

Partitioning Analysis in Temporal Decomposition for Security-Constrained Economic Dispatch

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Abstract— Distributed optimization algorithms are proposed to, potentially, reduce the computational time of large-scale optimization problems, such as security-constrained economic dispatch (SCED). While various geographical decomposition strategies have been presented in the literature, we proposed a temporal decomposition strategy to divide the SCED problem over the considered scheduling horizon. The proposed algorithm breaks SCED over the scheduling time and takes advantage of parallel computing using multi-core machines. In this paper, we investigate how to partition the overall time horizon. We study the effect of the number of partitions (i.e., SCED subproblems) on the overall performance of the distributed coordination algorithm and the effect of partitioning time interval on the optimal solution. In addition, the impact of system loading condition and ramp limits of the generating units on the number of iterations and solution time are analyzed. The results show that by increasing the number of subproblems, the computational burden of each subproblem is reduced, but more shared variables and constraints need to be modeled between the subproblems. This can result in increasing the total number of iterations and consequently the solution time. Moreover, since the load behavior affects the active ramping between the subproblems, the breaking hour determines the difference between shared variables. Hence, the optimal number of subproblems is problem dependent. A 3-bus and the IEEE 118-bus system are selected to analyze the effect of the number of partitions.

Index Terms— Temporal decomposition, distributed optimization, security-constrained economic dispatch, partitioning.

I. INTRODUCTION

Large optimization problems are constantly being solved for operation, planning, and energy management in power systems [1-7]. Computational burden of power system optimization problems will increase by growing the system size [8-10]. For problems such as security-constrained economic dispatch (SCED), the computation time increment depends on the number of variables and constraints [11]. Decomposition techniques have been presented to divide large optimization problems into several small subproblems and solve them in an iterative manner [12-19]. There are several ways to decompose such problems and each way of decomposition follows a

particular goal. Reference [16] surveys the literature of distributed algorithms with applications to optimization and control of power systems. Reference [15] reviews distributed/decentralized algorithms to solve optimal power flow (OPF). Geographical-based decomposition strategies, which divide a power system into several smaller zones, are most common. These methods formulate a subproblem for each zone and solve the subproblems iteratively either in a sequential manner or a parallel fashion [20-23]. Although geographical decompositions have shown desirable performance for large power system problems, they might not be effective for computation time reduction for optimization problems with multiple time intervals, e.g., SCED, especially when the focus is on a part of the system that has one owner. In such a multi-interval scheduling problem, the computational burden not only depends on the size of the system but also depends on the number of intertemporal constraints such as ramp limits of generating units.

We proposed a time decomposition strategy in our previous work [24]. The optimization problem was decomposed into seven equal sub-horizons, and a SCED problem was formulated for each sub-horizon. To model the ramping up and ramping down constraints, an extra time interval was added to the end of each sub-horizon, and a coordination algorithm was designed to make the amount of power generated by units in this coupling time interval (i.e., shared variables) equal to the value of power generated by units in the first time interval of the next sub-horizon. These shared time intervals between the subproblems were referred to as overlapping time intervals. A distributed coordination algorithm was developed to make the shared variables equal from the perspective of each two consecutive subproblems.

As discussed in [25], the number of subproblems have a significant impact on a geographical decomposition strategy. Similarly, the performance of temporal decomposition strategy depends on the number of subproblems. The optimal number of subproblems to minimize the solution time while obtaining accurate results depends on many factors. The solution time does not always decrease by increasing the number of subproblems since increasing the number of subproblems increases the required number of shared variables and the number of iterations for the coordination algorithm to converge.

In this paper, we analyze the partitioning of a SCED problem

over the considered time horizon. We study the effect of the number of subproblems on the performance of the time decomposition strategy, the solution time, the number of iterations, and accuracy of the obtained results. Since it is preferred to have equal sub-horizons to take the most advantage of parallel computing, the number of subproblems determines the breaking intervals. The power demand, the rate of change of load between two consecutive time intervals, and the ramp limits of the generating units are three features that are studied to provide useful insights on the application of temporal decomposition on SCED.

