Observer-Based Leader-Follower Tracking Control for High-Order Multi-Agent Systems with Limited Measurement Information

Chuan Yan and Huazhen Fang

Abstract— **This paper investigates high-order leader-follower tracking by a multi-agent system (MAS) when only very limited measurement information is available to an agent. We specifically consider the setting where an agent, either leader or follower, only has its first state measured. To address this challenge, we propose to use observers to reconstruct unmeasured quantities and perform observer-based control. We develop a series of novel observers that can allow a follower to estimate the leader's states, even when they cannot communicate with each other, and all of its own unmeasured states. We rigorously prove the convergence of these observers and the resultant distributed tracking control. A simulation result further illustrates the effectiveness of the proposed design.**

I. Introduction

MAS-based cooperative autonomy is finding ever-increasing application across a variety of sectors. This has driven a surge of research interest on distributed cooperative control for different MAS tasks, including consensus, leader-follower tracking, synchronization, rendezvous, flocking, and coverage control [1–8]. Most of the current literature in this vibrant area considers agents governed by first- or second-order models. Despite their utility, such low-order models are inadequate for characterizing more complex agents that demonstrate high-order dynamics. Moreover, it is known as a non-trivial problem to extend a first- or second-order cooperative control design to high-order systems. Recent years hence have witnessed a growing amount of work devoted to control synthesis for high-order MAS cooperation [9].

In the literature, high-order leader-follower tracking is emerging as the problem of great interest and importance. It is in [10] that a basic form of this problem is introduced, which assumes that the leader agent continually broadcasts its state information to all the followers. A consensus-based control algorithm is then developed to make the followers achieve consensus with, i.e., track, the leader. The study [11] considers a general setting where only a subset of the followers can receive information from the leader, proposing a leader-follower tracking control method. The analysis shows that followers with small degrees must be informed by the leader to ensure tracking convergence. High-order nonlinear agents constitute a stronger challenge for leader-follower tracking. A study of this problem is offered in [12], which adaptively estimates the nonlinearity involved in an agent's dynamics using neural networks and offsets it when applying control. In [13], a finite-time tracking control approach is developed for a high-order nonlinear MAS subject to actuator saturation. The problem of finite-time leader-follower higher-order tracking with mismatched disturbances is studied in [14].

C. Yan and H. Fang are with the Department of Mechanical Engineering, University of Kansas, Lawrence, KS, 66045 USA {{cyan, fang}@ku.edu}.

It is noted that the above studies about high-order MAS tracking generally assume that a large amount of information is available to a follower to make control decisions. For example, a follower agent must know all of its own states, all of the states of its neighbors, and if connected with the leader, all of the leader's states [10–13]. This assumption is highly demanding from a real-world perspective, due to the possible unavailability of relevant sensors and limited communication capacity [11].

In an attempt to address this limitation, we propose this work to investigate the tracking problem when only the first state of the high-order leader and followers is measured. Our contribution is the development of an observer-based tracking control design. In this regard, we propose a set of distributed observers, which, namely, allows a follower to estimate the leader's input, the leader's states, and its own states. The observers make up for the limited information and combine with a tracking controller. We then prove the convergence of tracking under the proposed controller. It is worth pointing out that this work is related to several studies about distributed tracking control based on state-space models, e.g., [15–18]. In particular, state observers are used in [15–17] to achieve output-feedback-based tracking. These studies, however, require either that the leader is input-free or that the followers have knowledge of the leader's input. By comparison, our work removes this restrictive requirement through the novel distributed observer design.

The notation used throughout this paper is standard. The n-dimensional Euclidean space is denoted as $\mathbb{R}^n$. For a vector, $\|\cdot\|_1$ denotes the 1-norm. We denote 2-norm by $\|\cdot\|$. We let $\mathbf{1}$ denote a column vector with all elements equal to 1. The notation $\mathrm{diag}(\ldots)$ and $\det(\cdot)$ represent a block-diagonal matrix and the determinant of a matrix, respectively. The eigenvalues of a $N \times N$ matrix are $\lambda_i(\cdot)$ for $i = 1, 2, \ldots, N$. The minimum and maximum eigenvalues of a real, symmetric matrix are denoted as $\underline{\lambda}(\cdot)$ and $\bar{\lambda}(\cdot)$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

