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Distributed optimisation-based collaborative security-constrained transmission expansion planning for multi-regional systems

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Abstract: This study presents a distributed collaborative transmission expansion planning (TEP) algorithm for interconnected multi-regional power systems. The proposed algorithm is a multi-agent-based TEP. A local TEP is formulated for each region (agent) with respect to the region's local characteristic and interactions (i.e. tie-line flows) with its neighbours. Nodal power balances at border buses are modified to model the interactions. Realistic planning constraints and objectives such as budget constraints, operational costs, $N-1$ security criterion, and uncertainties are modelled in the local TEPs. The information privacy is respected as each local planner needs to share limited information related to cross-border tie lines with other planners. To coordinate the local planners, a two-level distributed optimisation algorithm is proposed based on the concept of analytical target cascading (ATC) for multidisciplinary design optimisation. While the upper level solves the local TEPs in parallel, the lower level seeks to coordinate neighbouring regions. The lower-level problem is further replaced in the upper-level optimisation by Karush–Kuhn–Tucker conditions to relax the need for any form of central coordinator. This makes the proposed ATC-based TEP a fully parallelised distributed optimisation algorithm. An initialisation strategy is suggested to enhance the performance of the distributed TEP.

Nomenclature

Indices and sets

C	index for contingencies
d	index for demand
g	index for generating units
k	index for iterations
l	index for transmission lines
i, j	index for buses
m, n	index for regions (networks)
w	index for wind farms
$r(l)$	receiving-end node of transmission line l
$s(l)$	sending-end node of transmission line l
$\Omega_I^{\rm D}$	set of all demands located at node i
$\Omega_I^{\rm G}$	set of all generating units located at node i
$\Omega_I^{\rm W}$	set of all wind farms connected to node i
$\Omega^{\rm L}$	set of all existing transmission lines
$\Omega^{\rm L+}$	set of all candidate transmission lines
Ω_I^m	set of all buses of region m
Ω_I^{m+}	set of border buses of region m
Ω_I^+	set of border buses of all networks
Γ	set of decision variables in disjunctive TEP model
$\Delta_g^{\max}$	maximum adjustment capability of unit g

Parameters

$A_g^{\rm C}$	parameter that is equal to 0 if unit g is unavailable under contingency C and 1 otherwise
$A_{\rm l}^{\rm C}$	parameter that is equal to 0 if line l is unavailable under contingency C and 1 otherwise
$B_{\rm l}$	susceptance of transmission line l
$C_{\rm d}$	load-shedding cost of demand d
$C_{\rm d}^{\rm c}$	load-shedding cost of demand d in contingency C
C_g	production cost of generating unit g
$F_l^{\max}$	capacity of transmission line l
$\tilde{I}_l$	investment cost of candidate transmission line l
$I^{\max}$	investment budget for building candidate lines
M	large enough number, called big-M
$P_{\rm d}$	expected power demand d
P_w	expected power produced by wind farm w
$P_g^{\max}$	expected production capacity of generating unit g
π^m	penalty function corresponding to consistency constraints of area m
α, β	penalty multipliers
λ	tuning parameter

Variables

$x_{\rm l}$	binary decision variable to indicate whether candidate line l is constructed
P_g	power produced by unit g
f_l	power flow through line l
f_{ij}^m	power flow in tie line ij determined by region m
f_{ij}^n	power flow in tie line ij determined by region n
$P_{\rm d}^{\rm LS}$	load shedding of demand d
$r_{ji}^{cm,k}$	coordinating (or response) variables corresponding to tie line ji sent by the central coordinator to region m in iteration k
θ_i	voltage angle at node i

1 Introduction

In electric power industries, new transmission lines are required to support load growth, remove transmission congestion, support the integration of distributed renewable energy sources, provide non-discriminatory transmission access for all market participants, and support system reliability [1]. The location, the number, and the installation time of new transmission lines are determined by long-term transmission expansion planning (TEP) problem. The TEP problem is used by planning entities to expand the network topology optimally with the least investment costs. This is a very expensive, lengthy, and burdensome procedure [2].

Various approaches have been presented in the literature for modelling and implementation of TEP. The linearised DC TEP is the most popular model that is widely used in the literature, applied by system planners, and implemented in commercial software packages such as PLEXOS [3, 4]. Romero *et al.* [5] summarise and compare transportation, hybrid, disjunctive, and DC power flow models for TEP. Arabali *et al.* [6] present a multi-stage multi-objective TEP methodology taking into account three objectives: investment cost, absorption of private investment, and system reliability. In [7], based on the concept of the binary numeral system, strategies are introduced to reduce the number of variables related to the candidate transmission lines and network constraints. Bus splitting option considered in [8] changes flow path impedances and accordingly adjust short-circuit levels of the TEP solution. Incorporation of short-circuit level constraints and bus splitting decision variables in TEP decrease the total expansion planning costs.

TEP is a more challenging problem in interconnected multi-regional power systems including several independent networks. Each power system could have its own local transmission planner [9]. If the planners solve their TEP problem separately, the grid topology might not be optimal from the perspective of the whole grid. This imposes unnecessary investment costs and reduces the overall benefit of the system. Most of the existing literatures ignore interactions between the regions when solving for the local regional TEP. Several papers deal with TEP in the interconnected power systems, while most of them assume that there is an entity that has all grid information and formulates a centralised TEP problem for the whole grid [9]. Such a TEP framework potentially enhances system performance and reduces the overall investment/operational costs. However, in the privatised power sector, planners (or operators) are not willing to share their commercially sensitive information with other parties, and each transmission planner seeks to find its optimal TEP solution [10]. On the other hand, any decision made by a transmission planner affects TEP results of other planners, as the whole system is an interconnected grid. Therefore, implementing an individual TEP by a planner regardless of TEP results in other areas might cause higher-transmission planning costs and lower system reliability.

Few papers have been published on TEP in multi-regional power systems. In previous studies, two different approaches have been proposed for multi-regional TEP: cooperative approach and non-cooperative approach. In the cooperative approach, all planners work together to achieve the highest overall social welfare. One TEP problem can be formulated for the whole system. While, in the non-cooperative approach, each transmission planner seeks to maximise its social welfare considering the planning decisions of other planners [11]. In the non-cooperative approach, the social welfare of the whole interconnected system might not be obtained due to competition between different transmission planners [12]. According to these terminologies, the proposed algorithm in this paper, which aims at maximising the overall social welfare of the whole system, falls in the cooperative TEP category. However, despite conventional cooperative models in which the information privacy of independent planners is not met, the information privacy of the planners is respected in the proposed algorithm. Therefore, we use the term *collaborative* for the proposed algorithm. The concept of cooperative and non-cooperative solutions for TEP in the multilateral context is discussed in [13]. Papers that deal with the non-cooperative TEP consider, usually, the cooperative TEP's solution as benchmark results since it maximises the overall social welfare. Most of the existing works in the field of cooperative/non-cooperative are based on either a centralised optimisation or game theory. While several studies have been done on the distributed optimal power flow [14] and distributed unit commitment [15], due to the complex nature of TEP, no reference is reported on the domain of distributed TEP (DTEP) for interconnected power systems. We deal with the multi-regional TEP in the context of distributed optimisation.

