Cross-Network Prioritized Sharing: An Added Value MVNO's Perspective

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***Abstract*—We analyze the prioritized sharing between an *added value* Mobile Virtual Network Operator (MVNO) and multiple Mobile Network Operators (MNOs). An added value MVNO is one which earns added revenue from wireless users in addition to the revenue it directly collects for providing them wireless service. To offer service, an MVNO needs to contract with one or more MNOs to utilize their networks. Agreeing on such a contract requires the MNOs to consider the impact on their revenue from allowing the MVNO to enter the market as well as the possibility that other MNOs will cooperate. To further protect their customers, the MNOs may prioritize their direct customers over those of the MVNO. We establish a multi-stage game to analyze the equilibrium decisions of the MVNO, MNOs, and users in such a setting. In particular, we characterize the condition under which the MVNO can collaborate with the MNOs. The results show that the MVNO tends to cooperate with the MNOs when the band resources are limited and the added value is significant. When there is significant difference in band resources among the MNOs, the MVNO first considers cooperating with the MNO with a smaller band. We also consider the case when the users also have access to unlicensed spectrum.**

I. Introduction

The global MVNO market has been growing rapidly in recent years. According to [7], the size of the global MVNO market has been increasing since 2012 with a market value that is estimated to reach about 80 billion U.S. dollars in 2021. Traditionally, MVNOs have helped MNOs reach segments of the market that are not profitable to them without diluting their core band. As the MVNO market is evolving, several new features are emerging. First, while historically, MVNOs partnered with a single MNO, new MVNOs such as Google's Project-Fi [8] have partnered with multiple MNOs and operate across their networks. Second, as wireless technologies have evolved, it is possible for the MNOs to prioritize their own direct customers over those of an MVNO, which may make them more willing to allow an MVNO to use their network. Third, many recent MVNOs can be viewed as obtaining an "added value" beyond the money they collect directly for mobile service, which gives them additional incentives to enter the market. These three issues clearly impact the strategic interaction of MVNOs and MNOs and are tightly coupled. As such, they deserve further study. In this paper, we do this by developing and analyzing a game theoretic model for the interactions of an added value MVNO, multiple MNOs and their customers.

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Google's Project-Fi also serves as an example of such an added value MVNO. Google profits from expanding wireless data usage as it can earn more from other services such as online ads. Another example is that Tecent, one of the biggest internet companies in China, cooperates with China Unicom to offer new data plans. Becoming an MVNO is also increasingly attractive to traditional internet companies (e.g., Facebook and NetFlix) in that it gives them more control over the network infrastructure used to deliver their content [11], [12].

It is important for an MVNO to reach win-win agreements with MNOs. Although cooperating with the MVNO can potentially expand the MNOs' market share and bring new revenue streams outside the wireless service market, there are still challenges to reaching an agreement. The key challenge is to convince the MNOs that the entry of an MVNO will not become a threat to their core business. An MNO can ease these concerns by limiting the MVNO's priority of accessing the MNO's infrastructure, and getting the compensation from the MVNO. For example, the four major carriers in the U.S. may deprioritize MVNOs' traffic on their networks during times of heavy congestion [11]. Of course, offering a lower priority may also make entering into an agreement less attractive for an MVNO. Our model considers such prioritization and its impact on the competition between MVNOs and MNOs.

Another dimension of forming an agreement for the MVNO is determining which MNO(s) to cooperate with. Moreover, for a cross-network MVNO, once it is cooperating with one MNO, does this impact the incentives to cooperate with other MNOs? For example Google Fi in the U.S. first partnered with Sprint and T-Mobile (in 2015) and subsequently added U.S. Cellular in 2016. Our game theoretic model provides insights into these decisions.

A. Related work

There have been many recent works studying competition among wireless service providers, e.g., [3]–[6], [14]–[19]. These papers did not consider MVNOs but rather focused on competition among MNOs under different spectrum sharing scenarios. In this paper, we adopt a similar model to that in [16]–[18] to study the downstream competition between MNOs and the added value MVNO. Our model for MNO prioritization is similar to that in [16]. Specifically, [16] studied the competition with primary-secondary spectrum sharing, and the MVNO in our model is similar to a secondary spectrum user considered in [16].[1]

There has also been a number of works studying the economics of MNOs and MVNOs in the wireless market, e.g., [22]–[24]. Different from [23], which focused on the impact of the user type, our paper focuses on the impact of the MNOs' band resources and the MVNO's added value. As in our paper, [24] studied a cross-network MVNO but focused on how users' usage rates affect their rates of defecting from their current MNOs to join an MVNO as well as the resulting revenues. Our work instead focuses on an added value MVNO with MNO prioritization as well as modeling the strategic interactions among the MVNO and MNOs (these were assumed to be given in [24]). Reference [22] considered the price competition between an MNO and an MVNO. Different from our consideration of an added value MVNO, [22] focused on an MVNO that appeals to a different market segment of customers compared to the MNO. Also this paper did not consider cross-network sharing or MNO prioritization. To summarize, the novelty of our work is that it jointly considers MNO prioritization, cross-network sharing and an added value MVNO. We show that jointly treating these factors leads to non-trivial findings. Furthermore, we model the strategic interaction of the MVNO and MNOs to determine the prices they charge and the cooperation contracts that are signed, which are exogenously specified in most prior work.

B. Contributions

In this paper, we model the MVNO's cooperation with the MNOs and their price competition as a three-stage Stackelberg game. In stage I, the MVNO offers a cooperation contract to the MNOs. In stage II, the MNOs decide whether to cooperate with the MVNO. In stage III, all the operators in the market choose their prices and compete for the users. Given the characteristics of the MVNO and MNOs (e.g., the MVNO's added value and the MNOs' available band resources), we use this model to shed light on the following questions: (i) *will the MVNO enter the market?* (ii) *which MNOs will the MVNO cooperate with and what price should the contract specify?* (iii) *what will be the MVNO's and MNOs' prices and market shares in equilibrium?*

When there is an arbitrary number of MNOs, we prove the existence and uniqueness of the price equilibrium in stage III. We use this to then solve for the equilibrium of the three-stage game. We also provide more results for two special cases: (i) a monopolistic MNO and (ii) an MNO duopoly. We summarize our key results as follows:

- When there is an arbitrary number of MNOs, whenever the MNOs are willing to cooperate with the MVNO, increasing the MVNO's added value per user decreases all MNOs' prices and increases the number of users served. In particular, the MVNO's market share increases, while each MNO's market share decreases.
- When there is only a monopolistic MNO in the market, (i) the MVNO with a large added value per user can always cooperate with the MNO, and (ii) the MVNO with a small added value can cooperate with the MNO if and only if the band resource is limited.
- When there are duopolistic MNOs in the market, there might be multiple equilibria. We consider different equilibrium selection criteria, and fully characterize the conditions under which the MVNO cooperates with the MNOs. For example, when the MNOs prefer the equilibrium that leads to cooperation and the MVNO has a small added value, the MVNO will cooperate with at least one MNO if and only if both MNOs have limited band resources. Specifically, if their band resources are similar, the MVNO cooperates with both MNOs; otherwise, the MVNO cooperates with the MNO with a smaller band.
- When there is an additional unlicensed band available to the users, increasing the unlicensed bandwidth can increase the profit of an MVNO with a large added value per user, and lead to more cooperation between the MVNO and MNOs. However, increasing the unlicensed bandwidth can decrease the profit of an MVNO with a very small added value.

