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# The physics of the $\eta$–$\eta'$ system versus $B^0 \to J/\Psi\eta(\eta')$ and $B_s \to J/\Psi\eta(\eta')$ decays

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An approach to the properties of the $\eta$–$\eta'$ system developed to solve the famous $U(1)$ problem is used to calculate the partial widths ratios to $\eta$ and $\eta'$ in the $B^0 \to J/\Psi\eta(\eta', \pi^0)$ and $B_s \to J/\Psi\eta(\eta')$ decays. We obtain the results in agreement with the experimental data.

*Keywords*: Chiral anomaly; $B$-meson decays.

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## 1. Introduction

Solution of the $U(1)$ problem is an important achievement of QCD at low energies.[1–7] It provides a successful description of the properties of $\eta'$- and $\eta$-mesons. The results obtained in solution of the $U(1)$ problem will be used below to find the ratios of the $B^0 \to J/\Psi\eta$, $B^0 \to J/\Psi\eta'$ and $B^0 \to J/\Psi\pi^0$ decay probabilities as well as the ratio $\Gamma(B_s \to J/\Psi\eta)/\Gamma(B_s \to J/\Psi\eta')$.

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Relative probabilities of the $B^0 \to J/\Psi\eta$ and $B^0 \to J/\Psi\eta'$ decays were measured in Refs. 8 and 9, while the relative probability of the $B^0 \to J/\Psi\pi^0$ decay was measured in Ref. 10. The partial widths of the $B_s \to J/\Psi\eta$ and $B_s \to J/\Psi\eta'$ decays were measured in Refs. 9, 11 and 12 while the probabilities of the decays with $\psi(2S)$ in the final state were determined in Refs. 9 and 13. In what follows we will use the values of the ratios of these probabilities presented in the Review of Particle Physics.[14]

In the case of $B^0$-meson the decays which we are studying occur due to the $\bar{b} \to c\bar{c}\bar{d}$ quark transition. In the case of $B_s$ the quark transition $\bar{b} \to c\bar{c}\bar{s}$ is responsible for the decays to the same final states. The $c\bar{c}$ pair forms $J/\Psi$- or $\psi(2S)$-meson, while the remaining light quark combines with a spectator quark forming $d\bar{d}$ state in the case of $B^0$ decays or $s\bar{s}$ state in the case of $B_s$ decays.

We will investigate the consequences of the hypothesis that the probability amplitude of the $\eta$-meson production is proportional to the matrix element $\langle 0|\bar{d}\gamma_5 d|\eta\rangle$ in the case of $B^0$ decay and the matrix element $\langle 0|\bar{s}\gamma_5 s|\eta\rangle$ in the case of $B_s$ decay. Similar matrix elements with the substitution $\eta \to \eta'$ describe $J/\Psi\eta'$ production and with substitution $\eta \to \pi^0$ they describe $J/\Psi\pi^0$ production. In Sec. 2 we neglect the isospin symmetry violation. We will discuss possible consequences of the violation of isospin symmetry in Sec. 3.

## 2. Estimates of the Decay Probabilities

The naive wave functions of the isospin singlet pseudoscalar mesons in the framework of the quark model should be $\pi_1 = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$ and $\pi_2 = \bar{s}\gamma_5 s$. The mass of $\pi_1$ should not exceed that of $\pi$-meson in a stark contrast with the measured mass of the $\eta$-meson. This is the essence of $U(1)$ problem which in the framework of QCD is resolved due to account for the contribution of the $U(1)$ axial anomaly.[1–5] The anomaly contribution was deciphered in Refs. 6 and 7 as mixing of the massless in the chiral limit $m_u = m_d = 0$ $\pi_1$-meson with the massless ghost state made from gluons (see below). The state $\pi_2$ mixes with this ghost as well, while the $SU(3)$ octet superposition of $\pi_1$ and $\pi_2$ effectively decouples from the ghost in the limit of light $s$-quark $m_s = m_u = m_d \ll \Lambda_{\rm QCD}$. In this way, light $\eta_0 = (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s)/\sqrt{6}$ and heavy $\eta_0' = (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)/\sqrt{3}$ states are formed which mix due to the heaviness of $s$-quark, $m_s \gg m_u, m_d$. This is the way how physical $\eta$- and $\eta'$-mesons are formed and gluons are very important in this process.

