Real-Time Optimal Charging for Lithium-Ion Batteries *via* Explicit Model Predictive Control

Ning Tian
University of Kansas
Lawrence, KS 66045, USA
Email: ning.tian@ku.edu

Huazhen Fang
University of Kansas
Lawrence, KS 66045, USA
Email: fang@ku.edu

Yebin Wang
Mitsubishi Electric Research Laboratories
Cambridge, MA 02139, USA
Email: yebinwang@ieee.org

***Abstract*—The rapidly growing use of lithium-ion batteries across various industries highlights the pressing issue of optimal charging control. The literature increasingly adopts model predictive control (MPC) to perform charging control, taking advantage of its capability of performing optimization under constraints. However, the computationally complex online constrained optimization involved in MPC often hinders real-time implementation. This paper is thus motivated to develop a new charging control algorithm based on explicit MPC (eMPC). Leveraging the merits of eMPC, the new algorithm can shift the constrained optimization to offline by precomputing explicit solutions to an optimal charging control problem and expressing the control law as piecewise functions. This drastically reduces not only the online computational costs in the control run but also the difficulty to code the algorithm. Numerical simulation results verify the utility of the proposed charging control algorithm, which can potentially meet the needs for real-time battery management running on embedded hardware.**

I. Introduction

Finding the best way to charge lithium-ion batteries (LiBs) has attracted sustained attention in the past two decades. Currently, the most popular industrial practice is the so-called constant-current-constant-voltage (CC/CV) charging [1]. It applies a constant current to charge a LiB cell until it reaches a threshold voltage and then enforces a constant voltage to charge the cell at a gradually diminishing current. Another often endorsed practice is pulse charging that feeds energy into a battery using current pulses [2]. These methods, however, usually involve some heuristic determination of charging parameters, which hence has motivated researchers to develop optimal charging protocols by combining physics-based LiB models and optimization to meet certain objectives concerning LiB health and/or charging time. Model predictive control (MPC), a constrained optimal control strategy, holds considerable promise here for two reasons. First, it can handle hard state and input constraints. This gives a leverage to guarantee satisfaction of health- or safety-related constraints necessary for LiB operation. Second, it can optimize different kinds of objective functions to meet different charging needs or considerations. As another benefit, its formulation well admits nonlinear systems, thus bearing applicability to different types of nonlinear LiB models.

A lead is taken in [3] with the development of minimum-time charging control by applying nonlinear MPC to a 1-D electrochemical model of LiBs. However, a barrier in the way of MPC-based charging is the high computational complexity that results from the numerical constrained optimization procedure. This can be more serious in the context of nonlinear electrochemical models. Significant research hence has been devoted to computationally efficient MPC charging control design. The study in [4], [5] considers nonlinear MPC for single particle model and exploits the differential flatness of Fick's law of diffusion to reduce computational load. Besides, model reduction is often used to simplify an electrochemical model and make it amenable for the design of efficient MPC. For example, the approach in [6] linearizes a nonlinear electrochemical model successively along a reference SoC trajectory. Other examples, e.g., [7] develops input-output approximations of a pseudo 2-D (P2D) model such as step response models and autoregressive exogenous models, so that application of MPC to them causes less computation.

Equivalent circuit models (ECMs) represent another appealing choice for MPC-based charging control due to much less computation required. An early study is in [8], which yet adopts a genetic algorithm as the optimizer despite its costly computation. There is a consensus today that it is still a critical need to design fast MPC for ECMs for the sake of practical implementation. To this end, the literature derives either simpler models or computationally frugal control frameworks. The method in [9] proposes to identify a time-series model recursively as an input-output approximation of the Thevenin model and takes advantage of its simplicity to achieve efficient generalized predictive control. Optimal charging based on the Thevenin model is formulated as a standard linear MPC problem in [10] that eases computation. Similarly, a linear-time-varying MPC method is proposed in [11]. A hierarchical MPC design in [12] features the generation of reference current profiles at a slow time scale and the reference tracking at a faster time scale, which lowers the cost of computation.