We use a graph to describe the topological structure for information exchange among the leader and followers. First, consider a network composed of N independent followers. The interaction topology is modeled as an undirected graph. The follower graph is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set that contains unordered pairs of nodes. A path is a sequence of connected edges in a graph. The follower graph is connected if there is a path between every pair of vertices. The neighbor set of agent i is denoted as $\mathcal{N}_i$, which includes all the agents in communication with it. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$ where $i \neq j$. For the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, $l_{ij} = -a_{ij}$ if $i \neq j$ and $l_{ii} = \sum_{k\in\mathcal{N}_i} a_{ik}$. The leader is numbered as node 0 and can send information to its neighboring followers. Then, we have a graph $\bar{\mathcal{G}}$, which consists of graph $\mathcal{G}$, node 0 and edges from it to its neighbors. The leader is globally reachable in $\bar{\mathcal{G}}$ if there is a path in graph $\bar{\mathcal{G}}$ from every vertex i to vertex 0. In order to express the graph $\bar{\mathcal{G}}$ more precisely, we denote the leader adjacency matrix associated with $\bar{\mathcal{G}}$ by $B = \mathrm{diag}(b_1, \ldots, b_N)$, where $b_i > 0$ if the leader is a neighbor of agent i and $b_i = 0$ otherwise.

II. Leader-Follower Tracking

In this section, we first formulate the tracking problem for an MAS with high-order dynamics and then propose an observer-based tracking control strategy. Finally, we characterize the convergence properties of the proposed strategy.

A. Problem Formulation

Consider an MAS composed of $N + 1$ agents, among which agent 0 is the leader and and agents numbered from 1 to N are followers. The dynamics of an agent is lth-order ($l \geq 3$) and governed by

$$\begin{cases} \dot{x}_{i,m} = x_{i,m+1}, \quad m = 1, 2, \ldots, l-1, \\ \dot{x}_{i,m} = u_i, \quad m = l, \ i = 0, 1, \ldots, N, \end{cases} \tag{1}$$

where $x_{i,m} \in \mathbb{R}$ is the mth state of agent i, and u_i the input. The objective here is to design a distributed control law u_i such that follower i for $i = 1, 2, \ldots, N$ can convergently track the leader with $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for $m = 1, 2, \ldots, l$.

Here, we assume that only $x_{i,1}$ can be measured for agent i for $i = 0, 1, \ldots, N$. That is, only the first state of an agent is measured, regardless of whether it is the leader or a follower. This setting limits the information available to the MAS, which presents a significant challenge for the design of a distributed tracking controller.

B. Proposed Algorithm

This section develops an observer-based control strategy to enable effective tracking in the above setting. To begin with, we consider the following controller structure for i:

$$\begin{aligned} u_i = &- k_1 \left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_{i,1} - x_{j,1}) + b_i(x_{i,1} - x_{0,1}) \right] \\ &- \sum_{m=2}^{l} k_m(\hat{x}_{i,m} - \hat{x}_{0,i,m}) + \hat{u}_{0,i}, \end{aligned} \tag{2}$$

where k_m for $m = 1, 2, \ldots, l$ are gain parameters, $\hat{x}_{0,i,m}$ and $\hat{u}_{0,i}$ are follower i's estimates of the leader's state $x_{0,m}$ and input u_0, respectively, and $\hat{x}_{i,m}$ is follower's estimate of its own state $x_{i,m}$. The motivation behind (2) is to drive follower i toward its neighbors and the leader simultaneously, and when all the followers do this, they can hopefully track the leader in a collective manner. Next, we design the observers to obtain the estimates needed in (2).

An input observer is first introduced to enable follower i to estimate u_0, which is given by

$$\begin{aligned} \dot{\hat{u}}_{0,i} = &- \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) - b_i(\hat{u}_{0,i} - u_0) \\ &- d_i \cdot \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right], \end{aligned} \tag{3a}$$

$$\dot{d}_i = \tau_i \left| \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right|, \tag{3b}$$

where d_i is an adaptive gain, and τ_i is a positive scalar. Here, (3a) is meant to enable distributed and collective estimation of u_0 among the followers, and (3b) provides a mechanism for adaptively tuning the gain d_i.