Another problem that makes TEP more challenging is $N-1$ security criterion that is an essential constraint in the long-term planning problem [16]. On the basis of North American Electric Reliability Corporation standards, a planned network must be able to operate in a way that the outage of a single component does not interrupt supplying demands [17]. Therefore, the $N-1$ criterion must be taken into consideration in a cooperative/non-cooperative TEP framework.

This paper contributes to the literature by presenting a DTEP algorithm for interconnected multi-regional power systems in a collaborative framework taking into account $N-1$ security criterion for each region and uncertainties. The proposed DTEP is a multi-agent-based TEP. Each region has an independent planning entity. Interactions (i.e. power exchange) between the regions through tie lines are modelled by a set of pseudo generations and a set of pseudo loads, and the power balance equation at border buses are modified accordingly. Taking into account the information privacy of the planning entities and their mutual interactions, a local TEP problem is formulated for each region. Each local planner handles the TEP problem of its network while having access to the information of cross-border tie lines connecting that network to its neighbours. On the basis of the concept of analytical target cascading (ATC) technique for multidisciplinary design optimisation, a two-level distributed optimisation algorithm is developed. While the classical ATC is a sequential procedure, the developed ATC allows the parallel solution of local TEP problems in level one with the use of a central coordinator in level two. Since the level two's problem is a convex optimisation, it is further replaced in the level one's TEP problems by the Karush–Kuhn–Tucker (KKT) conditions. With this procedure, we eliminate the need for a central coordinator by introducing a set of coordinating variables and enforcing a set of constraints in the local TEP of each region. This makes the proposed algorithm fully parallelised. An initialisation strategy is suggested to reduce the number of iterations of the DTEP algorithm. The proposed collaborative DTEP is applied to the IEEE 24-bus and 118-bus test systems, and promising results are obtained.

The main contributions of this paper are summarised as follows:

- Multi-regional TEP is formulated in the content of distributed optimisation instead of the conventional centralised optimisation methods that might not be appropriate for collaborative expansion.
- An ATC-based distributed optimisation algorithm is presented to coordinate long-term planning.
- On the basis of the concept of bi-level optimisation, the distributed ATC-based TEP, which is a hierarchical approach, is transformed into a fully parallel approach with no need for any form of central coordinator.
- To enhance the performance of the proposed DTEP, an initiation strategy is suggested.

The remainder of this paper is organised as follows. The mathematical formulation of DTEP is explained in Section 2. The ATC-based distributed solution algorithm is presented in Section 3. Section 4 provides numerical results. Concluding remarks are discussed in Section 5.

2 Mathematical formulation of DTEP

The TEP problem is mostly modelled with DC power flow in the literatures [2, 3, 6, 7, 9]. Voltage angles of buses are used to calculate power flow in existing and candidate lines. Power flow of candidate lines is, usually, modelled through bilinear equations, where integer variables representing candidate lines are multiplied by bus voltage angles. Bilinear equations can be transformed into linear equations using disjunctive techniques [1]. The resultant TEP formulation is mixed-integer linear programming which is called the disjunctive TEP model. A complete formulation of the disjunctive TEP model considering $N-1$ security criterion is given in the Appendix. To account for uncertainties (which are power demand, the capacity of generating units, and wind power generation), a data-driven approach is applied [18].

In this section, we present the DTEP model for the system shown in Fig. 1, which can be extended for interconnected systems with multiple regions. The regions exchange power with each other through the tie lines. Consider that in a given time, the power in the

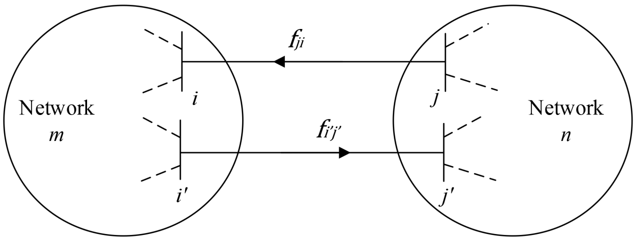

Fig. 1 *Two interconnected networks*

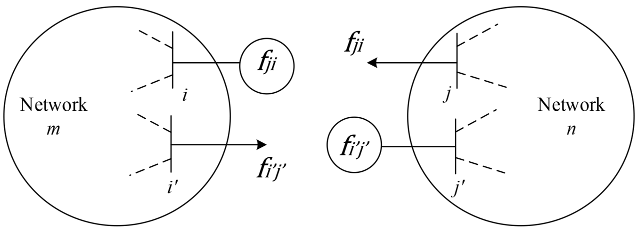

Fig. 2 *Modelling power exchange between interconnected networks m and n*

tie line ij flows from network n toward the network m, and the direction of power flow in tie line $i'j'$ is toward the network n. We model the tie line ij as a controllable pseudo generation in the network m and as a controllable pseudo load in the network n. The same concept is used for the tie line $i'j'$ as shown in Fig. 2. Values of the pseudo generations and loads are equal to the line flows. Following this approach, we can virtually disconnect networks m and n. As the pseudo generations and loads appear in both networks, we call them coupling (shared) variables between networks m and n. These coupling variables are not constant and can take any values between $-F_{ij}^{\max}$ and $F_{ij}^{\max}$.

Remark 1: In most applications of distributed optimisation on power system problems, e.g. distributed optimal power flow [14] and distributed unit commitment [15], voltage angles of cross-border buses (i.e. i, i', j, and j') are considered as coupling variables. That is, two coupling variables should be assumed for each tie line. We directly use tie-line flows as coupling variables to have less number of variables in the model. It is obvious that in the DC power flow, bus voltage angles are proportional quantities, which depend on the reference bus location. However, power flow through each line, which depends on the difference between voltage angles of sending and receiving buses [see (20)], is independent of the reference bus location. Therefore, it is better to select power flowing in tie lines as coupling variables. This enhances the convergence performance of the DTEP algorithm.

Remark 2: In the proposed method, the reference bus is selected for one region (e.g. region m). After convergence, the values of voltage angles for other regions can be obtained from the values of voltage angles of cross-border buses of the region m and power flow through tie lines [see (20)].

Now, each planning entity can formulate a local TEP problem with respect to its local parameters/variables and coupling variables (i.e. the pseudo generations and loads).