[1]An MVNO differs from a secondary spectrum user in that the MVNO is sharing both the MNO's spectrum and infrastructure.

II. MODEL

We assume that there is one potential MVNO and n existing MNOs in the market. The MVNO seeks to cooperate with the MNOs and get their band resources. We use SPg to denote the MVNO, and SPi to denote each MNO, where $i \in N, N \triangleq \{1, 2, \dots, n\}$. To characterize the MNOs' cooperation state with the MVNO, we define a partner MNO set $V \in \mathcal{P}(N)$, where $\mathcal{P}(N)$ is the power set of N. If SPi cooperates with SPg, then $i \in V$; otherwise, $i \notin V$.

We build a Stackelberg game to study the cooperation and competition between the MVNO and MNOs. The MVNO is the Stackelberg leader, offering separate cooperation contracts to the MNOs. In the contract offered to SPi, SPg will offer a flat-fee payment as compensation to SPi. We denote c_i as the payment SPg offers to SPi and $\mathbf{c} = [c_1, c_2, \cdots, c_n]$ as the MVNO's compensation vector. Each SPi decides whether to accept the contract offered by SPg. Next, all the SPs in the market decide their service prices for the users. We denote the service price vector by $\mathbf{p} = [p_1, p_2, \cdots, p_n, p_g]$, where p_i is SPi's price and p_g is SPg's price. We formulate the SPs' interactions by a three-stage game:

- Stage I: SPg offers compensation vector $\mathbf{c}$ to each SPi.
- Stage II: Each SPi decides whether to accept the contract, resulting in the cooperation state V.
- Stage III: Each SPi sets its service price p_i, and SPg sets its service price p_g if and only if it cooperates with at least one SPi. The SPs in the market then compete for a common pool of wireless users based in part on the announced prices.

In the following subsections, we continue to introduce the detailed modeling of each stage.

A. Price competition in stage III

We first model the price competition in stage III assuming that SPg enters the market. As in [17], [18], we assume that the SPs in the market compete for a common pool of non-atomic wireless users with a total mass of 1. Each user will choose an SP considering its *delivered price*, which is the sum of its service price and a congestion cost. Here, the congestion cost characterizes the SP's Quality of Service (QoS) and the delivered price models that users are sensitive to QoS in addition to the cost of service. If SPi cooperates with SPg, it serves its own users and also SPg's users. We call SPi's own users its *primary users* and SPg's users the *secondary users*. Because of MNO prioritization, the primary and secondary users experience different congestion costs.

As in [16], we assume that the congestion cost experienced by SPi's primary users is $g_i(x_i)$, where x_i is the mass of SPi's primary users. The congestion cost experienced by the secondary users is $g_i(x_i + x_{gi})$, where x_{gi} is the mass of secondary users served by SPi. This models the fact that the primary users do not experience any degradation due to the existence of the secondary users, while the secondary users do experience degradation incurred by the primary users. To simplify the analysis and derive engineering insights, we focus on the case where $g_i(x) = \frac{x}{B_i}$, like in [15], [16]. Here, B_i is a measure of the available bandwidth and technology used by SPi, where larger values of B_i result in lower congestion for the same mass of customers. Each user has a reserve price for using the wireless service. A user uses the wireless service if and only if the wireless service's delivered price (i.e., sum of the service price and congestion) is no greater than its reserve price. We assume that the users' reserve prices are uniformly distributed in $[0, 1]$. Therefore, if the fraction of users using the wireless service is x, then the delivered price of the wireless service is $1-x$. This is also known as a linear inverse demand function, which has been widely considered in the literature such as [15], [16].

We use p_i and p_g to denote SPi's and SPg's service prices, respectively. In stage III, the users select SPs based on their services' delivered prices, which leads to a Wardrop Equilibrium (WE) [25], [27]. Specifically, for the SPs having positive market shares, their delivered prices should be the same. If any SP has a higher delivered price than the other SPs, it serves no users. Thus, the delivered prices experienced by a user of any SP should be the same and also equal to the inverse demand of $1 - \sum_i x_i - x_g$. Here, $x_g = \sum_{i \in V} x_{gi}$ is SPg's market share. We denote the Wardrop Equilibrium constraints as $\mathrm{WE}(\mathbf{p}, \bar{\mathbf{x}}, V)$, and they are given as follows:

$$\begin{cases} \frac{x_i}{B_i} + p_i \geq 1 - \sum_i x_i - x_g, \quad x_i \geq 0, & \forall i \in N, \\ x_i(1 - \sum_i x_i - x_g - \frac{x_i}{B_i} - p_i) = 0, & \forall i \in N, \\ \frac{x_i + x_{gi}}{B_i} + p_g \geq 1 - \sum_i x_i - x_g, \quad x_{gi} \geq 0, & \forall i \in V, \\ x_{gi}(1 - \sum_i x_i - x_g - \frac{x_i + x_{gi}}{B_i} - p_g) = 0, & \forall i \in V. \end{cases} \tag{1}$$

Here, $\bar{\mathbf{x}} \triangleq [\mathbf{x}^{\mathbf{P}} | \mathbf{x}^{\mathbf{S}}]$, where $\mathbf{x}^{\mathbf{P}} = [x_1, x_2, \cdots, x_n]$ captures the market share of each SPi and $\mathbf{x}^{\mathbf{S}} = [x_{gi} : i \in V]$ captures the mass of users subscribing to SPg and being allocated to each SPi's network. In a Wardrop Equilibrium, SPg's users experience the same congestion no matter which network they are served on. This arises since SPg routes each user to whichever network gives that user the best experience. Note that the total mass of users is 1. We define D as the set of $\bar{\mathbf{x}}$ that satisfies $\bar{\mathbf{x}} \succeq \mathbf{0}$ and $\sum_{i \in N} x_i + x_g \leq 1$. We use $\bar{\mathbf{x}}^{\mathrm{WE}}(\mathbf{p}, V)$ to denote the set of $\bar{\mathbf{x}}$ in D that satisfies $\mathrm{WE}(\mathbf{p}, \bar{\mathbf{x}}, V)$, i.e., this is the set of market shares that can arise in a WE for a given choice of prices $\mathbf{p}$ and a given cooperation state V.