The next observer is designed to allow follower i to estimate $x_{0,m}$ for $m = 2, 3, \ldots, l$:

$$\begin{aligned} \dot{z}_{0,i,2} = &- b_i c_{0,2} z_{0,i,2} - b_i^2 c_{0,2}^2 x_{0,1} \\ &- c_{0,2} \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) + \hat{x}_{0,i,3}, \end{aligned} \tag{4a}$$

$$\hat{x}_{0,i,2} = z_{0,i,2} + b_i c_{0,2} x_{0,1}, \tag{4b}$$

$$\dot{z}_{0,i,m} = - c_{0,m} z_{0,i,m} - c_{0,m}^2 \hat{x}_{0,i,m-1} + \hat{x}_{0,i,m+1}, \tag{4c}$$

$$\hat{x}_{0,i,m} = z_{0,i,m} + c_{0,m}\hat{x}_{0,i,m-1}, \quad m = 3, 4, \ldots, l-1, \tag{4d}$$

$$\dot{z}_{0,i,l} = - c_{0,l} z_{0,i,l} - c_{0,l}^2 \hat{x}_{0,i,l-1} + \hat{u}_{0,i}, \tag{4e}$$

$$\hat{x}_{0,i,l} = z_{0,i,l} + c_{0,l}\hat{x}_{0,i,l-1}, \tag{4f}$$

where $z_{0,i,m}$ and $c_{0,m}$ for $m = 2, 3, \ldots, l$ are this observer's internal states and gain parameters, respectively. The development of (4) is inspired by [19], in which a centralized disturbance observer is designed for a single plant. Here, we transform the original design and introduce the above observer with a distributed, coupled structure suitable for the considered MAS setting.

Finally, we propose a follower observer such that follower i can estimate its own states $x_{i,m}$ for $m = 2, 3, \ldots, l$:

$$\dot{z}_{i,2} = -r_2 z_{i,2} - r_2^2 x_{i,1} + \hat{x}_{i,3}, \tag{5a}$$

$$\hat{x}_{i,2} = z_{i,2} + r_2 x_{i,1}, \tag{5b}$$

$$\dot{z}_{i,m} = -r_m z_{i,m} - r_m^2 \hat{x}_{i,m-1} + \hat{x}_{i,m+1}, \tag{5c}$$

$$\hat{x}_{i,m} = z_{i,m} + r_m \hat{x}_{i,m-1}, \ m = 3, 4, \ldots, l-1, \tag{5d}$$

$$\dot{z}_{i,l} = -r_l z_{i,l} - r_l^2 \hat{x}_{i,l-1} + u_i, \tag{5e}$$

$$\hat{x}_{i,l} = z_{i,l} + r_l \hat{x}_{i,l-1}, \tag{5f}$$

where $z_{i,m}$ and $r_{i,m}$ for $m = 2, 3, \ldots, l$ are this observer's internal states and gain parameters.

Putting together (2)-(5), we can obtain a distributed observer-based control algorithm for the considered problem of high-order leader-follower tracking. Its convergence is analyzed next.

C. Stability Analysis

This section characterizes the convergence property for the algorithm proposed above. Before proceeding further, we make the following assumption:

Assumption 1: The leader's input $u_0 \in \mathcal{C}^1$ with $|\dot{u}_0| \leq w < \infty$, where w is unknown.

This assumption can be well justified by the fact that control actuations are usually smooth and subject to ramp-down and ramp-up limits. For follower i's estimation of the leader's input, we define the error as $e_{u,i} = \hat{u}_{0,i} - u_0$. According to (3), its dynamics is

$$\begin{aligned}\dot{e}_{u,i} &= -b_i e_{u,i} - \sum_{j\in\mathcal{N}_i} a_{ij}(e_{u,i} - e_{u,j}) \\ &\quad - d_i \cdot \operatorname{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(e_{u,i} - e_{u,j}) + b_i e_{u,i}\right] - \dot{u}_0.\end{aligned} \tag{6}$$

Considering all the followers, we further define $e_u = \begin{bmatrix} e_{u,1} & e_{u,2} & \cdots & e_{u,N}\end{bmatrix}^\top$. It follows from (6) that

$$\dot{e}_u = -He_u - D\cdot \operatorname{sgn}(He_u) - \dot{u}_0\mathbf{1}, \tag{7}$$

where $H = B + L$ and $D = \operatorname{diag}\{d_1, \ldots, d_N\}$. It is seen that the signum function term in (7) is discontinuous, Lebesgue measurable and locally bounded. Therefore, (7) admits a Filippov solution, which is governed by the differential inclusion [20] as follows:

$$\dot{e}_u \in^{a.e.} \mathcal{K}\left[-He_u - D\cdot \operatorname{sgn}(He_u) - \dot{u}_0\mathbf{1}\right]. \tag{8}$$

The following lemma unveils its convergence property.

Lemma 1: If Assumption 1 holds, the observer in (3) then yields convergent estimation of u_0, with $\lim_{t\to\infty} e_u = 0$.