A complete set of anomalous and nonanomalous Ward identities for flavor axial current correlation functions was derived in Refs. 6 and 7. Topological susceptibility $\langle QQ\rangle$ $(Q = (\alpha_s/8\pi)G\tilde{G})$ that naturally arises in the Ward identities was considered as a zero-momentum limit of the correlation function $q^\mu q^\nu\langle K_\mu K_\nu\rangle$, where $K_\mu$ is the gauge noninvariant gluon axial current. Then nonzero topological susceptibility means that there is a massless ghost pole $a_\mu$ in the correlator $\langle K_\mu K_\nu\rangle$ (for more details see Refs. 6 and 7). The correlators in the Ward identities were approximated by the low-lying pseudoscalar pseudo-Goldstone bosons. As a result the relationships for the parameters of the pseudoscalar nonet were obtained. The matrix elements we are looking for are expressed through the following parameters:

$$f_1 = f_\pi = 130 \text{ MeV}\,, \quad f_K = 156 \text{ MeV}\,, \quad f_2 = 2f_K - f_\pi = 182 \text{ MeV}\,,$$
$$m_1 = m_\pi\,, \quad m_2^2 = 2m_K^2 - m_\pi^2\,, \quad \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}f_2}{f_1} = 1.98\,, \tag{1}$$
$$m_1^2 + m_2^2 + \mu_1^2 + \mu_2^2 = m_\eta^2 + m_{\eta'}^2\,, \quad \mu_1^2 = 0.57 \text{ GeV}^2\,, \quad \mu_2^2 = 0.16 \text{ GeV}^2\,,$$

where $\mu_i$ parametrize transition amplitudes of the ghost to $\pi_i$, $\langle a_\nu|\pi_{1,2}\rangle = -iq_\nu\mu_{1,2}$ and the numerical values of $f_\pi$ and $f_K$ are taken from Ref. 15.

Numerical values of $\mu_1$ and $\mu_2$ are determined by the masses of $\eta'$ and $\eta$ mesons:[6,7]

$$m_{\eta',\eta}^2 = \frac{1}{2}\left[m_1^2 + m_2^2 + \mu_1^2 + \mu_2^2 \pm \sqrt{\left(m_1^2 + \mu_1^2 - m_2^2 - \mu_2^2\right)^2 + 4\mu_1^2\mu_2^2}\right]. \tag{2}$$

According to Refs. 6 and 7 matrix elements of the divergence of the strange quarks axial current $P_2 = 2im_s\bar{s}\gamma_5 s$ are:

$$\langle 0|P_2|\eta\rangle = -\sqrt{\frac{f_2^2 m_2^4\left(m_1^2 + \mu_1^2 - m_\eta^2\right)}{m_{\eta'}^2 - m_\eta^2}} = -0.056 \text{ GeV}^3\,,$$
$$\langle 0|P_2|\eta'\rangle = \sqrt{\frac{f_2^2 m_2^4\left(m_{\eta'}^2 - m_1^2 - \mu_1^2\right)}{m_{\eta'}^2 - m_\eta^2}} = 0.062 \text{ GeV}^3\,. \tag{3}$$

Matrix elements of the isoscalar axial current divergence $P_1 = i\sqrt{2}[m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d]$ are:[6,7]

$$\langle 0|P_1|\eta\rangle = \sqrt{\frac{f_1^2 m_1^4\left(m_2^2 + \mu_2^2 - m_\eta^2\right)}{m_{\eta'}^2 - m_\eta^2}} = 1.9\cdot 10^{-3} \text{ GeV}^3\,,$$
$$\langle 0|P_1|\eta'\rangle = \sqrt{\frac{f_1^2 m_1^4\left(m_{\eta'}^2 - m_2^2 - \mu_2^2\right)}{m_{\eta'}^2 - m_\eta^2}} = 1.8\cdot 10^{-3} \text{ GeV}^3\,. \tag{4}$$

