

Absolute magnetometry with ^3He

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We report development of a highly accurate (ppb) absolute magnetometer based on ^3He NMR. Optical pumping polarizes the spins, long coherence times provide high sensitivity, and the ^3He electron shell effectively isolates the nuclear spin providing accuracy limited only by corrections for materials, sample shape and magnetization. Our magnetometer was used to confirm calibration, to 32 ppb, of the magnetic-field sensors used in recent measurements of the muon magnetic moment anomaly ($g_\mu - 2$), which differs from the Standard Model by 2.4 ppm. With independent determination of the magnetic moment of ^3He , this work will lead the way to a new absolute magnetometry standard.

Magnetometry, determining the intensity of a magnetic field over space and time, is crucial to many fields of applied and fundamental science including medicine, materials science, geology and geodesy, astronomy and fundamental physics. Often competing magnetometry requirements include sensitivity, stability, absolute accuracy, bandwidth and spatial resolution, and many applications require accurately determining the magnetic field intensity in Tesla ($\text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1}$). The most sensitive magnetometers, superconducting quantum-interference devices (SQUIDS) reach sub fT/ $\sqrt{\text{Hz}}$ sensitivity [1], but are not absolute. Optically pumped atomic magnetometers approach this sensitivity in small fields [2, 3], but are generally not highly accurate or stable. Accurate absolute magnetometers include rotating coils [4] and calibrated NMR [5].

Magnetic fields are most accurately determined by measuring the *frequency* corresponding to the energy difference of two quantum states. The frequency ω and magnetic field B are related by $B = \omega (\hbar J / |\mu|) = \omega / |\gamma|$, where $\hbar J$ is the total angular momentum, μ is the magnetic moment, and $\gamma = \mu / \hbar J$. $J = 1/2$ nuclei in diamagnetic materials are the most nearly ideal two-state quantum systems, and NMR of proton rich substances, e.g. H_2O , is most commonly encountered. For pure H_2O , the shielded proton magnetic moment for a spherical sample at 25° $\mu'_p(25^\circ)$ is known to 11.3 ppb, limited by measurement of $\mu'_p / \mu_e(H)$ [6, 7].

In practical magnetometry, temperature-dependent sample, shape and external material effects lead to perturbations of the magnetic field that must be calibrated or corrected. With much smaller intrinsic corrections illustrated by the practically engineered magnetometer demonstrated in this paper, a new more precise and more accurate standard for magnetometry can be established with ^3He . Unlike H_2O , for which the signal size is proportional to $1/T$, the ^3He is hyperpolarized by laser optical pumping techniques; thus ^3He magnetometry is applicable over a very broad range of magnetic fields using NMR

for fields >0.1 T and atomic sensors for lower fields [8].

Quantities relevant to determining the magnetic moment of a free ^3He atom are provided in the Supplement [9]. Diamagnetic shielding of the atomic electrons reduces the field at the ^3He nucleus, referred to as the helion. The shielded helion moment μ'_h has been measured to 4.3 ppb relative to the shielded proton magnetic moment in water [10] and to 4 ppb relative to protons in high-pressure H_2 [11], but the uncertainty on μ'_h , 11.4 ppb, is dominated by the uncertainty on $\mu'_p / \mu_e(H)$. To eliminate the proton from the chain, effort is underway [14, 15] to directly measure magnetic moment of the unshielded nucleus μ_h in a Penning trap with techniques similar to those used to measure the proton [16] and antiproton [17] g -factors. The high-precision theoretical calculation of the diamagnetic shielding [18], which is only weakly temperature dependent, would provide the shielded magnetic moment μ'_h at the few ppb level.