Proof: Let us consider a Lyapunov functional candidate, $V_1 = \bar{V}_1(e_u) + \tilde{V}_1$, where

$$\bar{V}_1(e_u) = \frac{1}{2}e_u^\top He_u,\ \tilde{V}_1 = \sum_{i=1}^N \frac{(d_i-\beta)^2}{2\tau_i},$$

with $\beta \geq w$. The set-valued Lie derivative of $\bar{V}_1(e_u)$ along with (8) is given by

$$\begin{aligned}\mathcal{L}\dot{\bar{V}}_1 &= \mathcal{K}\left[-e_u^\top H^2 e_u - e_u^\top HD\cdot\operatorname{sgn}(He_u) - e_u^\top H\dot{u}_0\mathbf{1}\right] \\ &= \mathcal{K}\Big[-\sum_{i=1}^N d_i\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right)^\top \\ &\quad \cdot \operatorname{sgn}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right) \\ &\quad - e_u^\top H^2 e_u - e_u^\top H\dot{u}_0\mathbf{1}\Big] \\ &\leq -\sum_{i=1}^N d_i\left|\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right| \\ &\quad - e_u^\top H^2 e_u + w\|He_u\|_1\end{aligned}$$

where the fact that $\mathcal{K}[f] = \{f\}$ if f is continuous is used. Invoking [20], it is noted that $\dot{\bar{V}}_1$ exists and $\dot{\bar{V}}_1 \in \mathcal{L}\dot{\bar{V}}_1$. Then, the derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= \dot{\bar{V}}_1 + \dot{\tilde{V}}_1 = \dot{\bar{V}}_1 + \sum_{i=1}^N \frac{(d_i-\beta)\dot{d}_i}{\tau_i} \\ &\leq -\sum_{i=1}^N d_i\left|\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right| \\ &\quad + \sum_{i=1}^N (d_i-\beta)\left|\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right| \\ &\quad - e_u^\top H^2 e_u + w\|He_u\|_1 \\ &= -e_u^\top H^2 e_u - (\beta - w)\|He_u\|_1.\end{aligned}$$

Note that H is positive definite [21]. This, together with $\beta \geq w$, indicates $\dot{V}_1 \leq 0$. Hence, $V_1(e_u)$ is non-increasing, which indicates that e_u and d_i are bounded. It follows from (3b) that d_i is monotonically increasing. This indicates that d_i should converge to some finite value. In the meantime, $V_1(e_u)$ reaches a finite limit as it is decreasing and lower-bounded by zero. If denoting $s(t) = \int_0^t e_u^\top(\tau)H^2 e_u(\tau)d\tau$, we see that $s(t) \leq V_1(0) - V_1(t)$ by integrating $\dot{V}_1(e_u) \leq -e_u^\top H^2 e_u$. Hence, $\lim_{t\to\infty} s(t)$ exists and is finite. Due to the boundedness of e_u and $\dot{e}_u$, $\ddot{s}$ is also bounded. This implies that $\dot{s}$ is uniformly continuous. Then, $\lim_{t\to\infty}\dot{s}(t) = 0$ by Barbalat's Lemma [22]. It is then obtained that $\lim_{t\to\infty} e_u = 0$. ■

Now, we investigate the convergence property for the observer in (4), which is designed to enable followers to estimate the leader's unmeasured states, $x_{0,m}$ for $m = 2, 3, \ldots, l$. Let us define the estimation error as $e_{0x,i,m} = \hat{x}_{0,i,m} - x_{0,m}$. According to (1) and (4), we have

$$\begin{aligned}\dot{e}_{0x,i,2} &= \dot{\hat{x}}_{0,i,2} - \dot{x}_{0,2} = \dot{z}_{0,i,2} + b_i c_{0,2}\dot{x}_{0,1} - \dot{x}_{0,2} \\ &= -c_{0,2}b_i e_{0x,i,2} - c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) \\ &\quad + \hat{x}_{0,i,3} - x_{0,3},\end{aligned} \tag{9a}$$

$$\begin{aligned}\dot{e}_{0x,i,m} &= \dot{\hat{x}}_{0,i,m} - \dot{x}_{0,m} = \dot{z}_{0,i,m} + c_{0,m}\dot{x}_{0,i,m-1} - \dot{x}_{0,m} \\ &= -c_{0,m}\hat{x}_{0,i,m} - c_{0,m}c_{0,2}b_i e_{0x,i,2} + c_{0,m}\hat{x}_{0,i,m} \\ &\quad - c_{0,m}c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) + \hat{x}_{0,i,m+1} \\ &\quad - x_{0,m+1},\quad m = 3, 4, \ldots, l-1,\end{aligned} \tag{9b}$$

$$\begin{aligned}\dot{e}_{0x,i,l} &= \dot{\hat{x}}_{0,i,l} - \dot{x}_{0,l} = \dot{z}_{0,i,l} + c_{0,l}\dot{x}_{0,i,m-1} - \dot{x}_{0,l} \\ &= -c_{0,l}\hat{x}_{0,i,l} - c_{0,l}c_{0,2}b_i e_{0x,i,2} \\ &\quad - c_{0,l}c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) \\ &\quad + c_{0,l}\hat{x}_{0,i,l} + \hat{u}_{0,i} - u_0.\end{aligned} \tag{9c}$$