Let us assume exact isospin symmetry and neglect $u$- and $d$-quark mass differences. Then, according to PCAC, the divergence of the isotriplet neutral axial current is proportional to $\pi^0$ field and its matrix element between the $\eta$-meson and vacuum is zero:

$$\langle 0|\bar{u}\gamma_5 u - \bar{d}\gamma_5 d|\eta\rangle = 0\,. \tag{5}$$

Exactly the same relationship holds for $\eta'$. All matrix elements we are looking for in the case of $B^0$ decays can be extracted from (4).

We are considering $p$-wave decays and their probabilities are proportional to the third power of the momentum of the produced particles:

$$|\bar{p}|^3 \sim \left[1 - \left(\frac{\mu + m}{M}\right)^2\right]^{3/2}\left[1 - \left(\frac{\mu - m}{M}\right)^2\right]^{3/2}. \tag{6}$$

Table 1. The numerical values of the right-hand part of Eq. (6).

| | |
|---|---|
| $B^0 \to J/\psi\eta$ | 0.25 |
| $B^0 \to J/\psi\eta'$ | 0.20 |
| $B^0 \to J/\psi\pi^0$ | 0.28 |
| $B_s \to J/\psi\eta$ | 0.27 |
| $B_s \to J/\psi\eta'$ | 0.23 |
| $B_s \to J/\psi\pi^0$ | 0.30 |
| $B_s \to \psi(2S)\eta$ | 0.12 |
| $B_s \to \psi(2S)\eta'$ | 0.08 |

Here $M$ is the mass of the decaying particle ($B_0$ or $B_s$) and $\mu$ and $m$ are the masses of decay products. The numerical values of this factor for the decays under consideration are given in Table 1.

For the ratios of the decay probabilities in case of $\eta$ and $\eta'$ production we obtain:

$$\begin{aligned}
\frac{\mathrm{Br}(B^0 \to J/\psi\eta)}{\mathrm{Br}(B^0 \to J/\psi\eta')} &= \left(\frac{p_\eta}{p_{\eta'}}\right)^3 \left(\frac{\langle 0|P_1|\eta\rangle}{\langle 0|P_1|\eta'\rangle}\right)^2 = 1.39\ [1.11 \pm 0.47]\,, \\
\frac{\mathrm{Br}(B_s \to J/\psi\eta)}{\mathrm{Br}(B_s \to J/\psi\eta')} &= \left(\frac{p_\eta}{p_{\eta'}}\right)^3 \left(\frac{\langle 0|P_2|\eta\rangle}{\langle 0|P_2|\eta'\rangle}\right)^2 = 0.96\ [1.15 \pm 0.08]\,,
\end{aligned} \tag{7}$$

where $p_\eta$ and $p_{\eta'}$ here and in the formulae below are the momenta of the final $\eta$ and $\eta'$ in each of the respective decays. In the brackets here and in the similar equations below are the results of measurements averaged according to Ref. 14.

We use the relationship ($2m_s/(m_u + m_d) = 27.3 \pm 0.7$, see Ref. 14)

$$\frac{\langle 0|\bar{s}\gamma_5 s|\eta\rangle}{\langle 0|\bar{d}\gamma_5 d|\eta\rangle} = \frac{\langle 0|P_2|\eta\rangle/(2m_s)}{\langle 0|P_1|\eta\rangle/(\sqrt{2}(m_u + m_d))} = -1.53 \pm 0.03 \tag{8}$$

to determine the ratios of the probabilities of $B_s$ and $B^0$ decays:

$$\begin{aligned}
\frac{\mathrm{Br}(B_s \to J/\psi\eta)}{\mathrm{Br}(B^0 \to J/\psi\eta)} &= \left(\frac{0.27}{0.25}\right) \frac{1}{\tan^2\theta_c} \left(\frac{\langle 0|\bar{s}\gamma_5 s|\eta\rangle}{\langle 0|\bar{d}\gamma_5 d|\eta\rangle}\right)^2 \\
&= 50.6 \left[\frac{(4.0 \pm 0.7)\cdot 10^{-4}}{(10.8 \pm 2)\cdot 10^{-6}} = 37 \pm 9\right].
\end{aligned} \tag{9}$$

The factor $\tan^2\theta_c$ takes into account the suppression of $\bar{c}d$ charged current by the tangent of the Cabibbo angle.