Considering the users' Wardrop Equilibrium, the SPs set prices that maximize their profits. For SPi, if it does not cooperate with SPg, its profit purely comes from serving its own users, i.e., $\pi_i \triangleq x_i p_i$; otherwise, its profit also contains the compensation c_i from SPg. Hence, SPi's profit Π_i is given as:

$$\Pi_i(\mathbf{c}, V, \mathbf{p}, \bar{\mathbf{x}}) = \begin{cases} x_i p_i, & \text{if } i \notin V, \\ x_i p_i + c_i, & \text{if } i \in V. \end{cases}$$

For SPg, its profit consists of three parts. The first part is the profit from serving the users. We denote it by π_g, which satisfies $\pi_g = x_g p_g$. The second part is the added value from the users served by all the SPs. We use $k \in [0, 1]$ to denote the added value from one user.[2] The third part is the compensation from SPg to SPi if $i \in V$. Hence, SPg's profit Π_g is as follows:

$$\Pi_g(\mathbf{c}, V, \mathbf{p}, \bar{\mathbf{x}}) = x_g p_g + k(\sum_i x_i + x_g) - \sum_{i \in V} c_i.$$

If SPg does not enter the market, the analysis above will still hold by letting $V = \emptyset$. In stage III, each SPi's problem is

$$\max_{p_i \in \mathbb{R}} \Pi_i(\mathbf{c}, V, \mathbf{p}, \bar{\mathbf{x}}) \quad \text{s.t. } \bar{\mathbf{x}} \in \bar{\mathbf{x}}^{WE}(\mathbf{p}, V),$$

and SPg's problem is:

$$\max_{p_g \in \mathbb{R}} \Pi_g(\mathbf{c}, V, \mathbf{p}, \bar{\mathbf{x}}) \quad \text{s.t. } \bar{\mathbf{x}} \in \bar{\mathbf{x}}^{WE}(\mathbf{p}, V).$$

We denote the resulting price equilibrium as $\mathbf{p}^{\mathrm{PE}}(V)$.[3] In Sect. III, we will prove that this price equilibrium is unique. For simplicity, we will also use the superscript PE when indicating the results at the price equilibrium. For example,

[2] Here, we assume that the added value MVNO achieves a (average) value of k for any wireless users regardless of whether it is served by the MVNO or another MNO. Again, this captures examples of Internet companies whose interest in serving as an MVNO is partly to expand wireless data usage so as to benefit other business lines. Next, we estimate a possible value of k by an example. In 2017, Google's global revenue is $77,788 million [28], and the US market occupies 47% of it [29]. There are around 200 million smartphone users in the U.S.. We assume that wireless market contributes 30% of the revenue and the highest reserve price for wireless service is $100 per month. In this case, Google's added value per user per month is around $5.00, which gives $k = 0.05$ after being normalized by the highest reserve price. Higher values of k could be possible for companies if the percentage of revenue from mobile traffic increases or the overall revenue grows.

[3] p_i and p_g can be any real numbers. However, for SPi, there is no reason to set a price lower than 0. For SPg, when the added value is large, p_g might be negative. This means SPg is willing to compensate the wireless users to enlarge the market size.

$\mathbf{x}^{\mathrm{PE}}(V)$ denotes the market shares at the price equilibrium given cooperation state V, and $\pi_i^{\mathrm{PE}}(V)$ denotes the SPi's profit from serving its own users at the price equilibrium given V.

B. Cooperation decisions in stage II

In this subsection, we model each SPi's cooperation decision. Let s_i denote SPi's decision: $s_i = 1$ if it cooperates; $s_i = 0$ otherwise. Given the other SPs' cooperation state $\bar{V}_i$, SPi's decision will be

$$s_i = \begin{cases} 1, & \text{if } c_i \geq \pi_i^{\mathrm{PE}}(\bar{V}_i) - \pi_i^{\mathrm{PE}}(\bar{V}_i \cup \{i\}), \\ 0, & \text{if } c_i < \pi_i^{\mathrm{PE}}(\bar{V}_i) - \pi_i^{\mathrm{PE}}(\bar{V}_i \cup \{i\}). \end{cases}$$

We call a set V as an *equilibrium cooperation state* if and only if the following conditions hold for all $i \in N$: if $i \in V$,

$$\pi_i^{\mathrm{PE}}(V) + c_i \geq \pi_i^{\mathrm{PE}}(V/\{i\}); \tag{2}$$

if $i \notin V$,

$$\pi_i^{\mathrm{PE}}(V) > \pi_i^{\mathrm{PE}}(V \cup \{i\}) + c_i. \tag{3}$$

We use $V^{\mathrm{NE}}(\mathbf{c})$ to denote the set of equilibrium cooperation states under SPg's contracts $\mathbf{c}$. $V^{\mathrm{NE}}(\mathbf{c})$ could contain zero, one or multiple elements. When there exist multiple equilibria, we assume that the equilibrium that all SPi eventually achieve is determined by an equilibrium selection function $\mathcal{F}$, which can be determined by external factors like policies and historical relationships between the SPs.

C. Contract design

In stage I, SPg decides the compensation vector $\mathbf{c}$ by anticipating each SPi's response in stage II. As mentioned in the last subsection, it is possible that there is no equilibrium cooperation state in stage II, given vector $\mathbf{c}$. However, SPg, who determines $\mathbf{c}$, always prefers a stable outcome in the following stages. Thus, we assume that SPg will choose $\mathbf{c} \in \mathbf{C}$, where $\mathbf{C} \subseteq \mathbb{R}^n$ includes all the compensation vectors under which there exists at least one equilibrium cooperation state in stage II.[4] The following lemma implies that $\mathbf{C}$ is non-empty.

Lemma 1. *For any $V \in \mathcal{P}(N)$, SPg can always find a compensation vector $\mathbf{c} \in \mathbb{R}^n$, such that $V \in V^{\mathrm{NE}}(\mathbf{c})$.*

Hence, for any cooperation state V, SPg can always properly choose the compensation vector to ensure that V becomes an equilibrium in stage II. Thus, given $\mathbf{c} \in \mathbf{C}$ and a selection function $\mathcal{F}$, there will be a unique cooperation state in stage II.

We formulate SPg's problem in stage I as:

$$\max_{\mathbf{c} \in \mathbf{C}} \ \Pi_g(\mathbf{c}, V, \mathbf{p}, \bar{\mathbf{x}}) \tag{4}$$
$$\text{s.t.} \ V = \mathcal{F}(V^{\mathrm{NE}}(\mathbf{c})), \tag{5}$$
$$\mathbf{p} = \mathbf{p}^{\mathrm{PE}}(V), \tag{6}$$
$$\bar{\mathbf{x}} = \bar{\mathbf{x}}^{WE}(\mathbf{p}, V). \tag{7}$$

[4]Without this assumption, SPg's payoff for selecting a $\mathbf{c}$ not in $\mathbf{C}$ will not be defined.

Here, (5) implies that when there exist multiple equilibrium cooperation states under $\mathbf{c}$, the eventual equilibrium cooperation state is the one determined by the function $\mathcal{F}$. Some specific examples of such a function will be given in later sections.

III. GENERAL CASE

In this section, we analyze the three-stage game in a general case, where n can be an arbitrary positive integer. In Sects. IV and V, we will focus on the monopoly case ($n = 1$) and duopoly case ($n = 2$), respectively.