2.1 Local objective and constraints

The objective function of the local TEP problem of network m, i.e. (1a), is to minimise the investment and operational costs of the region m. The sets of equality and inequality constraints (1b) are local constraints that include only local variables and parameters of region m. These constraints are given in the Appendix

$$\min_{\Gamma} F^m(\Gamma) \tag{1a}$$

$$\text{s.t.} \quad g^m(\Gamma) = 0; \quad h^m(\Gamma) \le 0 \tag{1b}$$

2.2 Constraints of border buses

In addition to constraints (1b), we formulate constraints (2a)–(2e) that not only consist of local variables/parameters but also include coupling variables between the network m and its neighbours. Constraint (2a) indicates that if bus i of network m is connected to a tie line between this network and its neighbours, and if the tie line is modelled as a pseudo generation from the perspective of network m, power injected to bus i is equal to power output of the pseudo generation plus power generated by actual units on this bus. A similar discussion is valid for (2b), where the tie line is modelled as a pseudo load. Inequalities (2c) and (2d) ensure that the pseudo generations and loads are within their limits. These limits are defined according to the capacity of tie lines. Note that f_{ji}^m and $f_{i'j'}^m$ can be negative. In this case, f_{ji}^m becomes a pseudo load and $f_{i'j'}^m$ becomes a pseudo generation. The nodal power balance at the border buses is modelled by equality (2e)

$$P_g^m = \begin{cases} P_g^m; & g \in \Omega_I^{\mathrm{G}},\ i \in \Omega_I^m,\ i \notin \Omega_I^{m+} \\ P_g^m + f_{ji}^m; & g \in \Omega_I^{\mathrm{G}},\ i \in \Omega_I^{m+} \end{cases} \tag{2a}$$

$$P_{\mathrm{d}}^m = \begin{cases} P_{\mathrm{d}}^m; & d \in \Omega_I^{\mathrm{D}},\ i' \in \Omega_I^m,\ i' \notin \Omega_I^{m+} \\ P_{\mathrm{d}}^m + f_{i'j'}^m; & d \in \Omega_I^{\mathrm{D}},\ i' \in \Omega_I^{m+} \end{cases} \tag{2b}$$

$$-f_{ji}^{\max} \le f_{ji}^m \le f_{ji}^{\max}; \quad i \in \Omega_I^{m+},\ j \in \Omega_I^{n+} \tag{2c}$$

$$-f_{i'j'}^{\max} \le f_{i'j'}^m \le f_{i'j'}^{\max}; \quad i' \in \Omega_I^{m+},\ j' \in \Omega_I^{n+} \tag{2d}$$

$$\begin{aligned} &\sum_{g \in \Omega_I^{\mathrm{G}}} P_g - \sum_{d \in \Omega_I^{\mathrm{D}}} \left(P_{\mathrm{d}} - P_{\mathrm{d}}^{\mathrm{LS}}\right) - \sum_{l \mid s(l) = i} f_1 + \sum_{l \mid r(l) = i} f_1 \\ &= P_{\mathrm{d}}^m - P_g^m; \quad \forall i \in \Omega_I^{m+} \end{aligned} \tag{2e}$$

2.3 Consistency constraints

Consider tie line ij in Fig. 1, which is modelled by a pseudo generation and a pseudo load in Fig. 2. If planner m separately solves its local TEP regardless of the TEP of a planner n, the solution procedure might result in different values for f_{ji}^m and f_{ji}^n that is not a feasible solution. As f_{ji}^m and f_{ji}^n refer to the tie-line flow, their values need to be the same to reach a consistent and feasible solution for the whole grid. We introduce new sets of equality constraints (3a) and (3b), named consistency constraints, and enforce them in local TEPs m and n. The consistency constraints enforce pseudo generations to be equal to their corresponding pseudo loads. In other words, the consistency constraints ensure that the values of coupling variables are the same in the local TEP problems

$$\mathrm{CC1}: f_{ji}^m - f_{ji}^n = 0; \quad i \in \Omega_I^{m+},\ j \in \Omega_I^{n+} \tag{3a}$$

$$\mathrm{CC2}: f_{i'j'}^m - f_{i'j'}^n = 0; \quad i' \in \Omega_I^{m+},\ j' \in \Omega_I^{n+} \tag{3b}$$

Finally, in the DTEP model, the local TEP for the region m is modelled as follows:

$$\begin{aligned} &\min_{\Gamma} F^m(\Gamma) \\ &\text{s.t.}\ (1b),\ (2a)-(2e),\ (3a),\ \text{and}\ (3b) \end{aligned}$$

3 Solution algorithm for DTEP

Although the consistency constraints ensure the feasibility of TEP results, these hard constraints are barriers for the separate solution of local TEPs. A strategy is needed to coordinate the planners' decisions for the values of pseudo generations and loads.

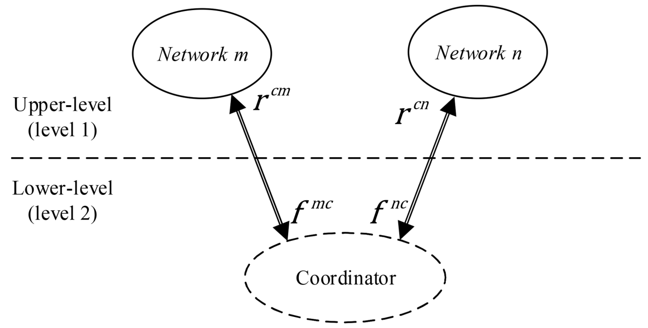

Fig. 3 *Bi-level ATC structure for networks m and n*

3.1 ATC technique for DTEP implementation

On the basis of the concept of ATC, we develop an algorithm to coordinate the planners' decisions and solve the local TEPs in a distributed manner. ATC is a model-based method for multilevel hierarchical optimisation problems [19–21]. The general concept of ATC is similar to other popular algorithms that are based on augmented Lagrangian relaxation such as the auxiliary problem principle and the alternating direction method of multipliers [22]. However, ATC has a hierarchical structure and works based on propagating target values from upper-level systems toward the lower-level systems, and passing the response variables from lower levels to upper levels. In addition, we have flexibility in the choice of penalty function in the ATC method and can select, for instance, a quadratic function, or an exponential function [21].

Consider the local TEP for the region m. We penalise violations of the consistency constraints (3a) and (3b) into the objective function using two penalty functions

$$\min_{\Gamma} F^m(\Gamma) + \pi_1^m(\text{CC1}) + \pi_2^m(\text{CC2}) \tag{4}$$

Here, we applied second-order penalty functions for $\pi_1^m(.)$ and $\pi_2^m(.)$. Accordingly, the objective function of TEP m is rewritten as follows:

$$\begin{aligned}\min_{\Gamma} & F^m(\Gamma) \\ & + \sum_{ij \in \Omega_I^+} \{\alpha_{ji}^m(f_{ji}^m - f_{ji}^n) + \| \beta_{ji}^m \circ (f_{ji}^m - f_{ji}^n) \|_2^2 \\ & + \alpha_{i'j'}^m(f_{i'j'}^m - f_{i'j'}^n) + \| \beta_{i'j'}^m \circ (f_{i'j'}^m - f_{i'j'}^n) \|_2^2 \}\end{aligned} \tag{5}$$

where the symbol $\circ$ represents the Hadamard product. The parameter α is the Lagrangian multiplier and β is a penalty parameter. A similar relaxed TEP problem is formulated for the network n, and accordingly, local TEP problems of all interconnected networks will be formulated. Decision variables of TEP m include local variables of network m and the coupling variables denoted by superscript m. The pseudo generations and loads denoted by superscript n are constants received from the neighbour n.

