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Complexity analysis of serrated flows in a bulk metallic glass under constrained and unconstrained conditions

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ABSTRACT

For the present work, the $\rm Zr_{55}Cu_{30}Ni_5Al_{10}$ (atomic percent) bulk metallic glass underwent room-temperature compression experiments in both the constrained and unconstrained conditions. For the constrained condition, samples were exposed to strain rates ranging from $2\times 10^{-5}~\rm s^{-1}$ to $2\times 10^{-3}~\rm s^{-1}$, while for the unconstrained case, the rates varied between $5\times 10^{-5}~\rm s^{-1}$ and $2\times 10^{-3}~\rm s^{-1}$. The serrated flow was modeled and analyzed using the refined composite multiscale entropy algorithm. Findings indicate that the complexity of the serration behavior increased with respect to the strain rate for both conditions. The increase in the complexity with increasing the strain rate is thought to be linked to a rise in the number of defect interactions that occur in the alloy during plastic deformation. Results also indicate that for a strain rate of $2\times 10^{-3}~\rm s^{-1}$, the complexity of the serrated flow was the highest in the constrained condition, which corresponds to higher spatiotemporal correlations between slipping weak spots during a serration event. The microscopy results combined with the complexity analysis support the idea that the serration events, which correspond to defect interactions, are more spatially correlated in the constrained condition.

1. Introduction

Bulk metallic glasses (BMGs) have attracted a great deal of interest during the past several decades due to their excellent combinations of mechanical, physical, and chemical properties, such as exceptional hardness, room-temperature irradiation resistance, good fatigue resistance, large elastic limits, high strength, favorable magnetic performance, wear resistance, formability, and good corrosion resistance [1–9]. Plastic strain during inhomogeneous deformation in BMGs is typically localized into a thin nanoscale region that is referred to as a shear band [10]. These types of defects are the primary force governing the mechanical behavior of this alloy system. It has been suggested that a key issue in designing ductile metallic glasses will lie in the ability to control the shear-band nucleation and propagation [10,11].

In terms of the atomic theory in BMGs, Spaepen has proposed a model where the plastic deformation results from a large amount of the stress-driven creation of free volumes by atomic jumps [12]. The local distribution of free volumes is thought to control the deformation, and the fertile sites with a higher free volume content can easily accommodate local shear strains [13]. Subsequently, Argon developed a

shear-transformation theory, which hypothesized that atomic-cluster rearrangements were the basic carriers of plasticity in BMGs [14].

During compression tests, BMGs may undergo what is known as the serrated flow, which is characterized by jerky oscillations in the stress-strain graph [15]. This type of behavior is significant because it is associated with plastic instabilities that lead to considerable alterations in the microstructure [16,17]. The serrated flow has been observed in BMGs [18–23], as well as in numerous other structural materials, such as steels [15,24–26], Al–Mg alloys [15,27–33], and high entropy alloys [15,34–39].

Complexity-based measurements, such as the multiscale entropy (MSE) algorithm, have been used to model and analyze the dynamical behavior of the serrated flow [27,40]. As a successor to the MSE method, the refined composite multiscale entropy (RCMSE) algorithm can also model and analyze time series data [41,42]. The RCMSE technique has two distinct advantages over its predecessor, namely that it can achieve more accurate results given the same amount of data, and it has a significantly lower chance of producing undefined entropy values [41].

The present investigation applies the RCMSE method to model and analyze the serrated flow in BMGs to achieve a link between the free-

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volume defects and the dynamics of the serrated flow. In doing so, it is expected that a greater understanding of the effects of strain rate and geometric constraint on the serration behavior of BMGs will be achieved. Therefore, it is anticipated that the present work will advance our fundamental understanding of the serration phenomena in BMGs.

2. Materials and methods

The alloy mixture of Zr, Cu, Ni, and Al with purity higher than 99.9 wt percent (wt. %) was arc-melted in a Ti-gettered high-purity argon atmosphere to fabricate the $\rm Zr_{55}Cu_{30}Ni_5Al_{10}$ (atomic percent, at. %) sample. Then the melted mixture was suction cast into a water-cooled copper mold to form a rectangular bar with a cross section of 3×3 mm. The cast rods for compression tests were prepared with a length of 6 mm (unconstrained) or 3 mm (constrained) and diameter of 3 mm. The two compression faces of each sample were then carefully polished such that they were parallel to one other.