A. Price competition

We start the analysis from stage III, where the compensation vector $\mathbf{c}$ and the cooperation state V are given. We first prove that there exists a unique $\bar{\mathbf{x}}^{WE}(\mathbf{p}, \bar{\mathbf{x}}, V)$.

Lemma 2. *There exists a unique Wardrop Equilibrium $\bar{\mathbf{x}}$ in D.*

Proof. We can show that D is convex and $\mathbf{g}(\bar{\mathbf{x}}) \triangleq [g_i(x_i) : i \in N | g_i(x_i + x_{gi}) : i \in V]$ is continuous. Based on Theorem 1 in [19], this implies the existence of Wardrop equilibrium in D. For any $\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2 \in D$, it is easy to verify that

$$(\mathbf{g}(\bar{\mathbf{x}}_1) - \mathbf{g}(\bar{\mathbf{x}}_2)) \cdot (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) > 0.$$

Based on Theorem 3 in [25], there exists a unique Wardrop Equilibrium. □

Next, we show that when the SPs' service prices are at the equilibrium, the Wardrop equilibrium constraints for $\bar{\mathbf{x}}$ can be simplified to a set of linear equations.

Lemma 3. *If $\mathbf{p} \in \mathbf{p}^{\mathrm{PE}}(V)$, then $\mathrm{WE}(\mathbf{p}, \bar{\mathbf{x}}, V)$ can be simplified as:*

$$\begin{cases} \frac{x_i}{B_i} + p_i = 1 - \sum_i x_i - x_g, & \forall i \in N, \\ \frac{x_i + x_{gi}}{B_i} + p_g = 1 - \sum_i x_i - x_g, & \forall i \in V. \end{cases} \tag{8}$$

Solving (8) gives the SPs' market shares $\mathbf{x} = [x_1, x_2, \cdots, x_n, x_g]$ under a given price vector $\mathbf{p} = [p_1, p_2, \cdots, p_n, p_g]$. Specifically, we have:[5]

$$x_i(\mathbf{p}) = \begin{cases} B_i(\frac{1 + \sum_{j \in N \setminus V} B_j p_j + B_V p_g}{1 + B_T} - p_i), & i \in N, \\ \sum_{i \in V} B_i p_i - B_V p_g, & i = n+1, \end{cases} \tag{9}$$

$$x_{gi}(\mathbf{p}) = \quad B_i(p_i - p_g), \qquad i \in V. \tag{10}$$

Here, $B_V \triangleq \sum_{i \in V} B_i$ is the total amount of band resource SPg has access to. It can be verified that at a price equilibrium, the number of customers of each SPi given the service prices is feasible, i.e., $[x_i(\mathbf{p}) : i \in N \mid x_{gi}(\mathbf{p}) : i \in V] \in D$ if $\mathbf{p} \in \mathbf{p}^{\mathrm{PE}}(V)$.

[5]Here, x_g and p_g are the $(n+1)$th elements in $\mathbf{x}$ and $\mathbf{p}$, respectively. Thus, we will also use the notations x_{n+1} and p_{n+1} in (9) and (11) referring to x_g and p_g, respectively.

Theorem 1. *There exists a unique Nash Equilibrium for the pricing game given the cooperation state V:*

$$p_i^{\mathrm{PE}}(V) = \begin{cases} p_c, & \text{if } i \in V, \\ \frac{2(1+B_T)p_c}{2(1+B_T)-B_i}, & \text{if } i \in N \backslash V, \\ \frac{p_c}{2} - \frac{k}{2(1+B_T)}, & \text{if } i = n+1, \end{cases} \tag{11}$$

where $B_T \triangleq \sum_{i \in N} B_i$ is the total band resource in the market and $p_c = \frac{1}{1-(\sum_{i\in N\backslash V}\frac{B_i}{2(1+B_T-B_i)}+\frac{B_V}{4(1+B_T)})}(\frac{1}{2(1+B_T)} - \frac{kB_V}{4(1+B_T)^2})$.

One interesting insight is that the SPs cooperating with SPg have the same service price (even though they may have different values of B_i). Moreover, the remaining SPs' prices increase with B_i and decrease with k, which is SPg's added value. Several other impacts of k are characterized in the following lemma.

Lemma 4. *Given V, $p_i^{\mathrm{PE}}(V)$, $x_i^{\mathrm{PE}}(V)$ and $p_g^{\mathrm{PE}}(V)$ are decreasing with k for all $i \in N$, while $x_g^{\mathrm{PE}}(V)$ and $x_T^{\mathrm{PE}}(V)$ are increasing with k. Here, $x_T^{\mathrm{PE}}(V) \triangleq \sum_{i \in V} x_i^{\mathrm{PE}}(V) + x_g^{\mathrm{PE}}(V)$ is the total number of users in the price equilibrium under cooperation set V.*

Insights: In the price equilibrium stage, if the MVNO's added value per user increases, all wireless service prices will decrease, and more users are served. The market shares of the MNOs will decrease, while the market share of the MVNO and total number of wireless users increase. Intuitively, the larger added value motivates the MVNO to decrease its price to increase the market size, and the MNOs then need to decrease their prices as well to compete with the MVNO.

B. Cooperation decisions and compensation design

In this subsection, we analyze each SPi's cooperation decision in stage II and SPg's compensation design in stage I. With the assumption that $\mathbf{c} \in \mathbf{C}$ and the predetermined equilibrium selection function $\mathcal{F}$, there will be a unique resulting cooperation state in $\mathcal{V}^{\mathcal{F}}$ given $\mathbf{c}$. Recall that when there are multiple equilibrium cooperation states in stage II, the equilibrium selection function $\mathcal{F}$ gives the eventual outcome of the cooperation state. Hence, given $\mathcal{F}$, SPg can choose $\mathbf{c}$ to determine the cooperation state $\mathcal{F}(V^{\mathrm{NE}}(\mathbf{c}))$. For a particular cooperation state V that SPg wants to achieve, there can be infinitely many possible choices of $\mathbf{c}$ such that $V = \mathcal{F}(V^{\mathrm{NE}}(\mathbf{c}))$. Among these choices, SPg always wants to choose the one that generates the least total compensation since this compensation is a loss against SPg's revenue. Once V is determined, the revenue SPg gains from serving users and the added value are independent of $\mathbf{c}$.