The decay $B^0 \to J/\Psi\pi^0$ can be considered similarly. Using the PCAC relationship

$$i\langle 0|2m_u\bar{u}\gamma_5 u - 2m_d\,\bar{d}\gamma_5 d|\pi^0\rangle = \sqrt{2} f_\pi m_\pi^2 \tag{10}$$

and taking into account that $\langle 0|\bar{u}\gamma_5 u + \bar{d}\gamma_5 d|\pi^0\rangle = 0$ we obtain

$$i\langle 0|\bar{d}\gamma_5\, d|\pi^0\rangle = -\frac{f_\pi m_\pi^2}{\sqrt{2}(m_u + m_d)}\,. \tag{11}$$

Then the ratio of the decay probabilities is

$$\begin{aligned}\frac{\mathrm{Br}\big(B^0 \to J/\psi\pi^0\big)}{\mathrm{Br}\big(B^0 \to J/\psi\eta\big)} &= \left(\frac{p_\eta}{p_{\eta'}}\right)^3 \left(\frac{f_\pi m_\pi^2}{\langle 0|P_1|\eta\rangle}\right)^2 \\ &= 1.8\left[\frac{(1.7 \pm 0.1)\cdot 10^{-5}}{(1.08 \pm 0.23)\cdot 10^{-5}} = 1.6 \pm 0.4\right].\end{aligned} \tag{12}$$

Comparison of the theoretical and experimental (in square brackets) results in Eqs. (7), (9) and (12) shows a satisfactory agreement.

These decays were analyzed in Ref. 9 with the help of the wave functions of $\eta$- and $\eta'$-mesons. Exploiting the observation[16,17] that the gluon admixture in $\eta$ is negligible the authors of Ref. 9 come to the conclusion that gluon admixture in $\eta'$ is small (see also Ref. 18). These conclusions are confirmed in Refs. 19–21. In all these works the experimental results for the decay branching ratios and cross sections as well as Fock space quark and glueball states are used to obtain the mixing angles.

In this work we follow Refs. 6 and 7, and rely only on the model-independent QCD results and the phenomenological values of the decay constants $f_\pi$, $f_K$, etc. and obtain a satisfactory description of the experimental decay ratios.[a] Our considerations show that one cannot neglect the importance of gluons for the description of both $\eta$ and $\eta'$. To clarify this point let us remind that the large mass of $\eta'$ is explained by its mixture with the ghost state made from gluons. In the $SU(3)$ limit $m_u = m_d = m_s \ll \mu_i$ decoupling of $\eta$-meson from the ghost state really occurs. In this limit instead of Eqs. (1) and (2) we obtain[b]

$$\begin{gathered} f_K = f_\pi = f_1 = f_2\,, \quad m_K = m_\pi = m_1 = m_2\,, \quad \mu_1 = \sqrt{2}\mu_2\,, \\ m_\eta^2 = \frac{\big(m_1^2 + 2m_2^2\big)}{3} = m_\pi^2\,, \quad m_{\eta'}^2 = \mu_1^2 + \mu_2^2 = 3\mu_2^2\,. \end{gathered} \tag{13}$$

We see that in the case of exact $SU(3)$ symmetry and tiny quark masses all mass of the $\eta'$-meson is due to coupling with the ghost state.