Define $e_{0x,m} = \begin{bmatrix} e_{0x,1,m} & e_{0x,2,m} & \cdots & e_{0x,N,m}\end{bmatrix}^\top$ and $e_{0x} = \begin{bmatrix} e_{0x,2}^\top & e_{0x,3}^\top & \cdots & e_{0x,l}^\top\end{bmatrix}^\top$. Then, (9) can be written into a compact form as below:

$$\dot{e}_{0x} = F_1 e_{0x} + \ell_1, \tag{10}$$

where

$$F_1 = \begin{bmatrix} -c_{0,2}H & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -c_{0,l-1}c_{0,2}H & 0 & \cdots & 0 & I \\ -c_{0,l}c_{0,2}H & 0 & \cdots & \cdots & 0 \end{bmatrix}, \ \ell_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ e_u \end{bmatrix}.$$

Lemma 2: If there exists $c_{0,2}, c_{0,3}, \ldots, c_{0,l} > 0$ such that the polynomial

$$h_i(s) = s^{l-1} + c_{0,2}s^{l-2}\lambda_i(H) + c_{0,2}\lambda_i(H)\sum_{z=0}^{l-3} c_{0,l-z}s^z$$

for $i = 1, 2, \ldots, N$ are Hurwitz stable, then the system in (10) is asymptotically stable with $\lim_{t\to\infty} e_{0x} = 0$.

Proof: Based on Schur complement, we can derive the characteristic polynomial of F_1 as $\det(sI - F_1) = \prod_{i=1}^{N} h_i(s)$. In addition, $\lim_{t\to\infty} \ell_1 = 0$ by Lemma 1. The theory of Input-to-State Stability (ISS) then implies $\lim_{t\to\infty} e_{0x} = 0$. ■

Consider the observer in (5), which is run by a follower to estimate its own states, $x_{i,m}$ for $i = 1, 2, \ldots, N$ and $m = 2, 3, \ldots, l$. Let us define the estimation error as $e_{x,i,m} = \hat{x}_{i,m} - x_{i,m}$. Using (1) and (5), we can derive

$$\dot{e}_{x,i,2} = -r_2 e_{x,i,2} + \hat{x}_{i,3} - x_{i,3}, \quad (11a)$$

$$\dot{e}_{x,i,m} = -r_m\hat{x}_{i,m} - r_m r_2 e_{x,i,2} + r_m\hat{x}_{i,m} + \hat{x}_{i,m+1} - x_{i,m+1}, \quad m = 3, 4, \ldots, l-1, \quad (11b)$$

$$\dot{e}_{x,i,l} = -r_l\hat{x}_{i,l} - r_l r_2 e_{x,i,2} + r_l\hat{x}_{i,l}. \quad (11c)$$

Further, define $e_{x,m} = \begin{bmatrix} e_{x,1,m} & e_{x,2,m} & \cdots & e_{x,N,m} \end{bmatrix}^\top$ for $m = 2, 3, \ldots, l$ and $e_x = \begin{bmatrix} e_{x,2}^\top & e_{x,3}^\top & \cdots & e_{x,l}^\top \end{bmatrix}^\top$. Recalling (11), it then follows that

$$\dot{e}_x = F_2 e_x, \quad (12)$$

where

$$F_2 = \begin{bmatrix} -r_2 I & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -r_{l-1}r_2 I & 0 & \cdots & 0 & I \\ -r_l r_2 I & 0 & \cdots & \cdots & 0 \end{bmatrix}.$$

We can obtain the next lemma along the lines in Lemma 2 and skip the proof.

Lemma 3: If there exist observer gains $r_2, r_3, \ldots, r_l > 0$ such that the polynomial

$$s^{l-1} + r_2 s^{l-2} + r_2 \sum_{z=0}^{l-3} r_{l-z}s^z$$

is Hurwitz stable, the the system in (12) is asymptotically stable with $\lim_{t\to\infty} e_x = 0$.