The muon magnetic moment anomaly: A spin $\frac{1}{2}$ -particle (lepton) with charge q_l and mass m_l has a magnetic moment $\mu_l = g_l \frac{q_l}{2m_l} \frac{\hbar}{2}$. For a structureless particle with no radiative corrections $g_l=2$ [19], however due to interactions with the virtual fields of the quantum vacuum corrections to $g_l = 2(1 + a_l)$ [20–22]. The largest correction, $\approx 0.1\%$, is due to QED with the strong and weak interactions entering at three and six orders of magnitude less, respectively. For the muon, the most recent Standard Model determinations of a_μ^{SM} are summarized in [23, 24]. The muon magnetic-moment anomaly has been measured by producing muons in accelerators at CERN [25] and Brookhaven [26]. The most recent measurements a_μ^{exp} from Brookhaven (E821) for μ^+ [27] and μ^- [28] reveal $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.0 \pm 7.4) \times 10^{-10}$ or (2.40 ± 0.63) ppm, using a_μ^{SM} from [24]. A new measurement with higher statistical precision and smaller systematic errors is underway at Fermilab and is expected to improve the uncertainty by a factor of four [29].

Experimentally, muons are confined in a 7.1 m radius magnetic storage ring with weak vertical focusing by elec-

tric quadrupole fields. The anomaly frequency, determined from the variation of the rate of μ decays into positrons or electrons that exceed a threshold energy, is $\omega_{a_\mu} \approx a_\mu \frac{e}{m_\mu} \tilde{B}$ for muon momentum near 3.094 GeV/c, where the electric field effects are effectively cancelled. The field averaged over the muon trajectories and time is $\tilde{B} \approx 1.45$ T. A chain of measurements using proton-NMR magnetometers (probes) calibrated with high-purity H₂O yields the frequency $\tilde{\omega}'_p = \frac{2|\mu'_p|}{\hbar} \tilde{B}$. In terms of ω_{a_μ} and $\tilde{\omega}'_p$,

$$a_\mu = \frac{g_e}{2} \frac{\omega_{a_\mu}}{\tilde{\omega}'_p} \left| \frac{\mu'_p}{\mu_e} \right| \frac{m_\mu}{m_e}. \quad (.1)$$

where μ'_p/μ_e is determined to 10.5 ppb [9], g_e has been determined to 0.28 ppt [30], and m_μ/m_e is determined to 22 ppb from the muonium hyperfine splitting [31] and QED [7]. In this work, the ^3He magnetometer independently determined and confirmed the systematic corrections to the standard H₂O reference probes used in E821. The ^3He Magnetometer illustrated in Fig. 1 is a compact device designed to operate at 1.45 T providing polarization and NMR magnetometry *in-situ*. The required elements are the ^3He sample contained in a glass cell, coils to excite a discharge in the ^3He , the NMR coil for excitation and pickup of free-precession, optical fibers to guide the incident optical pumping light and to monitor transmission through the cell for laser tuning, circular polarization optics, and the mounting structure. The design principles require removing all ferromagnetic materials including electronic components and cables, minimizing materials near the ^3He sample and fabricating as closely as possible cylindrically or spherically symmetric distributions of materials.

The magnetometer was assembled in a 3-D printed mount with grooves that aligned the components. In our initial (Mark-I) assemblies the mount was polylactic acid thermoplastic (PLA) with a square cross section “clamshell” for ease of mounting, alignment and access for cell orientation and material perturbation studies. The plastic mount was enclosed in 0.002-inch thick copper folded from a flat sheet and soldered at the joints. The ^3He cells were blown borosilicate (PYREX) glass, approximately spherical with 2.5 cm diameter and a stem several mm long (see Supplement [9]). Several ^3He pressures were investigated, and 10-torr (20°C) provided the best combination of polarization lifetime, free-precession relaxation time (T_2^*) and signal size.

Optical pumping of metastable ^3He or MEOP has been extensively studied, most recently at high magnetic fields relevant to this work [9, 32–34]. A discharge was excited by radio frequency (4-6 MHz) applied to a 2-cm-diameter coil pair in contact with the outside of the glass cell. The input fiber provided optical pumping light from a 1083 nm, 2W fiber-amplified laser [35]. For development studies at 2-3 mT an optical polarimeter [36] monitored the

circular polarization of the 668 nm fluorescence indicating 30-50% ^3He polarization. We did not directly measure the ^3He polarization at 1.45 T, however NMR signal sizes and system parameters were consistent with 4-8% in the 10-torr cell. The polarization (discharge on) time was typically 1-2 minutes, and the polarization lifetime was greater than five hours.