The above survey highlights the main advances in the development of computationally efficient MPC charging control. However, the existing methods still demand a relatively large amount of online computation as required by solving a constrained optimization problem, which is rarely available for the hardware on which a battery management system runs. A central challenge then lies in how to offload the primary part of the computational effort offline and run a

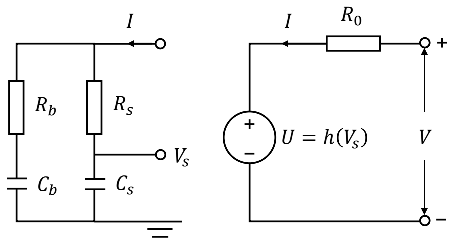


Figure 1: The nonlinear double-capacitor model.

lean controller online. The explicit MPC (eMPC) strategy, pioneered in [13]–[15], is set to address this challenge. It pre-computes the control law offline by deriving explicit solutions to an MPC problem. The control law is composed of piecewise affine functions of the system's current state, which can be run online through only straightforward arithmetic operations. The advantages of eMPC are remarkable. First, it can achieve MPC functionality with microsecond-millisecond online computational efficiency. Second, it is easy to code and executable on cheap embedded control hardware. Therefore, eMPC can hopefully provide a solution to bridge the gaps in computation and execution facing the current breed of MPC charging methods.

The contribution of this paper is twofold. 1) This work presents the first study of eMPC-based real-time optimal charging control. It considers the nonlinear double-capacitor (NDC) model [16] and formulates a general nonlinear MPC charging problem, which is then simplified as a combination of approximate linear MPC problems by piecewise linear approximation. On such a basis, synthesis of eMPC-based control is performed to build an optimal charging control algorithm, which constructs on a set of piecewise affine functions over a parameter space. 2) The proposed charging control algorithm is validated through simulation.

The paper is organized as follows. Section II introduces the NDC model along with the charging-related constraints. Section III contains the statement of an MPC-based health-aware charging control problem, the piecewise linearization of the model, and the formulation of eMPC-based charging control. Section IV evaluates the proposed charging control law using simulation. Finally, Section V gathers concluding remarks and discussions of future research.

II. The NDC Model for LiBs

This section describes the NDC model for LiBs and the constraints to be enforced during charging.

A. Model Description

Developed in [16], the NDC model is shown in Figure 1, which includes two main parts. The first part (left) includes two capacitors in parallel, C_b and C_s, each serially connected with a resistor, R_b and R_s, respectively. The two capacitors play the role of an electrode, providing storage for electric charge. The parallel connection between them allows the distribution and migration of charge within the electrode to be simulated. To be specific, the R_s-C_s circuit conceptually corresponds to the electrode's surface region; the R_b-C_b circuit represents the electrode's bulk inner part analogously. It generally holds that $C_b \gg C_s$ and that $R_b \gg R_s$. The second part (right) has a voltage source $U = h(V_s)$, where V_s is the voltage across C_s. Hence, U is similar to an open-circuit voltage but depends on V_s rather than SoC in a conventional sense. It also contains R_0, which is the internal resistance. The NDC model can well capture a LiB's electrical behavior, as is shown in [16], with its construction and justification detailed therein.

The NDC model's dynamics is governed by the following state-space equations:

$$\begin{cases} \begin{bmatrix} \dot{V}_b(t) \\ \dot{V}_s(t) \end{bmatrix} = A \begin{bmatrix} V_b(t) \\ V_s(t) \end{bmatrix} + BI(t), & \text{(1a)} \\ V(t) = h(V_s(t)) + R_0 I(t), & \text{(1b)} \end{cases}$$

where V_b is the voltage across C_b, I is the applied current with $I > 0$ for charging and $I < 0$ for discharging, and

$$A = \begin{bmatrix} -\frac{1}{C_b(R_b+R_s)} & \frac{1}{C_b(R_b+R_s)} \\ \frac{1}{C_s(R_b+R_s)} & -\frac{1}{C_s(R_b+R_s)} \end{bmatrix}, \; B = \begin{bmatrix} \frac{R_s}{C_b(R_b+R_s)} \\ \frac{R_b}{C_s(R_b+R_s)} \end{bmatrix}.$$