With the above results, we are now in a good position to investigate the global tracking error. When all the followers with high-order dynamics in (1) run the control law (2), the tracking errors are given by

$$\dot{x}_{i,m} - \dot{x}_{0,m} = x_{i,m+1} - x_{0,m+1}, \quad (13a)$$

$$\dot{x}_{i,l} - \dot{x}_{0,l} = -k_1\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_{i,1} - x_{j,1}) + b_i(x_{i,1} - x_{0,1})\right] - \sum_{m=2}^{l} k_m(x_{i,m} - x_{0,m}) + \hat{u}_{0,i} - u_0 - \sum_{m=2}^{l} k_m(\hat{x}_{i,m} - x_{i,m} + x_{0,m} - \hat{x}_{0,i,m}), \quad (13b)$$

for $m = 1, 2, \ldots, l-1$ and $i = 1, 2, \ldots, N$. Define $e_{i,m} = x_{i,m} - x_{0,m}$, $e_m = \begin{bmatrix} e_{1,m} & e_{2,m} & \cdots & e_{N,m} \end{bmatrix}^\top$, and $e = \begin{bmatrix} e_1^\top & e_2^\top & \cdots & e_l^\top \end{bmatrix}^\top$ for $m = 1, 2, \ldots, l$ and $i = 1, 2, \ldots, N$. Then, according to (13), the dynamics of the global tracking error e can be expressed as

$$\dot{e} = F_3 e + \ell_3,$$

where

$$F_3 = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ -k_1 H & -k_2 I & \cdots & \cdots & -k_l I \end{bmatrix},$$

$$\ell_3 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\sum_{m=2}^{l} k_m(e_{x,m} - e_{0x,m}) + e_u \end{bmatrix}.$$

The following theorem shows that $\lim_{t\to\infty} e(t) = 0$. The proof is similar to that of Lemma 2 and thus omitted here.

Theorem 1: For the considered leader-follower tracking, the dynamics of the global tracking error is asymptotically stable with $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for $m = 1, 2, \ldots, l$, if there exist controller gains k_m for $m = 1, 2, \ldots, l$ such that the polynomial

$$s^l + k_1\lambda_i(H) + \sum_{z=2}^{l} s^{z-1}k_z$$

is Hurwitz stable.

Remark 1: For all these observers, it should be noted that one can usually find out gain parameters to satisfy the conditions in Lemmas 2-3 and Theorem 1 according to the properties of the roots of polynomials and ensure the convergence of estimation. This implies that the proposed tracking controller can be effective under only mild conditions.

III. NUMERICAL STUDY

In this section, we offer an illustrative example to show the effectiveness of the proposed approach. Consider a third-order MAS including one leader and five followers. The agents interchange information according to