The ^3He NMR system consisted of a saddle coil surrounding the ^3He cell that produced a field along the y -axis. The NMR coil, matched to 50 Ω and tuned to the ^3He resonant frequency $\omega_h/(2\pi) \approx 47.1$ MHz, provided both the NMR pulse and inductive pickup. The coil was connected to the NMR controller described in the Supplement. The mixer reference frequency was set below the precession frequency, and the 100-300 Hz mixed-down signal was digitized. An identical NMR controller for the H₂O probes was tuned to 61.7 MHz and interfaced to the same data acquisition system. The NMR tip angle was $\approx 23^\circ$ for ^3He leaving about 90% of the longitudinal polarization after each pulse and providing variation of the longitudinal and transverse magnetization for systematic studies. The 90° pulse for H₂O protons maximized the signal size. The pulse and mixer reference frequencies for both the ^3He and proton (H₂O) channels and the data acquisition sample trigger were generated by separate function generators all locked to a single rubidium clock. Details of the NMR signal processing are discussed in the Supplement [9].

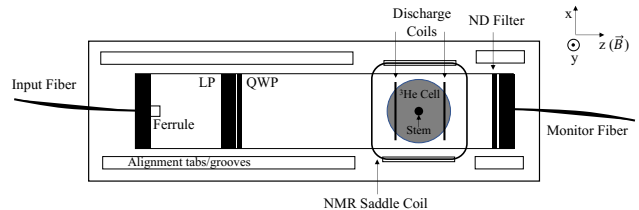


FIG. 1: The ^3He magnetometer assembly described in the text. The assembly is 5 cm square in cross section and 15 cm long. The distance from the input-fiber ferrule to the cell is approximately 8 cm. LP and QWP indicate the linear polarizer and quarter wave plate.

Calibration of ^3He and E821 H₂O probes: The two H₂O probes used in E821, one with a spherical sample and one with a cylindrical sample, are described in detail in [37]. The calibration of each H₂O probe to ^3He used a precision-shimmed superconducting solenoid magnet. The magnetic field drift was dominated by a diurnal cycle and was less than 60 ppb over the five hour calibration measurements. The ^3He magnetometer and H₂O probes were mounted on a translation stage, and a positive or negative linear gradient along each axis was applied by rapidly ($\lesssim 1$ min) reversing the gradient-coil currents to determine a unique position (x_0, y_0, z_0) , the $\Delta B = 0$ position. Adjusting each probe to find x_0 and y_0 then required only translation along the z axis to position each probe at the $\Delta B = 0$ position indicated in Fig. 2.

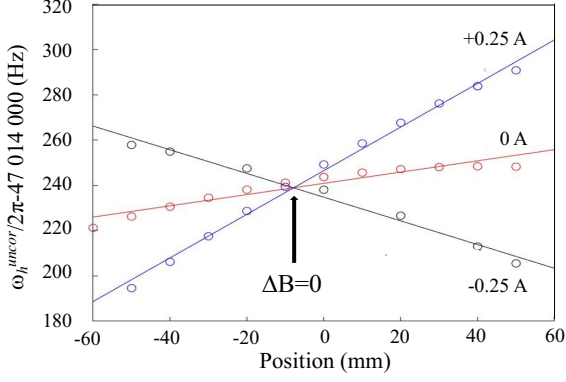


FIG. 2: (Color on line) Scan of the ^3He frequency as the magnetometer was translated along z with positive and negative gradient currents showing the $\Delta B = 0$ position z_0 . The solenoid's higher order (quadratic) gradients are evident; straight lines are provided to guide the eye.