Here, $h(V_s)$ can be parameterized as a fifth-order polynomial:

$$h(V_s) = \sum_{i=0}^{5} \alpha_i V_s^i,$$

where α_i for $i = 0, 1, \dots, 5$ are coefficients. Note that V_b and V_s should be limited to a range. For both, the lower bound of the range is set to be $V_{s,\min} = 0$ V, and the upper bound $V_{s,\max} = 1$ V for simplicity. In other words, $V_b = V_s = 0$ V when the LiB is fully depleted (SoC $= 0\%$), and $V_b = V_s = 1$ V when it is fully charged (SoC $= 100\%$). Furthermore, the following shows the relation between SoC and V_b and V_s:

$$\text{SoC} = \frac{C_b V_b + C_s V_s}{C_b + C_s} \times 100\%,$$

where $C_b + C_s$ is equal to to a battery's total capacity, and $C_b V_b + C_s V_s$ the available capacity.

B. Constraints

To ensure health-conscious and safe charging, some constraints must be imposed during a charging process. First, SoC must lie between 0% and 100% to avoid overcharging, i.e.,

$$0 \le \text{SoC} \le 1. \tag{2}$$

The charging current and terminal voltage must also be subject to limitations to circumvent safety issues, implying

$$I_{\min} \le I \le I_{\max}, \tag{3}$$
$$V_{\min} \le V \le V_{\max}. \tag{4}$$

In addition, V_b and V_s should be bounded as aforementioned. Given that $V_s \ge V_b$ always holds during charging if the LiB is at equilibrium initially, one only needs to limit V_s by

$$0 \le V_s \le 1. \tag{5}$$

The final constraint to add concerns $V_s - V_b$. It is seen that $V_s - V_b$ drives the migration of charge from C_s to C_b during charging. The study in [16] points out that this variable is comparable to the lithium-ion concentration gradients within an electrode. The gradients are a cause for internal stress buildup, heating, formation of solid-electrolyte interphase film, and polarization, which eventually will degrade the capacity, cycle life and thermal stability of LiBs [17]–[19]. As such, too steep gradients should be circumvented during charging, implying a necessity for restricting $V_s - V_b$. Besides, the restriction should be increasingly stricter as SoC grows. Therefore, the constraint on $V_s - V_b$ is designed as an affine decreasing function of SoC:

$$V_s - V_b \leq \beta_1 \mathrm{SoC} + \beta_2,$$

where β_1 and β_2 are two coefficients. It can be rewritten as

$$\eta \leq \beta_2, \tag{6}$$

where

$$\eta = -\frac{C_b + \beta_1 C_b + C_s}{C_b + C_s} V_b + \frac{C_b + C_s - \beta_1 C_s}{C_b + C_s} V_s.$$

III. Health-Aware Battery Charging via Explicit Model Predictive Control

This section states an MPC-based charging control problem and then develops an eMPC-based charging control law.

A. Health-Aware Charging Problem Formulation

To proceed, let us define

$$x = \begin{bmatrix} V_b & V_s & I \end{bmatrix}^\top,$$
$$y = \begin{bmatrix} \mathrm{SoC} & V_s & I & V & \eta \end{bmatrix}^\top.$$

Considering x as a state vector and y as an output vector, one can derive from (1) an extended model as follows

$$\begin{cases} x_{k+1} = \mathcal{A} x_k + \mathcal{B} u_k, & \text{(7a)} \\ y_k = g(x_k), & \text{(7b)} \end{cases}$$

where $u_k = I_{k+1} - I_k$,

$$\mathcal{A} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ 0 & 1 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and $g(\cdot)$ represents the nonlinear mapping from x to y. In above, $\tilde{A}$ and $\tilde{B}$ are discrete counterparts of A and B. Besides, the constraints in (2)-(6) can be put in a compact form:

$$y_{\min} \leq y \leq y_{\max}.$$

Thus, the health-aware charging problem can be formulated as the following nonlinear MPC problem at time step k:

$$\min_z \sum_{k=0}^{N-1} \frac{1}{2} (\mathrm{SoC}_k - \breve{r})^\top Q (\mathrm{SoC}_k - \breve{r}) + \frac{1}{2} \Delta u_k^\top R \Delta u_k, \tag{8}$$
$$\text{s.t. } (7),\ x_0 = \breve{x},\ u_{-1} = \breve{u},$$
$$u_k = u_{k-1} + \Delta u_k, k = 0, \ldots, N-1,$$
$$\Delta u_k = 0, k = N_u, \ldots, N-1,$$
$$y_{\min} \leq y_k \leq y_{\max}, k = 0, \ldots, N_c - 1,$$

where $z = \begin{bmatrix} \Delta u_0 & \ldots & \Delta u_{N_u - 1} \end{bmatrix}^\top$ is the future input sequence to be optimized, $\breve{x}$ the model state vector at current time instant, $\breve{u}$ the control input applied in the previous time interval, and $\breve{r}$ the target SoC as reference, respectively. Besides, N represents the prediction horizon, N_u the input horizon, N_c the constraint horizon, $Q = Q^\top \geq 0$ and $R = R^\top > 0$. The problem (8) can be solved using nonlinear programming at each time step. When the optimal solution z^* is found, i.e.,

$$z^* = \begin{bmatrix} \Delta u_0^* & \ldots & \Delta u_{N_u - 1}^* \end{bmatrix}^\top, \tag{9}$$

the charging current then is given by its first element Δu_0^* as

$$I_1 = \Delta u_0^* + \breve{u} + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \breve{x}. \tag{10}$$

After this, the entire optimization problem is resolved at the next time step with a new starting point.

The problem statement in (8) gives a complete description of MPC-based charging control based on the NDC model. However, a relatively large deal of online computation is needed to address the nonlinear programming at each time step, let alone a need for a powerful computing workhorse for the execution of complex code. This would limit its applicability to embedded charging control. Hence, it is our aim to address (8) *via* eMPC for easy online computation.

As eMPC is designed for linear systems, one must first linearize the model in (8). Here, the linearization is concerned only with the nonlinear mapping $h(V_s)$. Note that a single linear function is not accurate enough to approximate it, which motivates us to adopt multi-segment linear approximation to enhance the accuracy of approximation.

Proceeding to show this idea, consider linearizing $h(V_s)$ around a general fixed operating point, V_s^{op}, as a first step:

$$h(V_s) \approx \gamma_1 V_s + \gamma_2, \tag{11}$$

where

$$\gamma_1 = \left. \frac{\partial h(V_s)}{\partial V_s} \right|_{V_s = V_s^{\mathrm{op}}},\ \gamma_2 = h(V_s^{\mathrm{op}}) - \gamma_1 V_s^{\mathrm{op}}.$$

By (11), one can modify (7b) into a linear form as follows:

$$y_k = \mathcal{C} x_k + \mathcal{D}, \tag{12}$$

where

$$\mathcal{C} = \begin{bmatrix} \frac{C_b}{C_b + C_s} & \frac{C_s}{C_b + C_s} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \gamma_1 & R_0 \\ -\frac{C_b + \beta_1 C_b + C_s}{C_b + C_s} & \frac{C_b + C_s - \beta_1 C_s}{C_b + C_s} & 0 \end{bmatrix}, \ \mathcal{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_2 \\ 0 \end{bmatrix}.$$

Accordingly, the original nonlinear MPC problem (8) would

Table I: Battery model parameters.

Name	C_b/F	C_s/F	R_b/Ω	R_s/Ω	R_0/Ω	α_0	α_1	α_2	α_3	α_4	α_5	β_1	β_2
Value	9,878	921.5	0.04	0	0.0823	3.2	3.4656	-13.4144	26.4186	-22.4962	7.0264	-0.05	0.125

Table II: Linearization setting.