II. CENTRALIZED SECURITY-CONSTRAINED ECONOMIC DISPATCHING MODEL

SCED is solved to find the optimal power output of generating units at each time interval (i.e., hour) of a considered operation horizon. The objective function is to minimize operation cost while the generation-demand is satisfied at every hour. The optimization constraints under normal conditions and $N - 1$ contingency criteria are formulated by (2)-(16) [26, 27].

$$\min \sum_t \sum_u \frac{a_u \cdot p_{ut}^2 + b_u \cdot p_{ut} + C_u}{f(p_{ut})} \quad (1)$$

s. t.

$$p_{it} - pd_{it} = \sum_{i=1, i \neq j}^N \frac{\delta_{it} - \delta_{jt}}{X_{ij}} \quad \forall i, \forall t \quad (2)$$

$$p_{ijt} = \frac{\delta_{it} - \delta_{jt}}{X_{ij}} \quad \forall ij, \forall t \quad (3)$$

$$\delta_{ref,t} = 0 \quad \forall t \quad (4)$$

$$\underline{P}_u \leq p_{ut} \leq \overline{P}_u \quad \forall u, \forall t \quad (5)$$

$$p_{ut} - p_{u(t-1)} \leq UR_u \quad \forall u, \forall t \quad (6)$$

$$p_{u(t-1)} - p_{ut} \leq DR_u \quad \forall u, \forall t \quad (7)$$

$$\underline{P}_{ij} \leq p_{ijt} \leq \overline{P}_{ij} \quad \forall ij, \forall t \quad (8)$$

$$\underline{\delta}_i \leq \delta_{it} \leq \overline{\delta}_i \quad \forall i, \forall t \quad (9)$$

$$p_{it}^c - pd_{it}^c = \sum_{i=1, i \neq j}^N \frac{\delta_{it}^c - \delta_{jt}^c}{X_{ij}} \quad \forall i, \forall t, \forall c \quad (10)$$

$$p_{ijt}^c = \frac{\delta_{it}^c - \delta_{jt}^c}{X_{ij}} \quad \forall ij, \forall t, \forall c \quad (11)$$

$$\delta_{ref,t}^c = 0 \quad \forall t, \forall c \quad (12)$$

$$\underline{P}_{ut} \leq p_{ut}^c \leq \overline{P}_{ut} \quad \forall u, \forall t, \forall c \quad (13)$$

$$\underline{P}_{ij} \leq p_{ijt}^c \leq \overline{P}_{ij} \quad \forall ij, \forall t, \forall c \quad (14)$$

$$\underline{\delta}_i \leq \delta_{it}^c \leq \overline{\delta}_i \quad \forall i, \forall t, \forall c \quad (15)$$

$$|p_{ut} - p_{ut}^c| \leq \Delta \quad \forall u, \forall t, \forall c \quad (16)$$

where a_u , b_u , and C_u are cost coefficients for generating unit u , i and j are indices for buses, t is the time interval index, u

refers to generating units, c is the index for contingency (line outage), ij is the index for lines, f is the generation cost function, p_{ut} is power generated by unit u at time t , p_{ut}^c is power generated by unit u at time t after contingency c , p_{ij} denotes power flow in line ij , δ_i is the voltage angle of bus i , δ_i^c is the voltage angle of bus i after contingency c , X_{ij} is the reactance of line ij , \overline{P}_u and \underline{P}_u refer to the maximum and minimum limits of generating unit u , \overline{P}_{ij} and \underline{P}_{ij} refer to the maximum and minimum limits of line ij , UR_u and DR_u show the ramping up and ramping down limits of unit u , and Δ is the allowable change in generated power by units after the occurrence of a contingency.