The local TEPs can be assumed to be connected hierarchically, and a sequential, iterative procedure as presented in [19, 20] can be applied to coordinate the subproblems. While a planner is solving its TEP, other planners should stay idle. This increases the overall computational time of the solution process. We will further propose a fully parallel, scalable solution procedure.

3.2 Decentralised parallel TEP implementation

3.2.1 Partially parallel solution algorithm: We introduce a coordinator to enable a parallel solution of the local TEPs. This coordinator virtually disconnects the TEPs of neighbouring planners as shown in Fig. 3. The local TEPs are at the upper level, and the coordinator is at the lower level. Instead of direct interaction between the neighbouring planners, each planner only communicates with the coordinator. Therefore, regions can solve their local TEPs in parallel. Note that there is no need to have an entity to play the role of a coordinator. Any of local planners may handle the role of the coordinator (we will relax the need for such a virtual coordinator in Section 3.2.2).

According to the concept of ATC, information sent by local regions to the central coordinator has named targets, and information sent by the coordinator toward the regions is called responses. The communication direction is indicated in superscripts. For instance, f_{ji}^{mc} denotes the target variable that is calculated in the region m and sent to the coordinator. We introduce *coordinating variables* r_{ji}^{cm} that are the responses sent from the central coordinator to the region m. A set of consistency constraints is formulated to make the solution of the central coordinator and the local regions consistent

$$r - f = 0 \tag{6}$$

These constraints are relaxed in the objective function of local planners by augmented Lagrangian penalty functions (see [15] for more details). This procedure leads to the following TEP problem for the region m in iteration k (see (7)) . A local TEP is formulated for each region. The coordinator has no role, except coordinating local TEPs. Thus, its objective function includes only a set of penalty functions for relaxing the consistency constraints. For the system shown in Fig. 2, the coordinator's objective function is formulated as (8a). Four quadratic augmented Lagrangian penalty functions appear in (8a). The first two functions are related to relaxation of the consistency constraints ($r_{ji}^{cm} - f_{ji}^{mc}$ and $r_{i'j'}^{cm} - f_{i'j'}^{mc}$) between the coordinator and TEP of the region m, and the last two penalty functions relax the consistency constraints ($r_{ji}^{cn} - f_{ji}^{nc}$ and $r_{i'j'}^{cn} - f_{i'j'}^{nc}$) between the coordinator and TEP of the region n

$$\begin{aligned} r^k = \operatorname{argmin} \sum_{ij \in \Omega_I^+} \{ & \\ & \alpha_{ji}^{m,k}\left(r_{ji}^{cm,k} - f_{ji}^{mc,k}\right) + \| \beta_{ji}^{m,k} \circ \left(r_{ji}^{cm,k} - f_{ji}^{mc,k}\right) \|_2^2 \\ & + \alpha_{i'j'}^{m,k}\left(r_{i'j'}^{cm,k} - f_{i'j'}^{mc,k}\right) + \| \beta_{i'j'}^{m,k} \circ \left(r_{i'j'}^{cm,k} - f_{i'j'}^{mc,k}\right) \|_2^2 \\ & + \alpha_{ji}^{n,k}\left(r_{ji}^{cn,k} - f_{ji}^{nc,k}\right) + \| \beta_{ji}^{n,k} \circ \left(r_{ji}^{cn,k} - f_{ji}^{nc,k}\right) \|_2^2 \\ & + \alpha_{i'j'}^{n,k}\left(r_{i'j'}^{cn,k} - f_{i'j'}^{nc,k}\right) + \| \beta_{i'j'}^{n,k} \circ \left(r_{i'j'}^{cn,k} - f_{i'j'}^{nc,k}\right) \|_2^2 \}\end{aligned} \tag{8a}$$

$$r_{ji}^{cm,k} = r_{ji}^{cn,k}; \quad i \in \Omega_I^{m+},\ j \in \Omega_I^{n+} \tag{8b}$$

$$r_{i'j'}^{cm,k} = r_{i'j'}^{cn,k}; \quad i' \in \Omega_I^{m+},\ j' \in \Omega_I^{n+} \tag{8c}$$

The response values determined by the central coordinator must be consistent for both regions m and n. This is reflected in (8b) and (8c). With the above procedure, the local TEPs become virtually separated, and a two-level ATC can be applied to solve TEPs in parallel. However, the main drawback is the need for a central coordinator that is responsible for coordinating the coupling variables between the regions.

$$\begin{aligned}\min_{(\Gamma, f^{mc})} & F^{m,k}(\Gamma) + \sum_{ij \in \Omega_I^+} \{\alpha_{ji}^{m,k}\left(r_{ji}^{cm,k-1} - f_{ji}^{mc,k}\right) + \| \beta_{ji}^{m,k} \circ \left(r_{ji}^{cm,k-1} - f_{ji}^{mc,k}\right) \|_2^2 \\ & + \alpha_{i'j'}^{m,k}\left(r_{i'j'}^{cm,k-1} - f_{i'j'}^{mc,k}\right) + \| \beta_{i'j'}^{m,k} \circ \left(r_{i'j'}^{cm,k-1} - f_{i'j'}^{mc,k}\right) \|_2^2 \} \\ & \text{s.t. } (1b),\ (2a) - (2e)\end{aligned} \tag{7}$$

3.2.2 Fully parallel solution algorithm: Assume that region m and the coordinator are interacting as shown in Fig. 3. We can form a bi-level optimisation problem to model their interactions in which TEP m is the leader's problem and the coordinator's optimisation is the follower's problem. The leader's problem includes (1a), (1b), and (2a)–(2e), and the follower's problem consists of (8a)–(8c). Note that in iteration k, the region m uses values of the coordinated shared variables determined by the coordinator in the iteration $k-1$. Thus, while the leader's problem deals with variables of iteration k, we use the coordinating variable r^{k-1} in the follower's problem. Note that we use only the coordinating values of the coordinator that are needed for the region m (i.e. $r^{cm,k-1}$) and ignore other coordinating values (that is, any coupling variables that have no direct impact on region m do not appear in the bi-level optimisation of this region). The bi-level model of TEP m is formulated as follows:

$$\text{upper}-\text{level (leader)}: \begin{cases} \min\limits_{(\Gamma, f^{mc})} \; (7) \\ \text{s.t. } (1b),\ (2a)-(2e) \end{cases} \tag{9a}$$

$$\text{lower}-\text{level (follower)}: \begin{cases} \min\limits_{r^{cm,k-1}} \; (8a) \\ \text{s.t. } (8b),\ (8c) \end{cases} \tag{9b}$$