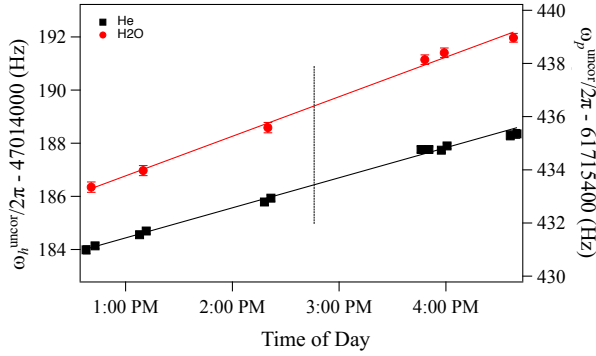


FIG. 3: (Color on line) Uncorrected frequencies for ^3He (left axis) and the spherical H_2O probe (right axis); the solid lines are linear fits vs time. The χ^2_ν , 1.0 and 1.2, respectively for ^3He and H_2O , are accounted for in the magnetic drift uncertainties. The vertical dotted line indicates the time to which all frequencies are interpolated. Both scales span 10 Hz. The different slopes are due to the different magnetic moments.

Calibration studies of the two H_2O probes were undertaken on separate days, with small modifications to the ^3He setup between the two studies. Fig. 3 shows the uncorrected frequencies ω_3^{uncor} and ω_p^{uncor} as a function of time during calibration of the cylindrical H_2O probe showing the effect of the magnetic-field drift. All frequency measurements were interpolated to the average time of the ^3He measurements indicated by the vertical dotted line in Fig. 3. Corrections were applied to both species' interpolated frequencies to provide the two shielded frequencies ω'_3 and ω'_p for each H_2O probe at the specified position and time.

Corrections and uncertainties: Corrections, determined independently within one day of each calibration study, are presented in Table I. Corrections fall into four categories: materials external to the sample, sample container materials, sample-material magnetization effects, and temperature dependence. Though all of these are in

principle applicable to both species, the size and method of determining the corrections differed significantly.

External materials include all components of a probe, cables, translation stage and mounting structure other than the sample and glass sample container. All effects of external materials were measured with an auxiliary vasoline-sample NMR probe at the $\Delta B = 0$ position with/without the ^3He sample or H_2O probe. The auxiliary probe temperature was not monitored (temperature dependence 2.5 ppb/ $^\circ\text{C}$ [38]) and uncertainties including those due to temperature variations were estimated from the scatter of multiple measurements. Due to the NMR coil inhibiting positioning of the auxiliary probe at the exact position of the ^3He cell, and because the $\Delta B = 0$ positioning could not be checked with the ^3He cell removed, the auxiliary probe was moved back and forth a few mm over cell position to estimate the uncertainty due to position misalignment.

For the H_2O probe ± 10 ppb variations when rotating the probe around the probe axis were reported by [37] and confirmed for this work. Rotations of the stem with respect to the axis of the magnet with polar angle $\theta=0$ and 90° and with $\phi=0$ and 90° were consistent with modeling the stem as a magnetic dipole, which predicted $P_2(\cos\theta)$ dependence at the cell center. For the H_2O calibration measurements, the stem was oriented along y ($\theta = 90^\circ$); extrapolating to the magic angle $\theta = 54^\circ$ resulted in the correction of -0.61 ± 0.20 Hz. The stem also caused magnetic gradients, which resulted in a factor of two shorter T_2^* with $\theta = 0^\circ$. Rotation of the cell around the stem axis showed variations of 0.35 Hz, included as an uncertainty. The same ^3He cell and stem orientation corrections were applied to both H_2O probes. In principle, there was also a correction for the different displacement of air ($\approx 20\%$ paramagnetic O_2) by the sample and the auxiliary probe. For a spherical ^3He sample, this effect would vanish. Modeling the air displaced by the stem as a dipole, the magnetic field at the center of the cell has an amplitude less than about 20 fT. For the H_2O probes, the sample was not removed, and there was no correction.