The uniaxial compression tests were performed at room temperature using a computer-controlled MTS 810 materials testing machine at a constant strain rate. The samples compressed in the constrained condition were subjected to strain rates ranging from $2\times 10^{-5}~\text{s}^{-1}$ - $2\times 10^{-3}~\text{s}^{-1}$ while the specimens in the unconstrained condition underwent strain rates varying from $5\times 10^{-5}~\text{s}^{-1}$ - $2\times 10^{-3}~\text{s}^{-1}$. The time, stress, and strain data were recorded, using a data-acquisition rate of 100 Hz. The data was analyzed using the RCMSE method.

Scanning electron microscopy (SEM) was performed to characterize the surface morphology of the fractured samples. The microscopy was achieved via a LEO Gemini 1525 field-emission scanning-electron microscope. To better understand the effect of the sample geometry on the shear-band interactions during compression, microscopy was conducted on the samples compressed in both the constrained and unconstrained conditions.

To examine the complexity of the serrated flow, one employs the RCMSE technique. One begins this process by first eliminating the underlying strain-aging trend that occurs during plastic deformation by fitting the stress versus time data using a moving average method [24]. The trend is eliminated from the original data, and then the coarse-grained time series, y_k^r [43], is constructed:

$$y_{kj}^{\tau} = \frac{1}{\tau} \sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_i \qquad ; 1 \le j \le \frac{N}{\tau} \qquad 1 \le k \le \tau$$
 (1)

where N is the total number of data points from the original time series, x_i is the ith point from the given stress-time series, k is an index, which denotes where to begin the algorithm in the original series, and τ is the scale factor, which ranges from 1 to 20 ($\tau_{\rm max} = 20$ allows for sufficient analysis of data) [44–46].

One then makes the template vectors of dimensions, m and m + 1:

$$\mathbf{y}_{k,i}^{\tau,m} = \left\{ y_{k,i}^{\tau} \quad y_{k,i+1}^{\tau} \dots y_{k,i+m-1}^{\tau} \right\}; \ 1 \le i \le N - m; \ 1 \le k \le \tau$$
 (2)

Here *m* was given a value of 2 to reduce the standard error in the sample-entropy results [44]. Next, determine the number of matching template vectors [44]:

$$d_{ab}^{\tau,m} = \left| \left| \mathbf{y}_{a}^{\tau,m} - \mathbf{y}_{b}^{\tau,m} \right| \right|_{\infty} = \max \left\{ \left| \mathbf{y}_{1,a}^{\tau} - \mathbf{y}_{1,b}^{\tau} \right| \dots \left| \mathbf{y}_{i+m-1,a}^{\tau} - \mathbf{y}_{i+m-1,b}^{\tau} \right| \right\} < r$$
 (3)

where r is typically chosen as 0.15σ (σ is the standard deviation of the data) to ensure that the sample entropy does not depend on the variance of the original time series [44].

The refined composite multiscale entropy values (also known as the sample entropy values) for the stress-time series are then found by Ref. [41]:

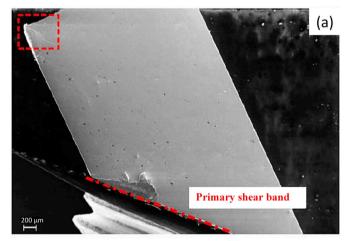
$$RCMSE(y, \tau, m, r) = Ln\left(\frac{\sum_{k=1}^{\tau} n_{k,\tau}^{m}}{\sum_{k=1}^{\tau} n_{k,\tau}^{m+1}}\right)$$
(4)

where the parameters in Eq. (4) are the same as defined above.