The follower problem includes only a set of convex quadratic penalty functions and linear constraints. Therefore, it is a convex optimisation problem that can be replaced in the leader problem by applying KKT conditions and solving the resulting mathematical programming problem with equilibrium constraints. Consider penalty terms corresponding to a shared variable between regions m and n (i.e. power flow in tie line ji) shown in (10). From the constraints of the follower problem, we have $r_{ji}^{cm,k-1} = r_{ji}^{cn,k-1}$. Taking the partial derivative of π with respect to r_{ji}^{cm} results in the formulation (11) for $r_{ji}^{cm,k-1}$

$$\begin{aligned} \pi = {} & \alpha_{ji}^{m,k-1}\left(r_{ji}^{cm,k-1} - f_{ji}^{mc,k-1}\right) \\ & + \| \beta_{ji}^{m,k-1} \circ \left(r_{ji}^{cm,k-1} - f_{ji}^{mc,k-1}\right) \|_2^2 \\ & + \alpha_{ji}^{n,k-1}\left(r_{ji}^{cn,k-1} - f_{ji}^{nc,k-1}\right) \\ & + \| \beta_{ji}^{n,k-1} \circ \left(r_{ji}^{cn,k-1} - f_{ji}^{nc,k-1}\right) \|_2^2 \end{aligned} \tag{10}$$

$$r_{ji}^{cm,k-1} = \frac{2\left(\beta_{ji}^{m,k-1}\right)^2 f_{ji}^{mc,k-1} + 2\left(\beta_{ji}^{n,k-1}\right)^2 f_{ji}^{nc,k-1} - \alpha_{ji}^{m,k-1} - \alpha_{ji}^{n,k-1}}{2\left(\beta_{ji}^{m,k-1}\right)^2 + 2\left(\beta_{ji}^{n,k-1}\right)^2} \tag{11}$$

If each region directly calculates the values of the central coordinator's coordinating variables $r_{ji}^{cm,k-1}$, the coordinator can be omitted. However, to calculate $r_{ji}^{cm,k-1}$, the equivalents of $\alpha_{ji}^{m,k-1}$, $\alpha_{ji}^{n,k-1}$, $\beta_{ji}^{m,k-1}$, $\beta_{ji}^{n,k-1}$, $f_{ji}^{mc,k-1}$, and $f_{ji}^{nc,k-1}$ are needed. If regions m and n directly exchange information without a central coordinator and multipliers α and β are updated as (12a) and (12b), then we can replace $\alpha_{ji}^{m,k}$, $\alpha_{ji}^{n,k}$, $\beta_{ji}^{m,k}$, and $\beta_{ji}^{n,k}$ by their equivalents $\alpha_{ji}^{mn,k}$, $\alpha_{ji}^{nm,k}$, $\beta_{ji}^{mn,k}$, and $\beta_{ji}^{nm,k}$, respectively. Expressions (12a) and (12b) are used to update α and β with respect to the violation of consistency constraints. This updating process is based on the concept of the method of multipliers [23]

$$\alpha^k = \alpha^{k-1} + 2\left(\beta^{k-1}\right)^2\left(r^{k-1} - f^{k-1}\right) \tag{12a}$$

$$\beta^k = \lambda\,\beta^{k-1} \tag{12b}$$

The value $r_{i'j'}^{cm,k-1}$ can is obtained in a similar manner. Finally, by replacing the follower optimisation with its KKT conditions in the leader optimisation, the decentralised formulation for local TEP of the region m is modelled as shown by the equation below:

(see (13))

Variables $f_{ji}^{mc,k}$ and $f_{i'j'}^{mc,k}$ are replaced by $f_{ji}^{mn,k}$ and $f_{i'j'}^{mn,k}$, which shows that region m exchanges values of coupling variables directly with its neighbours instead of the central coordinator. Therefore, the local TEP of the region m only relies on the local information of the region m and coupling variables between this region and its immediate neighbouring regions. Meaning, it can be solved in a decentralised manner with no need for a central coordinator. Moreover, in iteration k, each region needs values of coupling variables calculated by its neighbours in the iteration $k-1$. That is, the regions do not need to stay idle and can solve their local TEP problems concurrently. In other words, the local TEPs can be solved in a parallel way with peer-to-peer communications only among the immediate neighbouring regions. We propose the iterative coordination algorithm shown in Fig. 4, which is a fully parallel ATC technique, to solve the local TEPs of the regions.

3.3 Notes on convergence property

ATC is proven to converge for convex problems if responses r^{k-1} are used in the subproblem in which targets f_{ij}^k are computed (i.e. subproblems are solved sequentially) [19]. Although the proposed distributed algorithm solves the problem in a parallel manner, r^{k-1} is computed analytically and used in the TEP subproblem in which f_{ij}^k is to be determined. That is, all features of the sequential ATC are preserved, and the convergence proof of ATC provided in [19] is valid for the proposed parallel ATC. Although DTEP deals with non-convex optimisations, our simulations show that the algorithm converges to a high-quality solution. The quality of the solution enhances if good initial guess for the coupling variables and multipliers are available. In addition, adding convex quadratic penalty terms in the objective function acts as local convexifiers and enhances the algorithm behaviour.

Note that one might use convexification techniques such as those presented in [24], to convexify local TEP problems. In this

$$\begin{aligned} & \min_{\left(\Gamma, f^{mn,k}\right)} \; F^{m,k}(\Gamma) \\ & \qquad + \sum_{ij \in \Omega_1^+} \{\alpha_{ji}^{mn,k}\left(r_{ji}^{cm,k-1} - f_{ji}^{mn,k}\right) + \| \beta_{ji}^{mn,k} \circ \left(r_{ji}^{cm,k-1} - f_{ji}^{mn,k}\right) \|_2^2 \\ & \qquad + \alpha_{i'j'}^{mn,k}\left(r_{i'j'}^{cm,k-1} - f_{i'j'}^{mn,k}\right) + \| \beta_{i'j'}^{mn,k} \circ \left(r_{i'j'}^{cm,k-1} - f_{i'j'}^{mn,k}\right) \|_2^2 \} \\ & \text{s.t.} \quad (1b),\ (2a)-(2e) \\ & r_{ji}^{cm,k-1} = \frac{2\left(\beta_{ji}^{mn,k-1}\right)^2 f_{ji}^{mn,k-1} + 2\left(\beta_{ji}^{nm,k-1}\right)^2 f_{ji}^{nm,k-1} - \alpha_{ji}^{nm,k-1} - \alpha_{ji}^{mn,k-1}}{2\left(\beta_{ji}^{mn,k-1}\right)^2 + 2\left(\beta_{ji}^{nm,k-1}\right)^2} \\ & r_{i'j'}^{cm,k-1} = \frac{2\left(\beta_{i'j'}^{mn,k-1}\right)^2 f_{i'j'}^{mn,k-1} + 2\left(\beta_{i'j'}^{nm,k-1}\right)^2 f_{i'j'}^{nm,k-1} - \alpha_{i'j'}^{nm,k-1} - \alpha_{i'j'}^{mn,k-1}}{2\left(\beta_{i'j'}^{mn,k-1}\right)^2 + 2\left(\beta_{i'j'}^{nm,k-1}\right)^2} \end{aligned} \tag{13}$$

case, the convergence of the proposed ATC-based TEP algorithm is guaranteed. However, such convexification techniques potentially come with the cost of degrading the accuracy of TEP results.