No.	V_s	V_s^{op}	γ_1	γ_2	N_{CR}
I	$[0.00, 0.50]$	0.3	0.4050	3.4591	13
II	$[0.50, 0.70]$	0.6	1.0168	3.1774	15
III	$[0.70, 0.90]$	0.8	1.0441	3.1663	16
IV	$[0.90, 0.93]$	0.92	0.9634	3.2347	15
V	$[0.93, 0.97]$	0.96	0.9785	3.2204	15
VI	$[0.97, 1.00]$	0.99	1.0192	3.1807	15

reduce to a linear one, which can be expressed as

$$\min_{z} \sum_{k=0}^{N-1} \frac{1}{2}(\mathrm{SoC}_k - \breve{r})^\top Q(\mathrm{SoC}_k - \breve{r}) + \frac{1}{2}\Delta u_k^\top R \Delta u_k, \quad (13)$$
$$\text{s.t. (7a)},\ (12),\ x_0 = \breve{x},\ u_{-1} = \breve{u},$$
$$u_k = u_{k-1} + \Delta u_k, k = 0, \ldots, N-1,$$
$$\Delta u_k = 0, k = N_u, \ldots, N-1,$$
$$y_{\min} \le y_k \le y_{\max}, k = 0, \ldots, N_c - 1.$$

Next is to extend this procedure to multi-segment approximation. Specifically, one can select multiple linearization points, denoting them as $V_{s,i}^{\text{op}}$ for $i = 1, 2, \cdots, N_{\text{op}}$. The range of V_s then is subdivided into N_{op} partitions. The same procedure as in (11)-(13) can be repeated for each $V_{s,i}^{\text{op}}$. Finally, a set of linear MPC subproblems akin to (13) will be obtained.

B. Charging Control Based on eMPC

Consider the linear MPC charging control problem (13), and define the following vector of parameters:

$$\theta = \begin{bmatrix} \breve{x}^\top & \breve{r}^\top & \breve{u}^\top \end{bmatrix}^\top \in \mathbb{R}^m.$$

In light of the linearity in (7a) and (12), (13) can be recast as a convex quadratic program (QP) taking a standard form:

$$\min_{z} \frac{1}{2} z^\top \Sigma z + (F\theta)^\top z, \quad (14a)$$
$$\text{s.t. } Gz \le S\theta + W, \quad (14b)$$

where $z \in \mathbb{R}^n$, $\Sigma > 0 \in \mathbb{R}^{n\times n}$ and $F \in \mathbb{R}^{n\times m}$. The QP problem (14) is also a multi-parametric QP (mpQP), as the characterization of its solution fundamentally involves the parameter vector θ. The solution can be described as a set-valued function $Z^*(\theta) : \Theta \to 2^{\mathbb{R}^n}$, where $Z^*(\theta)$ is a set of optimizer functions $z^*(\theta)$, Θ the set of feasible parameters, and $2^{\mathbb{R}^n}$ the set of subsets of $\mathbb{R}^n$. It has been proven in [13] that Θ can be partitioned into convex polyhedral regions, also referred to as critical regions and denoted as CR_i for $i = 1, 2, \cdots, N_{CR}$. For critical region CR_i, we have

$$z^*(\theta) = K^i\theta + g^i,\ \forall\theta \in CR_i.$$

In other words, $Z^*(\theta)$ is piecewise affine and continuous over Θ. As can be observed from (10), the charging control laws are also piecewise affine functions of θ, i.e., the present model state, the target SoC and the input in last time instant.

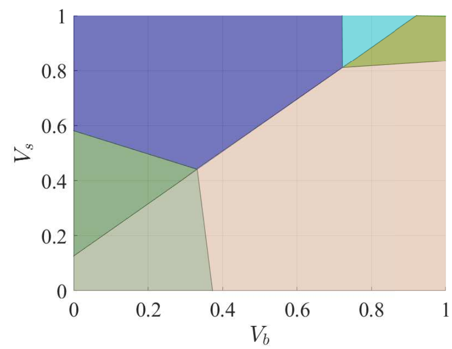


Figure 2: V_b-V_s plane with $I = 2$ A, $\breve{r} = 1$, $\breve{u} = 0$ A.