Lemma 5. *Given $\mathcal{F}$, for every $V \in \mathcal{P}(N)$, there exists a $\mathbf{c}^{\mathcal{F}}(V)$ such that $\sum_{i\in V} c_i^{\mathcal{F}}(V) = \min\{\sum_{i\in V} c_i : \mathcal{F}(V^{\mathrm{NE}}(\mathbf{c})) = V, \mathbf{c} \in \mathbf{C}\}$, and $c_i^{\mathcal{F}}(V) = -\infty$ for all $i \in N\backslash V$.*

For any cooperation state V that SPg wants to achieve, we can assume that it always chooses the compensation vector $\mathbf{c}^{\mathcal{F}}(V)$, since it has the least total compensation cost.[6] This narrows down the search for the optimal $\mathbf{c}$ to simply searching the 2^n cooperation states (n is usually small in typical markets for wireless service). The value of $\mathbf{c}^{\mathcal{F}}(V)$ for a given V can be determined by (2) and (3) if V is always chosen as the cooperation state when it arises as a possible equilibrium. For a V that is not always selected in that case of multiple equilbria, determining $c_i^{\mathcal{F}}(V)$ is more subtle. We will show an example of how to derive $\mathbf{c}^{\mathcal{F}}(V)$ when $n = 2$ in Sect. V. In the following theorem, we characterize SPg's optimal compensation vector $\mathbf{c}^*$ under a given equilibrium selection function $\mathcal{F}$.

Theorem 2. *In any equilibrium, SPg offers $\mathbf{c}^* = \mathbf{c}^{\mathcal{F}}(V^*)$ to the remaining SPs, where*

$$V^* = \arg\max_{V \in \mathcal{P}(N)} \Pi_g(V),$$

and $\Pi_g(V) \triangleq \pi_g^{\mathrm{PE}}(V) - \sum_{i\in V} c_i^{\mathcal{F}}(V)$ is the profit of SPg under the cooperation state V.

Similar to the definition of $\Pi_g(V)$ in this theorem, we use $\Pi_i(V) \triangleq \pi_i^{\mathrm{PE}}(V) + c_i^{\mathcal{F}}(V)$ to denote the profit of SPi under the cooperation state V, considering the price competition in stage III.

IV. Monopoly case

In this section, we discuss the case where there is only one MNO with band B_1 in the market. In this case, there are only two possible cooperation states: $V = \{1\}$ and $V = \emptyset$, and there will be no multi-equilibria problem.

If SPg's offered contract cannot compensate for SP1's loss in revenue, SP1 will not sign and their resulting profits will be

$$\Pi_1(\emptyset) = \frac{B_1}{4(1+B_1)}, \qquad \Pi_g(\emptyset) = \frac{kB_1}{2(1+B_1)}.$$

If SP1 accepts the contract and cooperates, SP1's and SPg's revenues from the wireless service are

$$\pi_1^{\mathrm{PE}}(\{1\}) = \frac{B_1(2+B_1(2-k))^2}{(1+B_1)^2(4+3B_1)^2},$$
$$\pi_g^{\mathrm{PE}}(\{1\}) = B_1\frac{B_1^2k + (1+B_1)(1+4k(3+k)+8B_1k)}{(1+B_1)(4+3B_1)^2}.$$

It can be verified that SP1 always loses profit from wireless service even if SPg only has secondary access to SP1's band, i.e., $\pi_1^{\mathrm{PE}}(\emptyset) > \pi_1^{\mathrm{PE}}(\{1\})$. This is due to more competition caused by the entry of SPg. Thus, SPg has to compensate SP1 with at least $\pi_1^{\mathrm{PE}}(\emptyset) - \pi_1^{\mathrm{PE}}(\{1\})$. Hence, SP$g$'s profit under the smallest compensation that leads to the cooperation is $\Pi_g(\{1\}) = \pi_g^{\mathrm{PE}}(\{1\}) - (\pi_1^{\mathrm{PE}}(\emptyset) - \pi_1^{\mathrm{PE}}(\{1\}))$. SP$g$ compares this to $\Pi_g(\emptyset)$, which is $\frac{kB_1}{2(1+B_1)}$, to decide whether to offer this contract price so that SP1 will cooperate.

[6] Note that the compensation for SPi not in V is not unique, as SPg simply needs to announce a low enough price so that these SPs do not cooperate. However, any such choice of compensation generates the same revenue and hence there is no loss in focusing on the choice in Lemma 5.

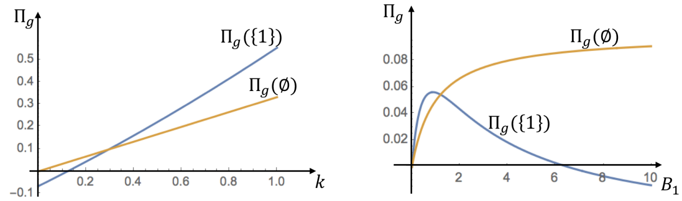

Fig. 1. A comparison between SPg's profits under the cooperation and non-cooperation cases. In the left figure, $B_1 = 2$, and it shows that $\Pi_g(\{1\})$ increases with k. In the right figure, $k = 0.2$, and it shows that when B_1 is big, cooperation decreases SPg's profit.

Two examples are shown in Fig. 1. The blue line gives SPg's total profit if it compensates SP1 enough to cooperate, and the orange line gives SPg's total profit if it does not enter the market. It can be observed that SPg and SP1 tend to cooperate when the added value k is large and the band resource B_1 is limited. Formally, we introduce the following two lemmas.

Lemma 6. *There exits a* $k^* \in [0, 1)$ *such that* $V^* = \{1\}$ *if* $k > k^*$.

Insights: This lemma shows that SPg cooperates with SP1 if the added value is large enough. A larger added value increases SPg's willingness to cooperate with SP1, because the cooperation can increase the mass of users using wireless service.

Lemma 7. *When* $k \geq \frac{1}{2}$, *SPg and SP1 will always cooperate; when* $k < \frac{1}{2}$, *there exists a* B_1^* *such that they cooperate if and only if* $B_1 \leq B_1^*$.

Insights: If the added value is large enough, SPg is always willing to cooperate with SP1 to expand their total market share. However, when the added value is small, SP1 and SPg will cooperate if and only if the band resource is limited. The reason is that, when the band resource is limited, the loss of SP1's revenue due to the entry of SPg is smaller compared with the case where the band resource is abundant. Hence, the compensation that SPg needs to offer to SP1 in the limited band case is smaller.

V. Duopoly case

In Sect. IV, we showed the impact of the added value and the band resource in the monopoly case. In this section, we will study the case where there are two competing MNOs. This will give us insight into how the MVNO chooses between different MNOs.

We first derive the price equilibrium under every cooperation state according to (11) in Theorem 1. In the equilibrium cooperation state, no SPi is willing to unilaterally deviate from its decision. Hence, we have the following relation between the equilibrium cooperation state and the compensation vector:

- $\{1,2\} \in V^{\text{NE}}(\mathbf{c})$ if $c_1 + \pi_1^{\text{PE}}(\{1,2\}) \geq \pi_1^{\text{PE}}(\{2\})$ and $c_2 + \pi_2^{\text{PE}}(\{1,2\}) \geq \pi_2^{\text{PE}}(\{1\})$;

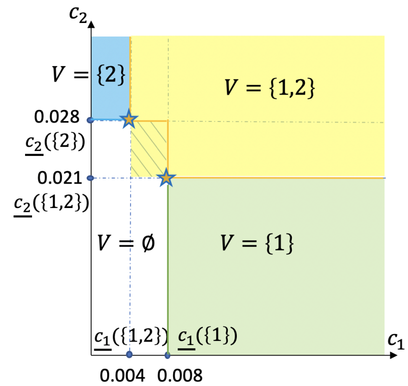

Fig. 2. An example of SP1's and SP2's cooperation decisions.