4 Numerical results

Two popular test systems for TEP studies including the IEEE RTS 24-bus system and the IEEE 118-bus system are used to evaluate the performance of the proposed method. All simulations are carried out using GAMS and ILOG CPLEX MIQP solver [25]. A PC with an Intel(R) Xeon(R) CPU @2.6 GHz including eight cores and 16 GB of RAM is used.

```
Initialize coupling variables, α, β, and set λ, k = 0
While {not converged} do
    k = k + 1
    Solve (13) for all regions in parallel and determine the optimal values of coupling variables
    Exchange the values of coupling variables between neighboring regions
    Update α^k = α^{k-1} + 2(β^{k-1})^2 (r^{k-1} − f^{k-1})
    Update β^k = λ.β^{k-1}
    If { |f^k_mn − f^k_nm| ≤ ϵ_1 and |F^{m,k}(Γ) − F^{m,k-1}(Γ)| / F^{m,k}(Γ) ≤ ϵ_2 } then
        Declare convergence
    End if
End while
```

Fig. 4 *The proposed parallel ATC-based coordination strategy with no coordinator for decentralised TEP implementation*

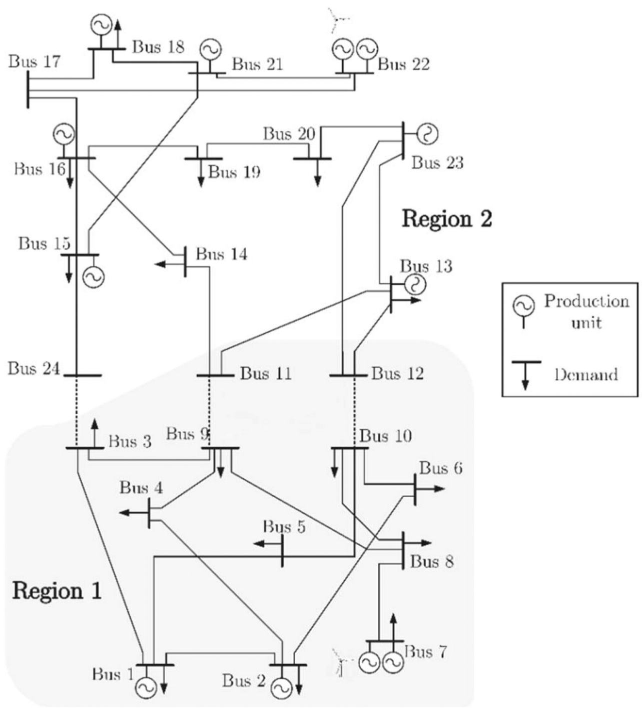

Fig. 5 *One-line diagram of a modified 24-bus system*

4.1 IEEE 24-bus system

The one-line diagram of the modified 24-bus system is shown in Fig. 5. This system is modified by Ruiz and Conejo for TEP studies, and the list of candidate lines are suggested accordingly [26]. We replace half of the conventional generation capacity of units connected to buses 7 and 22 by wind farms with parameters given in [27]. The system includes two regions, which are connected via tie lines 3-24, 9-11, and 10-12. Each region contains five candidate lines listed in Table 1.

Three scenarios are studied:

- *Scenario 1*: Each region implements its local TEP regardless of its interactions with its neighbouring regions.
- *Scenario 2*: It is assumed that a centralised planning entity exists and gathers the information of all regions and aims at maximising the social welfare (or minimising the total costs) for the whole power system. Since this fictitious centralised entity has information of all regions, it can provide the optimal TEP solution from the perspective of the whole system.
- *Scenario 3*: The proposed collaborative DTEP is implemented taking into account interactions between the regions and the information privacy.

We consider the results of scenario 2 (that provides the best results from the perspective of the whole grid) as the benchmark and use the following convergence measure to evaluate the performance of the proposed DTEP:

$$\text{rel} = \frac{\left| f^{*} - f^{\text{d}} \right|}{f^{*}} \tag{14}$$

where f^{*} is the cost function of the centralised TEP and f^{d} is the cost function determined by the proposed collaborative DTEP. For scenario 3, the initial value of penalty multipliers and tuning parameter λ are set to one, and the convergence thresholds ϵ_1 and ϵ_2 are 0.05 and 0.01, respectively. Simulation results of all scenarios are summarised in Table 2. When each local TEP is solved individually regardless of its interactions with its neighbouring regions, the total cost is \$553.12 M (costs of regions one and two are \$43.58 and \$172.87 M, respectively), and candidate lines one, eight, and nine are selected to be installed. The annual investment cost of installing new lines is higher than the two other scenarios. Moreover, region two cannot support the forecasted load and has to shed 74.8 MW of the load at bus 15. For the centralised TEP, the total cost goes down to \$231.53 M, and candidate lines one and eight are selected to be installed. Although the cost decreases because of interactions between the regions, the information privacy of the regions is not preserved. The proposed collaborative DTEP algorithm converges after 27 iterations. Figs. 6 and 7 show the coordinating variables ($r_{ji}^{cm,k}$) and the rel index over the course of iterations. On convergence, the rel index and differences between the share variables are within an acceptable threshold. The rel index is rough 1×10^{-6}. The total cost of DTEP is the same as the centralised TEP. The costs of regions one and two are \$58.86 and \$ 171.17 M, respectively, and candidate lines one and eight are planned for installation. The proposed collaborative DTEP could

Table 1 Candidate lines' parameters for the IEEE 24-bus system

Number of lines	From bus	To bus	Region	Annual investment cost, M$	Number of lines	From bus	To bus	Region	Annual investment cost, M$
1	2	7	1	0.7	6	20	22	2	1.1
2	6	8	1	0.8	7	14	15	2	0.9
3	2	5	1	1.0	8	15	19	2	0.8
4	1	4	1	1.2	9	13	14	2	1
5	5	7	1	1.3	10	19	21	2	1

Note: All candidate lines are assumed to have a susceptance of 500 Ω^{-1} and a capacity of 500 MW.

