $p_{\ell c}$, of the common target at the ℓ-th view is given by

$$p_{\ell c} = (\mathbf{a}_R^H(\phi_c)\mathbf{a}_\ell(\phi^{(\ell)}))^2 \tag{13}$$

where ϕ_c is the DOA of the common target. Notice that in order to accurately estimate the Doppler frequency of the common target using CCA, one needs to choose $\phi^{(\ell)}$ to be close to ϕ_c for $\ell = 1,2$, thus obtaining high correlation coefficient which in turn allows an accurate estimation of the common target speed. However, since the angle of arrival, ϕ_c, is not known *a priori*, the receiver will perform a DOA scan for weak targets, and for each scan angle ϕ_s set $\phi^{(1)} = \phi_s + \epsilon$ and $\phi^{(2)} = \phi_s - \epsilon$, with fixing ϵ to few degrees, i.e., $\epsilon \in [1,3]$ degrees. For each value of ϕ_s, we solve problem (10) and track the value of the resulting correlation coefficient. Observing a high correlation coefficient at any ϕ_s indicates the presence of a target with its angle of arrival close to this ϕ_c, thereby enabling weak target *detection*. Upon defining the vector $\mathbf{g} := \mathbf{X}_1^H\mathbf{q}_1$, we have the following result.

Proposition 1. *If the matrices* $\mathbf{B}$ *and* $\mathbf{H}$ *are full column rank, then upon finding the optimal solution* $\mathbf{q}_1^*$ *of* (10)*, we obtain* $\mathbf{g} = \sqrt{\gamma_c}\mathbf{b}_c e^{j\phi_e} + \mathbf{w}_c$*, where* γ_c *is the received SNR of the common target,* ϕ_e *is a phase ambiguity and* $\mathbf{w}_c$ *is the residual noise with variance much smaller than that of the noise in* (7).

Proof. The above result follows from the proof of Proposition 1 in [21] albeit with slight modifications to accommodate the complex representation of the matrix $\mathbf{B}$. □

Fig. 1: Power spectrum of the original and estimated weak signal

V. Experimental Results

To evaluate the performance of our proposed BCC method, we consider a radar system with the following parameters; $M = N = 64$, $Q = 300$, $T = 256$, $f_c = 5$GHz, $T_p = 1\mu s$, $d_T = d_R = \lambda/2$ where $\lambda = c/f_c$ and $c = 3 \times 10^8$m/s. We consider $K = 4$ closely-spaced targets with DODs $\{\theta_k\}_{k=1}^K = (-80°, -75°, -72°, -70°)$, DOAs $\{\phi_k\}_{k=1}^K = (63°, 57°, 55°, 60°)$, and speeds $\{v_k\}_{k=1}^K = (200, 240, 285, 330)$ m/s. Additive white Gaussian noise was added with $\sigma^2 = 0$ dBm, and the target steering vectors were scaled such that the resulting SNR for each of the first three targets is in the range $[15\ 25]$ dB while the last target SNR is -15 dB unless stated otherwise. The value of ϵ was set to $2°$ and the angle ϕ_s was scanned over a range of angles, however, the reported results are for $\phi_s = 60°$. All results were averaged over 500 noise realizations.

In order to benchmark the performance of the proposed method, we used a method developed in [8], which first decomposes the 3-D tensor in (2) using CPD, and then uses 1-D ESPRIT [11] to estimate the Doppler/speed of all targets from one of the CPD factor matrices. Note that neither the proposed BCC method nor this baseline exploits the Vandermonde structure of the transmit and receive arrays (both methods are more generally applicable to arbitrary array geometries) so the comparison is fair. Another possible baseline is to apply a single beamformer centered at the scan angle, and then compute the Doppler frequency from the resulting 2-D model in (8), and we implemented this method as well.

In the first experiment, we plot the spectra of the estimated Doppler signal using our proposed CCA method, the original Doppler signal of the weak target, and the signal resulting from applying a single narrow beamformer in (8). Figure 1 shows that our proposed method can perfectly identify the Doppler frequency of the weak target. On the other hand, the single beamformer method returns multiple peaks in the frequency domain. This suggests that after applying one beamformer we obtain a linear combination of the signals of the strong targets. Recall that in each of the two data views in (7) or (8), the received power of the common target is very low compared to the strong K_ℓ targets in the ℓ-th view, and hence, the results in Fig. 1 suggest that using the 2-D harmonic retrieval methods on (7) or (8) will recover the Doppler frequencies of the strong targets and treat the weak target as noise. This is indeed our experience from simulations, thus we drop this baseline from further consideration.

We carried out another experiment where we varied the SNR of the weak target, and we observed the absolute value of the speed error after averaging over 500 noise realizations for each SNR value. Figure 2 demonstrates the high accuracy of

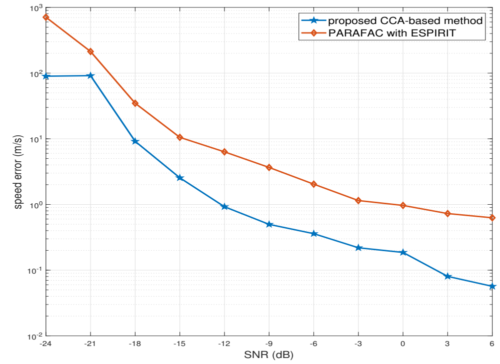

Fig. 2: Speed error vs. SNR of the weak target

the speed estimated by the proposed BCC method compared to the 3-D CPD method. For instance, our proposed method attains an approximately order of magnitude improvement in the speed estimation at SNR $= -12$ dB. This is in fact reflects the power of the proposed method in accurately estimating the Doppler of the weak target at extremely low SNR.

VI. Conclusions

We studied the problem of weak target detection and speed estimation under strong interference from nearby targets in a bistatic MIMO radar system. Our proposed solution is based on creating two different beamdspace views of the received signal space, using two judiciously misaligned beamformers. Applying CCA to these output views, we showed that the proposed BCC method can accurately detect and estimate the speed weak targets buried under much stronger reflections from nearby targets and clutter. Our method is backed up by both analysis and experiments, revealing that BCC can achieve an approximately order of magnitude reduction in the relative speed error compared to other radar techniques. Whereas our presentation focused DOA scanning and spatial beamforming to create the two views, it is also possible to flip the dimensions and perform Doppler-domain (slow-time) scanning and beamforming, and thus estimate the transmit and receive steering vectors of weak targets at a given Doppler. This and more additional results will be included in the journal version of this work.

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