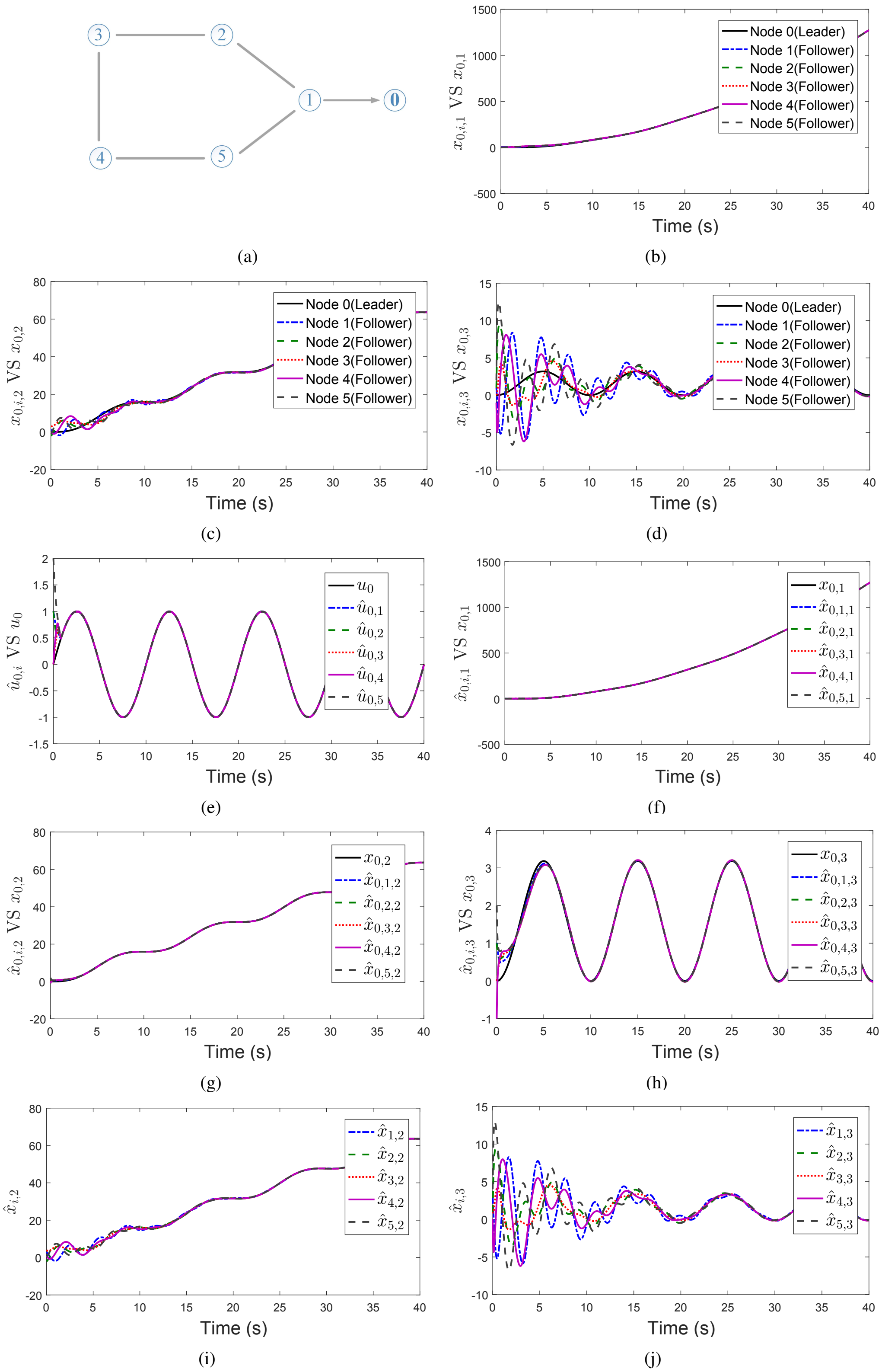

Fig. 1: Third-order MAS tracking: (a) communication topology; (b) $x_{i,1}$ for $i = 0, 1, \ldots, N$; (c) leader's and followers' state trajectory profiles of $x_{i,2}$ for $i = 0, 1, \ldots, N$; (d) leader's and followers' state trajectory profiles of $x_{i,3}$ for $i = 0, 1, \ldots, N$; (e) leader's input profile and the estimation by each follower; (f) leader's state trajectory profile of $x_{0,1}$ and the estimation by each follower; (g) leader's state trajectory profile of $x_{0,2}$ and the estimation by each follower; (h) leader's state trajectory profile of $x_{0,3}$ and the estimation by each follower; (i) followers' estimation of their own state trajectories of $x_{i,2}$ for $i = 1, 2, \ldots, N$; (j) followers' estimation of their own state trajectories of $x_{i,3}$ for $i = 1, 2, \ldots, N$.

the communication topology shown in Figure 1(a). Here, node 0 is the leader, and nodes 1 to 5 are followers. The leader transmits data to only follower 1, and the followers maintain bidirectional communication with their neighbors. We initialize the first states of the leader and followers as $\begin{bmatrix} 0 & 3 & 0 & -2 & 1 & -1 \end{bmatrix}^\top$, the second states as $\begin{bmatrix} 0 & 1 & -2 & 3 & 0 & -1 \end{bmatrix}^\top$, and the third states as $\begin{bmatrix} 0 & 1 & 1 & 0 & -1 & 2 \end{bmatrix}^\top$, respectively. The input driving the leader is set to be $u_0(t) = \sin(0.2\pi t)$. We further choose $c_{0,2} = c_{0,3} = 5$, $r_2 = r_3 = 5$, $k_1 = k_2 = k_3 = 6$, $l = 1$ and $\tau_i = 1$ for $i = 1, 2, \ldots, 5$ to apply the observer-based controller proposed in in Section II. The simulation result is summarized in Figure 1. Figures 1(b)-1(d) show the state tracking performance. It is seen that all the states of a follower can catch up with the leader's despite the state differences and then keep accurate tracking afterwards. Figures 1(e)-1(j) illustrate the estimation performance of the observers. We see from Figures 1(e)-1(h) that the distributed observers for the leader's input and states can produce estimation that gradually converges to the truth. The local observers for followers to estimate their own unmeasured states are also convergent, as shown in Figures 1(i)-1(j). We hence can conclude that these observers well overcome the issue of limited measurements by estimating the quantities unmeasured but needed for tracking, which paves a foundation for successful tracking.