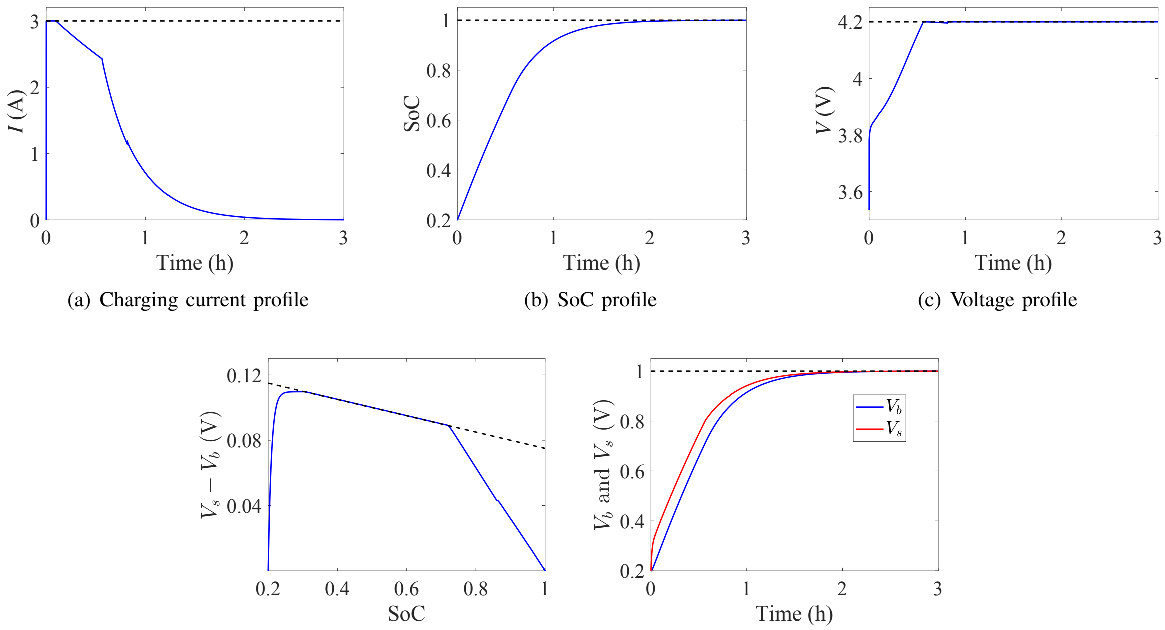
Putting together the above developments, the eMPC-based charging control algorithm is summarized as follows:

- *Offline mpQP computation*
  - Consider the first linear model
    - Select a parameter θ_0 and determine CR_0
    - Solve (14) to obtain $z^*(\theta) = K^0\theta + g^0$
    - Partition the parameter space Θ outside CR_0 and determine $z^*(\theta)$ for a new critical region
    - Repeat until all space in Θ has been explored
    - Store in a table all (K^i, g^i) for $i = 1, 2, \cdots, N_{CR}$
  - Repeat the procedure for all the other linear models
- *Online eMPC-controlled charging*
  - Determine the governing linear model at time k
  - Search for CR_j in (K, g) table such that $\theta_k \in CR_j$
  - Determine $z^*(\theta) = K^j\theta_k + g^j$ and I_{k+1} as in (10)
  - Repeat the charging control procedure until the condition for charging completion is satisfied

The above assumes state-feedback design. As in practice internal states cannot be measured, output-feedback control is more desired. To attain this end, one can just use a nonlinear Kalman filter [20] or some other observer to perform real-time state estimation. Then, the charging control setup is a closed loop between a LiB, the Kalman filter and eMPC controller. The design can also be applied to a more sophisticated NDC model like an SoC-dependent R_0 in [21] by linearization.