III. TIME DECOMPOSITION AND COORDINATION STRATEGY

The number of variables and constraints directly affects the solution time of optimization problems. To speed up the solution procedure for SCED, a time decomposition strategy is proposed based on our previous work [24] to divide the problem into several sub-horizons each presenting a subproblem (i.e., a subset of variables and constraints) of the scheduling horizon. To model the inequality constraints pertaining to generators ramping in a simple way, we add an extra time interval (i.e., one hour) to the end of each sub-horizon and formulate a SCED subproblem for each sub-horizon. We call these extra time intervals *overlapping* intervals. The power generated by units at this overlapping hour are shared variables between these subproblems and must be equal from the perspective of neighboring subproblems. If an extra constraint is added to the neighboring subproblem to force the value of power in the first time interval to be equal to the output power achieved from the previous subproblem, the solution would be suboptimal. In addition, if the subproblems are solved independently, there may be two different values for the output power of some generating units in these shared intervals. Since this is not possible in reality, a coordination strategy is needed to coordinate the consecutive subproblems to make their shared variables equal in a way that is optimal for the whole problem in the overall scheduling time horizon. Since the main goal is to decrease the solution time and the computational burden, we prefer to solve all subproblem at the same time in parallel computers, and therefore we propose a coordination algorithm that is suitable for parallel computing.

A. Auxiliary Problem Principle

Auxiliary problem principle (APP) is implemented as a suitable coordination algorithm for parallel computing. APP is an iterative method, based on augmented Lagrangian relaxation, which tries to find the optimal solution of several coupled optimization subproblems [28]. A sequence of auxiliary problems with special features is solved to coordinate subproblems. This is a practical approach for optimizing in parallel by approximating the shared variables in each iteration based on the previous iteration.

Consider that the overall scheduling horizon of one week is decomposed into NS subproblems. For the sake of explanation, we focus on two consecutive subproblems n and $n+1$. The

power output of all generating units at the last time interval of subproblem n is linked to time interval one of subproblem $n+1$ through ramping up/down limitations of generating units. In order to model this constraint, we add an extra (overlapping) time interval, tc , to the end of subproblem n . For each unit, the optimal amount of generated power in overlapping time interval tc must be equal to the amount of generated power in time interval 1 of subproblem $n+1$. Thus, generating powers at the overlapping hour tc are shared with generating powers of hour one of the next sub-horizon. The shared variables of subproblem n are shown by ϕ_n and shared variables of subproblem $n+1$ by ϕ_{n+1} . Since ϕ_n and ϕ_{n+1} are physically referring to the same concept, the following consistency constraint needs to be satisfied for all generating units.

$$\phi_n - \phi_{n+1} = 0 \quad (17)$$

However, (17) is a hard constraint that may make the overall solution suboptimal which may be different than the centralized algorithm and is not desirable for distributed optimization. We relax the hard constraint (17) by adding a penalty function to the objective function (1) using the concept of augmented Lagrangian relaxation. To do so, we formulate (1-16) for subproblem n at iteration k as (18).

$$\begin{aligned} \min_{(x_n^k, \phi_n^k)} \quad & \sum_{u,t} f(p_{u,t}^k) \\ & + \left(\frac{\rho}{2} \left\| \Phi_n^k - \Phi_n^{*k-1} \right\|^2 + \gamma \Phi_n^{k\top} \left(\Phi_n^{*k-1} - \Phi_{n+1}^{*k-1} \right) \right. \\ & \left. + \lambda^{(k-1)\top} \Phi_n^k \right) \\ \text{s.t.} \quad & h_n(x_n^k, \Phi_n^k) = 0 \\ & g_n(x_n^k, \Phi_n^k) \leq 0 \end{aligned} \quad (18)$$

$$x_n^k = \{p_{u,t,n}^k\}, \Phi_n^k = \{p_{u,tc,n}^k\}, \Phi_{n+1}^{*k-1} = \{p_{u,tc,n+1}^{*k-1}\}$$

where \top is a transpose operator, x_n is the set of output power of generating units during sub-horizon n , ϕ_n is the set of output power of generating units in the overlapping time interval tc , λ^k is the vector of Lagrange multipliers at iteration k , ρ and γ are suitable positive constants. Φ_n^{*k-1} and Φ_{n+1}^{*k-1} indicate the values of the shared variables of subproblems n and $n+1$ that are determined at iteration $k-1$, and Φ_n^k is the shared variable of subproblem n that needs to be determined in the iteration k . In fact, Φ_n^{*k-1} and Φ_{n+1}^{*k-1} in (18) are known values while Φ_n^k is a decision variable.