- $\{1\} \in V^{\text{NE}}(\mathbf{c})$ if $c_1 + \pi_1^{\text{PE}}(\{1\}) \geq \pi_1^{\text{PE}}(\emptyset)$ and $c_2 + \pi_2^{\text{PE}}(\{1,2\}) < \pi_2^{\text{PE}}(\{1\})$;
- $\{2\} \in V^{\text{NE}}(\mathbf{c})$ if $c_1 + \pi_1^{\text{PE}}(\{1,2\}) < \pi_1^{\text{PE}}(\{2\})$ and $c_2 + \pi_2^{\text{PE}}(\{2\}) \geq \pi_2^{\text{PE}}(\emptyset)$;
- $\emptyset \in V^{\text{NE}}(\mathbf{c})$ if $c_1 + \pi_1^{\text{PE}}(\{1\}) < \pi_1^{\text{PE}}(\emptyset)$ and $c_2 + \pi_2^{\text{PE}}(\{2\}) < \pi_2^{\text{PE}}(\emptyset)$.

If SPg wants to make $\{1,2\}$ become an equilibrium cooperation state, it will have to pay SP1 at least $\underline{c_1}(\{1,2\}) \triangleq \pi_1(\{2\}) - \pi_1(\{1,2\})$ and SP2 $\underline{c_2}(\{1,2\}) \triangleq \pi_2^{\text{PE}}(\{1\}) - \pi_2^{\text{PE}}(\{1,2\})$. If SP$g$ wants to make $\{1\}$ become an equilibrium cooperation state, it will have to pay SP1 $\underline{c_1}(\{1\}) \triangleq \pi_1(\emptyset) - \pi_1(\{1\})$. If SP$g$ wants to make $\{2\}$ become an equilibrium cooperation state, it will have to pay SP2 $\underline{c_2}(\{2\}) \triangleq \pi_2^{\text{PE}}(\emptyset) - \pi_2^{\text{PE}}(\{2\})$. The following lemma characterizes some relationships among these values and the corresponding profits.

Lemma 8. *(1) It always holds that* $0 < \underline{c_1}(\{1,2\}) < \underline{c_1}(\{1\})$, *and* $0 < \underline{c_2}(\{1,2\}) < \underline{c_2}(\{2\})$.
(2) If $\mathcal{F}(\{\{1,2\},\emptyset\}) = \{1,2\}$, *then* $\Pi_1(\{1,2\}) < \Pi_1(\emptyset)$ *and* $\pi_2^{\text{PE}}(\{1,2\}) < \Pi_2(\emptyset)$.

This lemma is illustrated in Fig. 2, which shows the equilibrium cooperation state under different regions of $[c_1, c_2]$. The yellow region, where $c_1 \geq \underline{c_1}(\{1,2\})$ and $c_2 \geq \underline{c_2}(\{1,2\})$, $\{1,2\}$ is one equilibrium cooperation state. In the white region and the shadowed yellow region, where $c_1 < \underline{c_1}(\{1\})$ and $c_2 < \underline{c_2}(\{2\})$, $\emptyset$ is one equilibrium cooperation state. According to (1) of Lemma 8, when $c_1 \in [\underline{c_1}(\{1,2\}), \underline{c_1}(\{1\})]$ and $c_2 \in [\underline{c_2}(\{1,2\}), \underline{c_2}(\{2\})]$, (i.e., the shadowed yellow area in Fig. 2), the equilibrium cooperation state could either be $\{1,2\}$ or $\emptyset$. Thus, there could be two equilibrium selection functions. The first possibility is that $\mathcal{F}(\{\{1,2\},\emptyset\}) = \emptyset$, which is the **entry prohibitive (EP)** case. According to (2) of Lemma 8, the equilibrium state $\emptyset$ leads to higher profits for SP1 and SP2, compared with the equilibrium state $\{1,2\}$. Hence, if SP1 and SP2 can coordinate their decisions, the eventual equilibrium will be in the entry prohibitive case. The second possibility is that $\mathcal{F}(\{\{1,2\},\emptyset\}) = \{1,2\}$, which is the **cooperation friendly (CF)** case. This case could occur if the SPs cannot coordinate their decisions, and it might also be

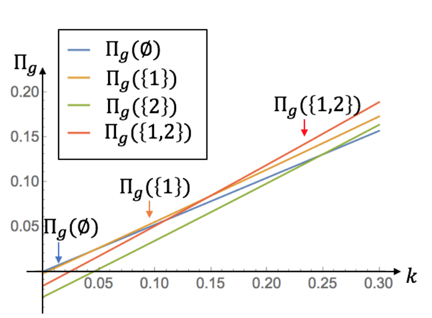

Fig. 3. SPg's profit under different cooperation states versus k. Here, $B_1 = 1$ and $B_2 = 2$.

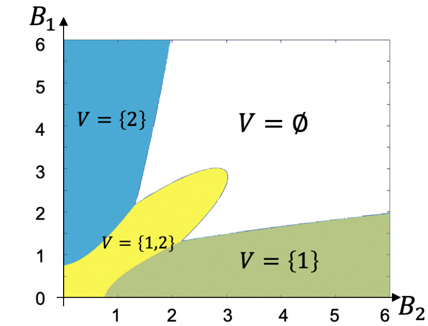

Fig. 4. SPs' cooperation states under different band resources. Here, $k = 0.1$.

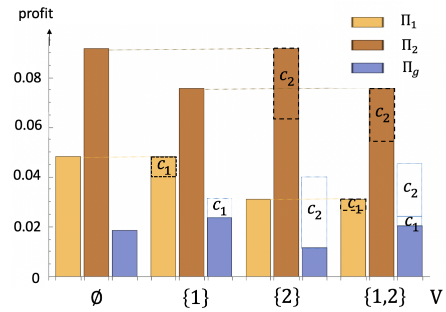

Fig. 5. SPs' profits under different cooperation states ($B_1 = 0.5$, $B_2 = 1$, and $k = 0.05$).

preferred by a regulator as it may bring more social welfare.

Recall that under a general equilibrium selection function $\mathcal{F}$, we use $\mathbf{c}^{\mathcal{F}}(V)$ to denote the compensation vector that SPg should choose to achieve the cooperation state V. In the cooperation friendly case, we use $\mathbf{c}^{\mathrm{CF}}(V)$ to denote this compensation vector. Specifically, we have
(i) for $i \in V$, $c_i^{\mathrm{CF}}(V) = \underline{c_i}(V)$;
(ii) for $i \in N \backslash V$, $c_i^{\mathrm{CF}}(V) = -\infty$.
In the entry prohibitive case, we use $\mathbf{c}^{\mathrm{EP}}(V)$ to denote the compensation vector that SPg should choose to achieve the cooperation state V. Apparently, SPg needs to compensate more, compared with the cooperation friendly case. We next introduce Lemma 9, and then characterize the compensation vector. Without losing generality, we assume that $B_1 \leq B_2$ in the rest of the paper.