3. Results

Fig. 1(a)-(b) presents the SEM images for the sample compressed at a strain rate of $2\times 10^{-4}~{\rm s}^{-1}$ in the unconstrained condition. Here, the primary shear bands can be clearly observed on the lateral surface of the sample. Fig. 1(b) displays the amplified region, which gives a better visualization of the shear-band interactions. The same surfaces for the samples tested in the constrained condition (same strain rate) are displayed in Fig. 2(a)-(b). The specimens compressed in this condition show more shear-band intersections, as compared to the unconstrained condition

The detrended stress vs. time data for the constrained condition with strain rates varying from $2\times 10^{-5}~\text{s}^{-1}$ to $2\times 10^{-3}~\text{s}^{-1}$ (room temperature) is presented in Fig. 3(a). During the experiment, samples that were compressed in the unconstrained condition fractured earlier than the samples compressed in the constrained condition. From the results, there was an apparent decrease in the time between successive avalanches as the strain rate was increased. Furthermore, the stress



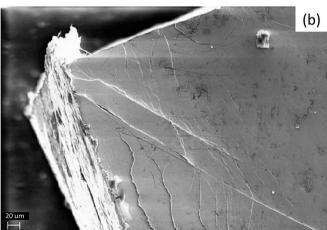
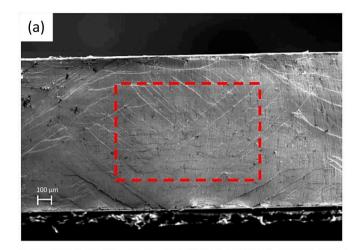


Fig. 1. (a) SEM image showing the primary shear bands (fracture plane, indicated as a red dash-dot line) on the lateral surface after the unconstrained compression at the strain rate of $2\times10^{-4}/s$, and (b) the amplified region showing the interaction of shear bands (From Ref. [47]). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)



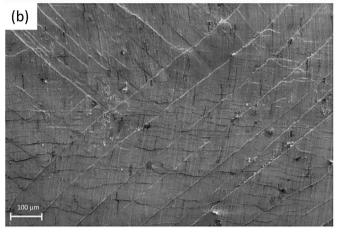


Fig. 2. (a) SEM image showing the multiple shear bands on the lateral surface after constrained compression at a strain rate of 2×10^{-4} s⁻¹ and (b) the amplified region (red box) showing the interaction of shear bands (From Ref. [47]). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

fluctuations displayed more irregular behavior with an increase in the strain rate. For the lowest strain rate, the magnitude of the large stress drops is significantly greater, as compared to the other strain rates.

Fig. 3(b) displays the fluctuations for the unconstrained condition with strain rates of $5 \times 10^{-5} \, \mathrm{s^{-1}} \cdot 2 \times 10^{-3} \, \mathrm{s^{-1}}$. For this set of experiments, the serrations appear to have a more irregular structure as compared to the constrained condition. However, like the constrained condition, the time between successive drops observes a decreasing trend as the strain rate is increased.

Fig. 4(a) presents the results of the sample entropy (complexity) modeling and analysis for the Zr BMG compressed in the constrained condition. As observed, the sample-entropy curve increases with increasing strain rate. For a strain rate of $2\times 10^{-3}~s^{-1}$, the sample entropy monotonically increased with increasing the scale factor, $\tau.$ In contrast, the samples compressed at strain rates ranging from $2\times 10^{-5}~s^{-1}$ - $2\times 10^{-4}~s^{-1}$ initially exhibited decreasing complexity for lower scale factors ($\tau<\sim 3$) and subsequently showed increasing complexity with increasing $\tau.$ These results indicate that the samples exhibited complex serration behavior during compression. Finally, the sample entropy for the highest strain rate was significantly larger at all scale factors, relative to the other strain-rate conditions.

The sample-entropy curves for the specimens compressed in the unconstrained condition for strain rates of $5 \times 10^{-5} \, \mathrm{s^{-1}} \cdot 2 \times 10^{-3} \, \mathrm{s^{-1}}$ are displayed in Fig. 4(b). Similar to the constrained condition, the sample-entropy curves generally increased with an increasing strain

rate. However, the curve for the sample compressed at a strain rate of $1\times 10^{-4}~s^{-1}$ exhibited greater values (for $\tau>1$) as compared to the sample compressed at $2\times 10^{-4}~s^{-1}$. Also similar to the constrained condition, the increasing trend in the sample entropy at larger scales indicates that the serrated flow exhibited complex dynamics during compression. On the other hand, the decreasing trend for the complexity values at lower scale factors (strain rates less than $2\times 10^{-3}~s^{-1}$) is significantly more pronounced, relative to the results for the constrained sample geometry, (see Fig. 4(a)). Lastly, for the highest strain rate, the sample entropy values for the constrained condition were greater by a factor of ~ 2 as compared to the unconstrained condition when $\tau>10$.