A similar ramp-constrained SCED is formulated for subproblem $n+1$ in (19).

$$\min_{x_{n+1}^k, \Phi_{n+1}^k} \sum_{u,t} f(p_{u,t}^k) \quad (19)$$

$$+ \left(\frac{\rho}{2} \left\| \Phi_{n+1}^k - \Phi_{n+1}^{*k-1} \right\|^2 + \gamma \Phi_{n+1}^{k\top} \left(\Phi_{n+1}^{*k-1} - \Phi_n^{*k-1} \right) - \lambda^{(k-1)\top} \Phi_{n+1}^k \right)$$

s. t.

$$h_{n+1}(x_{n+1}^k, \Phi_{n+1}^k) = 0$$

$$g_{n+1}(x_{n+1}^k, \Phi_{n+1}^k) \leq 0$$

$$x_{n+1}^k = \{p_{u,t,n+1}^k\}, \Phi_{n+1}^k = \{p_{u,tc,n+1}^k\}, \Phi_n^{*k-1} = \{p_{u,tc,n}^{*k-1}\}$$

where Φ_n^{*k-1} and Φ_{n+1}^{*k-1} are known values whereas Φ_{n+1}^k is a decision variable. The SCED subproblems are solved iteratively. The penalty multiplier λ needs to be updated at the end of each iteration as:

$$\lambda^k = \lambda^{k-1} + \alpha \left(\Phi_{n+1}^{*k} - \Phi_n^{*k} \right) \quad (20)$$

where α is a suitable positive constant, which is selected based on experimental results. Note that the value of the Lagrange multiplier λ in each iteration corresponds to the cost of maintaining the consistency constraint. It should be mentioned that since the considered SCED problem is inherently convex, APP is proven to converge [28, 29].

IV. PARTITIONING ANALYSIS AND MOTIVATING EXAMPLE

The number of variables and constraints (for both normal and contingency conditions) at each interval is multiplied by the number of scheduling intervals. The less the number of variables and constraints of a subproblem is, the less the computational time would be. However, because of increasing the number of shared variables, the number of iterations and consequently the total solution time might go up. Thus, there should be a trade-off between the number of sub-horizons and the solution time.

Consider a week-ahead SCED problem for the IEEE 118-bus test system that ten contingency scenarios are assumed in each hour. This optimization problem includes roughly 656,000 variables and 1,541,000 constraints (including the limitation of generating units, power balance, preventing action, line flow limits, angle of reference bus, voltage angles limitations and ramping up/down constraints). As shown in Table I, the solution time for this problem in a centralized manner is 33 seconds. If we decompose the problem into three subproblems, the solution time decreases to 7 seconds, which is 78.79% faster than the centralized SCED. Table I shows the solution time for this problem in centralized and distributed manners. Note that this is a relatively small system, and the time saving (in terms of seconds) is much more significant for larger systems.

After decomposing the problem into three subproblems, the number of variables in each subproblem is 223,896 for the first and third subproblems, and 227,824 for the second subproblem considering shared variables (explained further in section IV). In addition, the number of constraints in the first and third subproblems is 522,480, and it is 531,544 for the second

TABLE I
COMPARING THE SOLUTION TIME OF CENTRALIZED AND
DECENTRALIZED METHODS

	Centralized	Distributed		
		2 subproblems	3 subproblems	4 subproblems
Time (s)	33	10	7	45
Iterations	1	2	2	5

subproblem. This difference is due to the fact the middle subproblems have shared variables with their two neighboring subproblems, but the first and last subproblems have only one neighbor. Decomposing the problem into three subproblems reduces the number of variables and constraints in each subproblem. Further, partitioning the problem into four subproblems will reduce the size of each subproblem. However, it increases the number of active consistency constraints and thus, the difference between shared variables of consecutive subproblems, and this may result in more computational complexity.

Figure 1 shows that partitioning the overall time horizon (i.e., a week) into two and three equal sub-horizons and solving the subproblems in parallel decreases the solution time. However, partitioning the problem into four subproblems does not improve the solution time. The solution time goes up by 36% as compared to the case with three subproblems.

Hence, it is crucial to determine the optimal number of subproblems and decide from which intervals to break the scheduling horizon. Based on the results, the load pattern, the rate of the load change from one interval to another interval, and the ramping capability of the generating units are critical factors that must be taken into consideration for the scheduling horizon partitioning.