IV. Validation Results and Discussions

This section presents simulation results to validate the proposed eMPC-based charging control algorithm. Given a LiB cell governed by the NDC model with the parameters shown in Table I, consider its optimal charging following the problem formulation in (8). Suppose that the objective is to



(a) Charging current profile (b) SoC profile (c) Voltage profile

(d) $V_s - V_b$ (e) V_b and V_s

Figure 3: Simulation of charging control based on the proposed algorithm.

raise the SoC from 20% to 100%. The following constraints are imposed to ensure health and safety:

$$V_s \leq 1 \text{ V}, \; 0 \text{ A} \leq I \leq 3 \text{ A}, \; V \leq 4.2 \text{ V}, \; \eta \leq 0.125 \text{ V}.$$

The weighting matrices are set to be $Q = 1$ and $R = 0.1$. The sampling period is 1 s. The prediction horizon is $N = 10$, the control horizon is $N_u = 1$, and the constraint horizon for η is $N_c = 2$ and for other constraints $N_c = 1$.

The nonlinear MPC problem (8) is then broken down into six linear MPC problems through multi-segment approximation. Table II summarizes the linearization points of V_s, segment ranges, and coefficients of the linear approximation of $h(V_s)$. Each linear MPC problem is characterized in the form of (13). One can then compute the explicit solution to every problem by conveniently resorting to the MATLAB® MPC Toolbox [22], which leads to six eMPCs that combine to make up the charging control algorithm. Note that the eMPCs have different numbers of critical region partitions, as they are based on different models and operating ranges. The number of critical regions, N_{CR}, for each component eMPC is shown in Table II. To give the reader a flavor of the control law, let us consider the second eMPC. It involves 15 critical regions. Figure 2 shows the critical region partitions on the two-dimensional V_b-V_s plane when $I = 2$ A, $\breve{r} = 1$, and $\breve{u} = 0$ A. For the right-bottom critical region (marked in orange), the charging control law is piecewise affine as follows:

$$I_{k+1} = \begin{bmatrix} -26.42 \\ 26.42 \\ 0.03 \\ 0 \\ 0 \end{bmatrix}^\top \begin{bmatrix} V_{b,k} \\ V_{s,k} \\ I_k \\ \breve{r} \\ u_{k-1} \end{bmatrix},$$

which is easy to code and implement. The control laws for other critical regions are also piecewise affine and omitted here due to limited space.

Running the simulation, the eMPC charging control algorithm yields a current profile as shown in Figure 3(a). If compared with the CC/CV charging, this design introduces an intermediate stage, which is partially attributed to the constraint regarding η. Figure 3(d) displays $V_s - V_b$ during charging, which never exceeds the pre-set limit (dashed line). Further, Figure 3(e) shows the profiles of V_s and V_b, which demonstrate a gradually slowing pace of growth to protect the LiB. The above results indicate the amenability of the proposed eMPC charging control algorithm to practical execution and its promise for health-aware charging.

V. Conclusion

Charging control is essential for the health and safety of LiBs. This paper exploited eMPC to enhance charging control design, which inherits all the merits of MPC but enables highly efficient computation. Our design started with formulating an MPC charging control problem based on the NDC model. As the model is nonlinear, multi-segment linearization was developed to approximate the original MPC problem by a combination of multiple linear MPC problems. The solutions to the linear MPCs were computed offline and explicitly expressed as piecewise affine functions, which made up an eMPC charging control algorithm. Contrasting the previous counterparts, the obtained charging control algorithm is tremendously easy to code and fast to run online, potentially applicable to embedded computing hardware. The simulation results verified the efficacy of the proposed algorithm. Our future work will include extensive experimental validation of the charging algorithm.

References

[1] A. A.-H. Hussein and I. Batarseh, “A review of charging algorithms for nickel and lithium battery chargers,” *IEEE Transactions on Vehicular Technology*, vol. 60, no. 3, pp. 830–838, 2011.

[2] H. Fang, C. Depcik, and V. Lvovich, “Optimal pulse-modulated lithium-ion battery charging: Algorithms and simulation,” *Journal of Energy Storage*, vol. 15, pp. 359–367, 2018.

[3] R. Klein, N. A. Chaturvedi, J. Christensen, J. Ahmed, R. Findeisen, and A. Kojic, “Optimal charging strategies in lithium-ion battery,” in *Proceedings of American Control Conference*. IEEE, 2011, pp. 382–387.

[4] J. Liu, G. Li, and H. K. Fathy, “A computationally efficient approach for optimizing lithium-ion battery charging,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 138, no. 2, p. 021009, 2016.