4. Discussion

As mentioned previously, the sample-entropy curves for the constrained condition were characteristically different, relative to the unconstrained condition [see Fig. 4(a)-(b)]. In contrast to the constrained condition, the sample entropy exhibited a decreasing trend in the unconstrained condition for lower τ at strain rates less than $2 \times 10^{-3} \text{ s}^{-1}$. For white noise, the sample entropy values also significantly decreases at lower scales [46]. Since this type of noise is comprised of the highly-uncorrelated behavior, this trend suggests that in the unconstrained condition, the Zr BMG undergoes serrations that behave more randomly. Furthermore, the decreasing trend in the complexity may arise from a lower signal to noise ratio due to a reduced number of data points caused by lower fracture times for the unconstrained condition [44]. This relatively-shorter time until failure may arise from the absence of a boundary effect in the unconstrained condition, where the successive triggering of weak spots can occur at a relatively-faster rate [47]. The rise in the sample entropy for the unconstrained condition [see Fig. 4(b)] at larger τ indicates that despite the pronounced decrease in the sample-entropy at lower values, the serrated flow still exhibits the complex behavior across multiple time-scales [44,45]. Interestingly, the decreasing-then-increasing trends, as discussed above, is much less pronounced for the constrained condition, indicating that the serrated flow for this condition is more complex in nature.

As discussed in Refs. [48–50], metallic glasses are regarded as a topologically-disordered material that contains soft-zone defects. These defects are similar to liquid-like sites, which are lower in density and contain a higher free volume content [51] as compared to the glassy matrix. These lower-density regions are coupled with more condensed regions that are known as the anti-free volume, or p-type defects [51, 52]. Furthermore, during compression, the free volume content (and soft zone defects) in an amorphous alloy increases with an increase in the strain rate [53]. To compensate for an increase in the quantity of soft zones, there will also be an increase in the p-type defects in the matrix.

Importantly, an increase in the local concentration of free volumes will eventually lead to the creation of voids that can spontaneously coalesce into microcracks [54]. Thus, with an increasing deformation rate, there should be a rise in the variety of interactions between adjacent microcracks in the matrix. This increase in the variety of interactions should consequently lead to serrated flow that exhibits more complex behavior.

Therefore, an increase in the dynamical complexity of the serration behavior with increasing the strain rate appears to be linked to an enhanced frequency of defect creation at the higher strain rate [53]. Thus, a higher strain rate will lead to a greater number of both soft-zone and p-type defects that can interact with propagating microcracks and shear bands during the serrated flow. This enhancement in the variety of interactions is reflected by the results of the RCMSE modeling and analysis, where the sample entropy is greater for the higher compression rates (see Fig. 4(a)–(b)). Therefore, as the density of defects increases in the alloy, the serrations will contain underlying dynamics that are of a more complex nature.

At a strain rate of 2×10^{-4} s⁻¹, SEM revealed a higher density of

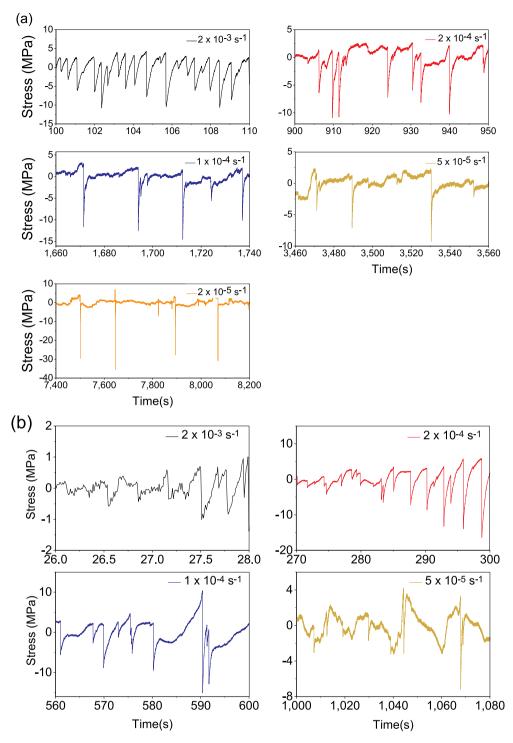
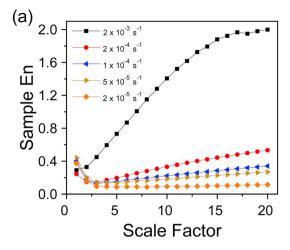