Sample magnetization dependence arises due to magnetic susceptibility and due to ^3He nuclear polarization. For perfectly spherical samples the average long-range contribution to the field anywhere in the sample vanishes [39]. Modeling the hyperpolarized ^3He gas in the stem as a magnetic dipole predicts ≈ 0.5 pT for 10% ^3He polarization, corresponding to a shift less than 15 μHz . The H_2O shape correction is given by $\delta_{\text{shape}} = -\Delta\omega'_p/\omega'_p = \chi^{\text{H}_2\text{O}}(T)(\epsilon - 1/3)$ (SI units), where $\chi^{\text{H}_2\text{O}}(T)$ is the susceptibility. For an infinitely-long cylinder perpendicular to $\vec{B}$, $\epsilon = 1/2$ and $\Delta\omega'_p = 93.10$ Hz; for a sphere, $\epsilon = 1/3$. Additional magnetization-dependent shifts have been revealed in recent studies with hyperpolarized ^3He - ^{129}Xe mixtures used in a comagnetometer configuration [40–43]. Scaling these effects from [44] suggests shifts much

	Spherical H ₂ O (Hz)				Cylindrical H ₂ O (Hz)			
	H ₂ O		³ He		H ₂ O		³ He	
Source	Corr.	Unc.	Corr.	Unc.	Corr.	Unc.	Corr.	Unc.
External materials	10.61	0.49	21.38	1.22	13.82	0.49	23.64	1.13
Probe materials	2.71	0.62	-	-	2.90	0.62	-	-
H ₂ O Probe-material asymmetry*	0	1.24	-	-	0	1.24	-	-
³ He glass stem*	-	-	-0.61	0.20	-	-	-0.61	0.20
³ He cell rotation*	-	-	0	0.35			0	0.35
Sample magnetization	0	0	0	0.20	-93.10	0.26	0	0.18
Radiation damping*	-	-	0	0.18	-	-	0	0.18
σ _p ^{H₂O} Temperature dependence	-0.48	0.64	-	-	-1.27	0.64	-	-
χ ^{H₂O} Temperature dependence	0	0	-	-	0	0.02	-	-
Total of corrections	12.84	1.60	20.77	1.31	- 77.65	1.62	23.05	1.23
Frequency extraction unc.	0.01		-		0.10		-	
Magnet drift uncertainty	0.05		0.09		0.18		0.12	
Position uncertainty*	0.07		0.05		0.07		0.05	
Total unc.	1.61		1.32		1.64		1.23	
ω'/2π	61,710,229.90		47,009,998.24		61,715,758.45		47,014,209.45	
B−1,449,000,000 (nT)	400419 (41)		400448 (44)		530269(41)		530287 (42)	
R' _{hp} = ω' ₃ /ω' _p	0.761786147 (29)				0.761786141 (28)			
Combined R' _{hp}	0.761786144(20)							
μ' _h (10 ^{−27} J T ^{−1})	1.074553107(31)							

TABLE I: Corrections and uncertainties for cross-calibration studies of both H₂O probes. For entries with correction < 0.01 Hz, the correction, is indicated with a 0; entries that do not apply for a specific probe are indicated with a -; the * indicates that one correction/uncertainty applies to both studies. The last two lines provide new determinations from this work to be compared to $|R'_{hp}| = 0.7617861313(33)$ and $|\mu'_{hp}| = 1.074553090(13) \times 10^{-27}$ J T⁻¹ based on [10]. To convert frequency uncertainties to approximate ppb, divide by 47.1×10^{-3} for ³He and 61.7×10^{-3} for H₂O.

less than 1 mHz.

Effects of the FID frequency evolution due to dephasing in the non-uniform field are discussed by [45, 46]. Studies with simulated FIDs using the measured magnetic field gradients indicate that these effects were less than 0.1 Hz and less than 0.01 Hz for the cylindrical and spherical H₂O probes and negligible for ³He. Radiation damping shifts due to the current induced in the pickup coil was studied through the frequency-dependence of ω_3^{uncor} over a set of five 23° pulses. Extrapolating the the frequency dependence to zero longitudinal polarization sets an upper limit of 0.18 Hz on the shift. Radiation damping for the H₂O probes, studied by the longitudinal magnetization dependence over the five-pulse sequences and as the probe-tuning drifted, was negligible.