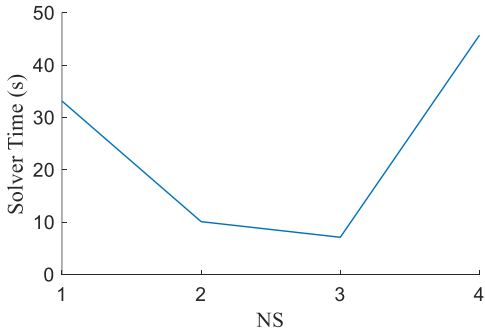


Fig. 1. The effect of the number of subproblems (NS) on the solution time.

V. NUMERICAL ANALYSIS AND DISCUSSIONS

Although time decomposition is proposed to solve the problem faster, the number of subproblems is a key factor that determines the solution time and accuracy of results. The optimal number of subproblems is problem dependent, and it is not possible to find a general pattern to address this challenge for all problems with different load patterns. We study the effect of the number of subproblems to give insights on how to partition the problem. We aim at solving a week-ahead SCED problem. The overall time interval for one week is divided into several smaller and equal subproblems that are solved in parallel. Consistency constraints are modeled as shared

variables to form the connections between subproblems to optimize the system over the entire time span. In this way, we achieve the overall optimal cost for the whole system. All simulations are carried out on MATLAB using YALMIP [30] as modeling software and Gurobi solver on a 3.7 GHz personal computer with 16GB of RAM.

A. A 3-Bus Test System

We have applied the proposed methodology on a 3-bus test system to solve the SCED problem. The total cost using both centralized and decentralized methods is \$1,909,498. This system has two generating units whose ramping up/down limitations are 50 and 100 MW. However, according to Fig. 2, the maximum load change is 40 MW. Therefore, the distributed algorithm converges just after one iteration since shared variables are equal from the very first iteration, in this simple case. The load change used in Fig. 2 is calculated from (21).

$$\text{Load Change} = \frac{\text{Load}(t) - \text{Load}(t-1)}{1} \quad (21)$$

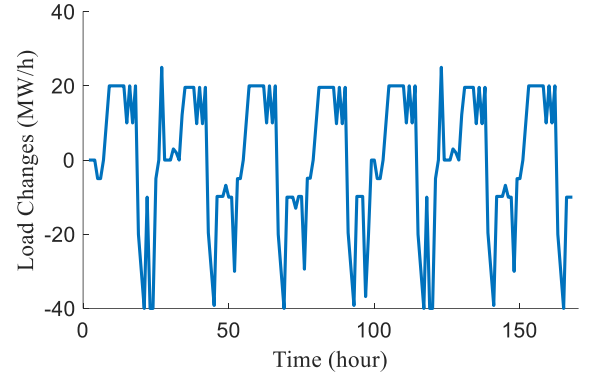


Fig. 2. Changes of the load in each hour for the 3-bus test system.

Therefore, in this case, we have several subproblems that there is no connection between them. Indeed, the related intertemporal constraints between the subproblems are not active. Figure 3 illustrates the solution time for this case. Since subproblems are independent, by increasing the number of subproblems, the size of each subproblem will be smaller. Therefore, the solution time decreases. However, since the system is very small, by decomposing the problem to more than 20 subproblems the solution time does not change considerably.

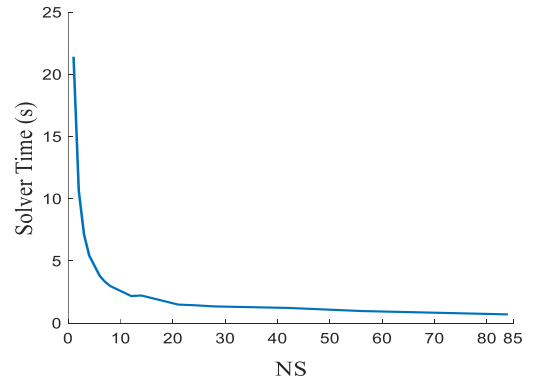


Fig. 3. The effect of the number of subproblems (NS) on the solution time for the 3-bus test system.