Fig. 3. The detrended stress vs. time for the $Zr_{55}Cu_{30}Ni_5Al_{10}$ BMG compressed in the (a) constrained condition with strain rates ranging from 2×10^{-5} s⁻¹ to 2×10^{-3} s⁻¹ and (b) unconstrained condition with strain rates varying from 5×10^{-5} s⁻¹ to 3×10^{-3} s⁻¹.

shear-band intersections in multiple directions for the sample compressed in the constrained condition, relative to the unconstrained condition [47]. Combining the above result with the data corresponding to Fig. 4(a)–(b) suggests that at the given strain rate, the increased density of shear bands in the constrained condition is associated with a higher frequency of relatively-larger stress drops. This prolonged drop in the stress may be linked to an avalanche event where a weak spot may trigger other weak spots to slip [55] for a longer period of time. Thus, the microscopy results combined with the complexity analysis support the idea that the serration events, which correspond to defect interactions,

are more spatially correlated in the constrained condition.

Therefore, in this context, the higher sample entropy of the serrated flow at a strain rate of $2\times 10^{-3}\,\mathrm{s}^{-1}$ may correspond to weak spots that are more spatially linked, in general. In this sense, the complexity may gauge the ability of weak spots to "communicate" with one another during an avalanche. The significantly higher sample-entropy curve in the constrained condition at a rate of $2\times 10^{-3}\,\mathrm{s}^{-1}$ suggests that the defect interactions have a greater effect on the complexity of the serrated flow at higher strain rates. Furthermore, the samples also underwent compression for a longer time before rupture [see Fig. 3(a)–(b)]



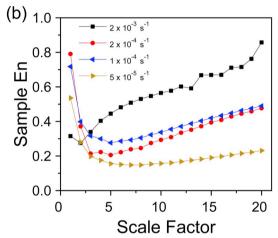


Fig. 4. Sample-entropy results for the $Zr_{55}Cu_{30}Ni_5Al_{10}$ BMG compressed in the (a) constrained condition with strain rates ranging from $2\times 10^{-5}~s^{-1}$ to $2\times 10^{-3}~s^{-1}$ and (b) unconstrained condition with strain rates varying from $5\times 10^{-5}~s^{-1}$ to $3\times 10^{-3}~s^{-1}$.

in the constrained condition. The combination of generally-higher sample entropy values, increased shear-band densities, and longer sample lifetimes, suggests that an increase in the complexity may correspond to an increased ability of the alloy to adapt [44] to an applied load in the constrained condition.

5. Conclusions

For the present work, the Zr₅₅Cu₃₀Ni₅Al₁₀ bulk metallic glass underwent room-temperature compression tests in both constrained and unconstrained conditions at strain rates ranging from 2×10^{-5} s⁻¹ to 2×10^{-3} s⁻¹. The complexity of the serration behavior was found to increase with increasing the strain rate for both conditions. This increase in the sample entropy with increasing the strain rate is thought to be due to an increase in the number of interactions among the free volumes, anti-free volume defects, cracks, and shear bands in the alloy during plastic deformation. Furthermore, the complexity of the serrated flow was the highest in the constrained condition for a strain rate of $2\times 10^{-3}\,$ s^{-1} , which indicates higher spatiotemporal correlations between slipping weak spots during an avalanche at this condition. The SEM imaging revealed a higher density of shear-band interactions during the serrated flow for the sample compressed in the constrained condition. The microscopy results combined with the complexity analysis, therefore, support the idea that the serration events (which correspond to defect interactions) are more spatially correlated in the constrained condition.

Author contributions

All authors contributed extensively to the work presented in this manuscript. X.X. and Z.W. prepared the samples and performed the compression testing. The analysis of the stress-time data was done by J. B. J.B., X.X., J.Q., and P.K.L. wrote the main components of the manuscript while all authors discussed the results and commented on the manuscript at all stages of preparation.

Data availability

The raw/processed data required to reproduce these findings cannot be shared at this time due to legal or ethical reasons.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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