Table 2 Simulation results for the IEEE 24-bus system

	Cost function, M$	Annual cost of candidate lines, M$	Number of candidate lines for installation	Annual cost of units, M$	Annual cost of load shedding, M$
scenario 1	553.12	2.5	1, 8, 9	216.45	334.17
scenario 2	231.53	1.5	1, 8	230.03	0
scenario 3	231.53	1.5	1, 8	230.03	0

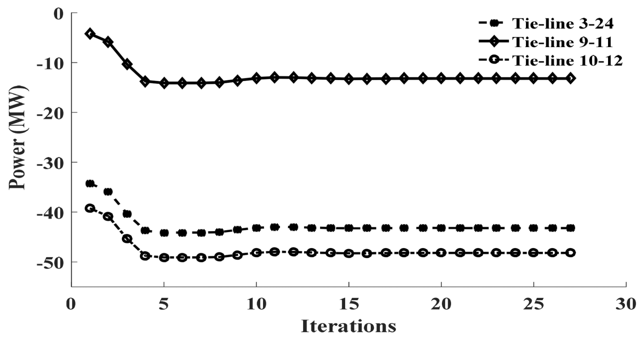

Fig. 6 *Coordinating (response) values in scenario 3*

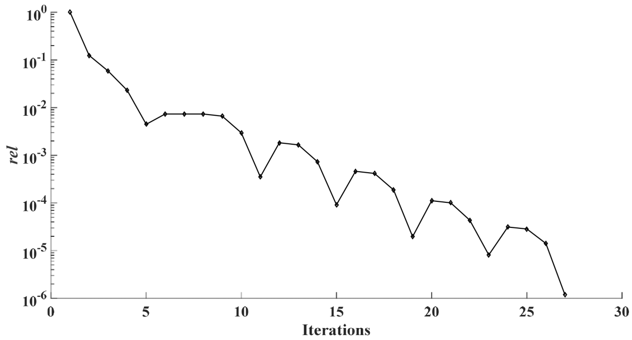

Fig. 7 *rel Index obtained by DTEP (scenario 3) for the IEEE 24-bus system*

provide the benchmark results obtained in scenario 2 while preserving the information privacy of the local planning entities.

One may terminate the DTEP algorithm before iteration 27th (e.g. 19) and obtain acceptable results (rel is around 1×10^{-5}). However, a trade-off should be considered between the optimality (i.e. the rel index), feasibility (i.e. $\left|f_{mn}^{k} - f_{nm}^{k}\right|$), and the number of iterations. Although the convergence measure rel decreases overall, in several iterations it goes up. This is because of the feasibility criterion. In those iterations, for instance in the transition from iteration 19 to iteration 20, the distributed algorithm wants to reduce the feasibility gap $\left|f_{mn}^{k} - f_{nm}^{k}\right|$, and this may lead to a slightly larger rel index.

4.2 IEEE 118-bus system

The system information is provided in [28]. For this system, we have considered a set of candidate lines that geographically make sense and decrease the operational costs if installed. The system is modified by adding three wind farms on buses 36, 69, and 77 [29]. The system includes three regions. Seven coupling variables exist. Regions one and two are connected through five tie lines (15-33, 19-34, 30-38, 75-77, and 75-188) and regions two and three are connected through two tie lines (77-82 and 80-96). Each region includes ten candidate lines as listed in Table 3.

The same three scenarios as in case 1 are considered. Simulation results are summarised in Table 4. In scenario 1, the operation costs of regions one, two, and three are \$236.28, \$298.30, and \$ 214.07 M, respectively. Two more candidate lines (lines 8 and 27) are selected to be installed compared with the benchmark results, and the total cost is larger than the benchmark cost. The proposed collaborative DTEP of scenario 3 converges after 129 iterations. Fig. 8 shows the rel index, which is small enough (almost 1×10^{-4}) on the convergence. The annual planning cost of installing new lines is the same as that obtained in scenario 2. The proposed DTEP provides planning decisions similar to the benchmark results while respecting the information privacy of planning entities. Note that in TEP, the cost of generation units (operation cost) might not realise in the real-time operation. The main goal of solving TEP problem is to decide about installing new transmission lines. Therefore, the slight difference (this 0.015% error is because of the considered acceptable gap for the distributed algorithm) between operational costs of scenarios 2 and 3 does not affect planning decisions.

4.2.1 DTEP with initialisation: In the previous DTEP scenario, the initial values of coupling variables were set to zero. To reduce the number of iterations, the initial value of coupling variables in the decentralised ATC technique can be selected wisely. For this purpose, we fixed the decision variable of installing new lines to zero ($x_l = 0$) and simplify DTEP to a distributed OPF problem, which is a convex problem. The solution of the distributed OPF is selected to initialise the coupling variables in DTEP. In this case, DTEP converges after 62 iterations, which almost 50% less than that for DTEP with flat start (i.e. initialising the coupling variables to zero). The rel index value is depicted in Fig. 8 and the planning results are the same as them for scenario 3 of Table 4.

5 Conclusion

A distributed collaborative TEP algorithm was presented for interconnected multi-regional power systems. Realistic planning constraints and objectives such as budget constraints, operational costs, and $N-1$ security criterion were taken into account. A data-driven approach was applied to account for uncertainties of power demand, the capacity of generating units, and wind power generation. While each region handles its local planning problem, it collaborates with other regions to achieve the optimal and

Table 3 Candidate lines' parameters for the IEEE 118-bus system

Number of lines	From bus	To bus	Annual cost, M$	Region	Number of lines	From bus	To bus	Annual cost, M$	Region
1	1	12	0.406	1	16	61	58	0.446	2
2	7	117	0.324	1	17	65	81	0.324	2
3	13	16	0.406	1	18	65	68	0.365	2
4	5	16	0.507	1	19	69	66	0.324	2
5	10	29	0.284	1	20	116	62	0.527	2
6	22	26	0.446	1	21	90	84	0.104	3
7	24	71	0.324	1	22	91	93	0.054	3
8	113	115	0.365	1	23	90	93	0.104	3
9	26	24	0.629	1	24	103	110	0.104	3
10	24	25	0.527	1	25	111	93	0.054	3
11	52	54	0.406	2	26	111	94	0.104	3
12	59	62	0.324	2	27	107	93	0.104	3
13	99	63	0.406	2	28	107	94	0.104	3
14	66	49	0.507	2	29	87	84	0.104	3
15	61	51	0.284	2	30	103	84	0.054	3

Note: All candidate lines are assumed to have a susceptance of 30 Ω^{-1} and a capacity of 390 MW.

Table 4 Simulation results for the IEEE 118-bus system

	Cost function, M $	Annual cost of candidate lines, M$	Number of candidate lines for installation	Annual cost of units, M$	Annual cost of load shedding, M$
scenario 1	751.24	2.59	5, 7, **8**, 9, 13, 17, 22, **27**, 29	748.65	0
scenario 2	735.43	2.13	5, 7, 9, 13, 17, 22, 29	733.30	0
scenario 3	735.54	2.13	5, 7, 9, 13, 17, 22, 29	733.41	0

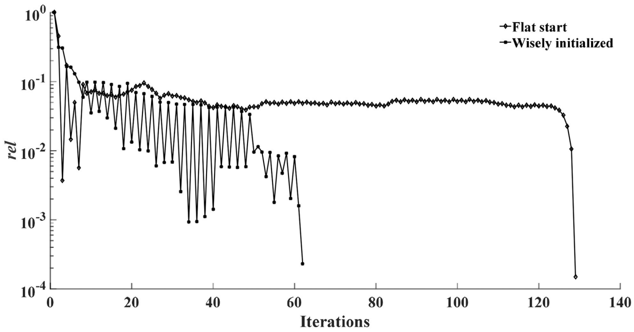

Fig. 8 *rel Index in scenario 3 with a flat start and with the suggested initialisation strategy (for the IEEE 118-bus system)*

feasible planning schemes for the whole interconnected grid. A two-level decentralised solution algorithm was developed based on the concept of ATC. The proposed distributed algorithm allowed the parallel implementation of TEP subproblems with no need for a central coordinator.