In the real world the $SU(3)$ flavor symmetry is violated and even $\eta$-meson does not decouple from the gluons. The decays $J/\psi \to \eta(\eta')\gamma$ were considered in Ref. 23. The authors calculated the ratio $\langle 0|Q|\eta\rangle/\langle 0|Q|\eta'\rangle \approx 0.46$. This result is close to 0.36 obtained in Refs. 6 and 7. Using this ratio the ratio of the $J/\Psi$-meson decay

[a] For the pseudoscalar mixing angle in the quark-flavor basis from (3) we obtain $\tan\varphi_P = 0.056/0.062$, $\varphi_P = 42.1^0$ in perfect agreement with the values obtained in Refs. 9, 19–21. For a detailed discussion of $\eta$–$\eta'$ mixing see review paper of Ref. 22.

[b] We correct some misprints in Refs. 6 and 7 in the expressions for the $\eta$ and $\eta'$ masses in the $SU(3)$ limit.

probabilities was obtained:

$$\frac{\Gamma(\psi \to \eta\gamma)}{\Gamma(\psi \to \eta'\gamma)} = \left|\frac{\langle 0|Q|\eta\rangle}{\langle 0|Q|\eta'\rangle}\right|^2 \left|\frac{p_\eta}{p_{\eta'}}\right|^3 = 0.16 \div 0.25\,, \tag{14}$$

to be compared with the experimental result $\left[1.10(3)\cdot 10^{-3}\right]/\left[5.2(2)\cdot 10^{-3}\right] = 0.21$.[14] This result confirms large admixture of gluons in the $\eta$-meson. It is wrong to rely here on the $\eta$–$\eta'$ mixing model, see the arguments in the appendix of Ref. 23.

In the same way as above, we calculate the ratio of $B_s \to \psi(2S)\eta(\eta')$ decay probabilities

$$\begin{aligned}\frac{\mathrm{Br}(B_s \to \psi(2S)\eta)}{\mathrm{Br}(B_s \to \psi(2S)\eta')} &= \left(\frac{p_\eta}{p_{\eta'}}\right)^3 \left(\frac{\langle 0|P_2|\eta\rangle}{\langle 0|P_2|\eta'\rangle}\right)^2 \\ &= 1.22\left[\frac{(3.3 \pm 0.9)\cdot 10^{-4}}{(1.29 \pm 0.35)\cdot 10^{-4}} = 2.6 \pm 1\right].\end{aligned} \tag{15}$$

We also calculate the ratio of the decay probabilities for the decays $B_s \to \psi(2S)\eta'$ and $B_s \to J/\psi\eta'$. The probabilities of the charmonium states production in the weak $b \to c\bar{c}q$ decays are proportional to the $c\bar{c}$ wave function squared at zero charm quarks separation. The probabilities of charmonium decay to $e^+e^-$ pair are also proportional to the wave function squared at zero and we obtain:

$$\begin{aligned}\frac{\mathrm{Br}(B_s \to \psi(2S)\eta')}{\mathrm{Br}(B_s \to J/\psi\eta')} &= \left(\frac{p_{\psi(2S)}}{p_{J/\psi}}\right)^3 \frac{\Gamma(\psi(2S) \to e^+e^-)}{\Gamma(\psi \to e^+e^-)} \\ &= 0.17\left[\frac{1.29 \pm 0.35}{3.3 \pm 0.4} = 0.4 \pm 0.1\right],\end{aligned} \tag{16}$$

where $p_{\psi(2S)}$ and $p_{J/\psi}$ are the momenta of the final $\psi(2S)$ and $J/\psi$, respectively. Naive factorization could be the reason for considerable deviation of the theoretical result from the experimental number.

Consider finally radiative decays of the $\phi$-meson with $\eta$ and $\eta'$ in the final state. The ratio of the partial widths $R_\phi = BR(\phi \to \eta'\gamma)/BR(\phi \to \eta\gamma)$ was measured in Refs. 19 and 20. Again using the matrix elements from (3) to describe these $p$-wave decays we obtain:

$$R_\phi = \left(\frac{\langle 0|P_2|\eta'\rangle}{\langle 0|P_2|\eta\rangle}\right)^2 \frac{p_{\eta'}^3}{p_\eta^3} = 5.4 \cdot 10^{-3}\,, \tag{17}$$

to be compared with the experimentally measured ratio $(4.8 \pm 0.2)\cdot 10^{-3}$.[19,20]

## 3. The Deviations from Isospin Symmetry

We used isospin symmetry calculating the matrix elements and now we would like to address corrections due to violation of the isospin symmetry. There are two sources of isospin symmetry violation, QED corrections and $u$- and $d$-quark mass differences. The QED corrections are very small numerically and we will not consider them here. The situation with the quark mass differences is more involved.