Lemma 9. *Given $B_1 \leq B_2$, we have $\underline{c_1}(\{1\}) + \underline{c_2}(\{1,2\}) \leq \underline{c_2}(\{2\}) + \underline{c_1}(\{1,2\})$.*

As shown in Fig. 2, $\mathbf{c}^{\mathrm{EP}}(\{1,2\})$ could be $[\underline{c_1}(\{1\}), \underline{c_2}(\{1,2\})]$ or $[\underline{c_2}(\{2\}), \underline{c_1}(\{1,2\})]$, indicated by the two orange stars. Lemma 9 implies that $\mathbf{c}^{\mathrm{EP}}(\{1,2\})$ should be $[\underline{c_1}(\{1\}), \underline{c_2}(\{1,2\})]$. Note that $\underline{c_1}(\{1\})$ is larger than $\underline{c_1}(\{1,2\})$. Therefore, SPg should offer a larger compensation to SP1 compared to the cooperation friendly case to make cooperating the dominant strategy of SP1, who has a smaller band. Since SP1 will always cooperate, SP2 will accept the compensation $\underline{c_2}(\{1,2\})$, which is lower than $\underline{c_2}(\{2\})$. In this case, $\mathbf{c}^{\mathrm{EP}}(V)$ is given by:

$$\mathbf{c}^{\mathrm{EP}}(\{1,2\}) = [\underline{c_1}(\{1\}), \underline{c_2}(\{1,2\})],$$
$$\mathbf{c}^{\mathrm{EP}}(\{1\}) = [\underline{c_1}(\{1\}), -\infty],$$
$$\mathbf{c}^{\mathrm{EP}}(\{2\}) = [-\infty, \underline{c_2}(\{2\})],$$
$$\mathbf{c}^{\mathrm{EP}}(\emptyset) = [-\infty, -\infty].$$

With $\mathbf{c}^{\mathrm{CF}}(V)$ and $\mathbf{c}^{\mathrm{EP}}(V)$, we can completely solve the three-stage game based on Theorem 2. In the following subsections, we will analyze the cooperation state, the SPs' profits, and market shares at the equilibrium, and study their dependence on k, B_1, and B_2. The main difference between the cooperation friendly case and the entry prohibitive case is that SPg needs to compensate more to cooperate with both SPs in the entry prohibitive case. As a result, there exist conditions under which SPg cooperates with both SPs in the cooperation friendly case but not in the entry prohibitive case. For succinctness, we will only discuss the cooperation friendly case in detail. The entry prohibitive case can be analyzed in a similar manner.

A. Cooperation state

We will first discuss how the cooperation state changes with the added value k and the MNOs' band resources. First, we consider the impact of k in the following lemma.

Lemma 10. *For any $V, U \in \mathcal{P}(N)$ and $i \in N$, if $U \subset V$, $\Pi_g(V) - \Pi_g(U)$ is increasing with k.*

Insights: This lemma shows that SPg's willingness to cooperate with the SPs in the market increases with the added value k.[7] The intuition is that, when k increases, SPg can get more profit from all users using wireless service. Thus, SPg will cooperate with more SPs in the market to increase the mass of users using the wireless service. In Fig. 3, we plot Π_g against k under different parameter settings. It can be observed that when k increases, Π_g increases faster when more SPs cooperate. In the case shown in Fig. 3, when k is small, SPg does not enter the market. When k increases, SPg cooperates with the SP with a smaller band and when k is large, SPg cooperates with both SPs. The next lemma illustrates this behavior in some special settings.

Lemma 11. *For a duopoly market, (1) there exits a $k^* \in [0, 1)$ such that $V^* = \{1,2\}$ if $k > k^*$;*
(2) when $k \to 0$, $B_1 \to 0$, and $B_2 \to 0$, we have $V^* = \{1,2\}$;
(3) when $k \to 0$, $B_1 \to \infty$, and $B_2 \to \infty$, we have $V^* = \emptyset$.

The first part of this lemma shows that when the added value is large, SPg will cooperate with both SPs in equilibrium. The reason is similar to that in Lemma 10. A large added value motivates SPg to cooperate with more SPs and serve

[7] According to [28], Google's revenue and the number of smart phone users have been increasing since 2013. Hence, it is likely that Google's added value k also increased. Hence, this result could explain why Google's Project Fi partnered with U.S. Cellular in 2016 after its collaboration with Sprint and T-Mobile in 2015.

more users in the market. This is shown in Fig. 3. The second part of this lemma shows that when the added value is small, and the band resource is extremely limited for SP1 and SP2, SPg will cooperate with both SPs. When the band resource is extremely limited, even if the extra profit of SPg is small, the tiered use of the SPs' networks can increase the utilization of these resources, making it profitable for SPg to cooperate with both SPs. This is shown in the yellow area of Fig. 4. The final part of this lemma shows that when the added value is small and band resources are abundant, SPg will not enter the market. This is illustrated in Fig. 4.

In Fig. 4, we also illustrate the case where the added value is small and the band resource is relatively scarce. When the two SPs in the market have similar bands (i.e., in the yellow area of Fig. 4), we can observe that SPg always achieves the highest profit when it cooperates with both SPs. When there is a large difference between the bands of the two SPs in the market (i.e., in the green and blue areas in Fig. 4), we can observe that SPg always achieves the highest profit by only cooperating with the SP with the smaller band.

As we can observe from Fig. 4, SPg never exclusively cooperates with the SP with a larger band.[8] This result follows directly from the following lemma.

Lemma 12. *If $B_2 > B_1$, $\Pi_g(\{1,2\}) - \Pi_g(\{2\}) > 0$.*

Insights: This lemma implies that it is always better for SPg to cooperate with both SPs, compared with only cooperating with SP2. We use Fig. 5 to explain the reason. In Fig. 5, we illustrate the SPs' profits as well as SPg's compensations under different cooperation states. We can see that when $V = \{2\}$ (i.e., only cooperating with SP2), SPg's profit is much lower than that under $V = \{1,2\}$. This is because if SPg only cooperates with SP2, the compensation to SP2 (i.e., c_2) is large.

B. Market shares

In Lemma 4, we characterized the dependence of the SPs' market shares on the added value k. Here, we characterize their dependence on the bands of the SPs in the market. Recall that given the cooperation state V, $x_g^{\mathrm{PE}}(V)$ and $x_T^{\mathrm{PE}}(V)$ are SPg's equilibrium market share and the overall equilibrium market share of all SPs (i.e., the mass of users using the wireless service), respectively. The following lemma shows how these depend on available band resources and the cooperation set.