IV. Conclusion

We studied the problem of leader-follower tracking control for high-order MASs in this paper. Here, we considered the challenging yet realistic setting where only the first state of every agent is measured. We proposed to build a distributed observer-based control approach. Along this line, we designed multiple observers to reconstruct a few quantities, by which a follower can become aware of not only the leader's input and states but also their own unmeasured states, and combined them with a distributed tracking controller. We conducted the design for high-order MASs and characterized the convergence properties. A simulation result was provided to show the effectiveness of the proposed design.

References

[1] J. Shamma, *Cooperative Control of Distributed Multi-Agent Systems*. New York, NY, USA: Wiley-Interscience, 2008.

[2] Y. Wang, E. Garcia, D. Casbeer, and F. Zhang, Eds., *Cooperative Control of Multi-Agent Systems: Theory and Applications*. Wiley, 2017.

[3] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. London: Springer-Verlag, 2011.

[4] J. Qin, Q. Ma, Y. Shi, and L. Wang, "Recent advances in consensus of multi-agent systems: A brief survey," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 4972–4983, 2017.

[5] M. E. J. Cortés, "Coordinated control of multi-robot systems: A survey," *SICE Journal of Control, Measurement, and System Integration*, vol. 10, no. 6, pp. 495–503, 2017.

[6] R. M. Murray, "Recent research in cooperative control of multivehicle systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 129, no. 9, pp. 571–583, 2007.

[7] C. Yan and H. Fang, "Observer-based distributed leader-follower tracking control: a new perspective and results," *International Journal of Control*, in press.

[8] ——, "A new encounter between leader–follower tracking and observer-based control: Towards enhancing robustness against disturbances," *Systems & Control Letters*, vol. 129, pp. 1–9, 2019.

[9] J. Huang, H. Fang, L. Dou, and J. Chen, "An overview of distributed high-order multi-agent coordination," *IEEE/CAA Journal of Automatica Sinica*, vol. 1, no. 1, pp. 1–9, 2014.

[10] W. Ren, K. L. Moore, and Y. Chen, "High-order and model reference consensus algorithms in cooperative control of multivehicle systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 129, no. 5, pp. 678–688, 2007.

[11] W. Yu, G. Chen, W. Ren, J. Kurths, and W. X. Zheng, "Distributed higher order consensus protocols in multiagent dynamical systems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 8, pp. 1924–1932, 2011.

[12] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, 2012.

[13] S. Mondal and R. Su, "Finite time tracking control of higher order nonlinear multi agent systems with actuator saturation," in *Proceedings of 14th IFAC Symposium on Control in Transportation Systems*, vol. 49, no. 3, 2016, pp. 165–170.

[14] X. Wang, S. Li, X. Yu, and J. Yang, "Distributed active anti-disturbance consensus for leader-follower higher-order multi-agent systems with mismatched disturbances," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5795–5801, 2017.

[15] H. Chu, "Observer-based adaptive consensus tracking for linear multi-agent systems with input saturation," *IET Control Theory & Applications*, vol. 9, pp. 2124–2131(7), September 2015.

[16] J. Sun, Z. Geng, and Y. Lv, "Adaptive output feedback consensus tracking for heterogeneous multi-agent systems with unknown dynamics under directed graphs," *Systems & Control Letters*, vol. 87, pp. 16 – 22, 2016.

[17] K. You and L. Xie, "Coordination of discrete-time multi-agent systems via relative output feedback," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 13, pp. 1587–1605, 2011.

[18] J. Sun and Z. Geng, "Adaptive output feedback consensus tracking for linear multi-agent systems with unknown dynamics," *International Journal of Control*, vol. 88, no. 9, pp. 1735–1745, 2015.

[19] J. Yang, S. Li, and X. Yu, "Sliding-mode control for systems with mismatched uncertainties via a disturbance observer," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 160–169, 2013.

[20] F. Clarke, *Optimization and Nonsmooth Analysis*. John Wiley & Sons New York, NY, USA, 1983.

[21] J. Hu and Y. Hong, "Leader-following coordination of multi-agent systems with coupling time delays," *Physica A: Statistical Mechanics and its Applications*, vol. 374, no. 2, pp. 853–863, 2007.

[22] H. K. Khalil, *Noninear Systems*. Prentice-Hall, New Jersey, 2002.