The corrections of the order of $(m_d - m_u)/m_s$ or $(m_d - m_u)/\Lambda_{\rm QCD}$ are also well below the level of accuracy to which we may pretend. The question is if the corrections of the order of $(m_d - m_u)/(m_d + m_u)$ do exist. They would be important numerically and are interesting from the theoretical point of view.

The difference of the masses of $u$- and $d$-quarks leads to $\eta^0$–$\pi^0$ mixing (in this section the superscript "0" mean the isospin symmetric case). The $SU(2)$ violating potential in the QCD Hamiltonian is

$$V = \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d)\,. \tag{18}$$

We use nonrelativistic perturbation theory to obtain the first-order correction to the $\eta$-meson wave function:

$$|\eta\rangle = |\eta^0\rangle + \frac{\langle\pi_0|V|\eta^0\rangle}{m^2_{\eta^0} - m^2_{\pi^0}}|\pi^0\rangle \approx |\eta^0\rangle + \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s}|\pi^0\rangle\,, \tag{19}$$

where we used the soft-pion theorem to calculate ($\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle \approx (-250\ \text{MeV})^3$ is the $SU(3)$ symmetric quark condensate)

$$\langle\pi_0|V|\eta^0\rangle = -\frac{1}{f_\pi^2}\frac{m_d - m_u}{\sqrt{3}}\langle\bar{u}u + \bar{d}d\rangle\,, \quad m^2_{\eta^0} - m^2_{\pi^0} \approx -\frac{1}{f_\pi^2}\frac{8m_s}{3}\langle\bar{s}s\rangle\,, \tag{20}$$

for more details see Refs. 6, 7 and 24.

Then the correction to the matrix element under discussion is

$$\begin{aligned}\langle 0|\bar{d}\gamma_5 d|\eta\rangle &= \langle 0|\bar{d}\gamma_5 d|\eta^0\rangle + \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s}\langle 0|\bar{d}\gamma_5 d|\pi^0\rangle \\ &= \langle 0|\bar{d}\gamma_5 d|\eta^0\rangle\left[1 + O\left(\frac{m_d - m_u}{m_s}\right)\right].\end{aligned} \tag{21}$$

First-order correction to the $\pi$-meson wave function is

$$|\pi\rangle = |\pi^0\rangle - \frac{\langle\pi_0|V|\eta^0\rangle}{m^2_{\eta^0} - m^2_{\pi^0}}|\eta^0\rangle = |\pi^0\rangle - \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s}|\eta^0\rangle\,, \tag{22}$$

and similarly to (21) we obtain a very small correction to the matrix element

$$\langle 0|\bar{d}\gamma_5 d|\pi\rangle = \langle 0|\bar{d}\gamma_5 d|\pi^0\rangle\left[1 + O\left(\frac{m_d - m_u}{m_s}\right)\right]. \tag{23}$$

Now we can also estimate the relative probability of the $B_s \to J/\Psi\pi$ decay

$$\frac{\text{Br}(B_s \to J/\Psi\pi)}{\text{Br}(B_s \to J/\Psi\eta)} = \left(\frac{p_\pi}{p_\eta}\right)^3 \frac{3}{16}\left(\frac{m_d - m_u}{m_s}\right)^2 \approx 1.5\cdot 10^{-4}\,, \tag{24}$$

where we used (22) to calculate

$$\langle 0|\bar{s}\gamma_5 s|\pi\rangle = -\frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s}\langle 0|\bar{s}\gamma_5 s|\eta\rangle\,, \tag{25}$$

and substitute $(m_d - m_u)/m_s \approx 0.027$ (Ref. 14) (according to Ref. 25 this ratio equals 0.022).