Temperature-dependence of the H₂O diamagnetic shielding was corrected to 25°C by measuring the temperature on the outside of the H₂O probe with a 1000Ω platinum resistor (PT1000). Conservatively estimating an uncertainty of 5 Ω corresponding to 1° resulted in 0.64 Hz uncertainty. From [47], $\chi^{\text{H}_2\text{O}}$ was measured to change by 0.5% over 40°, which we interpret as $\partial\chi^{\text{H}_2\text{O}}/\partial T = 1.1 \times 10^{-9}/^\circ\text{C}$, adding an uncertainty of 0.02 Hz for the H₂O cylindrical-probe shape correction.

Other Corrections and uncertainties including higher-order time dependence and field fluctuations, for example moving equipment or tools, was estimated as the standard deviation of the residuals of the linear fits in Fig 3.

Clock stability was found to be better than 0.01 Hz. The position reproducibility was 0.2 mm (200 μm) contributing an uncertainty 0.07 Hz for H₂O and 0.05 Hz for ³He.

Conclusions: The final results of measurements of the corrected frequencies and corresponding absolute magnetic field determined by the ³He and both H₂O probes are given in Table I. The absolute magnetic fields determined with ³He and H₂O differ by 20 ± 41 ppb and 12 ± 41 ppb for the spherical and cylindrical H₂O probes, respectively. Since the two H₂O probes with corrections consistent with those applied here [37] were used in the analysis of the E821 $g_\mu - 2$ measurements [27, 28] this can be interpreted as confirming the calibration of the E821 magnetic-field measurement system to 16 ± 29 ppb, i.e. agreement better than 32 ppb (68% C.L.) compared to the 2.4 ppm tension of a_μ^{exp} from E821 with the Standard Model a_μ^{SM} .

We can also use the corrected frequencies to determine $|R'_{hp}| = |\mu'_h/\mu'_p(25^\circ\text{C})|$ and $|\mu'_h| = |R'_{hp}||\mu'_p(25^\circ)|$. The combined results for the two probes presented in Table I are consistent with [10] but with a 6.8 times larger error on $|R'_{hp}|$. (Both determinations of $|\mu'_h|$ use the shielded proton moment introducing a common uncertainty.) Straightforward improvements to the ³He magnetometer materials, structure, and improved measurement of the corrections should lead to determination of $|R'_{hp}|$ at the few ppb level and provide a new method for

absolute calibration of H₂O probes.

Most importantly, this work and improvements to our first-generation absolute ³He magnetometer establish the technical basis for practical absolute magnetometry with ³He and the establishment of a new magnetic field standard. Though this new standard would currently trace to measurements of μ'_h/μ'_p , the anticipated independent measurement of the helion moment [14] would provide a completely new magnetometry standard.