B. IEEE 118-Bus System

The proposed method has been applied to the IEEE 118-bus test system. The total cost using both centralized and decentralized methods is \$11,094,780. In this case, although some ramping constraints for transition from one sub-horizon to another sub-horizon are active, we have changed the load to have considerable changes to increase the difference between shared variables to study the effect of load changes on the required number of iterations for the coordination algorithm to converge. Figure 4 shows (a) the load demand and (b) load changes over the one-week horizon, and Fig. 5 shows (a) the required number of iterations and (b) solver time to converge over the number of subproblems.

According to Fig. 5 (a), in this case, the general trend is the more subproblems we have, the more iterations are required for the coordinating algorithm to converge. This is mainly because the number of shared variables increases; hence, more shared variables need to be coordinated. As a result, the solution time increases. We can conclude that both the number of shared variables and subproblems effect on the convergence time. In addition, the number of subproblems determines the hours from which the problem is partitioned. If the ramping constraints of several generating units are active between the two consecutive subproblems, solving the subproblems independently results in different shared variables. Therefore, the coordination algorithm needs more iterations to decrease the difference between shared variables. In other words, it is more efficient to partition the problem from hours with the smallest change of load.

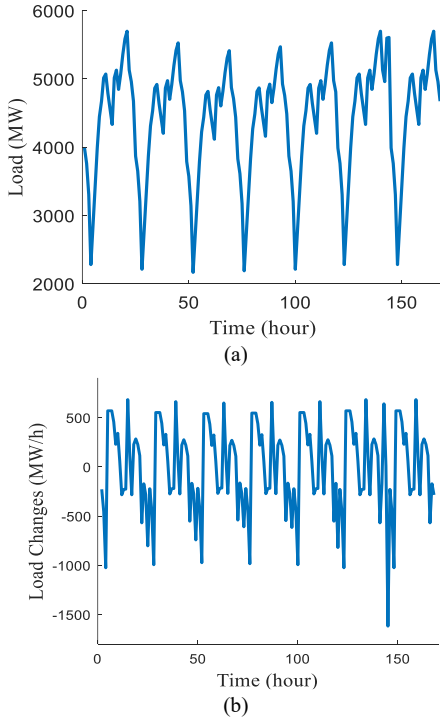


Fig. 4. (a) The load demand and (b) load change in each hour for the IEEE 118-bus test system.

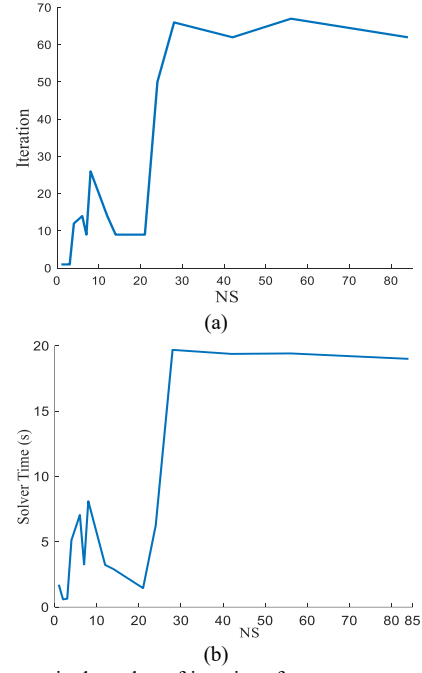


Fig. 5. (a) The required number of iterations for convergence of coordination algorithm and (b) the solution time over the number of subproblems for the IEEE 118-bus test system.

VI. CONCLUSION

This paper presented a temporal decomposition strategy to speed up the solution procedure of SCED. The partitioning of the scheduling horizon was discussed, and a coordination strategy was developed to solve the SCED formulated for each sub-horizon in a distributed manner. The impact of the number of sub-horizons on the number of iterations and the solution speed was discussed. It was analyzed that the load pattern, the rate of change of load, and ramp limits of the generating units have a significant impact on the optimal number of sub-horizons and time-saving. The results show that increasing the number of sub-horizons over a certain limit will increase the number of shared variables, the required number of iterations for the distribution algorithm to converge, and the total solution time. An important observation is that if we break the problem from hours that ramping of most generating units are not active, less number of iterations is required for the coordination algorithm to converge.

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