## 4. Conclusions

We considered above the $B^0(B_s) \to J/\Psi\ (\eta, \eta', \pi^0)$ decays and described the qualitative pattern of these decays using the methods developed in the late 1970s and in early 1980s for solution of the $U(1)$ problem.[c] Moreover, these methods allowed us to obtain quantitative description of the ratios of the partial widths that is in a satisfactory agreement with the experimental data.

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## References

1. J. Kogut and L. Susskind, *Phys. Rev. D* **11**, 3583 (1975).
2. G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976).
3. G. 't Hooft, *Phys. Rev. D* **14**, 3432 (1976).
4. E. Witten, *Nucl. Phys. B* **156**, 269 (1979).
5. G. Veneziano, *Nucl. Phys. B* **159**, 213 (1979).
6. D. I. Diakonov and M. I. Eides, *JETP* **54**, 232 (1981), www.jetp.ac.ru/cgi-bin/e/index/e/54/2/p232?a=list.
7. D. I. Diakonov and M. I. Eides, *XVI LIYAF Winter School* (1981), p. 121.
8. BELLE Collab. (M.-C. Chang *et al.*), *Phys. Rev. D* **85**, 091102 (2012).
9. LHCb Collab. (R. Aaij *et al.*), *J. High Energy Phys.* **1501**, 024 (2015), arXiv:1411.0943.
10. BELLE Collab. (B. Pal *et al.*), *Phys. Rev. D* **98**, 112008 (2018).
11. LHCb Collab. (R. Aaij *et al.*), *Nucl. Phys. B* **867**, 547 (2013), arXiv:1210.2631.
12. BELLE Collab. (J. Li *et al.*), *Phys. Rev. Lett.* **108**, 181808 (2012).
13. LHCb Collab. (R. Aaij *et al.*), *Nucl. Phys. B* **871**, 403 (2013), arXiv:1302.6354.
14. Particle Data Group (M. Tanabashi *et al.*), *Phys. Rev. D* **98**, 030001 (2018).
15. S. Aoki *et al.*, arXiv:1902.08191 [hep-lat].
16. C. Di Donato, G. Ricciardi and I. I. Bigi, *Phys. Rev. D* **85**, 013016 (2012).
17. R. Fleischer, R. Knegjens and G. Ricciardi, *Eur. Phys. J. C* **71**, 1798 (2011).
18. S. M. Polikarpov, I. M. Belyaev, V. J. Egorychev and D. V. Savrina, *Phys. Atom. Nucl.* **79**, 288 (2016).
19. KLOE Collab. (F. Ambrosino *et al.*), *Phys. Lett. B* **648**, 267 (2007).
20. KLOE Collab. (F. Ambrosino *et al.*), *J. High Energy Phys.* **0907**, 105 (2009).
21. S. V. Donskov *et al.*, *Eur. Phys. J. C* **73**, 2614 (2013).
22. T. Feldmann, *Int. J. Mod. Phys. A* **15**, 159 (2000).
23. V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **165**, 55 (1980).
24. D. J. Gross, S. B. Treiman and F. Wilczek, *Phys. Rev. D* **19**, 7 (1979).
25. J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).

[c]These decays were also discussed in Refs. 27–30 where an insufficient attention was paid to the axial anomaly contribution.

26. M. A. Andreichikov, M. I. Vysotsky and V. A. Novikov, *JETP Lett.* **110**, 635 (2020).
27. P. Z. Scands, *J. High Energy Phys.* **0101**, 008 (2001), arXiv:hep-ph/0010115.
28. P. Colangelo, F. De Fazio and W. Wang, *Phys. Rev. D* **83**, 094027 (2011), arXiv:1009.4612.
29. S. Dubnicka *et al.*, *Phys. Rev. D* **87**, 074021 (2013), arXiv:1301.0738.
30. J. V. Balitsky *et al.*, *Int. J. Mod. Phys. A* **31**, 1650106 (2016), arXiv:1505.07750.