Three scenarios were simulated. If each region solves its local TEP regardless of interactions with its neighbours (scenario 1), the overall investment planning (and operational) costs go up as compared with those costs obtained by taking into account the coordination of local TEPs (scenarios 2 and 3). Although the centralised TEP with the coordination of planners provided the optimal results, information of the local planners must be gathered in the centre. The proposed collaborative DTEP algorithm provided the benchmark results as the centralised TEP while respecting the information privacy of the independent planners. The suggested initialisation strategy reduced the number of iterations of DTEP by 50%.

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6 Appendix

The disjunctive TEP model with $N-1$ security criterion is formulated by (15)–(37). The objective function (15) consists of the costs of installing new lines and expected operational costs Q. Set of decision variables Γ is expressed in (16). Constraints of normal operation and $N-1$ security criterion are, respectively, shown by (17)–(27) and (28)–(37). Budget constraints (18), nodal power balance constraints (19), and flow constraints of the existing and candidate lines (20)–(23) must be satisfied in both normal and contingency conditions. Adjustment capabilities of generating units to provide either preventive or corrective action in response to a contingency are expressed by (37). More details are provided in [1]

$$\begin{aligned} &\min_{\Gamma} F(\Gamma); \\ &F(\Gamma) = \sum_{l \in \Omega^{L+}} \tilde{I}_l x_l + E\left[Q\left(p_g,\ p_d^{LS},\ p_d^{LS,c}\right)\right]; \\ &Q = \sum_{g \in \Omega^G} C_g p_g + \sum_{d \in \Omega^D} C_d p_d^{LS} + \sum_{c \in \Omega^C} \sum_{d \in \Omega^D} C_d^c p_d^{LS,c} \end{aligned} \tag{15}$$

$$\Gamma = \left\{x_l,\ f_l,\ P_g,\ P_d^{LS},\ \theta_i,\ f_l^c,\ P_g^c,\ P_d^{LS,c},\ \theta_i^c\right\} \tag{16}$$

$$x_l = \{0,\ 1\};\quad \forall l \Omega^{L+} \tag{17}$$

$$\sum_{l \in \Omega^{L+}} \tilde{I}_l x_l, \le I^{max};\quad \forall l \Omega^{L+} \tag{18}$$

$$\sum_{g \in \Omega_i^G} P_g + \sum_{w \in \Omega_i^W} P_w - \sum_{l|s(l)=i} f_l + \sum_{l|r(l)=i} f_l = \sum_{d \in \Omega_i^D} \left(P_d - P_d^{LS}\right);\quad \forall i \tag{19}$$

$$f_l = B_l\left(\theta_i - \theta_j\right);\quad \forall l \in \Omega^L,\ s(l) = i,\ r(l) = j \tag{20}$$

$$-F_l^{max} \le f_l \le F_l^{max};\quad \forall l \in \Omega^L \tag{21}$$

$$\begin{aligned} &-(1-x_l) M \le f_l - B_l\left(\theta_i - \theta_j\right) \le (1-x_l) M \\ &\forall l \in \Omega^{L+},\ s(l) = i,\ r(l) = j \end{aligned} \tag{22}$$

$$-x_l F_l^{max} \le f_l \le x_l F_l^{max};\quad \forall l \in \Omega^{L+},\ s(l) = i,\ r(l) = j \tag{23}$$

$$0 \le P_g \le P_g^{max};\quad \forall g \tag{24}$$

$$0 \le P_d^{LS} \le P_d;\quad \forall d \tag{25}$$

$$-\pi \le \theta_i \le \pi;\quad \forall i \tag{26}$$

$$\theta_i = 0;\quad \forall i = \text{ref} \tag{27}$$

$$\sum_{g \in \Omega_i^G} P_g^c + \sum_{w \in \Omega_i^W} P_w - \sum_{l|s(l)=i} f_l^c + \sum_{l|r(l)=i} f_l^c = \sum_{d \in \Omega_i^D} \left(P_d - P_d^{LS,c}\right);\quad \forall i,\ C \tag{28}$$

$$f_l^c = A_l^c B_l\left(\theta_i^c - \theta_j^c\right);\quad \forall C,\ l \in \Omega^L,\ s(l) = i,\ r(l) = j \tag{29}$$

$$-F_l^{max} \le f_l^c \le F_l^{max};\quad \forall C,\ l \in \Omega^L \tag{30}$$

$$\begin{aligned} &-\left(1 - A_l^c x_l\right) M \le f_l^c - B_l\left(\theta_i^c - \theta_j^c\right) \le \left(1 - A_l^c x_l\right) M \\ &\forall C,\ l \in \Omega^{L+},\ s(l) = i,\ r(l) = j \end{aligned} \tag{31}$$

$$\begin{aligned} &-A_l^c x_l F_l^{max} \le f_l^c \le A_l^c x_l F_l^{max}; \\ &\forall C,\ l \in \Omega^{L+},\ s(l) = i,\ r(l) = j \end{aligned} \tag{32}$$

$$0 \le P_g^c \le P_g^{max};\quad \forall g,\ C \tag{33}$$

$$0 \le P_d^{LS,c} \le P_d;\quad \forall d,\ C \tag{34}$$

$$-\pi \le \theta_i^c \le \pi;\quad \forall i,\ C \tag{35}$$

$$\theta_i^c = 0;\quad \forall C,\ i = \text{ref}. \tag{36}$$

$$A_g^c\left(P_g - \Delta_g^{max}\right) \le P_g^c \le A_g^c\left(P_g + \Delta_g^{max}\right);\quad \forall g,\ C \tag{37}$$

The available production capacity of generating units (P_g^{max}), power demand (P_d), and generation of wind farms (P_w) are uncertain parameters in TEP. We use a data-driven approach, presented in [18], to model these uncertain parameters in TEP. Consider that the true distribution X is unknown. The confidence set D is defined in a way to minimise the tolerance level of the distance between the reference distribution X' and the true distribution X [18]. That is, $D = \{\forall X : d(X, X') \le \alpha\}$, where parameter α represents a tolerance level of the distance and $d(X, X')$ is the predefined probability distance between X and X'. A histogram of historical data is used as the reference distribution. The data domain is divided into N bins, and the probability distribution of each bin is determined. The reference distribution is $X' = (X_1', X_2', \dots, X_N')$. Generally, having more information on the true distribution leads to more accurate estimated distribution. Two probability metrics are selected. Norm one as the probability distance and $\alpha_1 = (N/2S)\log(2N/(1-\beta))$ as a tolerance level [30]. The confidence set is constructed as $D = \left\{X \in R^N : \sum_{n=1}^{N} |\chi_n - \chi_n'| \le \alpha_1\right\}$ for each uncertain parameter (i.e. P_g^{max}, P_d, and P_w), and the objective function is minimised subject to all constraints and under the worst-case distribution in D. Since the data-driven approach is not one of the contributions of this paper, we see [18] for more details.