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- [1] R. Körber *et al.*, Superconductor Science and Technology **29**, 113001 (2016).
 - [2] D. Budker and M. V. Romalis, Nature Phys. **3**, 227 (2007).
 - [3] I. K. Kominis, T. W. Kornack, J. C. Allred, and M. V. Romalis, Nature **422**, 596 (2003).
 - [4] P. Arpaia, G. Caiafa, and S. Russenschuck, Scientific Reports **9**, 1491 (2019).
 - [5] Metrolab, <https://www.metrolab.com>.
 - [6] W. Phillips, W. Cooke, and D. Kleppner, Metrologia **13**, 179 (1977).
 - [7] P. J. Mohr, D. B. Newell, and B. N. Taylor, Rev. Mod. Phys. **88**, 035009 (2016), (2018 CODATA recommended values).
 - [8] H. Koch, G. Bison, Z. Grujić, W. Heil, M. Kasprzak, P. Knowles, A. Kraft, A. Pazgalev, A. Schnabel, J. Voigt, and A. Weis, Eur. Phys. J. D **69**, 262 (2015).
 - [9] See Supplemental Material [url], which includes Refs. [48–68].
 - [10] J. Flowers, B. Petley, and M. Richardson, Metrologia **30**, 7 (1993).
 - [11] Y. Neronov and N. Seregin, J. of Exp. and Theo. Phys. **115**, 115 (2012).
 - [12] B. Petley and R. Dondaldson, Metrologia **20**, 21 (1984).
 - [13] D. Sundholm, J. Gauss, and A. Schäfer, The Journal of Chemical Physics **105**, 11051 (1996).
 - [14] A. Mooser, A. Rischka, A. Schneider, K. Blaum, S. Ulmer, and J. Walz, Journal of Physics: Conference Series **1138**, 012004 (2018).
 - [15] A. Schneider, A. Mooser, A. Rischka, K. Blaum, S. Ulmer, and J. Walz, Annalen der Physik **531**, 1800485 (2019).
 - [16] G. Schneider *et al.*, Science **358**, 1081 (2017).
 - [17] C. Smorra *et al.*, Nature **550**, 371 (2017).
 - [18] A. Rudzinski, M. Puchalski, and K. Pachucki, Journal of Chemical Physics **130**, 244102 (2009).
 - [19] P. A. M. Dirac, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **117**, 610 (1928).
 - [20] J. Schwinger, Phys. Rev. **73**, 416 (1948).
 - [21] T. Aoyama, T. Kinoshita, and M. Nio, Phys. Rev. D **97**, 036001 (2018).
 - [22] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012).
 - [23] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, arXiv **hep-ph**, 1908.00921 (2019).
 - [24] A. Keshavarzi, D. Nomura, and T. Teubner, arXiv **hep-ph**, 1911.00367 (2019).
 - [25] J. Bailey *et al.* (CERN-Mainz-Daresbury), Nucl. Phys. **B150**, 1 (1979).
 - [26] G. W. Bennett *et al.* (Muon g-2), Phys. Rev. **D73**, 072003 (2006).
 - [27] G. W. Bennett *et al.* (Muon g-2), Phys. Rev. Lett. **89**, 101804 (2002), [Erratum: Phys. Rev. Lett. **89**, 129903 (2002)].
 - [28] G. W. Bennett *et al.* (Muon g-2), Phys. Rev. Lett. **92**, 161802 (2004).
 - [29] I. Logashenko *et al.* (Muon g-2), J. Phys. Chem. Ref. Data **44**, 031211 (2015).
 - [30] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008).
 - [31] W. Liu, M. G. Boshier, S. Dhawan, O. vanDyck, P. Egan, X. Fei, *et al.*, Phys. Rev. Lett. **82**, 711 (1999).
 - [32] M. Abboud, A. Sinatra, X. Maître, G. Tastevin, and P.-J. Nacher, Europhysics Letters (EPL) **68**, 480 (2004).
 - [33] A. Nikiel, P. Blümli, W. Heil, M. Hehn, S. Karpuk, A. Maul, E. Otten, L. Schreiber, and M. Terekhov, Eur. Phys. J. **D68**, 330 (2014).
 - [34] W. Heil, in *High Sensitivity Magnetometers*, Vol. 19, edited by A. Grosz, M. Haji-Sheikh, and S. Mukhopadhyay (Springer, Cham, Switzerland, 2017) pp. 493–521.
 - [35] Keopsys.com Model CYFL-GIGA.
 - [36] J. Maxwell, C. Epstein, and R. Milner, Nuc. Inst. and Meth. **A764**, 215 (2014).
 - [37] X. Fei, V. W. Hughes, and R. Prigl, Nucl. Inst. and Meth. **A394**, 349 (1997).
 - [38] M. Ferli, Fermilab E989 internal (2014).
 - [39] J. H. Wesenberg and K. Mølmer, Phys. Rev. Lett. **93**, 143903 (2004).
 - [40] M. V. Romalis, D. Sheng, B. Saam, and T. G. Walker, Phys. Rev. Lett. **113**, 188901 (2014).
 - [41] D. Sheng, A. Kabcenell, and M. V. Romalis, Phys. Rev. Lett. **113**, 163002 (2014).
 - [42] M. E. Limes, N. Dural, M. V. Romalis, E. L. Foley, T. W. Kornack, A. Nelson, L. R. Grisham, and J. Vaara, Phys. Rev. A **100**, 010501(R) (2019).
 - [43] W. A. Terrano, J. Meinel, N. Sachdeva, T. E. Chupp, S. Degenkolb, P. Fierlinger, F. Kuchler, and J. T. Singh, Phys. Rev. A **100**, 012502 (2019).
 - [44] N. Sachdeva, I. Fan, E. Babcock, M. Burghoff, T. E. Chupp, S. Degenkolb, *et al.*, Phys. Rev. Lett. **123**, 143003 (2019).
 - [45] J. Flowers, P. Franks, and B. Petley, IEEE Trans. on Inst. and Meas. **44**, 488 (1995).
 - [46] B. Cowan, Meas. Sci. and Tech. **7**, 690 (1996).
 - [47] J. S. Philo and W. M. Fairbank, J. of Chem. Phys. **72**, 4429 (1980).
 - [48] S. G. Karshenboim and V. G. Ivanov, Phys. Lett. **B566**, 27 (2003).
 - [49] Y. I. Neronov and N. Seregin, Metrologia **51**, 54 (2014).
 - [50] N. Nevo Dinur, O. J. Hernandez, S. Bacca, N. Barnea, C. Ji, S. Pastore, M. Piarulli, and R. B. Wiringa, Phys. Rev. C **99**, 034004 (2019).
 - [51] M. A. Bouchiat, T. R. Carver, and C. M. Varnum, Phys. Rev. Lett. **5**, 373 (1960).
 - [52] T. E. Chupp and K. P. Coulter, Phys. Rev. Lett. **55**, 1074 (1985).