[5] ——, “An extended differential flatness approach for the health-conscious nonlinear model predictive control of lithium-ion batteries,” *IEEE Transactions on Control Systems Technology*, vol. 25, no. 5, pp. 1882–1889, 2017.

[6] C. Zou, C. Manzie, and D. Nešić, “Model predictive control for lithium-ion battery optimal charging,” *IEEE/ASME Transactions on Mechatronics*, vol. 23, no. 2, pp. 947–957, 2018.

[7] M. Torchio, N. A. Wolff, D. M. Raimondo, L. Magni, U. Krewer, R. B. Gopaluni, J. A. Paulson, and R. D. Braatz, “Real-time model predictive control for the optimal charging of a lithium-ion battery,” in *Proceedings of American Control Conference*. IEEE, 2015, pp. 4536–4541.

[8] J. Yan, G. Xu, H. Qian, Y. Xu, and Z. Song, “Model predictive control-based fast charging for vehicular batteries,” *Energies*, vol. 4, no. 8, pp. 1178–1196, 2011.

[9] K. Liu, K. Li, and C. Zhang, “Constrained generalized predictive control of battery charging process based on a coupled thermoelectric model,” *Journal of Power Sources*, vol. 347, pp. 145–158, 2017.

[10] M. A. Xavier and M. S. Trimboli, “Lithium-ion battery cell-level control using constrained model predictive control and equivalent circuit models,” *Journal of Power Sources*, vol. 285, pp. 374–384, 2015.

[11] C. Zou, X. Hu, Z. Wei, and X. Tang, “Electrothermal dynamics-conscious lithium-ion battery cell-level charging management via state-monitored predictive control,” *Energy*, vol. 141, pp. 250–259, 2017.

[12] Q. Ouyang, J. Chen, J. Zheng, and H. Fang, “Optimal multi-objective charging for lithium-ion battery packs: A hierarchical control approach,” *IEEE Transactions on Industrial Informatics*, 2018.

[13] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, “The explicit linear quadratic regulator for constrained systems,” *Automatica*, vol. 38, no. 1, pp. 3–20, 2002.

[14] A. Alessio and A. Bemporad, “A survey on explicit model predictive control,” in *Nonlinear Model Predictive Control*. Springer, 2009, pp. 345–369.

[15] A. Bemporad *et al.*, “A multiparametric quadratic programming algorithm with polyhedral computations based on nonnegative least squares,” *IEEE Trans. Automat. Contr.*, vol. 60, no. 11, pp. 2892–2903, 2015.

[16] N. Tian, H. Fang, and J. Chen, “A new nonlinear double-capacitor model for rechargeable batteries,” in *Proceedings of the 44th Annual Conference of the IEEE Industrial Electronics Society*. IEEE, 2018, pp. 1613–1618.

[17] M. B. Pinson and M. Z. Bazant, “Theory of SEI formation in rechargeable batteries: Capacity fade, accelerated aging and lifetime prediction,” *Journal of the Electrochemical Society*, vol. 160, no. 2, pp. A243–A250, 2013.

[18] W. H. Woodford IV, “Electrochemical shock: Mechanical degradation of ion-intercalation materials,” Ph.D. dissertation, Massachusetts Institute of Technology, 2013.

[19] T. M. Bandhauer, S. Garimella, and T. F. Fuller, “A critical review of thermal issues in lithium-ion batteries,” *Journal of the Electrochemical Society*, vol. 158, no. 3, pp. R1–R25, 2011.

[20] H. Fang, Y. Wang, and J. Chen, “Health-aware and user-involved battery charging management for electric vehicles: Linear quadratic strategies,” *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 911–923, 2017.

[21] N. Tian, Y. Wang, J. Chen, and H. Fang, “On parameter identification of an equivalent circuit model for lithium-ion batteries,” in *2017 IEEE Conference on Control Technology and Applications (CCTA)*. IEEE, 2017, pp. 187–192.

[22] A. Bemporad, N. L. Ricker, and M. Morari, “Model Predictive Control Toolbox User’s Guide R2018b,” *The MathWorks*, 2018.