Lemma 13. *(1) For any $V \in \mathcal{P}(N)$, $x_g^{\mathrm{PE}}(V)$ and $x_T^{\mathrm{PE}}(V)$ increase with B_V, where $B_V = \sum_{i\in V} B_i$.*
(2) $x_g^{\mathrm{PE}}(\{1,2\}) > x_g^{\mathrm{PE}}(\{1\})$ and $x_T^{\mathrm{PE}}(\{1,2\}) > x_T^{\mathrm{PE}}(\{1\})$.

Insights: The first part of this lemma shows that SPg's market share and the overall market share increase with the overall available band resources in the market, regardless of the cooperation state. The second part of the lemma shows that SPg's market share and the overall market share increase when SPg cooperates with more SPs in the market.

[8]This appears to hold in practice. In the U.S., Google Fi does not cooperate with the two main MNOs in the U.S., i.e., AT&T and Verizion [10]. In the UK, Google started its service by partnering with Three, instead of BT and O2 (the two main MNOs in the UK [9]).

C. SPs' profits

In this subsection, we investigate the relation between SPg's entry and the loss in the revenues that SP1 and SP2 extract from the users. Recall that $\pi_i^{\mathrm{PE}}(V)$ is the profit that SPi obtains by providing the wireless service to its own users under cooperation state V considering the price equilibrium.

Lemma 14. *(1) For any $V \in \mathcal{P}(N)$, $\pi_i^{\mathrm{PE}}(V)$ decreases with k for each SPi.*
(2) For any $V, U \in \mathcal{P}(N)$ and $i \in N$, if $U \subset V$, $\pi_i^{\mathrm{PE}}(U) > \pi_i^{\mathrm{PE}}(V)$.

The first part of this lemma shows that the increase of the added value always intensifies the competition in the market, and hence reduces the profit that SPi obtains from its own users. The second part of the lemma shows that when SPg cooperates with more SPs in the market, each SPi will always get a smaller profit from its own users. Fig. 5 illustrates the SPs' total profits under different cooperation states. However, the total profit Π_i includes the compensation it receives from SPg, which compensates for the loss in π_i given by this lemma. Indeed as shown in Fig. 5, when we subtract the compensation c_i, the SPs' profits from their own users decrease as SPg cooperates with more SPs. The second part of this lemma also implies that, if some SPs have already accepted the contract of SPg, SPg can offer less to an SP it wants to cooperate with compared to the case when no one cooperates with SPg. In Fig. 5, the profit that SPi extracts from its users can be obtained by subtracting c_i from Π_i. Therefore, SP1's and SP2's profits from their own users always decrease as SPg cooperates with more SPs.

Next, we compare the difference between the changes of SP1's and SP2's profits from their users when SPg only cooperates with one SP in the market.

Lemma 15. *If $B_1 = B_2$, $\pi_2(\emptyset) - \pi_2(\{1\}) > \pi_1(\emptyset) - \pi_1(\{1\})$.*

Lemma 15 implies that when SP1 and SP2 have the same band resource and SPg only cooperates with SP1, SP1's revenue loss is smaller than SP2's. The intuition is that by cooperating with SPg, SP1 gets a better position in the market compared to SP2. The reason is that by cooperating, SP1 can better influence SPg's decisions, giving it more control than SP2 on the price equilibrium that emerges.

VI. Influence of Unlicensed Spectrum

In the previous sections, we assumed that all the users need to subscribe to an SP to get wireless services. In practice, the presence of free public WiFi that utilizes unlicensed spectrum provides users another option for local wireless service. In this section, we briefly consider the impact of the free unlicensed band on SPg's profit. Here, we focus on an idealized scenario where users have the option of using WiFi for free instead of subscribing to any SP's service.

The addition of the unlicensed band does not affect the structure of stage I and stage II of our three-stage game. It

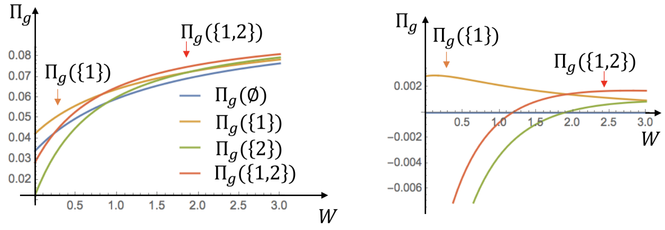

Fig. 6. In the left figure, $B_1 = 0.5$, $B_2 = 1.5$ and $k = 0.1$. In the right figure, $B_1 = 0.5$, $B_2 = 1.5$ and $k = 0$.

mainly changes the users' Wardrop equilibrium in stage III. Specifically, we use W to denote the unlicensed band and x_w to denote the mass of users using this band. The new Wardrop equilibrium constraints are as follows:

$$\begin{cases} \frac{x_i}{B_i} + p_i \geq 1 - \sum_i x_i - x_g - x_w, \quad x_i \geq 0, & \forall i \in N, \\ x_i(1 - \sum_i x_i - x_g - x_w - \frac{x_i}{B_i} - p_i) = 0, & \forall i \in N, \\ \frac{x_i + x_{gi}}{B_i} + p_g \geq 1 - \sum_i x_i - x_g - x_w, \quad x_{gi} \geq 0, & \forall i \in V, \\ x_{gi}(1 - \sum_i x_i - x_g - x_w - \frac{x_i + x_{gi}}{B_i} - p_g) = 0, & \forall i \in V, \\ \frac{x_w}{W} = 1 - \sum_i x_i - x_g - x_w, \quad x_w \geq 0. \end{cases}$$

Due to the space limit, we only use the duopoly case to illustrate the impact of W on SPg's profit. As shown in Fig. 6, when $B_1 = 0.5$, $B_2 = 1.5$, and $k = 0.1$, SPg's profit increases with W. In Fig. 6, we also illustrate the case where $B_1 = 0.5$, $B_2 = 1.5$, and $k = 0$. We can see that for an MVNO without added value, its profit may decrease with more unlicensed band W. Moreover, the unlicensed band affects the cooperation between SPg and the SPs in the market. When W is small, SPg only cooperates with SP1, who has a smaller band. When W is large, SPg should cooperate with both SPs.

The amount of unlicensed band affects SPg's revenue structure from two aspects. First, it reduces the number of users that are willing to pay for wireless service. Second, it enlarges the total market share. If SPg does not have added value, the first effect may decrease SPg's revenue. If SPg has added value, the second effect will increase SPg's profit. Also, the unlicensed band brings more competition in the wireless market, increasing the SPs' incentives to cooperate.

VII. Conclusions

An emerging trend in the wireless market is that firms extracting added value from wireless users become MVNOs, and partner with multiple MNOs. We analyzed the cooperation and competition between one such added value MVNO and multiple MNOs based on a three-stage game. For an arbitrary number of MNOs, we gave an approach to find the equilibrium of this game. In particular, we comprehensively studied the monopoly and duopoly cases. Our results show that the MVNO's willingness to enter the wireless market increases with its added value. Furthermore, the MVNO prefers to cooperate with the "small" MNOs. For example, in the duopoly case, the MVNO does not enter the wireless market when both MNOs have large bands.

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