- [53] F. D. Colegrove, L. D. Schearer, and G. K. Walters, Phys. Rev. Lett. **8**, 439 (1962).
- [54] T. R. Gentile, P. J. Nacher, B. Saam, and T. G. Walker, Rev. Mod. Phys. **89**, 045004 (2017).
- [55] K. Coulter *et al.*, Nucl. Inst. and Meth. in Phys. Res. A **288**, 463 (1990).
- [56] E. Babcock *et al.*, Journal of Physics: Conference Series **711**, 012008 (2016).
- [57] T. E. Chupp, R. A. Loveman, A. K. Thompson, A. M. Bernstein, and D. R. Tieger, Phys. Rev. **C45**, 915 (1992).
- [58] A. Camsonne *et al.*, Phys. Rev. Lett. **119**, 162501 (2017).
- [59] T. E. Chupp, P. Fierlinger, M. J. Ramsey-Musolf, and J. T. Singh, Rev. Mod. Phys. **91**, 015001 (2019).
- [60] D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecky, and C. D. Lane, Phys. Rev. Lett. **85**, 5038 (2000), [Erratum: Phys. Rev. Lett. **89**, 209902(E) (2002)].
- [61] R. Surkau *et al.*, Nuc. Inst. and Meth. **A384**, 444 (1997).
- [62] F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Yu. Sobolev, and K. Tullney, Phys. Rev. Lett. **112**, 110801 (2014).
- [63] J. M. Wild *et al.*, Physics in Medicine and Biology **47**, N185 (2002).
- [64] J. Mulger and T. Altes, J. Magn. Reson. Imaging **37**, 313 (2013).
- [65] G. D. Cates, S. R. Schaefer, and W. Happer, Phys. Rev. **A37**, 2877 (1988).
- [66] D. D. McGregor, Phys. Rev. A **41**, 2631 (1990).
- [67] T. E. Chupp, R. J. Hoare, R. L. Walsworth, and B. Wu, Phys. Rev. Lett. **72**, 2363 (1994).
- [68] Y. Chibane, S. K. Lamoreaux, J. M. Pendlebury, and K. F. Smith, Measurement Science and Technology **6**, 1671 (1995).